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Winfried Koeniger  
Omar Licandro

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**Winfried Koeniger**

*IZA Bonn*

**Omar Licandro**

*European University Institute and FEDEA*

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IZA

P.O. Box 7240  
53072 Bonn  
Germany

Phone: +49-228-3894-0  
Fax: +49-228-3894-180  
Email: [iza@iza.org](mailto:iza@iza.org)

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## ABSTRACT

### **Substitutability and Competition in the Dixit-Stiglitz Model\***

The effects of competition on growth are analyzed in the recent literature by comparing economies with the same market structure but different degrees of substitutability. In this note, we show that in a general equilibrium model with monopolistic competition à la Dixit-Stiglitz the effect of substitutability on the allocation of resources is independent of the associated change in competition. Higher substitutability increases welfare, output and productivity because resources shift towards the most productive sectors. However, since markups are equal across sectors, changes in market power do not affect the relative price of consumption goods, implying that the induced changes in market power do not have any direct effect on equilibrium allocations.

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Corresponding author:

Winfried Koeniger  
IZA  
P.O. Box 7240  
53072 Bonn  
Germany  
Email: [koeniger@iza.org](mailto:koeniger@iza.org)

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# 1 Introduction

The effects of product market competition on economic growth and welfare have been analyzed substantially in recent years. See, for example, Aghion et al. (2001). Since the study of exit and entry becomes quite intricate, especially in macroeconomic models, the literature often has used changes in the degree of substitution between goods as an indicator of changes in competition, holding the number of market participants constant.<sup>1</sup> This seems attractive because the elasticity of substitution is negatively related to the markup, a standard measure of competition.

In this note, we point out some limitations of this approach. A change in substitutability has other implications, apart from affecting the degree of competition in oligopolistic markets. In particular, the environment of the economy is modified after a change in the degree of substitution, which may shift the allocation of resources even under perfect competition. For this motive, we claim that a correct measurement of the effects on growth of changes in competition through market power and substitutability needs a careful analysis.

In this note, we solve a general equilibrium model with monopolistic competition à la Dixit-Stiglitz. We show that the decentralized equilibrium attains an optimal allocation, since relative prices are not distorted by imperfect competition, when the markup is equal across sectors; and the distortion of the price of goods in terms of labor has no effect as long as labor is inelastically supplied. Consequently, the benchmark result in this case is that changes in substitutability only affect the decentralized equilibrium through an optimal reallocation of resources. The associated change in market power and competition has no direct effect on the equilibrium. The nature of our results is related to Denicolò and Zanchettin (2003) in that changes in substitutability have an efficiency and a price effect. However, in Denicolò and Zanchettin (2003) these effects are the result of a switch from Cournot to Bertrand competition in a dynamic Neo-Schumpeterian growth model.

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<sup>1</sup>An exception is, e.g., Syverson (2003) who examines the effects of substitutability on productivity through exit and entry.

## 2 The model

We build on a static Dixit-Stiglitz framework without taste for variety, where only labor is employed in the production of goods. There is a mass of consumer-workers of measure one, each of them endowed with one unit of time. Let us call  $\rho$  the marginal productivity of labor in a particular sector and let  $F(\rho)$  be a continuous cumulative distribution function representing the distribution of sectors across  $\rho$  in the support  $\Gamma \subset \mathfrak{R}^+$ . The mean of  $\rho$  is normalized to one.

The representative consumer derives utility from the consumption of a continuum of goods according to the following utility function:

$$\left( \int_{\Gamma} c(\rho)^\alpha \, dF \right)^{\frac{1}{\alpha}}, \quad (1)$$

where  $c(\rho)$  is consumption of the good produced with productivity  $\rho$ , and  $\alpha \in ]0, 1[$ . The elasticity of substitution is  $\frac{1}{1-\alpha}$ . The industry with productivity  $\rho$  employs the technology

$$c(\rho) = \rho l(\rho), \quad (2)$$

where  $l(\rho)$  is labor.

### 2.1 The social planner

A social planner maximizes (1) subject to the feasibility constraint

$$\int_{\Gamma} \frac{c(\rho)}{\rho} \, dF = 1. \quad (3)$$

Restriction (3) represents the allocation of the labor endowment across industries, after replacing  $l(\rho)$  from (2). From the first order condition and some algebra, we get

$$c(\rho) = \frac{\rho^{\frac{1}{1-\alpha}}}{\int_{\Gamma} \rho^{\frac{\alpha}{1-\alpha}} \, dF}. \quad (4)$$

Given that  $1 - \alpha > 0$ , sectorial consumption depends positively on relative efficiency.

Equations (1) and (4) imply

$$V_p = \left( \int_{\Gamma} \rho^{\frac{\alpha}{1-\alpha}} \, dF \right)^{\frac{1-\alpha}{\alpha}}, \quad (5)$$

where  $V_p$  is the optimal utility level for a given value of  $\alpha$ .  $V_p$  is a true index of output and, given that the labor endowment is equal to unity, it also measures average labor productivity. From (5) it follows that average productivity is a weighted average of sectorial productivity. In a symmetric economy,  $\rho = 1$  for all industries, implying that  $V_p = 1$ , for any  $\alpha \in ]0, 1[$ . As soon as industries have different productivity, the average labor productivity depends on the degree of substitution, parametrized by  $\alpha$ . As shown in Proposition 1 below, an increase in the degree of substitution moves resources to the most efficient sectors, increasing output and productivity.

**Proposition 1**  $V_p$  is monotonically increasing in  $\alpha$ , for  $\alpha \in ]0, 1[$

**Proof.** Let us define  $\eta \equiv \frac{\alpha}{1-\alpha} > 0$  where  $\partial\eta/\partial\alpha > 0$ , and rewrite (5) as

$$(V_p)^\eta = \mathbb{E}(\rho^\eta). \quad (6)$$

By differentiating (6), we get the implicit derivative of  $V_p$  w.r.t.  $\eta$ :

$$\frac{dV_p}{d\eta} = \frac{\mathbb{E}[\ln(\rho) \rho^\eta] - \frac{1}{\eta} \mathbb{E}[\rho^\eta] \ln(\mathbb{E}[\rho^\eta])}{\eta (V_p)^{\eta-1}}.$$

The denominator of the r.h.s. is strictly positive. Let us define  $z \equiv \rho^\eta$ . The numerator becomes

$$\frac{1}{\eta} \{ \mathbb{E}[\ln(z) z] - \mathbb{E}[z] \ln(\mathbb{E}[z]) \} > 0$$

by the Jensen's inequality, since the function  $\ln(z) z$  is strictly convex for  $z > 0$ , which completes the proof. ■

## 2.2 The decentralized economy

The representative agent in the decentralized economy maximizes (1) subject to

$$\int_{\Gamma} p(\rho) c(\rho) dF = 1 + \Pi, \quad (7)$$

where aggregate income equals the sum of aggregate profits,  $\Pi$ , plus the value of the labor endowment, which is normalized to unity. Since leisure is not in the utility function, the representative agent offers inelastically one unit of the labor endowment. Additionally,

we take the labor endowment as numeraire. Consequently, prices  $p(\rho)$  are measured in units of the labor endowment.

Monopolistic producers exploit their market power and sell goods at the monopolistic price

$$p(\rho) = (\alpha\rho)^{-1}, \quad (8)$$

where  $\frac{1}{\alpha}$  measures the markup. At equilibrium, it is easy to show that  $1 + \Pi = \frac{1}{\alpha}$ , since profits measured in labor units are equal to  $\frac{1}{\alpha} - 1$ . Additionally, welfare at equilibrium is equal to the social optimum  $V_p$ . In a general equilibrium economy with monopolistic competition à la Dixit-Stiglitz, all monopolies fix the same markup, which implies that relative prices of consumption goods do not change at equilibrium. The only relative price affected by market power is the price of the labor endowment. But the labor supply is infinitely inelastic, which implies that this distortion does not affect the allocation of labor. Notice that, under homothetic preferences, this result is invariant to changes in the distribution of wealth. Finally, this result would also apply to any imperfectly competitive economy where markups are equal across sectors, or change proportionally to changes in substitutability.

## 2.3 Conclusions

We have shown that higher substitutability increases welfare, output and productivity because resources shift towards the most productive sectors (see Proposition 1). Moreover, since markups are equal across sectors, changes in market power do not affect the relative price of consumption goods, implying that the decentralized equilibrium is optimal. Consequently, in the decentralized equilibrium changes in substitutability only affect output and productivity through their effect on the optimal allocation of resources. It implies that the induced changes in market power do not have any direct effect on equilibrium allocations. Thus, changes in the degree of substitutability *cannot* be used to analyze the effect of competition on output. Of course, the assumption of completely inelastic labor supply and symmetric mark-ups is far from realistic. It is true that these assumptions are necessary to derive the extreme result that changes in substitutability between goods only affect the equilibrium through changes in the optimal allocation of

resources. Although relaxing these assumptions would allow for additional effects on the equilibrium through changes in market power, we claim that our result is still important for the recent literature on competition and growth: as long as the effect of changes in substitutability on output cannot be assigned completely to changes in competition, it results, at least partially, from an optimal reallocation of resources.

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