DISCUSSION PAPER SERIES

IZA DP No. 1024

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February 2004

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zur Zukunft der Arbeit
Institute for the Study
of Labor

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Discussion Paper No. 1024
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IZA Discussion Paper No. 1024
February 2004

ABSTRACT

Noncognitive Abilities and Within-Group Wage Inequality*

This paper argues that endogenous restructuring processes within firms towards productivity-enhancing human resource activities, triggered by advances in information and communication technologies (ICT) and rising supply of educated workers, are typically associated with higher demand for noncognitive abilities. Consistent with the evolution of the distribution of wages in advanced countries, this raises within-group wage inequality, possibly accompanied by a decline or stagnation of between-group wage dispersion. The mechanisms proposed in this research are consistent with empirical evidence on both the evolution of work-force composition in firms and the complementarity between skill-upgrading, new technologies and knowledge-based work organization.

JEL Classification: D20, J31

Keywords: human resource activities, noncognitive abilities, within-group wage inequality

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* We are grateful to Josef Falkinger for detailed comments on an earlier version. Moreover, we are indebted to Dennis Snower and seminar participants at Econometric Society Annual Meeting (ESEM) and the University of Zurich for helpful discussions.

1 Introduction

This paper develops a model in which individuals are heterogeneous ex ante in two dimensions: First, they differ in formal education levels which may be thought of reflecting differences in cognitive abilities. Second, they are heterogeneous with respect to non-cognitive abilities like social adaptability to new environments, management skills, the ability to communicate with coworkers, and other social skills necessary for modern firm organizations which rely on interaction among workers.

We argue that endogenous restructuring processes within firms towards productivity-enhancing *human resource* activities, triggered by advances in information and communication technologies (ICT) and rising supply of educated workers, are associated with higher demand for *non-cognitive abilities* of both educated and less educated workers. Consequently, *within-group wage inequality* typically rises, as observed in most advanced countries over the last decades.

Moreover, our analysis suggests that there can be quite complex interactions between the demand for non-cognitive abilities and the demand for educated labor. As a result, rising within-group wage dispersion is not necessarily related to rising between-group wage inequality. Interestingly, whereas an increase in the college premium has been largely confined to the US and UK (from the 1980s onwards until recently), inequality within education groups has risen substantially also in other advanced countries, albeit to a lesser degree than in the US and UK (see e.g. the survey by Gottschalk and Smeeding, 1997).[1] In addition to this evidence, it is a well-known fact that the US has experienced a decline in the college premium in the 1970s, although residual wage inequality has risen considerably at the same time (e.g. Juhn et al., 1993; Katz and Autor, 1999; Acemoglu, 2002). Our model is consistent with a weak or even adverse relationship between wage inequality across and within education groups.

Within-group wage inequality as found in standard estimations of Mincer equations

[1] Moreover, most empirical studies conclude that even for the US the rise in within-group wage inequality accounts for more than half of the rise in total wage inequality (e.g. Juhn et al., 1993; Katz and Autor, 1999).

is related to unobserved abilities of workers. In principle, there are two candidates of those abilities, cognitive and non-cognitive skills. The few studies which control for measures of cognitive skills like scores from IQ and other mental ability tests argue that - because of the high correlation between cognitive skills and education levels - it is very difficult to separate the earning effects of cognitive ability from those of schooling (e.g. Cawley et al., 2001). This evidence suggests that differences in cognitive skills are of limited value to explain wage inequality within education groups. In contrast, as forcefully discussed in e.g. Heckman (2000) and Bowles et al. (2001), differences in non-cognitive abilities play a major role in understanding earnings differentials among individuals.[2] Our paper takes this hypothesis as a starting point by analyzing the impact of technological change and the relative supply of educated workers on the allocation of labor towards human resource activity and the demand for non-cognitive abilities, respectively.

The mechanisms proposed in this study are consistent with empirical evidence on both the evolution of workforce composition in firms and the complementarity between skill-upgrading, new technologies and knowledge-based work organization. First, the employment share of workers in routine tasks like administrative work and mere machine operating dramatically declined over the last decades in favor of managing and professional tasks (e.g. Berman et al., 1994; Bresnahan, 1999; Falkinger and Grossmann, 2003). Evidence by Autor et al. (2003) suggests that computerization has caused this shift in the job composition from routine to non-routine tasks. Brynjolfsson and Hitt (2000) and Bresnahan et al. (2002) find a positive relationship between computerization, organizational change and training provision. Our hypothesis that new ICT has raised the demand for non-cognitive skills by inducing firms to restructure towards human resource management is consistent with these kinds of evidence.[3]

[2]Consistent with this hypothesis, recent evidence from the US Bureau of the Census (1998) suggests that personal attributes like "attitude" and "communication skills" are much more important for the hiring decisions of employers than "years of schooling" or "academic performance".

[3]For instance, human resource management includes the regular updating of workers about changes in work procedures, the organizational structure and employers' goals. Such activities provide workers with the ability to solve problems autonomously and to bear responsibility, which have been major innovations in work organization practices (e.g. OECD, 1999, ch. 4). Also consistent with an increased

Second, evidence by Caroli and van Reenen (2001) for both France and UK suggests a strongly positive effect of changes in the relative supply of skilled labor (proxied by regional skill price differentials) on restructuring of firms towards such knowledge-based organizational forms, as predicted by our theory.[4]

Although many studies have addressed earnings dispersion between education groups (for a survey, see e.g. Acemoglu, 2002), there is still a small but growing theoretical literature on within-group wage inequality. For instance, Acemoglu (1998) argues that technological change directed to skilled workers in response to an increase in the relative supply of educated labor may raise within-group inequality if some educated workers are actually unskilled and vice versa. Galor and Moav (2000) suggest that an increase in the rate of technological progress raises educational attainment of workers with relatively low learning abilities, thereby raising inequality within the group educated workers. Aghion (2002) and Aghion et al. (2002) show that within-group inequality may arise even among (with respect to their abilities) identical workers with different opportunities to adapt to the most recent vintages of machines. Thus, inequality rises with the speed of diffusion of new general purpose technologies. In another interesting contribution, Dalmazzo (2002) argues that within-group wage inequality is related to differences *across* firms in the adoption of complex technologies. In contrast but complementary to these papers, we argue that endogenous restructuring processes *within* firms raise the return to non-cognitive ability, thus fostering within-group wage

importance of human resource activity, Barron et al. (1999) find for a random sample of 3600 US businesses from the Comprehensive Business Database in 1992 that the average time a worker is in "informal management training" is threefold the time she is in "formal training", and that off-site training programs are by far less important than on-site training in the firm. Intuitively, it is not surprising that restructering processes which induce informal training are related to the demand for non-cognitive skills. For instance, autonomous decision-making and problem-solving presume interaction among workers in teams (Lazear, 1999), performance and coordination of multiple tasks (Lindbeck and Snower, 1996, 2000) and the need to gather relevant information from co-workers (Garicano, 2000). Our paper provides a model in which both informal mangement training and the demand for non-cognitive skills are endogenous.

[4] Also Acemoglu (1999) and Thesmar and Thoenig (2000) provide models in which an increase in the relative supply of skilled labor induces some kind of organizational change. The mechanisms in these models are rather different to ours. Thesmar and Thoenig argue that higher skill supply induces a shift from from high-sunk cost to low-sunk cost firms, whereas firms create more vacancies for skilled workers in Acemoglu's model. Within-group wage inequality is not considered in these papers.

inequality. Moreover, we show that, at the same time, between-group wage dispersion may fall, as especially observed in Continental Europe.

The paper is organized as follows. Section 2 sets up the model. It builds on an earlier framework (Egger and Grossmann, 2004), which, however, does not allow for heterogeneity of workers *within* education groups. Section 3 analyzes the equilibrium and provides comparative-static results. Section 4 summarizes and concludes with some remarks. All proofs are relegated to an appendix.

2 The Model

We begin with a description of the labor supply side. Ex ante, individuals differ in both formal education levels and non-cognitive (i.e., social) skills necessary for modern firm organizations which rely on interactions among individuals. We assume that non-cognitive abilities are relevant only if workers are assigned to non-standard tasks like decentralized decision-making or human resource management, but not if they are engaged in routinized administrative tasks, operating of machinery or the like. Human resource management may be thought of a continuous provision of relevant knowledge, supervising, counseling, and motivating commitment to employers' goals. This reflects the idea that, when performing non-routinized tasks, production workers have to be continuously informed about (changes in) production processes, products, employers' goals, work procedures, customer feedbacks, legal regulations etc.[5] Note that in contrast to one-shot formal training programmes which improve workers' human capital stock, this kind of informal training or *support* requires ongoing human resource activity and thus is reflected in variable (rather than fixed) non-production costs.[6]

[5] As Batt (1999) points out, under new organizational forms, "...'learning' ... is a continuous process of using new ideas and information as sources of innovation" (p. 541f.). "[V]irtually all training and work related information (work procedures, system capabilities, product information, legal regulations) are on-line; employees receive eight to ten e-mail messages per day advising them of any updates in any of their systems" (p. 558).

[6] Porter (1986) uses the term "support activities" for this kind of (informal) training provision. Note that "hold-up" problems which may arise from standard forms of (more formal) firm-specific training are not an issue in this context (see Egger and Grossman, 2004, for a discussion).

Moreover, we assume that workers who are assigned towards routinized tasks do not require support by human resource managers.

Regarding formal education, there are two types of workers, highly educated and less educated labor. Labor supply of both types is inelastic in segmented labor markets and denoted by H and L, respectively.[7] Within the group of less educated ($L-$) labor, individuals differ in non-cognitive ability $\beta \in B = \{\beta^1, ..., \beta^K\}$, $0 \leq \beta^1 < ... < \beta^K < \infty$, which is related to the productivity gain from assignment to non-standard tasks (as described below). The supply of type β is denoted by $l^S(\beta)$, i.e., $\sum_{\beta \in B} l^S(\beta) = L$. $L-$workers can only be employed in production-related activities. In contrast, highly educated ($H-$) workers (e.g. university graduates) may also be assigned to productivity-enhancing human resource management tasks. They differ in human resource management abilities, denoted $\gamma \in \Gamma = \{\gamma^1, ..., \gamma^J\}$, $0 \leq \gamma^1 < ... < \gamma^J < \infty$. The supply of type γ is denoted by $m^S(\gamma)$, i.e., $\sum_{\gamma \in \Gamma} m^S(\gamma) = H$. For simplicity, suppose that, when assigned to non-routinized *production* activities, the relevant non-cognitive abilities among $H-$workers are similar.[8]

Labor demand conditions are determined by the following assumptions. There is a unit mass of firms which produce a homogenous good. There are no market imperfections. Output y_i of firm i is produced according to the linearly homogenous function

$$y_i = F\left(\widetilde{h}_i, \widetilde{l}_i\right) \equiv \widetilde{l}_i f\left(\kappa_i\right), \; \kappa_i \equiv \widetilde{h}_i / \widetilde{l}_i, \tag{1}$$

(i.e., $f(\cdot) \equiv F(\cdot, 1)$), where $\widetilde{h}_i$ and $\widetilde{l}_i$ denote *efficiency units of $H-$ and $L-$labor in production*, respectively, i.e., κ_i is the *education-intensity of production labor*. $f(\cdot)$ is a

[7] Empirically, formal education levels are highly related to both cognitive skills and public education policy, providing some justification of treating the number of educated and less educated labor, respectively, as exogenous. For simplicity, we thus follow Acemoglu (1998, 1999) and Thesmar and Thoenig (2000), among others, in abstracting from the educational attainment decision.

[8] The assumption that educated workers only differ in managing abilities can be relaxed without affecting the main results in this paper. For instance, one may alternatively assume that non-cognitive abilities relevant for human resource management and production activities, respectively, are strongly positively correlated (as plausible), and that $H-$workers with high non-cognitive skills are more valuable for firms in management tasks. Such a modification would not yield much additional insight but implies significant costs regarding the expositional simplicity of the paper.

strictly increasing and strictly concave function which fulfills the boundary conditions $\lim_{\kappa \to \infty} f'(\kappa) = 0$ and $\lim_{\kappa \to 0^+} f'(\kappa) = \infty$.

$\widetilde{h}_i$ and $\widetilde{l}_i$ depend on the number of $H-$ and $L-$ workers employed in firm i, respectively, the number of workers assigned to non-standard tasks, and (regarding $L-$workers) non-cognitive abilities. The number (and efficiency units) of *unsupported* $H-$ and $L-$workers in firm i are denoted by h_i^1 and l_i^1, respectively, whereas *efficiency units of supported labor* (i.e., of workers assigned to non-routinized tasks) are denoted by $\tilde{h}_i^2$ and $\tilde{l}_i^2$. Efficiency units of production labor are then given by

$$\widetilde{h}_i = h_i^1 + \tilde{h}_i^2 \text{ and } \widetilde{l}_i = l_i^1 + \tilde{l}_i^2, \tag{2}$$

respectively. Let $\hat{l}_i(\beta)$ be the number of supported $L-$workers in firm i with non-cognitive ability β, i.e., the number of supported $L-$workers in firm i reads

$$l_i^2 = \sum_{\beta \in B} \hat{l}_i(\beta). \tag{3}$$

Moreover, let h_i^2 denote the number of supported $H-$workers in firm i. $\tilde{h}_i^2$ and $\tilde{l}_i^2$ are given by

$$\tilde{h}_i^2 = a h_i^2 \text{ and } \tilde{l}_i^2 = b \sum_{\beta \in B} \beta \hat{l}_i(\beta) \tag{4}$$

(recall that non-cognitive abilities of educated production workers are similar by assumption). To capture that supported production workers have higher productivity than those who are not supported except, possibly, the least able $L-$workers, suppose $a > 1$ and $b\beta > 1$ for all $\beta \in \{\beta^2, ..., \beta^K\}$.[9]

In order to support h_i^2 $H-$workers and l_i^2 $L-$workers (assigned to non-standard tasks), firm i needs to employ

[9]The support activity may be time-consuming for employees. Implicitly, we assume that workers receive wages during that time, i.e., firms bear the entire cost of providing informal training to workers, consistent with the findings by Barron et al. (1999). Moreover, we make the simplifying assumption that firms can perfectly screen workers with respect to their non-cognitive abilities, e.g., through job interviews and assessment centers.

$$\tilde{m}_i = G\left(h_i^2, l_i^2\right) \equiv l_i^2 g\left(\chi_i\right), \quad \chi_i \equiv h_i^2/l_i^2, \tag{5}$$

efficiency units of managerial H−labor, where G is a linearly homogeneous function. Note that the intensive form in (5) requires $l_i^2 > 0$. We exclusively focus on this case in the following (in order to avoid only mildly interesting borderline cases). χ_i is the *education-intensity of supported labor* in firm i. Let $g(\cdot)$ be a strictly increasing and strictly convex function. Thus, (5) may be viewed as joint production technology (e.g. Nadiri, 1987) with two outputs (h_i^2 and l_i^2) and one input ($\tilde{m}_i$), which has a strictly decreasing and strictly concave transformation curve. That is, the support technology exhibits complementarities among both types of labor (i.e., $G_{12} < 0$), in analogy to the standard assumption that $H-$ and $L-$labor are complements in the production technology F. (Note that $g''(\cdot) > 0$ is equivalent to $G_{12} < 0$ under linear homogeneity of G.) Let m_i be the total amount of managerial labor and $\hat{m}_i(\gamma)$ be the amount of type γ employed in firm i, respectively, i.e.,

$$m_i = \sum_{\gamma \in \Gamma} \hat{m}_i(\gamma). \tag{6}$$

For a given amount m_i, efficiency units $\tilde{m}_i$ depend on non-cognitive skills according to

$$\tilde{m}_i = \sum_{\gamma \in \Gamma} \gamma \hat{m}_i(\gamma). \tag{7}$$

In sum, the theoretical innovation of the model is to allow for a distinction between a 'Tayloristic' and modern production set up in firms, where the latter is characterized by two types of interaction among workers: first, between production workers and human resource managers as reflected by technology (5), and second, among supported production workers themselves as reflected by the cost-reducing complementarity $G_{12} < 0$ (in addition to the standard assumption $F_{12} > 0$). This enables us to study how the firms' decisions to restructure towards modern production, triggered by changes in technology conditions and the composition of labor supply in the economy, simultaneously affect wage inequality within and between education groups.

3 Equilibrium Analysis

This section provides the equilibrium analysis. After setting up the equilibrium conditions, we derive comparative-static results regarding technology parameters a, b, and changes in the supply of labor, H, L. First, it is plausible to argue that the introduction of ICT and advances in human resource management techniques can be reflected by an increase in a and b (see (4)). For instance, new ICT reduces the cost of lateral communication among workers and increases the ability to process information (e.g. Radner, 1993). Moreover, as well known, the share of educated workers has considerably increased in most advanced countries over the last decades, which is reflected by an increase in $\phi \equiv H/L$ in our model. We exclusively focus on the case in which the composition of non-cognitive abilities remain unchanged if labor supply changes. That is, $m^S(\gamma)/H$, $\gamma \in \Gamma$, and $l^S(\beta)/L$, $\beta \in B$, remain constant if H or L changes.

3.1 Equilibrium Conditions

Let w_h^1 and w_l^1 denote the wage rates of unsupported $H-$ and $L-$labor assigned to routinized production tasks, respectively, w_h^2 the wage rate of supported $H-$labor in non-standard production, and $w_m(\gamma)$ and $w_l(\beta)$ the wage rates of managerial labor of type γ and supported $L-$labor of type β, respectively. According to (1)-(5) and (7), the decision problem of firm i is given by

$$\max_{h_i^1, l_i^1, h_i^2, \hat{l}_i(\beta), \beta \in B, \hat{m}_i(\gamma), \gamma \in \Gamma} F\left(h_i^1 + a h_i^2, l_i^1 + b \sum_{\beta \in B} \beta \hat{l}_i(\beta)\right) - w_h^1 h_i^1 - w_l^1 l_i^1 - w_h^2 h_i^2 -$$

$$\sum_{\beta \in B} w_l(\beta) \hat{l}_i(\beta) - \sum_{\gamma \in \Gamma} w_m(\gamma) \hat{m}_i(\gamma) \quad \text{s.t.} \sum_{\gamma \in \Gamma} \gamma \hat{m}_i(\gamma) = G\left(h_i^2, \sum_{\beta \in B} \hat{l}_i(\beta)\right), \quad (8)$$

and subject to non-negativity constraints. If $l_i^2 > 0$, the first-order conditions from optimization problem (8) can be written as

$$
\begin{aligned}
f'(\kappa_i) &= w_h^1, && (9)\\
af'(\kappa_i) &\leq w_h^2 + \lambda_i g'(\chi_i), && (10)\\
f(\kappa_i) - \kappa_i f'(\kappa_i) &= w_l^1, && (11)\\
b\beta\left(f(\kappa_i) - \kappa_i f'(\kappa_i)\right) &\leq w_l(\beta) + \lambda_i\left(g(\chi_i) - \chi_i g'(\chi_i)\right), \quad \beta \in B, && (12)\\
\lambda_i \gamma &\leq w_m(\gamma), \quad \gamma \in \Gamma, && (13)
\end{aligned}
$$

holding with equality if the relevant non-negativity constraint is binding.[10] λ_i denotes the Lagrange multiplier associated with the constraint in (8). The left-hand sides of (9)-(12) are the marginal products of the respective types of labor, whereas the right-hand sides are the marginal costs (which, for supported labor, also contain the costs of human resource management).

We focus on a symmetric equilibrium and omit the firm index i from now on.[11] Also note that full employment in equilibrium with a unit mass of firms implies

$$h^1 + h^2 + m = H, \quad l^1 + l^2 = L. \tag{14}$$

Since $H-$labor assigned to production tasks is homogenous ex ante and the human resource management costs are entirely born by firms, we have $w_h^1 = w_h^2 \equiv w_h$ in equilibrium.[12]

Let the relative wage of unsupported labor, $\omega \equiv w_h / w_l^1$, be our measure of *between-*

[10] Note that (9) and (11) are equalities, according to the Inada conditions regarding f. Moreover, (12) is binding at least for one $\beta \in B$ under $l_i^2 > 0$.

[11] As often the case in models with identical firms, it is conceivable that asymmetric equilibria exist in addition to a symmetric one. However, note that $\kappa_i = \kappa$ is directly implied by (9) or (11), respectively.

[12] Moreover, as will become apparent below, the symmetric equilibrium is unique.

group wage inequality.[13] According to (9) and (11),

$$\omega = \frac{f'(\kappa)}{f(\kappa) - \kappa f'(\kappa)} \equiv \Omega(\kappa), \tag{15}$$

where $\Omega'(\kappa) < 0$. The following first result emerges.

Lemma 1 *In equilibrium, there exist threshold ability levels* $\widetilde{\beta} \in B$ *and* $\widetilde{\gamma} \in \Gamma$ *such that the following holds.*
(i) $\hat{l}(\beta) = l^S(\beta)$ *for all* $\beta > \widetilde{\beta}$, $\hat{l}(\widetilde{\beta}) > 0$, *and* $\hat{l}(\beta) = 0$ *for all* $\beta < \widetilde{\beta}$. *Moreover,* $w_l(\beta)/w_l^1 = b(\beta - \widetilde{\beta}) + 1$ *for all* $\beta \geq \widetilde{\beta}$ *if* $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$, *whereas* $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ *implies* $w_l(\widetilde{\beta}) \geq w_l^1$ *and, with* $\gamma \geq \widetilde{\gamma}$,

$$b\beta = \frac{w_l(\beta)}{w_l^1} + \frac{w_m(\gamma)}{w_h}\omega\frac{g(\chi) - \chi g'(\chi)}{\gamma} \text{ for all } \beta \geq \beta. \tag{16}$$

(ii) $\hat{m}(\gamma) = m^S(\gamma)$ *for all* $\gamma > \widetilde{\gamma}$, $\hat{m}(\widetilde{\gamma}) > 0$, *and* $\hat{m}(\gamma) = 0$ *for all* $\gamma < \widetilde{\gamma}$. *Moreover,* $w_m(\widetilde{\gamma}) = w_h$ *and* $w_m(\gamma)/w_h = \gamma/\widetilde{\gamma}$ *for all* $\gamma \geq \widetilde{\gamma}$ *if* $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$, *whereas* $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$ *implies* $w_m(\widetilde{\gamma}) \geq w_h$ *and*

$$a = 1 + \frac{w_m(\gamma)}{w_h}\frac{g'(\chi)}{\gamma} \text{ for all } \gamma \geq \widetilde{\gamma}. \tag{17}$$

Proof. See appendix. ■

Part (i) of Lemma 1 states that $L-$workers are supported up to a threshold level $\widetilde{\beta}$ of non-cognitive ability. Workers with ability above this threshold earn a wage premium, whereas $w_l(\widetilde{\beta}) = w_l^1$ if $L-$workers of type $\widetilde{\beta}$ are not a scarce resource (i.e., if $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$). Similarly, $H-$workers with ability $\gamma \geq \widetilde{\gamma}$ are assigned to human resource management, and always earn a wage premium if $\gamma > \widetilde{\gamma}$ (part (ii) of Lemma 1). Throughout the paper, *within-group* wage inequality is measured by relative wages of workers assigned to non-routinized and standard jobs, $w_m(\gamma)/w_h$, $\gamma \geq \widetilde{\gamma}$, and

[13] We suppose $h^1 > 0$ and $l^1 > 0$ to focus on interior solutions (which is the empirically relevant case). In fact, if the majority of low-educated workers still holds traditional jobs and the majority of educated labor are nonmanagerial workers (as plausible for the time periods which most empirical studies about the evolution of wage inequality have considered), ω represents the relative median wage of educated labor.

$w_l(\beta)/w_l^1$, $\beta \geq \widetilde{\beta}$, respectively.[14] Thus, (16) and (17) jointly give us a relationship between within-group wage inequality and between-group wage inequality (measured by ω). In particular, within-group inequality and between-group inequality may be negatively related. The left-hand side of (16) is the productivity of a supported $L-$worker with ability β relative to that of a unsupported $L-$worker, whereas the right-hand side is the respective relative cost. This relative cost consists of four (endogenous) components: within-group relative wage $w_l(\beta)/w_l^1$ of a $L-$worker of type β, within-group relative wage $w_m(\gamma)/w_h$ of a managerial $H-$worker of type γ, between-group relative wage ω, and the marginal physical cost of a managerial type γ for supporting a $L-$worker.[15] Similarly, the left-hand side of (17) is the relative productivity of a supported $H-$worker, whereas the right-hand side is the respective relative cost. The latter consists of three components: the relative wage of a supported $H-$worker in production (which equals unity since $w_h^1 = w_h^2 = w_h$), within-group relative wage $w_m(\gamma)/w_h$ of a managerial ($H-$)worker of type γ, and the marginal physical cost of a managerial type γ for supporting a $H-$worker.

Lemma 1 also indicates that there are potential differences regarding within-group wage inequality between scenarios with $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ (i.e., not all $L-$workers with threshold ability $\widetilde{\beta}$ are supported) or $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$, and scenarios with $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ or $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$, respectively. This plays an important role for the comparative static analysis presented in the next subsection.

3.2 Comparative-statics

According to Lemma 1, there are four possible scenarios for which we can study the *marginal* impact of changes in relative labor supply $\phi = H/L$ (holding the composition of abilities constant) and technology changes, reflected by changes in a or b:

1.) $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$,

[14] Suppose that sufficient shares of workers in the economy are in Tayloristic and modern jobs within each education group, respectively, and non-cognitive abilities are sufficiently dispersed. Then these inequality measures correspond for some particular ability type to the 90-10 wage differential within an education group. This measure is often used in empirical studies (e.g. Katz and Autor, 1999),

[15] Note that we decomposed $w_m(\gamma)/w_l^1$ into $w_m(\gamma)/w_h$ times ω (recall $\omega = w_h/w_l^1$).

2.) $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$,

3.) $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$, and

4.) $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$.

Lemma 2 *Comparative-static results for scenarios 1-4 are as shown in Table 1.*

Proof. See appendix. ■

<Please insert **Table 1** about here>

The remainder of this section discusses the intuition and derives implications of Lemma 2. We start with changes in the relative supply of educated labor, ϕ.

3.2.1 Relative Supply of Educated Labor

According to Table 1, whenever $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ (i.e., scenarios 1 and 4), $\hat{l}(\widetilde{\beta})/L$ is strictly increasing in the relative supply of $H-$workers, $\phi = H/L$. That is, an increase in ϕ induces firms to restructure in the sense that a higher share of the $L-$labor force is assigned to non-Tayloristic jobs. Moreover, for a certain range of relative supply ϕ, firms may choose not to support workers from a low-ability group, even though all workers from an adjacent group with higher non-cognitive ability is already assigned to modern jobs. Before discussing the intuition of these results, note that this implies following corollary.

Corollary 1 *For any $k = 2, ..., K$, there exist ϕ_1^k, ϕ_2^k, ϕ_3^k, with $0 < \phi_1^k < \phi_2^k < \phi_3^k$ and $\phi_1^{k-1} = \phi_3^k$, such that*
(i) $\widetilde{\beta} = \beta^k$ for all $\phi \in (\phi_1^k, \phi_3^k]$ and $\widetilde{\beta} < \beta^k$ for all $\phi > \phi_3^k$,
(ii) $\hat{l}(\widetilde{\beta}) < l^S(\beta)$ for all $\phi \in (\phi_1^k, \phi_2^k)$ and $\hat{l}(\widetilde{\beta}) = l^S(\beta)$ for all $\phi \geq \phi_2^k$.

<Please insert **Figure 1** about here>

Thus, according to Lemma 1, Lemma 2 and Corollary 1, our measure of wage inequality within the group of $L-$workers for a particular ability type $\beta^k > \widetilde{\beta}$, $w_l(\beta^k)/w_l^1$, evolves with increasing relative supply of $H-$workers, ϕ, as shown in Figure 1. The next result is thus directly implied by the preceding ones.

Proposition 1 *(Within-group wage inequality and education). (i) Wage inequality within the group of $L-$workers is a non-decreasing (and continuous) function of $\phi = H/L$, and strictly increasing in ϕ over some ranges. (ii) The impact of an increase in ϕ on wage inequality within the group of $H-$workers is ambiguous.*

Let us start with a discussion of part (i) of Proposition 1. The crucial insight is that an increase in relative supply of educated labor, $\phi = H/L$, raises the incentive of firms to reallocate $L-$labor towards non-routinized production tasks, i.e., support by human resource management becomes more attractive. To see this, suppose this would not be the case. Then an increase in ϕ unambiguously raises education-intensity of production labor, $\kappa = \widetilde{h}/\widetilde{l}$. Thus, all other things equal, between-group relative wage ω declines, according to (15). Ceteris paribus, this reduces the marginal costs to support $L-$workers, which is given by the right-hand side of (16). This gives firms an incentive to support more $L-$workers and thus raises the demand for non-cognitive skills. Thus, for given within-group wage inequality (i.e., $\hat{l}(\widetilde{\beta})/L < l^S(\widetilde{\beta})/L$, given threshold ability level $\widetilde{\beta}$), $\hat{l}(\widetilde{\beta})/L$ will rise (scenarios 1 and 4 in Table 1). However, as soon as $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ is reached (as some ϕ_2 in Fig. 1), a further marginal increase in ϕ does not make support of workers with lower ability than, say, $\beta^k = \widetilde{\beta}$ attractive (scenarios 2 and 3).[16] In this case, rising demand for all types $\beta \geq \beta^k$ meets "fixed supply" such that $w_l(\widetilde{\beta})/w_l^1$ rises with ϕ. At some level of wage inequality $w_l(\widetilde{\beta})/w_l^1$, support of $L-$labor with ability $\beta^{k-1} < \beta^k$ becomes attractive as ϕ increases so that $\widetilde{\beta}$ falls to β^{k-1}.[17] (Then we again start from scenario 1 or 4.)

Interestingly, regarding part (ii), the impact of an increase in ϕ on $\hat{m}(\gamma)/H$ and $w_m(\gamma)/w_h$, respectively, is less clear. This is due to our assumptions that human resource activity requires educated labor and $H-$individuals differ in managerial ability. To see this, first, note that between-group relative wage ω does not enter equation (17), which equates the relative benefit and relative costs of supporting educated production labor. According to the previous discussion, an increase in ϕ raises the incentive to

[16] In terms of Corollary 1 and Fig. 1, respectively, given that $\widetilde{\beta} = \beta^k$, scenarios 1 or 4 apply if ϕ rises in the interval (ϕ_1^k, ϕ_2^k), whereas scenarios 2 or 3 apply if ϕ rises in the interval $[\phi_2^k, \phi_3^k)$.

[17] At this point, $w_l(\beta^k)/w_l^1 = b(\beta^k - \beta^{k-1}) + 1$, according to Lemma 1.

reassign $L-$labor towards non-routinized jobs. This increases the labor requirement for human resource activities. Thus, if $\hat{l}(\widetilde{\beta})/L$ gradually increases with ϕ, then there are two possibilities. Either $\hat{m}(\widetilde{\gamma})/H$ rises without raising $w_m(\gamma)/w_h$, $\gamma \geq \widetilde{\gamma}$, which occurs as long as $\hat{m}(\widetilde{\gamma})/H < m^S(\widetilde{\gamma})/H$ (i.e., $H-$workers of type $\widetilde{\gamma}$ are not scarce yet) or $w_m(\gamma)/w_h$ rises for all $\gamma \geq \widetilde{\gamma}$ (scenarios 1 or 4, respectively). Moreover, as long as $\hat{l}(\widetilde{\beta})/L$ increases with ϕ for given $\widetilde{\beta}$, then threshold ability level $\widetilde{\gamma}$ may fall after an increase in ϕ from, say, type γ^j to γ^{j-1}. To the contrary, if $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$, then an increase in ϕ does not raise demand for managerial abilities. On the one hand, if $\hat{m}(\widetilde{\gamma})/H < m^S(\widetilde{\gamma})/H$, then an even lower share of managerial $H-$workers is needed to support the same fraction l^2/L of $L-$labor after a marginal increase in ϕ, i.e., $\hat{m}(\widetilde{\gamma})/H$ declines (scenario 2). On the other hand, if $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$, the reduction in demand for non-cognitive abilities after a marginal increase in ϕ is reflected by a decline in $w_m(\gamma)/w_h$, $\gamma \geq \widetilde{\gamma}$ (scenario 3). However, whenever m/H does *not* decline after a (marginal or non-marginal) increase in ϕ, then wage inequality within the group of $H-$labor does not decrease and, possibly, increases.

In sum, Proposition 1 is consistent with the empirical finding that both overall within-group wage inequality and the share of workers assigned to non-Tayloristic jobs have risen, and that these developments were accompanied by increased human resource activity within firms (e.g., OECD, 1999, ch. 4). Our analysis so far suggests that the observed increase in the relative supply of educated labor is a natural candidate for understanding these developments. Consistent with this hypothesis, Caroli and van Reenen (2001) find a significant impact of an increase in the relative supply of skilled labor on restructuring of firms towards knowledge-based organizational forms.

At the same time, however, many (in particular Continental European) countries have experienced stagnating or even declining between-group wage inequality. To address this fact, we now investigate the impact of $\phi = H/L$ on $\omega = w_h/w_l^1$.

Proposition 2 *(Between-group wage inequality and education). Between-group wage inequality is a non-increasing function of* $\phi = H/L$.

For the intuition of the ϕ-effect on $\omega = w_h/w_l^1$ we consider two different cases suggested by Table 1. First, if $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma})/H < m^S(\widetilde{\gamma})/H$ simultaneously hold (scenario 1), a marginal increase in ϕ raises the share of $L-$workers assigned to non-routinized production activities (as argued above). This stimulates demand for human resource managers, thereby raising m/H. Both the implied reallocation of $H-$labor away from production and the increase in efficiency units of $L-$labor reduce education-intensity of production labor $\kappa = \widetilde{h}/\widetilde{l}$. This is a counteracting effect of ϕ on κ.[18] In sum, both κ and thus between-group wage inequality ω remain unaffected.[19] Second, if one of the two conditions $\hat{l}(\widetilde{\beta}) \leq l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) \leq m^S(\widetilde{\gamma})$ is binding, a marginal increase in ϕ reduces (scenario 2) or does not affect (scenarios 3 and 4) the share of human resource managers in the total supply of $H-$labor. This gives rise to a positive impact of ϕ on κ and, therefore, reduces ω, according to (15).

Taken together, Propositions 1 and 2 demonstrate that within-group wage inequality and between-group wage inequality can be adversely related, as observed in some European economies. However, an increase in ϕ alone cannot explain the Anglo-American experience of rising wage inequality within *and* between groups in the 1980s and 1990s. To address this fact, we now turn to the impact of technological change.

3.2.2 New Information and Communication Technologies (ICT)

Most of the literature on the relationship between wage inequality and technological change has focussed on biased changes in the production technology, like (1) in our model, in the sense of an (exogenous or endogenous) increase in marginal productivity of educated relative to less educated labor.[20] In contrast, we consider a kind of tech-

[18] Note that such a counteracting effect is absent in a conventional model which does not distinguish between production and human resource activity.

[19] The effects regarding ω and the allocation of labor in scenario 1 are similar to those discussed in Egger and Grossmann (2004), in which, however, we did not allow for within-group heterogeneity (regarding non-cognitive abilities). Thus, scenarios 2-4 could not occur in this model.

[20] It has been established in a series of papers (focussing on different kinds of models and questions) that, somewhat surprisingly (and contrary to the common notion), such skill-biased technology change has an ambiguous effects on between-group wage inequality if one allows for some skill-intensive, productivity-enhancing technology like (5). Moreover, in such a model, biased technological change of this sort counterfactually leads to a decline in the non-production employment share. See Grossmann

nological change which raises the productivity gain from reallocating workers towards modern jobs that require support from human resource management. Such a technology change, which may be thought of being related to advances in ICT or management innovations (as argued above), is represented by parameters a and b in our model.

An increase in a raises the relative productivity of supported $H-$workers in production, according to (17), and thus has a positive effect on the demand of managers. This raises either the share of human resource managers in total supply of $H-$labor (scenarios 1 and 2) or $w_m(\widetilde{\gamma})/w_h^1$ for all $\gamma \geq \widetilde{\gamma}$ (scenarios 3 and 4). Thus, the impact of an increase in a on within-group wage inequality of $H-$labor is completely analogous to the impact of an increase in ϕ on that of $L-$labor (which is depicted in Fig. 1). Regarding between-group wage inequality, ω, however, the effect of a higher a is less clear-cut (i.e., is positive in scenario 1 but negative otherwise). The reason is that, holding everything else constant, a is positively related to the education-intensity of production labor, $\kappa = \widetilde{h}/\widetilde{l}$ (which in turn is negatively related to ω), according to (2) and (4).[21]

To the contrary, an increase in b raises ω in all scenarios, but the effects on within-group wage inequality are more ambiguous. First, a higher b has a direct negative effect on κ (analogous to the direct positive effect of a on κ), which ultimately gives rise to an increase in between-group wage inequality ω in all four scenarios. Second, since an increase in b raises the relative productivity of supported to unsupported $L-$workers, demand for non-cognitive ability of $L-$labor is raised, all other things equal. However, an increase of ω is associated with an increase in the marginal cost of human resource activity, according to (16), and therefore reduces the demand for non-cognitive abilities of $L-$labor. This counteracts the aforementioned effect, leaving the impact of an increase in b on within-group wage inequality $w_l(\beta)/w_l^1$, $\beta \geq \widetilde{\beta}$,

(2002), Falkinger and Grossmann (2003) and Egger and Grossmann (2004) for these kinds of reasoning.

[21] Only in scenario 1 in which a marginal increase in a induces a reallocation of workers towards non-routine jobs in both education groups, $\omega = w_h/w_l^1$ rises. This is because an increase in a raises the education-intensity of supported labor, $\chi = h^2/l^2$ (as formally shown in the proof of Lemma 2). In turn, this lowers the marginal costs to support $L-$workers since $g''(\cdot) > 0$. Hence, relative demand for all managerial workers rises. Since $w_m(\widetilde{\gamma}) = w_h$ in scenario 1, according to Lemma 1, ω rises despite the fact that a is positively related to κ for a given allocation of labor.

in general ambiguous. The impact of an increase in b on $w_l(\beta)/w_l^1$, $\beta \geq \widetilde{\beta}$, is only unambiguously positive if $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ (scenarios 1 and 4). We can thus conclude that the following robust relationships hold.

Proposition 3 *(Wage inequality and advances in ICT). (i) Within-group wage inequality of $H-$workers is a non-decreasing (and continuous) function of a, and strictly increasing in a over some ranges. (ii) Between-group wage inequality is strictly increasing in b.*

In sum, we may conclude that technological changes which raise a and b simultaneously (and thus raise the incentive of firms to reassign workers to non-Tayloristic tasks) can explain both rising within-group and rising between-group wage inequality.

4 Concluding Remarks

This paper has argued that both rising relative supply of educated labor and technological change has led to restructuring processes within firms which raised the demand for non-cognitive abilities. Our results are consistent with empirical evidence on a pervasive rise in within-group wage inequality and possibly stagnating or falling wage dispersion between education groups. The mechanisms suggested by the analysis contribute to an understanding of why reorganization of work towards non-Tayloristic jobs, requiring steady support from human resource management (i.e., informal training), has gone along with skill-upgrading and a pervasive surge in within-group inequality throughout the developed world. In fact, the empirical literature on so-called skill-biased technological change (e.g., Berman et al., 1994; Bresnahan, 1999; Bresnahan et al., 2002) has always pointed out that understanding changes in the demand for skills requires to take into account restructuring processes within firms. However, surprisingly little theoretical work has been done in this area so far.

Two final remarks are at order. The first is a brief comment on the ongoing debate why wage inequality in Continental Europe evolved so differently as opposed to the US and UK. On the one hand, our model is consistent with the view that declining growth of educated labor supply in the US in the 1980s, along with steady or even accelerating skill supply growth in many European countries can partly account for the fact that college premia have risen in the US but not, for instance, in Germany (e.g., Gottschalk and Smeeding, 1997).[22] On the other hand, our model suggests that this very fact can explain why within-group wage inequality has considerably risen also in Continental Europe at the same time.

The second remark is a tentative policy conclusion. Our model has emphasized the role of non-cognitive abilities for both wage patterns and the incentive of firms to restructure towards organizational forms which require these abilities. In other words, lack of these skills may be an impediment for firms to enhance productivity and may be the major source of low earnings individually, partly irrespective of formal education. In fact, as argued by Heckman (2000), labor market programmes aiming at raising qualifications of workers often turn out to be almost ineffective to boost earning prospects due to the lack of non-cognitive skills. Our model suggests that, if anything, this problem will become more severe in the future, and may require an increased emphasis on non-cognitive abilities in high-school education or even earlier.

Appendix

Proof of Lemma 1

Perfect competition in the labor market implies that $0 < \hat{l}(\beta) < l^S(\beta)$ $(0 < \hat{m}(\gamma) < m^S(\gamma))$ is only consistent with an equilibrium if $w_l(\beta) = w_l^1$, $\beta \in B$ $(w_m(\gamma) = w_h,$ $\gamma \in \Gamma)$. Hence, it is an immediate consequence of profit maximization that $0 < \hat{l}(\widetilde{\beta}) \leq l^S(\widetilde{\beta})$ $(0 < \hat{m}\left(\widetilde{\gamma}\right) \leq m^S\left(\widetilde{\gamma}\right))$ requires $\hat{l}\left(\beta\right) = l^S\left(\beta\right)$ for all $\beta > \widetilde{\beta}$ $(\hat{m}\left(\gamma\right) = m^S\left(\gamma\right)$ for all

[22] This is not to deny that also institutional factors (e.g., like strong unions or minimum wages) have contributed to stagnating college premia in Continental Europe, given similar technology-induced changes in the labor demand composition as in the US. However, as pointed out by Acemoglu (2002), this cannot explain why unemployment rates have risen almost proportionally for educated and less educated workers, contrary to the prediction of the institutional view.

$\gamma > \widetilde{\gamma}$) and $\hat{l}\,(\beta) = 0$ for all $\beta < \widetilde{\beta}$ ($\hat{m}\,(\gamma) = 0$ for all $\gamma > \widetilde{\gamma}$). Moreover, note from (13) that $\lambda = w_m\,(\gamma)\,/\gamma$ for all $\gamma \geq \widetilde{\gamma}$. Thus, (11) and (12) imply for all $\beta \geq \widetilde{\beta}$ and $\gamma \geq \widetilde{\gamma}$ that

$$b\beta w_l^1 = w_l\,(\beta) + \frac{w_m\,(\gamma)}{\gamma}\,(g\,(\chi) - \chi g'\,(\chi))\,, \tag{A.1}$$

whereas (9) and (10) imply

$$a w_h = w_h + \frac{w_m\,(\gamma)}{\gamma} g'\,(\chi) \tag{A.2}$$

for all $\gamma \geq \widetilde{\gamma}$. Equations (16) and (17) follow from (A.1), (A.2) and definition $\omega = w_h/w_l^1$. Moreover, using the facts that $w_l(\widetilde{\beta}) = w_l^1$ if $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $w_m(\widetilde{\gamma}) = w_h$ if $\hat{m}\,(\widetilde{\gamma}) < m^S\,(\widetilde{\gamma})$, (A.1) and (A.2) confirm $w_l(\beta)/w_l^1 = \beta - \widetilde{\beta} + 1$ for all $\beta \geq \widetilde{\beta}$ and $w_m(\gamma)/w_h = \gamma/\widetilde{\gamma}$ for all $\gamma \geq \widetilde{\gamma}$, respectively. However, note that $w_l(\widetilde{\beta}) > w_l^1$ and $w_m(\widetilde{\gamma}) > w_h$ is possible if $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ and $\hat{m}\,(\widetilde{\gamma}) = m^S\,(\widetilde{\gamma})$, respectively, since threshold ability types are a scarce resource in this case. ■

Proof of Lemma 2

First, note that using $\widetilde{h}^2 = ah^2$, (2), (14) and $\chi = h^2/l^2$, $\kappa = \widetilde{h}^2/\widetilde{l}^2$ can be written as

$$\kappa = \frac{h^1 + \widetilde{h}^2}{l^1 + \widetilde{l}^2} = \frac{\phi\left(1 - \frac{m}{H}\right) + (a-1)\,\chi \frac{l^2}{L}}{1 - \frac{l^2}{L} + \frac{\widetilde{l}^2}{L}} \tag{A.3}$$

(recall $\phi = H/L$). Also note that Lemma 1 implies

$$l^2 = \sum_{\beta > \widetilde{\beta}} l^S\,(\beta) + \hat{l}(\widetilde{\beta}), \tag{A.4}$$

$$\widetilde{l}^2 = b\left(\sum_{\beta > \widetilde{\beta}} \beta l^S\,(\beta) + \widetilde{\beta}\hat{l}(\widetilde{\beta})\right), \tag{A.5}$$

$$m = \sum_{\gamma > \widetilde{\gamma}} m^S\,(\gamma) + \hat{m}\,(\widetilde{\gamma})\,, \tag{A.6}$$

$$\widetilde{m} = \sum_{\gamma > \widetilde{\gamma}} \gamma m^S\,(\gamma) + \widetilde{\gamma}\hat{m}\,(\widetilde{\gamma})\,, \tag{A.7}$$

according to (3), (4), (6), (7). We now explore comparative-static effects for scenarios 1-4 separately.

Ad scenario 1: Recall from Lemma 1 that $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$ imply $w_l(\beta)/w_l^1 = b(\beta - \widetilde{\beta}) + 1$ for all $\beta \geq \widetilde{\beta}$ and $w_m(\gamma)/w_h = \gamma/\widetilde{\gamma}$ for all $\gamma \geq \widetilde{\gamma}$, respectively, thus confirming our results for within-group inequality. Moreover, recall $w_l(\widetilde{\beta}) = w_l^1$ and $w_m(\widetilde{\gamma}) = w_h$. Thus, (A.1) and (A.2) imply that $(g(\chi) - \chi g'(\chi))\omega = (b\widetilde{\beta} - 1)\widetilde{\gamma}$ and

$$g'(\chi) = (a-1)\widetilde{\gamma} \iff \chi = (g')^{-1}((a-1)\widetilde{\gamma}) \equiv \widetilde{\chi}(\underset{(+)}{a}), \tag{A.8}$$

respectively, where $\widetilde{\chi}(a)$ is increasing in a. Thus,

$$\omega = \frac{(b\widetilde{\beta} - 1)\widetilde{\gamma}}{[g(\chi) - \chi g'(\chi)]_{\chi = \widetilde{\chi}(a)}} \equiv \widetilde{\omega}(\underset{(+)}{a}, \underset{(+)}{b}), \tag{A.9}$$

where $\partial\widetilde{\omega}/\partial a > 0$,[23] $\partial\widetilde{\omega}/\partial b > 0$ and $\partial\widetilde{\omega}/\partial\phi = 0$. Using $\omega = \Omega(\kappa)$, according to (15), we find that $\kappa = \Omega^{-1}(\widetilde{\omega}(a,b)) \equiv \widetilde{\kappa}(a,b)$, where $\partial\widetilde{\kappa}/\partial a < 0$ and $\partial\widetilde{\kappa}/\partial b < 0$, since $\Omega'(\kappa) < 0$, and $\partial\widetilde{\kappa}/\partial\phi = 0$. Also note that combining $l^2 = \tilde{m}/g(\chi)$ from (5) with (A.4) and (A.7), and rearranging terms, yields

$$\frac{\hat{l}(\widetilde{\beta})}{L} = \frac{\phi\left(\sum_{\gamma > \widetilde{\gamma}} \gamma \frac{m^S(\gamma)}{H} + \widetilde{\gamma}\frac{\hat{m}(\widetilde{\gamma})}{H}\right)}{g(\chi)} - \sum_{\beta > \widetilde{\beta}} \frac{l^S(\beta)}{L}. \tag{A.10}$$

Substituting (A.4)-(A.6) into (A.3) and then using (A.8) and (A.10) yields

$$\frac{1 - [1 - \eta(\widetilde{\chi}(a))]\frac{\hat{m}(\widetilde{\gamma})}{H} + \sum_{\gamma > \widetilde{\gamma}}\left(\eta(\widetilde{\chi}(a))\frac{\gamma}{\widetilde{\gamma}} - 1\right)\frac{m^S(\gamma)}{H}}{\frac{1}{\phi} + \frac{b\widetilde{\beta} - 1}{g(\widetilde{\chi}(a))}\left(\sum_{\gamma > \widetilde{\gamma}} \gamma \frac{m^S(\gamma)}{H} + \widetilde{\gamma}\frac{\hat{m}(\widetilde{\gamma})}{H}\right) + \frac{b}{\phi}\sum_{\beta > \widetilde{\beta}}(\beta - \widetilde{\beta})\frac{l^S(\beta)}{L}} - \widetilde{\kappa}(a,b) = 0, \tag{A.11}$$

where $\eta(\chi) \equiv \chi g'(\chi)/g(\chi)$, $\chi = \widetilde{\chi}(a)$ and $\kappa = \widetilde{\kappa}(a,b)$ have been used. (A.11) defines $\hat{m}(\widetilde{\gamma})/H$ implicitly as function of (a,b,ϕ). Comparative-static results regarding $\hat{m}(\widetilde{\gamma})/H$ follow from applying the implicit function theorem to (A.11) and observing

[23] Use the fact that $[g(\chi) - \chi g'(\chi)]$ is strictly decreasing in χ together with the properties of $\widetilde{\chi}(a)$.

the properties of $\widetilde{\chi}(a)$ and $\widetilde{\kappa}(a,b)$.[24]

For the results regarding $\hat{l}(\widetilde{\beta})/L$, note that combining (5) with (A.7) yields

$$\frac{\hat{m}\left(\widetilde{\gamma}\right)}{H}=\frac{g\left(\widetilde{\chi}(a)\right)}{\widetilde{\gamma}\phi}\frac{l^2}{L}-\sum_{\gamma>\widetilde{\gamma}}\frac{\gamma}{\widetilde{\gamma}}\frac{m^S\left(\gamma\right)}{H}. \tag{A.12}$$

Next, substitute (A.5), (A.6) and (A.8) into (A.3) and use both $\chi=\widetilde{\chi}(a)$ and (A.12) to obtain

$$\kappa=\frac{\phi\left(1+\sum_{\gamma>\widetilde{\gamma}}\left(\frac{\gamma}{\widetilde{\gamma}}-1\right)\frac{m^S(\gamma)}{H}\right)-\frac{1}{\widetilde{\gamma}}\left[g\left(\chi\right)-\chi g'\left(\chi\right)\right]_{\chi=\widetilde{\chi}(a)}\frac{l^2}{L}}{1-\frac{l^2}{L}+b\sum_{\beta>\widetilde{\beta}}\beta\frac{l^S(\beta)}{L}+b\widetilde{\beta}\frac{l^S(\widetilde{\beta})}{L}}. \tag{A.13}$$

Finally, substitute (A.4) into (A.13) and use $\kappa=\widetilde{\kappa}(a,b)$ which leads to

$$\frac{\phi\left[1+\sum_{\gamma>\widetilde{\gamma}}\frac{\gamma-\widetilde{\gamma}}{\widetilde{\gamma}}\frac{\hat{m}(\widetilde{\gamma})}{H}\right]\frac{\hat{m}(\widetilde{\gamma})}{H}-\frac{1}{\widetilde{\gamma}}\frac{l^2}{L}\left[g\left(\chi\right)-\chi g'(\chi)\right]_{\chi=\widetilde{\chi}(a)}}{1+\sum_{\beta>\widetilde{\beta}}\left(b\beta-1\right)\frac{l^S(\beta)}{L}+\left(b\widetilde{\beta}-1\right)\frac{\hat{l}(\widetilde{\beta})}{L}}-\widetilde{\kappa}(a,b)=0. \tag{A.14}$$

(A.14) defines $\hat{l}(\widetilde{\beta})/L$ implicitly as function of (a,b,ϕ). Comparative-static results regarding $\hat{l}(\widetilde{\beta})/L$ follow from applying the implicit function theorem to (A.14) and, again, observing the properties of $\widetilde{\chi}(a)$ and $\widetilde{\kappa}(a,b)$.

Ad scenario 2: First, note that $\hat{m}\left(\widetilde{\gamma}\right)<m^S\left(\widetilde{\gamma}\right)$ implies that $w_m(\gamma)/w_h=\gamma/\widetilde{\gamma}$ for all $\gamma\geq\widetilde{\gamma}$, thus confirming our results regarding inequality within the group of $H-$workers. Moreover, (A.8) still holds. Also note that $\hat{l}(\widetilde{\beta})=l^S(\widetilde{\beta})$ implies that l^2/L is constant, according to (A.4). Thus, using (A.12) confirms the results regarding $\hat{m}\left(\widetilde{\gamma}\right)/H$. Next, use $\hat{l}(\widetilde{\beta})=l^S(\widetilde{\beta})$ in (A.13) and recall $\widetilde{\chi}'(a)>0$ to confirm that κ is strictly increasing in both a and ϕ, and strictly decreasing in b. Using $\omega=\Omega(\kappa)$ with $\Omega'(\kappa)<0$, according to (15), confirms the results regarding ω. Finally, use these results and substitute $\chi=\widetilde{\chi}(a)$ from (A.8) into (16) to confirm the results regarding $w_l\left(\beta\right)/w_l^1$, $\beta>\widetilde{\beta}$.

[24]Recall that $l^S\left(\beta\right)/L$, $\beta>\widetilde{\beta}$, and $m^S\left(\gamma\right)/H$, $\gamma>\widetilde{\gamma}$, are not affected by ϕ by the assumption of constant within-group compositions of noncognitive abilities. Moreover, note that $\eta'(\chi)>0$ since $g\left(\chi\right)-\chi g'\left(\chi\right)>0$ and $g''\left(\chi\right)>0$.

Ad scenario 3: First, note that $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ implies that l^2/L is constant and $\widetilde{l^2}/L = b\sum_{\beta\geq\widetilde{\beta}}\beta l^S(\beta)/L$, according to (A.4) and (A.5), respectively. Similarly, $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$ implies that m/H is constant and $\widetilde{m}/H = \sum_{\gamma\geq\widetilde{\gamma}}\gamma m^S(\gamma)$, according to (A.6) and (A.7), respectively. Thus, $\widetilde{m} = l^2 g(\chi)$ implies that

$$\chi = g^{-1}\left(\frac{\phi\sum_{\gamma>\widetilde{\gamma}}\gamma m^S(\gamma)/H}{l^2/L}\right) \equiv \widetilde{\widetilde{\chi}}(\underset{(+)}{\phi}), \tag{A.15}$$

where $\widetilde{\widetilde{\chi}}(\phi)$ is increasing in ϕ. Substituting both $\widetilde{l^2}/L = b\sum_{\beta\geq\widetilde{\beta}}\beta l^S(\beta)/L$ and (A.15) into (A.3) leads to

$$\kappa = \frac{\phi\left(1-\frac{m}{H}\right)+(a-1)\widetilde{\widetilde{\chi}}(\phi)\frac{l^2}{L}}{1-\frac{l^2}{L}+b\sum_{\beta\geq\widetilde{\beta}}\beta\frac{\hat{l}^S(\beta)}{L}} \equiv \widetilde{\widetilde{\kappa}}(\underset{(+)}{a},\underset{(-)}{b},\underset{(+)}{\phi}). \tag{A.16}$$

Thus, $\widetilde{\widetilde{\kappa}}(a,b,\phi)$ is increasing in a and ϕ, and decreasing in b. Noting that $\omega = \Omega(\widetilde{\widetilde{\kappa}}(a,b,\phi)) \equiv \widetilde{\widetilde{\omega}}(a,b,\phi)$ from (15) confirms the results regarding ω. Moreover, substituting $\chi = \widetilde{\widetilde{\chi}}(\phi)$ and $\omega = \widetilde{\widetilde{\omega}}(a,b,\phi)$ into (16) and observing functional properties confirms our results for $w_l(\beta)/w_l^1$, $\beta \geq \widetilde{\beta}$. Similarly, substituting $\chi = \widetilde{\widetilde{\chi}}(\phi)$ into (17) confirms the results regarding $w_m(\gamma)/w_h$, $\gamma \geq \widetilde{\gamma}$.

Ad scenario 4: First, note that $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ implies $w_l(\beta)/w_l^1 = b(\beta-\widetilde{\beta})+1$ for all $\beta \geq \widetilde{\beta}$, thus confirming our results regarding inequality within the group of $L-$workers. Substituting (A.4) and (A.7) into $\widetilde{m} = l^2 g(\chi)$ from (5) and observing $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$ implies that

$$\chi = g^{-1}\left(\frac{\phi\sum_{\gamma\geq\widetilde{\gamma}}\gamma\frac{m^S(\gamma)}{H}}{\sum_{\beta>\widetilde{\beta}}\frac{l^S(\beta)}{L}+\frac{\hat{l}(\widetilde{\beta})}{L}}\right) \equiv X\left(\underset{(-)}{\hat{l}(\widetilde{\beta})/L},\underset{(+)}{\phi}\right), \tag{A.17}$$

where $X\left(\hat{l}(\widetilde{\beta})/L,\phi\right)$ is decreasing in $\hat{l}(\widetilde{\beta})/L$, and increasing in ϕ. Next, recall that $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ implies $w_l(\widetilde{\beta}) = w_l^1$, i.e.,

$$\omega = \frac{b\widetilde{\beta}-1}{a-1}\frac{g'(\chi)}{g(\chi)-\chi g'(\chi)}, \tag{A.18}$$

according to (A.1). Combining (A.18) with $\omega = \Omega(\kappa)$ from (15) yields the relationship

$$\kappa = \Omega^{-1}\left(\frac{b\widetilde{\beta}-1}{a-1}\frac{g'(\chi)}{g(\chi)-\chi g'(\chi)}\right) \equiv K(\underset{(-)}{\chi}, \underset{(+)}{a}, \underset{(-)}{b}), \tag{A.19}$$

where $K(\chi, a, b)$ is decreasing in both χ and b, and increasing in a. Now, substituting $l^2 = \left(\sum_{\gamma\geq\widetilde{\gamma}} \gamma m^S(\gamma)\right)/g(\chi)$ into the numerator of (A.3) as well as substituting both (A.4) and (A.5) into the denominator of (A.3) leads to

$$\kappa = \frac{\phi\left[1 - \frac{m}{H} + \frac{(a-1)\chi}{g(\chi)}\sum_{\gamma\geq\widetilde{\gamma}}\gamma\frac{m^S(\gamma)}{H}\right]}{1 + \sum_{\beta>\widetilde{\beta}}(b\beta-1)\frac{l^S(\beta)}{L} + \left(b\widetilde{\beta}-1\right)\frac{\hat{l}(\widetilde{\beta})}{L}}. \tag{A.20}$$

Observing (A.17) and (A.19) then leads to

$$\begin{aligned} 0 \; = \; & K\left(X\left(\frac{\hat{l}(\widetilde{\beta})}{L}, \phi\right), a, b\right)\left[1 + \sum_{\beta>\widetilde{\beta}}(b\beta-1)\frac{l^S(\beta)}{L} + \left(b\widetilde{\beta}-1\right)\frac{\hat{l}(\widetilde{\beta})}{L}\right] - \\ & \phi\left[1 - \frac{m}{H} + \frac{(a-1)X\left(\frac{\hat{l}(\widetilde{\beta})}{L},\phi\right)}{g\left(X\left(\frac{\hat{l}(\widetilde{\beta})}{L},\phi\right)\right)}\sum_{\gamma\geq\widetilde{\gamma}}\gamma\frac{m^S(\gamma)}{H}\right]. \end{aligned} \tag{A.21}$$

Note that $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$ implies that m/H is constant, according to (A.6). Thus, (A.21) gives us $\hat{l}(\widetilde{\beta})/L$ implicitly as function of (a, b, ϕ). Hence, observing the properties of functions $X\left(\hat{l}(\widetilde{\beta})/L, \phi\right)$ and $K(\chi, a, b)$ confirms the results regarding $\hat{l}(\widetilde{\beta})/L$.[25]

We now turn to wage inequality. First, note that combining $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$ and (A.10) implies $\hat{l}(\widetilde{\beta})/L = \phi\left(\sum_{\gamma\geq\widetilde{\gamma}}\gamma m^S(\gamma)/H\right)/g(\chi) - \sum_{\beta>\widetilde{\beta}} l^S(\beta)/L$. Substituting this expression into (A.20), and using $\kappa = K(\chi, a, b)$ from (A.19), leads to

$$\frac{1 - \frac{m}{H} + \frac{(a-1)\chi}{g(\chi)}\sum_{\gamma\geq\widetilde{\gamma}}\gamma\frac{m^S(\gamma)}{H}}{\frac{1}{\phi} + \frac{b\widetilde{\beta}-1}{g(\chi)}\sum_{\gamma\geq\widetilde{\gamma}}\gamma\frac{m^S(\gamma)}{H} + \frac{b}{\phi}\sum_{\beta>\widetilde{\beta}}(\beta-\widetilde{\beta})\frac{l^S(\beta)}{L}} - K(\chi, a, b) = 0. \tag{A.22}$$

Thus, (A.22) gives us χ implicitly as function of (a, b, ϕ). Hence, observing $\partial K(\chi, a, b)/\partial\chi <$

[25] Note that $\chi/g(\chi)$ is strictly increasing in χ since $g(\chi) - \chi g'(\chi) > 0$.

0 reveals that χ is a decreasing function of ϕ.[26] (Moreover, it is easy to check that changes in a or b affect χ in an ambiguous way.) According to (A.18), this implies that ω decreases with ϕ, while within-group wage inequality $w_m(\gamma)/w_h$ for all $\gamma \geq \widetilde{\gamma}$ increases in ϕ, according to (17).

Next, we confirm that ω decreases with a. First, suppose χ is non-increasing in a. In this case, $\kappa = K(\chi, a, b)$ is increasing in a because of $\partial K(\chi, a, b)/\partial\chi < 0$ and $\partial K(\chi, a, b)/\partial a > 0$. Thus, since (15) implies $\omega = \Omega(\kappa)$ with $\Omega'(\kappa) < 0$, ω is decreasing in a if χ is non-increasing in a. Now suppose to the contrary that χ is increasing in a. In this case, (A.22) imposes $\partial\kappa/\partial a > 0$ and thus, $\partial\omega/\partial a < 0$, according to (15). In sum, we have shown that whatever the sign of $\partial\chi/\partial a$ is, $\partial\omega/\partial a < 0$.

In a similar fashion, we can show that $\partial\omega/\partial b > 0$. First, suppose χ is non-decreasing in b. In this case, $\kappa = K(\chi, a, b)$ is decreasing in b because of $\partial K(\chi, a, b)/\partial\chi < 0$ and $\partial K(\chi, a, b)/\partial b < 0$. Thus, since $\omega = \Omega(\kappa)$ with $\Omega'(\kappa) < 0$, ω is increasing in b if χ is non-decreasing in b. Now suppose to the contrary that χ is decreasing in b. In this case, (A.22) imposes $\partial\kappa/\partial b < 0$ and thus, $\partial w/\partial b > 0$, according to (15). In sum, we have shown that whatever the sign of $\partial\chi/\partial b$ is, $\partial\omega/\partial b > 0$.

Finally, we show that $w_m(\gamma)/w_h$ is increasing in a for all $\gamma \geq \widetilde{\gamma}$. To see this, first, solve (A.18) for $(a-1)/g'(\chi)$ and substitute the resulting expression into (17) to obtain

$$\frac{w_m(\gamma)}{w_h} = \frac{\gamma(b\widetilde{\beta} - 1)}{\omega\left[g(\chi) - \chi g'(\chi)\right]}, \ \gamma \geq \widetilde{\gamma} \tag{A.23}$$

Suppose that χ is increasing in a. Then $\partial\omega/\partial a < 0$ and $g''(\chi) > 0$ unambiguously imply that $w_m(\gamma)/w_h$ is increasing in a for all $\gamma \geq \widetilde{\gamma}$, according to (A.23). Now suppose to the contrary that χ is non-increasing in a. According to (17), also in this case $w_m(\gamma)/w_h$ is increasing in a for all $\gamma \geq \widetilde{\gamma}$. (The impact of an increase in b on inequality within the group of $H-$workers, however, is ambiguous, since its impact on χ is ambiguous.) This concludes the proof. ■

[26] Note that this is no contradiction to (A.17) since $\hat{l}(\widetilde{\beta})/L$ increases with ϕ in scenario 4.

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Table 1. Comparative-static results (marginal effects)

Scenario 1: $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$

	$\frac{\hat{l}(\widetilde{\beta})}{L}$	$\frac{\hat{m}(\widetilde{\gamma})}{H}$	ω	$\frac{w_l(\beta)}{w_l^1}$, $\beta \geq \widetilde{\beta}$	$\frac{w_m(\gamma)}{w_h}$, $\gamma \geq \widetilde{\gamma}$
ϕ	$+$	$+$	0	0	0
a	$+$	$+$	$+$	0	0
b	$+, 0, -$	$+, 0, -$	$+$	$+$	0

Scenario 2: $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) < m^S(\widetilde{\gamma})$

	$\frac{\hat{m}(\widetilde{\gamma})}{H}$	ω	$\frac{w_l(\beta)}{w_l^1}$, $\beta \geq \widetilde{\beta}$	$\frac{w_m(\gamma)}{w_h}$, $\gamma \geq \widetilde{\gamma}$
ϕ	$-$	$-$	$+$	0
a	$+$	$-$	$+$	0
b	0	$+$	$+, 0, -$	0

Scenario 3: $\hat{l}(\widetilde{\beta}) = l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$

	ω	$\frac{w_l(\beta)}{w_l^1}$, $\beta \geq \widetilde{\beta}$	$\frac{w_m(\gamma)}{w_h}$, $\gamma \geq \widetilde{\gamma}$
ϕ	$-$	$+$	$-$
a	$-$	$+, 0, -$	$+$
b	$+$	$+, 0, -$	0

Scenario 4: $\hat{l}(\widetilde{\beta}) < l^S(\widetilde{\beta})$ and $\hat{m}(\widetilde{\gamma}) = m^S(\widetilde{\gamma})$

	$\frac{\hat{l}(\widetilde{\beta})}{L}$	ω	$\frac{w_l(\beta)}{w_l^1}$, $\beta \geq \widetilde{\beta}$	$\frac{w_m(\gamma)}{w_h}$, $\gamma \geq \widetilde{\gamma}$
ϕ	$+$	$-$	0	$+$
a	$+, 0, -$	$-$	0	$+$
b	$+, 0, -$	$+$	$+$	$+, 0, -$

Figure 1: Wage dispersion within the group of *L*-workers