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for Pension Policy:
Period versus Cohort Approach**

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ABSTRACT

Getting Life Expectancy Estimates Right for Pension Policy: Period versus Cohort Approach¹

In many policy areas it is essential to use the best estimates of life expectancy, but such estimates are vital to most areas of pension policy – from indexed access age and the calculation of initial benefits to the financial sustainability of pension schemes and the operation of their balancing mechanism. This paper presents the conceptual differences between static period and dynamic cohort mortality tables, estimates the differences in life expectancy between both tables using data from Portugal and Spain, and compares official estimates of both life expectancy estimates for Australia, the United Kingdom, and the United States for 1981, 2010 and 2060. This comparison reveals major differences between period and cohort life expectancy in and between countries and across years. Using measures of period instead of cohort life expectancy creates an implicit subsidy for individuals of 30 percent or more, with potentially stark consequences on the financial sustainability of pension schemes. These and other implications for pension policy are explored and next steps suggested.

JEL Classification: D9, G22, H55, J13, J14, J16

Keywords: cross-country comparison, Lee-Carter, life expectancy indexation, balancing mechanism

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NON-TECHNICAL SUMMARY

While the discussion about life expectancy and its projected further increase has finally reached policy makers and the public at large, more technical and political attention is needed on the selection of the estimates and their application. Analysis of 5 countries data suggest that period life expectancy, even projected, substantially underestimates policy-relevant cohort life-expectancy by a margin of 30 and more percent. This has main implications for signalling financial stability and for many recent reforms in which pension variables have been linked to period life expectancy measures.

1. Motivation, approach, and structure

It is critical to know and apply the best estimate of individuals' remaining life expectancy in many areas of public policy, but this estimate is vital for pension policy in both the public and private sectors. Policy makers and private sector managers need to know with high confidence the median years that the newly born are expected to live and how this estimate will change in the decades to come. For pension policy, the best estimates of remaining life expectancy at retirement are crucial for determining the initial benefit or to price retirement income products. With the long-term decrease in age-specific mortality rates – the flip side of life expectancy estimates – the long-term increase in life expectancy at a specific age is increasingly anchored in public law and private sector contracts (for example, linking qualifying conditions, initial benefits, or retirement age to estimated life expectancy); in turn, the correct estimates are critical for establishing the financial sustainability of public and private sector schemes and for developing new retirement products.

Two main approaches are used to estimate life expectancy: one relies on period life tables; the other relies on cohort life tables. The period approach is simpler, as it uses mortality information across all ages for a recent period (e.g., a three-year average) to estimate mortality rates, and from these, life expectancy at a specific age. This approach ignores past and likely future improvements – i.e., the trend in the reduction of mortality rates and the consequent increase in life expectancy. The cohort approach incorporates the expected mortality improvement unique to each specific birth cohort, estimating the expected development in mortality rates and life expectancies for each birth cohort, by gender. This approach is much more ambitious and depends on many more assumptions. For this reason, most countries shy away from offering official cohort tables. But even reliable and country-produced official period tables are the exception rather than the rule for most countries across the world. Low-income and many emerging countries typically rely on United Nations (UN) estimates that are grounded in period tables; for projections, they apply a robust cohort-type approach adjusted to typically low-quality data.² Thus the differences between the conjectured higher and more reliable cohort life expectancies and their lower and biased period estimates are normally unknown.

This paper is structured as follows: Section 2 presents the conceptual difference between period and cohort mortality tables; the analytical underpinnings of cohort tables, including a brief overview of the different estimation methods used; and the authors' estimates of period and cohort life tables and life expectancy at birth and at assumed retirement age of 65 for Portugal

² See UN (2017) for the presentation of the methodology used in their most recent 2017 demographic projections to 2100, including the patterns of mortality decline (PMD) method. The latter is a dynamic version of the period table or a simplified cohort approach. A companion technical paper by Gu, Pelletier, and Sawyer (2017) compares the performance of the PMD method and the three variants of the modified Lee-Carter (MLC) method as applied to age- and sex-specific death rates (m_x) from 1950–1955 to 2010–2015 and used to project m_x from 2015–2020 to 2095–2100 for 155 countries. Overall, the MLC method regardless of its variants generally worked well for countries with good data quality, whereas the PMD method performed better for countries with lower data quality. Their study suggests that the MLC method produces less stable results for future age-sex-specific death rates for countries with relatively low-quality data.

and Spain. Section 3 broadens the international comparison to official estimates of both period and cohort life expectancy for distant past, recent, and future years for Australia, the United Kingdom (UK), and the United States (US). This comparison reveals major differences between estimates of period and cohort life expectancy in and between countries and across years that amounts to a subsidy rate on benefits of 30 and more percent if the inadequate lower estimate is chosen. Section 4 discusses the main implications of these differences: What is the scope of the differences in economic terms between individuals? What does this mean for public pension schemes' financial sustainability and their balancing mechanism? How do these differences add to the observed heterogeneity of longevity in period life considerations? How does this affect recent pension reforms that link life expectancy to scheme parameters? Section 5 summarizes the results and implications and proposes a simple way forward.

2. Period versus cohort approaches in measuring life expectancy

This section explains the difference between period and cohort life expectancy and highlights the importance of using the appropriate longevity measure in a context where past mortality improvements are projected to continue in the future. It briefly reviews the main methods used in practice by actuaries and demographers to forecast future mortality rates. Finally, it illustrates the use of these methods by computing period and cohort life expectancies using data from Portugal and Spain.

2.1. Explaining the difference in life expectancy measures

Life expectancy is the most common statistical measure of the average remaining lifetime an individual is expected to live, given his current age, year of birth, sex, and other demographic and socioeconomic factors, including education, income, and job (Ayuso, Bravo, and Holzmann 2017a). Life expectancy is critical in assessing a number of public policies, including pension schemes and health care systems. To compute life expectancy, the usual procedure involves building an ordinary life table, a tabular statistical tool that summarizes the survival and mortality experiences of a population and yields additional understanding about longevity prospects. In the past, analytical methods and mortality laws (e.g., De Moivre, Gompertz, Makeham, Weibull, logistic) were used to compute life expectancy estimates (Bravo 2007).

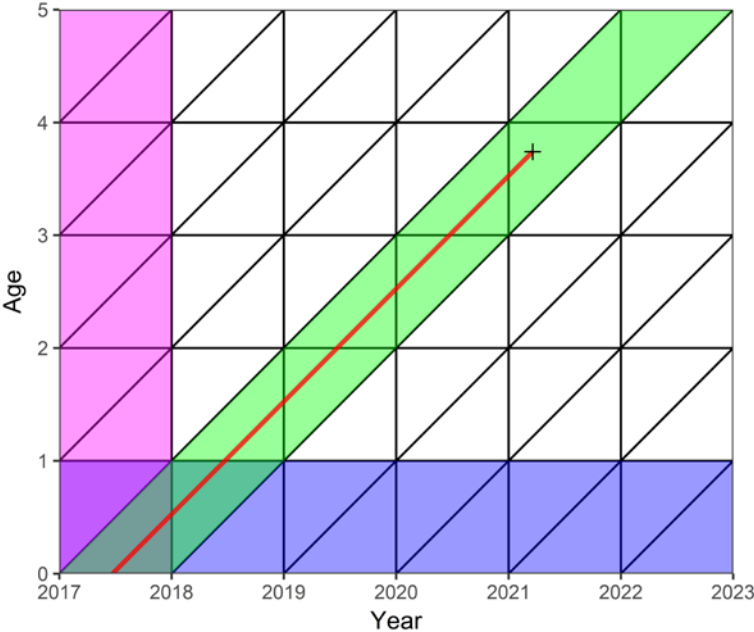
Many different types of life tables exist. Yet this paper focuses on the difference between period and cohort life tables since they represent the two main frameworks by which mortality can be analyzed. Period life tables represent the mortality risks experienced by the different cohorts of an entire population during a single, relatively short period of time, usually no longer than three years. The corresponding period life expectancies assume that the mortality rates observed at a given moment in time apply throughout the remainder of a person's life; i.e., they neglect any future expected changes in the longevity prospects of the population. Period life expectancies are useful if one wants to compare trends in mortality by gender, over time, by socioeconomic

risk factors, within regions of a country, or with other countries, but they do not actually represent the longevity prospects of individuals born in a given year.

Cohort or generation life tables represent the mortality experienced by a cohort of individuals born during a relatively short period of time (typically one year) over the course of their entire lifetime. They require age-specific probabilities of death computed using mortality data from the cohort only. Although cohort life tables based entirely on observed mortality are quite rare in practice since they require consistent quality data over more than a century, cohort life tables based on a combination of past and expected future mortality for the cohort are more common, particularly in actuarial practice and population projection exercises. Contrary to period life expectancy indicators, cohort life expectancy measures take into account both observed and projected longevity improvements for the cohort throughout its remaining lifetime and are therefore considered a more appropriate measure of an individual's future longevity prospects. Computing cohort life expectancy requires projecting future mortality rates using stochastic mortality models, thus introducing uncertainty (model and parameter risks) regarding the validity of the assumptions used to project future demographic trends.

A simple visual way of understanding the differences between the period and cohort life expectancy measures is through a traditional Lexis diagram (Figure 2.1).

Figure 2.1: Lexis diagram



Source: Authors' preparation.

This two-dimensional diagram is commonly used to represent events (e.g., deaths) related to individuals belonging to different generations. Age is usually represented on the vertical axis, and calendar time on the horizontal axis. Each demographic event corresponds to a point whose

coordinates are the time in which it occurred and the age of the individual. Points for demographic events are counted in triangles that represent the double classification by age and year of birth. Diagonals delimit an individual's lifeline in a Lexis diagram. The lifeline is a straight line that starts with an individual's birth at the respective point on the horizontal axis and ends with an individual's death (if observed).

Suppose the above Lexis diagram represents the mortality rates for a given population. To compute the period life expectancy at birth for a newborn in 2017, one would use the mortality rates for 2017 for ages 0, 1, 2, and 3 years and so on, up to the highest age attainable in the life table, usually around age 120 (vertical magenta column). In other words, in computing the period life expectancy for a newborn in 2017, it is assumed that this child will experience throughout its lifetime the same death probabilities that his mother, grandmother, and great-grandmother experienced in 2017, an extremely unlikely scenario given the dynamics of mortality observed in developed countries over the last decades. During the last two centuries, the life expectancy frontier of developed countries experienced a persistent linear increase in measured period life expectancy (Oeppen and Vaupel 2002). Past trends provide overwhelming evidence to suggest that declines in mortality rates are expected to continue in the future, making period life expectancies misleading in assessing longevity developments by systematically underestimating the remaining lifetime of individuals.

To compute the cohort life expectancy for a newborn in 2017, one would use the mortality rates at 0 in 2017, at age 1 in 2018, at age 2 in 2019, and so on, following the green diagonal band in Figure 2.1. Future mortality improvements are incorporated in the computation to better reflect the actual longevity prospects of this individual. As a result, period life expectancy would match cohort life expectancy only if no change occurred in age-specific mortality rates over time. In a scenario in which mortality is expected to decline (increase) over time, cohort life expectancy will always be higher (lower) than period life expectancy. Finally, the Lexis diagram can be used to highlight (in blue) the particular mortality profiles of a given age (age 0 in this example) over time.

2.2. Estimating the difference in life expectancy measures

Computing period and cohort life expectancy depends on forecasting age-specific mortality rates. Considerable attention has been paid to methods for forecasting mortality in recent years. Much of this work emerged from the seminal model proposed by Lee and Carter (1992). This model assumes that the force of mortality has a log-bilinear structure combining age and period parameters, the latter representing a general common time trend in mortality to be modelled using time series methods to produce mortality projections and generate prospective life tables. The Lee-Carter (LC) model motivated numerous variants and extensions. Alternative estimation approaches were developed by Lee and Miller (2001), Booth, Maindonald, and Smith (2002), and Brouhns et al. (2002a,b) to provide more robust statistical properties (Poisson regression setting and maximum likelihood (ML) estimation techniques) and to improve the model's goodness-of-fit and forecasting performance.

Renshaw and Haberman (2006) generalize the LC model by including cohort effects within generalized linear models (GLMs) with Poisson error structures. Currie (2006) develops a simplified version of the Renshaw-Haberman model with independent age, period, and cohort effects. Hyndman and Ullah (2007) propose a particular version of the LC model called the Functional Demographic Model (FDM), extending the original principal components approach by adopting a functional data paradigm. Currie et al. (2004) develop the P-spline approach, a penalized fitting process using basis splines.

Cairns, Blake, and Dowd (CBD) (2006) assume that the logit of the one-year death probability is a linear function of age, and treat the intercept and slope parameters across years as stochastic processes. Cairns et al. (2009) investigate three extensions to the original CBD model by incorporating combinations of a quadratic age term and a cohort effect term. Plat (2009) combines the LC and CBD models to generate a model that produces appropriate mortality estimates for all age ranges and is consistent with cohort effects.

Hunt and Blake (2014) present a general procedure for constructing mortality models that improves the goodness-of-fit of the model parsimoniously and exhibits demographic significance. The authors describe an age-period-cohort model structure that encompasses the vast majority of previous stochastic mortality models. Currie (2016) shows that many mortality models can be expressed in terms of GLMs or generalized nonlinear models (GNLMs). A number of papers develop multipopulation mortality modelling (e.g., Li and Lee 2005; Cairns et al. 2011; Dowd et al. 2011; Jarner and Kryger 2011; Zhu, Tan, and Wang 2017). Others focus on cause-of-death mortality modelling (e.g., Hanewald 2011; Gourieroux and Lu 2015). Other extrapolative approaches use Bayesian modelling, GLMs, and state-space approaches. An extensive review of mortality forecasting methods can be found in Bravo (2007), Booth and Tickle (2008), and Blake et al. (2017). A most recent and promising estimation approach takes account of the accelerating decrease in mortality rates across industrialized countries, particularly at higher ages (Palmer, Alho, and Zhao de Gosson de Varennes 2018). Their ex-post and ex-ante evaluations against 2600 birth cohort data of eight countries suggest a sizable and rising underestimation of cohort life expectancy using existing methods.

Nevertheless, to illustrate the use of mortality forecasting models for the computation of period and cohort life expectancies, the dynamics of mortality rates are modelled herein using the log-bilinear LC model under a Poisson setting (Brouhns et al. 2002a; Renshaw and Haberman 2003). This model assumes that:

$$D_{x,t} \sim \text{Poisson}(\mu_{x,t} E_{x,t}) \quad \text{with} \quad \mu_{x,t} = \exp(\alpha_x + \beta_x \kappa_t) \quad (1)$$

where $D_{x,t}$ is the number of deaths recorded at age x during year t , from those exposed to risk $E_{x,t}$; $\mu_{x,t}$ denotes the observed force of mortality at age x during year t ; α_x denotes the general

shape of the mortality schedule; β_x represents the age-specific patterns of mortality change; and κ_t represents the time trend. Additionally, the parameter estimates have to be constrained by:

$$\sum_{t=t_{\min}}^{t=t_{\max}} \kappa_t = 0 \quad \text{and} \quad \sum_{x=x_{\min}}^{x=x_{\max}} \beta_x = 1 \quad (2)$$

To forecast mortality rates, the LC model is first calibrated to Portugal and Spain's overall populations, using data from 1980 to 2015 and for ages 0–95. Data on deaths and exposures are obtained from the Human Mortality Database (2017). Parameter estimates are obtained using ML methods and an iterative method for estimating log-bilinear models developed by Goodman (1979), considering the usual identification constraints. It is then assumed that the age vectors α_x and β_x remain constant over time and forecast future values of κ_t using a standard univariate time series ARIMA model. Finally, to close the prospective life tables at high ages and to establish the highest attainable age ω , the simple and efficient method proposed by Denuit and Goderniaux (2005) is applied.

Once the matrix of observed and projected mortality rates $\{q_{x,t}\}$, $x \in [x_{\min}, x_{\max}]$, $t \in [t_{\min}, t_{\max}]$ is generated, complete period, $\dot{e}_{x,t}^p$, and cohort, $\dot{e}_{x,t}^c$, life expectancies can be computed by age and calendar year using:

$$\dot{e}_{x,t}^p = \frac{1}{2} + \sum_{k=1}^{\omega-x} \left(\prod_{j=0}^{k-1} (1 - q_{x+j,t}) \right) \quad (3)$$

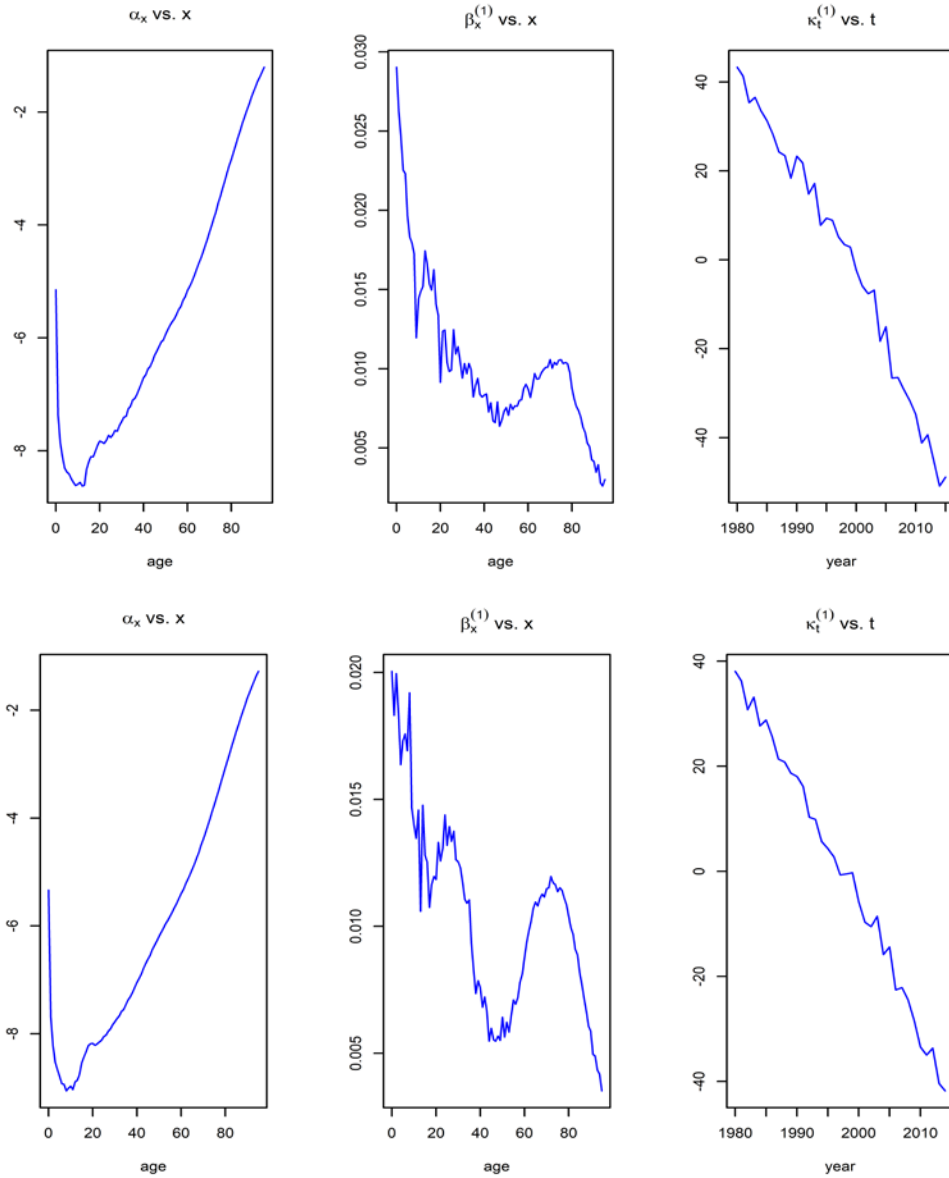
and

$$\dot{e}_{x,t}^c = \frac{1}{2} + \sum_{k=1}^{\omega-x} \left(\prod_{j=0}^{k-1} (1 - q_{x+j,t+j}) \right) \quad (4)$$

2.3. Results for Portugal and Spain

This section reports the LC parameter estimates (Figure 2.2), the forecasted period and cohort life expectancies (Figure 2.3) and the forecasted mortality rates for some representative ages (Figure 2.4) for the Portuguese and Spanish female populations. Figure 2.2 demonstrates that the general shape of mortality across ages (as represented by the α_x parameter estimates) exhibits similar patterns in Portugal and Spain between 1980 to 2015.

Figure 2.2: Lee-Carter parameter estimates for the Portuguese (top) and Spanish (bottom) female populations



Source: Authors' estimates.

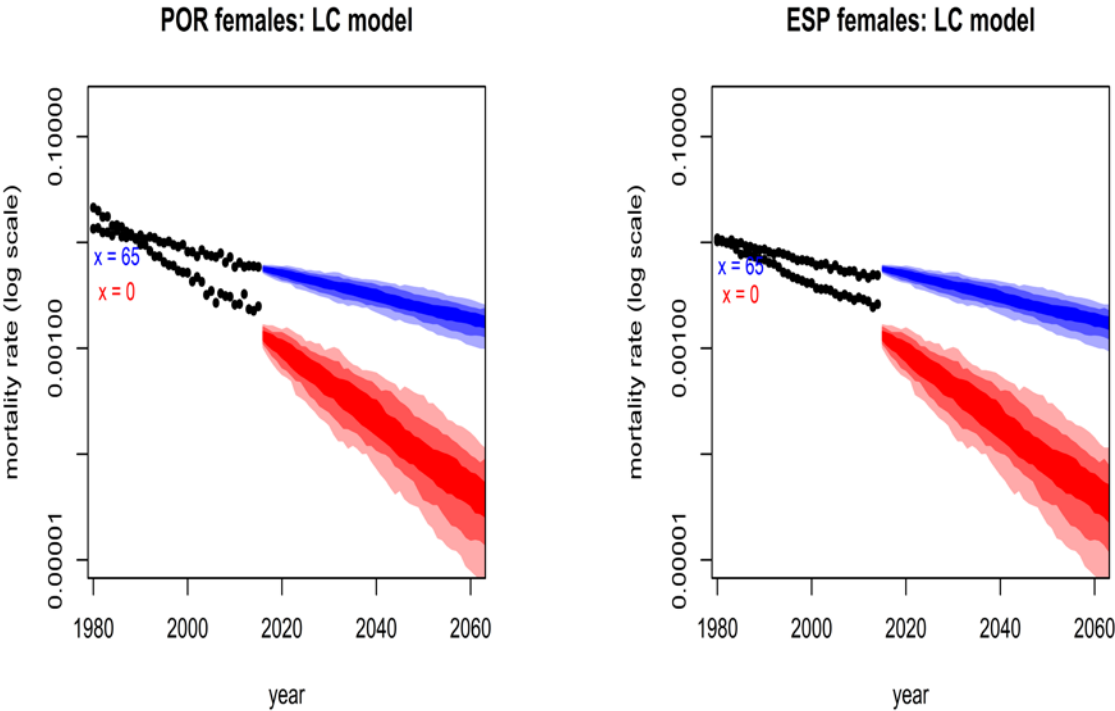
As is common in developed countries, average mortality rates are relatively high for newborns and children, then decrease rapidly toward their minimum (around age 12), increasing thereafter with age, reflecting higher mortality at older ages. The only minor exception is the well-known "mortality hump" around ages 20–25, normally more pronounced in the male population, a phenomena normally associated with accident- or suicide-related mortality.

The time trend parameter estimates κ_t exhibit a clear decreasing tendency (approximately linear) in both countries, indicating the significant mortality improvements registered for all ages and both sexes over the last 35 years. The rhythm at which mortality improvements have taken

place is not homogeneous across ages, however, as observed from the β_x parameter estimates. Observed mortality improvements have been more significant for youth, particularly in Portugal due to better infectious diseases control, better health care systems, and improved living conditions, but are also relevant for adults and the elderly.

The forecasted mortality rates project into the future past trends observed in mortality across all ages. Figure 2.3 uses bootstrap simulation methods to derive confidence bands for the mortality rates of Portuguese and Spanish female populations aged 0 and 65. As can be observed, the Poisson-Lee-Carter method projects a continued decline in mortality at these ages, with increased volatility around the general trend more significant at birth.

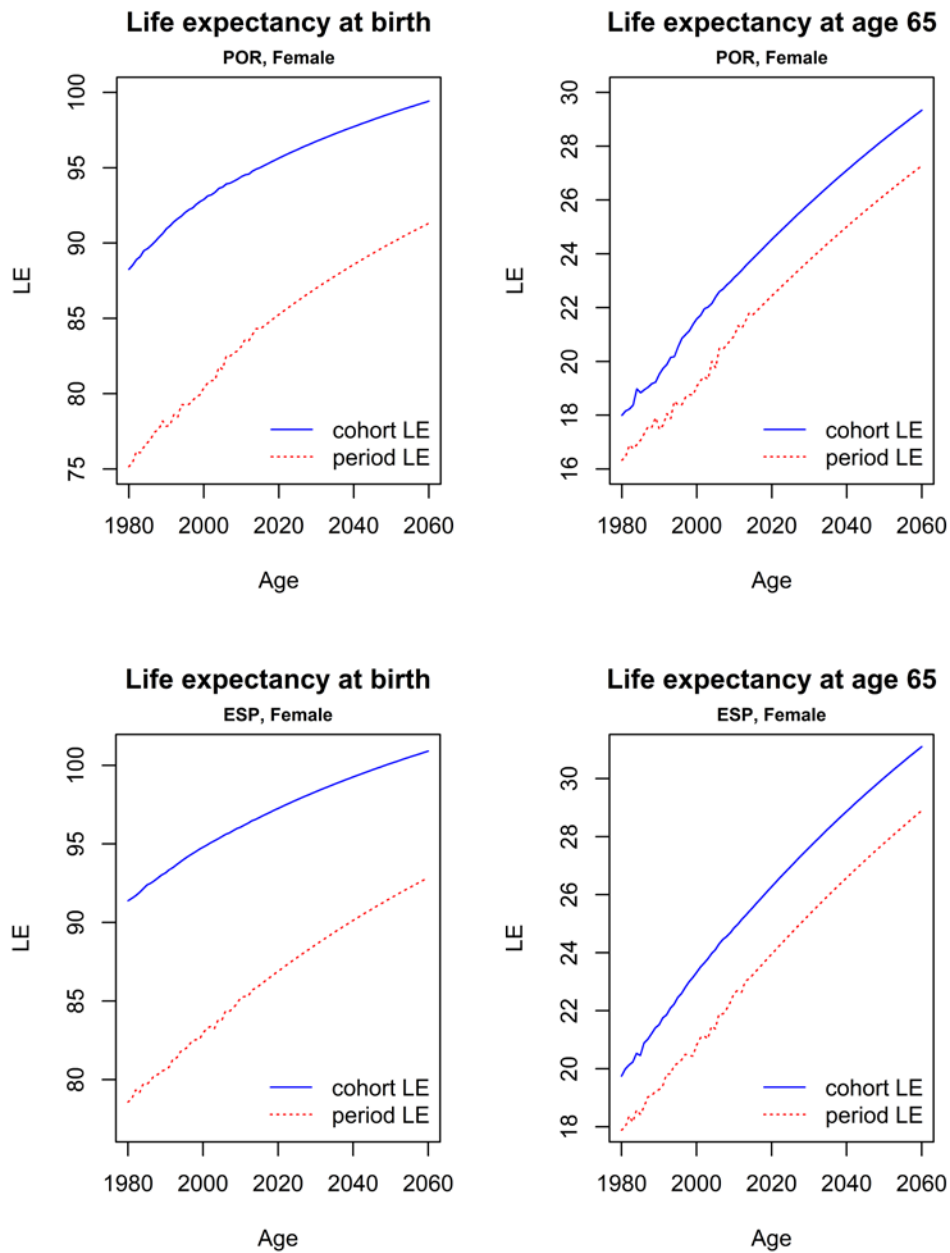
Figure 2.3: Forecasted mortality rates and confidence bands for the Portuguese and Spanish female populations



Source: Authors' estimates

Figure 2.4 reports period and cohort life expectancies computed at birth and at age 65 for Portuguese and Spanish females for the period between 1980 and 2060. In both populations the difference between the two approaches is significant, with period life expectancy indicators clearly underestimating future longevity prospects. The difference is, as expected, more significant at birth (13.1 years in 1980 in Portugal and 12.8 years in Spain) than at age 65 (1.7 years in 1980 in Portugal and 1.9 years in Spain).

Figure 2.4: Period and cohort life expectancies for the Portuguese and Spanish female populations



Source: Authors' estimates.

Similar results were obtained for the male populations of both countries. Differences in life expectancy indicators will likely continue to be noticeable in the future, although the gap is forecasted to be smaller at birth and slightly higher at age 65.

3. Period versus cohort life expectancy estimates: International results

Cohort life expectancies currently exceed period life expectancies, due to the observed decreases in mortality rates that started in the 18th century in some countries and continue in the 21st century worldwide (see Ayuso, Bravo, and Holzmann 2015). As explained in the preceding section, period tables are static tables built on the basis of the mortality behavior observed in the population during one period, while cohort tables incorporate projections of the future trend in mortality, taking into account observed changes over time, at birth and at different ages for different generations. The different demographic institutes across countries do not construct cohort tables as frequently as they do period tables. In fact, for most countries, information on observed and projected life expectancy based on static calculations (typically jointly collected for different countries by international organizations such as the UN,³ the World Bank,⁴ Eurostat,⁵ and OECD⁶) can be found, and is systematically used in calculations related to pensions, health, long-term care, and welfare status; on the contrary, it is rare to find life expectancy estimates based on cohort tables.

This section compares the limited comparable country data on period and cohort life expectancy that exist from official sources for Australia, the UK, and the US, supplemented by the estimates of cohort life expectancy for Portugal and Spain presented above.⁷ The three data points cover the years 1981, 2010, and 2060. The period and cohort life expectancy by sex, at birth and at age 65, for these countries and calendar years, and differences between them in absolute terms are presented in Table 3.1a, Table 3.2b, Table 3.2a, and Table 3.2b. Differences between gender estimates are presented in Table 3.1c and Table 3.2c. The results for the analyzed countries at birth and at age 65 are plotted in Figure 3.1 and Figure 3.2, respectively.

³ <http://www.un.org/en/development/desa/population/>

⁴ <http://databank.worldbank.org/data/reports.aspx?source=health-nutrition-and-population-statistics>

⁵ http://ec.europa.eu/eurostat/web/population-demography-migration-projections/deaths-life-expectancy-data/database?p_p_id=NavTreeportletprod_WAR_NavTreeportletprod_INSTANCE_OAgLk1BN22Jg&p_p_lifecycle=0&p_p_state=normal&p_p_mode=view&p_p_col_id=column-2&p_p_col_count=1

⁶ http://www.oecd-ilibrary.org/social-issues-migration-health/health-key-tables-from-oecd_20758480;jsessionid=1pv4a51h45r01.x-oecd-live-03

⁷ The period and cohort life expectancy estimates for New Zealand and their differences and trends support the analysis. Their results are not included here as the available years do not coincide with this analysis. For New Zealand estimates, see http://archive.stats.govt.nz/browse_for_stats/health/life_expectancy/cohort-life-expectancy.aspx.

Table 3.1a: Period and cohort life expectancy at birth: International comparison, by gender

	1981		2010		2060	
	<u>Period</u>	<u>Cohort</u>	<u>Period</u>	<u>Cohort</u>	<u>Period</u>	<u>Cohort</u>
Men						
UK	70.90	84.60	78.40	89.80	86.70	96.90
US	70.37	77.78	75.40	80.96	80.18	84.63
Australia	71.23	86.50	79.75	92.40	88.50	93.00
Spain	72.52	81.13	79.05	89.94	87.38	95.47
Portugal	68.30	81.13	76.76	88.33	85.82	94.29
Women						
UK	76.90	88.30	82.40	92.60	89.30	99.30
US	77.85	82.40	79.95	84.96	83.93	88.00
Australia	78.27	89.10	84.21	94.75	90.00	96.00
Spain	78.78	91.53	85.21	96.07	92.85	100.90
Portugal	75.43	88.53	83.14	94.41	91.31	99.42

Source: The UK (Office for National Statistics; 2014 Principal-based principal projection life expectancy variant); the US (Life Tables for the United States Social Security Area 1900–2100); Australia (PCPOP and ABS 2008, Australian Historical Population Statistics, Cat. No. 3105.0.65.001; and ABS (various issues), Life Tables, States, Territories and Australia, Cat. No. 3302055001DO001); Spain (Period life expectancies – Instituto Nacional de Estadística INE; Cohort life expectancies – Authors’ estimates); Portugal (Period life expectancies – Instituto Nacional de Estadística INE; Cohort life expectancies – Authors’ estimates).

Table 3.1b: Difference in years (absolute value) between the cohort and period life expectancy at birth: International comparison, by gender

	1981	2010	2060
Men			
UK	13.70	11.40	10.20
US	7.41	5.56	4.45
Australia	15.27	12.65	4.50
Spain	8.61	10.88	8.10
Portugal	12.82	11.56	8.47
Women			
UK	11.40	10.20	10.00
US	4.55	5.01	4.07
Australia	10.83	10.54	6.00
Spain	12.76	10.86	8.05
Portugal	13.10	11.27	8.11

Source: Authors’ calculations based on Table 3.1a.

Table 3.1c. Comparison between period and cohort life expectancy differences (years) at birth, by gender

	1981	2010	2060
	Cohort-Period	Cohort-Period	Cohort-Period
Men–Women			
UK	2.30	1.20	0.20
US	2.86	0.55	0.38
Australia	4.44	2.11	-1.50
Spain	-4.15	0.02	0.05
Portugal	-0.28	0.29	0.36

Source: Authors' calculations based on Table 3.1b.

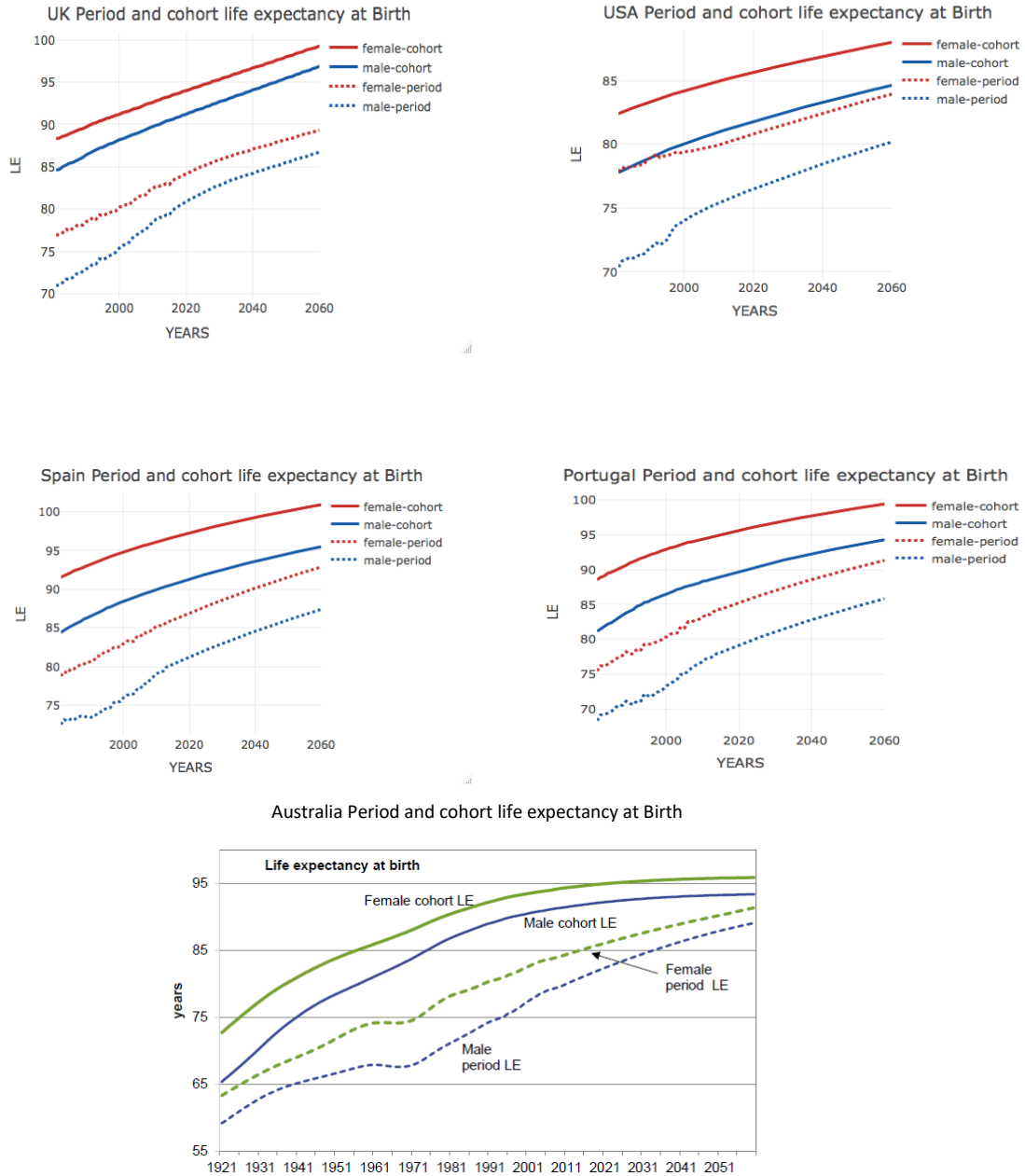
Table 3.1a confirms that cohort life expectancy estimates are always greater than those for period life expectancy for all compared countries and years. These differences are presented in Table 3.1b, with values higher than 10 years for several countries and periods analyzed.

This is the case for the UK, Australia, and Portugal, which have the greatest differences between the two estimates. Differences also exist in the US and Spain, but are less pronounced (e.g., in the US, the difference in life expectancy at birth between period and cohort estimates in 2010 is about 5.5 years). In all five countries, differences between the two values are projected to decrease over time, probably due to the smaller margin expected to improve survival probabilities (taking into account the high probabilities already reached in advanced ages).

Differences between countries are also observed by gender (Table 3.1c). In general, greater differences are seen between the estimates of period and cohort life expectancy at birth in men than in women. Only in Australia is a greater difference projected for women in 2060, a phenomenon that also occurred in Portugal and Spain in 1981. However, a reduction in the differences between men and women is generally projected over time (except in Spain and Portugal, where they remain essentially unchanged), which could be driven by the reduction in the gender gap in life expectancy in these countries.

A similar analysis for life expectancy at age 65 is presented in Table 3.2a, Table 3.2b, and Table 3.2c. Table 3.2a again shows that estimated life expectancies from cohort tables are higher than those obtained from period tables in all five countries. Differences in years are presented in Table 3.2b; the biggest values are observed for Australia (up to a 9 year difference in life expectancy), followed by the US (up to a 4 year difference). Again, projected values reflect a reduction in the (albeit still positive) differences between cohort and period estimates over time.

Figure 3.1: Period and cohort life expectancy at birth, by gender



Sources: See table 3.1a

Analyzing the differences between men and women (Table 3.2c), only small differences between genders tend to persist over time. In the UK, the US, and Australia, men show greater differences between cohort and period life expectancy estimates at 65 years; the opposite result is found in Spain and Portugal, which show slightly higher differences for women.

Table 3.2a: Period and cohort life expectancies at age 65: International comparison, by gender

	1981		2010		2060	
	<u>Period</u> (1989 for UK)	<u>Cohort</u> (1989 for UK)	<u>Period</u> (2014 for UK)	<u>Cohort</u> (2014 for UK)	<u>Period</u> (2039 for UK)	<u>Cohort</u> (2039 for UK)
Men						
UK	13.8	15.3	18.6	21.2	22.3	24.0
US	14.24	19.43	16.55	20.91	19.45	23.08
Australia	14.15	26.05	19.10	28.40	25.80	29.80
Spain	14.66	15.82	18.41	20.12	24.25	25.88
Portugal	13.40	16.46	17.21	18.90	23.34	24.96
Women						
UK	17.6	18.9	21.1	23.5	24.2	26.1
US	18.58	22.02	19.16	23.43	21.93	25.48
Australia	18.10	28.50	22.10	31.10	28.35	32.30
Spain	17.99	19.99	22.61	24.86	28.89	31.10
Portugal	14.37	18.16	20.95	23.13	27.28	29.34

Source: the UK (Office for National Statistics; 2014 Principal-based principal projection life expectancy variant); the US (Life Tables for the United States Social Security Area 1900–2100); Australia (PCPOP and ABS 2008, Australian Historical Population Statistics, Cat. No. 3105.0.65.001; and ABS (various issues), Life Tables, States, Territories and Australia, Cat. No. 3302055001DO001); Spain (Period life expectancies – Instituto Nacional de Estadística INE; Cohort life expectancies – Authors’ estimates); Portugal (Period life expectancies – Instituto Nacional de Estadística INE; Cohort life expectancies – Authors’ estimates).

Table 3.2b: Differences in years (absolute value) between period and cohort life expectancy at age 65: International comparison, by gender

	1981	2010	2060
	<u>Cohort-Period</u> (1989 for UK)	<u>Cohort-Period</u> (2014 for UK)	<u>Cohort-Period</u> (2039 for UK)
Men			
UK	1.50	2.60	1.70
US	5.19	4.36	3.63
Australia	11.90	9.30	4.00
Spain	1.16	1.71	1.63
Portugal	3.06	1.69	1.62
Women			
UK	1.30	2.40	1.90
US	3.44	4.27	3.55
Australia	10.40	9.00	3.95
Spain	2.00	2.25	2.21
Portugal	3.79	2.18	2.06

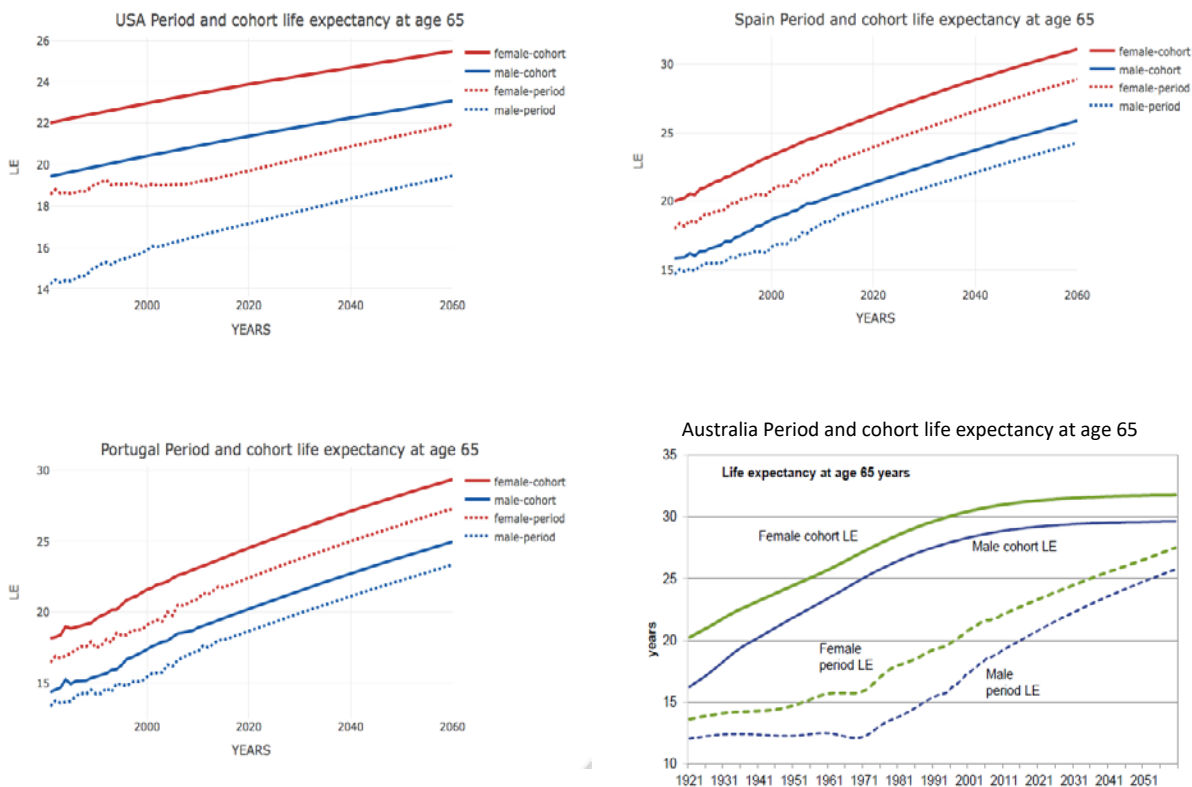
Source: Authors’ calculations based on Table 3.2a.

Tale 3.2c: Comparison between period and cohort life expectancy differences at age 65, by gender

	1981	2010	2060
	Cohort-Period	Cohort-Period	Cohort-Period
Men– Women			
UK	0.2	0.2	-0.2
US	1.75	0.09	0.08
Australia	1.5	0.3	0.05
Spain	-0.84	-0.54	-0.58
Portugal	-0.73	-0.49	-0.44

Source: Authors' calculations based on Table 3.2b.

Figure 3.2: Period and cohort life expectancy at age 65, by gender



Source: See Table 3.1a.

4. Implications for pension policy

Applying estimates of remaining life expectancy at retirement by using the lower period life expectancy instead of the higher cohort life expectancy has two key implications for pension policy.

First, at the individual level, doing so fails to establish an actuarially fair link between contributions and benefits, thus distorting individuals' labor supply and saving decisions (contrary to the goal of recent reform attempts). Assuming that cohort life expectancy is the correct estimate, using the lower period life expectancy to calculate the initial benefit implies a subsidy for individuals that may not only bias one's labor supply while young and one's retirement decision when older – it may also affect one's savings and dis-savings decisions over a lifetime. While the bias can go in both directions, the income effect is likely to dominate the substitution effect, which may lower one's labor supply when young and advance the retirement age; for saving, one may lower accumulation efforts when young and decumulate faster when retired.

Second, at the pension scheme level, use of the lower period life expectancy makes the pension scheme financially unsustainable, as it incorrectly signals solvency; i.e., that liabilities are smaller or at most equal to assets, while in reality this is not the case. This is valid for both unfunded and funded schemes: in funded schemes, assets are essentially unchanged by an underestimation of remaining life expectancy, while liabilities increase. In unfunded (nonfinancial) schemes, the contribution asset is negatively affected as it represents the present value of the difference between future contributions and the liabilities thereby created; if the life expectancies are actually higher, the pay-as-you-go (PAYG) asset must be lower and hence both liabilities and assets deteriorate, with liabilities higher and assets lower than assumed.

Of course, underestimation of actual life expectancy does not remain unnoticed, but emerges only gradually in periodic asset/liability checks or more often in the annual cash-flow comparison when expenditure due to longer periods of benefit payment exceed contribution revenues. The policy reaction is typically an ad hoc adjustment in nonfinancial defined benefit (NDB) schemes' parameters (such as increasing the contribution rate, playing with the indexation parameters of benefits under disbursement, or calling for an increase in the retirement age; else the government transfers are increased). In nonfinancial defined contribution (NDC) schemes, the key policy options are a lower notional interest rate for the annual account accumulation and a lower indexation for benefits in disbursement. In both cases, the consequences of systematically underestimated life expectancy are shared in an ad hoc manner between the working and retired populations. Such a disruptive approach does not create the confidence in the scheme that pension economists consider important for a smooth and successful operation.

This section explores four policy areas: (a) How important is the difference between period and cohort life expectancy for individual decisions and financial sustainability underestimation?; (b) What is the effect of the wrong life expectancy choice on the balancing mechanism recently

implemented in a number of NDB and NDC schemes, and is there a difference between these schemes with regard to the choice of life expectancy estimation? (c) What happens if heterogeneity in longevity exists?; and (d) How relevant are life expectancy changes as policy triggers after recent reforms?

4.1. How important is the difference between period and cohort life expectancy?

Section 3 offers the estimated magnitudes between period and cohort life expectancies at birth (age 0) and retirement (assumed at age 65). It is this latter age that matters most for pension policy considerations. Table 3.2a and Table 3.2b offer the scope of and differences between both approaches.

These magnitudes and their ratio can be given a simple welfare economic interpretation through the concept of pension wealth (see Ayuso, Bravo, and Holzmann 2017b). Pension wealth at any age is the present value of future benefit streams at this age. Assuming that the benefit indexation equals the discount rate (an assumption that broadly holds for wage-indexed pensions), then pension wealth at retirement is the pension benefit at retirement multiplied by life expectancy. If the scheme is actuarially fair, accumulation at retirement should equal pension wealth; i.e., the divisor of the accumulation needs to be the correct life expectancy.

$$[5] \quad PW_{65} = b_{65} LE_{65}$$

$$[6] \quad b_{65} = AK_{65} / LE_{65}$$

$$[7] \quad PW_{65} = AK_{65} \text{ for an actuarially fair scheme}$$

where PW is pension wealth, LE is life expectancy, AK is accumulated contributions (plus interest), and b is the initial pension benefit, all at retirement age 65.

If the initial benefit is calculated by using the too-low period life expectancy (PLE), then actual pension wealth exceeds the value of the accumulation by the ratio of the cohort life expectancy (CLE) to the PLE. Expressing the ratio as a change in the difference amounts to a subsidy that the generation would receive (unless corrective actions were undertaken).

$$[8] \quad \text{Subsidy rate} = PW_{65} [CLE] / PW_{65} [PLE] - 1 = ((AK_{65} / PLE_{65}) * CLE_{65}) / ((AK_{65} / PLE_{65}) * PLE_{65}) - 1 = CLE_{65} / PLE_{65} - 1$$

From Table 3.2b, one can calculate the implicit subsidy rate at retirement that is behind the differences between period and cohort life expectancy for the five countries and the three years of estimation (Table 4.1). The reason this can be called a subsidy rate is very simple: this is the rate at which own accumulations would need to increase to achieve the same benefit level as that derived from applying the period life expectancy rate to own accumulations.

Table 4.1: Implicit subsidy rates of applying period over cohort life expectancy in select countries, by gender

	1981	2010	2060
Men			
UK	10.9%	14.0%	7.6%
US	36.4%	26.3%	18.7%
Australia	84.1%	48.7%	15.5%
Spain	7.9%	9.3%	6.7%
Portugal	22.8%	9.8%	6.9%
Average	32.4%	21.6%	11.1%
Women			
UK	7.4%	11.4%	7.9%
US	18.5%	22.3%	16.2%
Australia	57.5%	40.7%	13.9%
Spain	11.1%	10.0%	7.6%
Portugal	26.4%	10.4%	7.6%
Average	24.2%	18.9%	10.6%

Source: Authors' calculations based on Table 3.2a.

Table 4.1 indicates both major differences and commonalities in the implicit subsidy rate between countries for which both period and cohort life expectancies are available. First, all countries have a declining subsidy rate and thus a shrinking relative gap between period and cohort life expectancy over time. While for the age cohort of 1981 the average difference is 32 percent and 24 percent for men and women, respectively, the difference reduces to slightly above 10 percent for both genders for the age cohort of 2060. Second, the differences between genders are reduced across 1981, 2010, and 2060 for Australia, the UK, and the US; in Portugal and Spain they remain broadly constant. Third, the differences between countries are also reduced. While the differences in 1981 between the highest and lowest country value were about 10:1, this ratio reduces to 2:1 in 2060.

It is not clear the extent to which these reduced differences are related to the application of similar or common estimation models or if these common trends actually constitute common underlying developments. In any case the scopes are relevant, and comparable in magnitude to the heterogeneity created by differences in lifetime income (see Section 4.3). For the current mid-career generation of 1981 or the current primary school generation of 2010, the subsidy created by applying period instead of cohort life expectancy is sizable and may distort their labor supply and savings decisions, working against the objectives of recent systemic and comprehensive parametric reforms in these five countries.

4.2. The balancing mechanism in defined benefit and defined contribution schemes

Most advanced and some emerging economies undertook systemic or comprehensive parametric reforms with the objective of making their pension system financially sustainable (or more specifically, their main earnings-related pension scheme(s)). To deal with future financial disequilibria, various countries introduced a balancing mechanism – i.e., a rule-bound mechanism of parametric adjustments to the scheme that is triggered when financial disequilibria emerge.⁸ The adjustment may be in the level of benefit indexation, a reduction in the nominal benefit level, a decrease in the annual accrual rate (in NDB schemes), or a change in the annual account indexation rate (in NDC schemes). What triggers the application of the balancing mechanism may simply be differences in expenditure and revenues of the scheme, or some measure of an actuarial imbalance based on the present value of deficits, or the application of more elaborate asset/liability comparisons. To establish financial (un-)soundness, some countries undertake annual or periodic actuarial assessments (e.g., Japan, Sweden, the UK, and the US).

How are these balancing mechanisms influenced if the “wrong” mortality/life expectancy data are selected? What is the scope of underestimated liabilities? Are assets also affected? Is there a difference between benefit type (DB/DC) and funding mechanism? These issues are discussed next in turn.

The scope of underestimated liabilities: For many policy makers and pension observers, this is quite likely the key question, as the size of underestimation of liabilities may determine the speed and type of corrective interventions. A full reply is, of course, country specific but the results of Table 4.1 suggest the magnitude. The birth cohort of 1981 (i.e., the cohort of those currently 37 years old) can be taken as the low boundary estimate for the current generation. If correct and without taking into account future corrective interventions, the subsidy rates can proxy the difference between liabilities estimated with cohort mortality rates/life expectancy and with the corresponding period estimates. The difference in the all-country average is well above 20 percent; for Australia, it is well above 50 percent; and for Spain, it is surprisingly low, at about 10 percent.

Clearly, the scope of underestimation of the true liabilities can be sizable, but the implications depend on the country and its scheme. In the US, the 30 percent underestimation is likely to prevent the adjustment mechanism established by the social security law from kicking in. As this scheme has a sizable trust fund that is running down, the policy reaction may only occur when the trust fund resources are closer to expiration (currently foreseen by 2034; see Board of Trustees 2017). In Australia, there are no direct consequences. Its earnings-related scheme is

⁸ A number of countries introduced balancing mechanisms to assure or at least support financial sustainability of the mandated schemes. Some are, in principle, automated through the move toward an NDC scheme and the choice of the account indexation rate (such as in Italy, Latvia, Norway, Poland, and Sweden), with only the latter country actually having a formal balancing mechanism. A number of NDB countries (e.g., Finland, Germany, Japan, Portugal, and Spain) introduced sustainability factors to the same end (see OECD 2012, 2015, and 2017). However, an assessment of their actual working and effectiveness is still outstanding.

funded and does not provide any annuity at all, nor is its purchase required, and purchase of voluntary life annuities is minimal. If individuals underestimate their remaining life expectancy and decumulate too fast, the consequences will be realized in the universal old-age pension, which is means- and asset-tested but accessed by only two-thirds of the eligible older population. In Spain, the (small) underestimation is only gradually felt by the cash-flow gap as no actuarial estimation reprocess and trigger is established.

The differences between DB/DC and funded/unfunded schemes: In the typical Bismarckian NDB schemes that still dominate much of the world, the differences between period and cohort life expectancy have little importance even if periodic actuarial assessments are undertaken. In almost all cases, it is the cash balance outcome (i.e., the difference between period revenue and period expenditure) that triggers actions (with or without a balancing mechanism in place). Actuarial assessments with the too-low life expectancy data may trigger a late and insufficient reform when the actual insolvency has already existed for some time. And actuarial assessments of NDB schemes are typically built on discounted cash balance approaches, not on asset/liability comparisons. Conceptually, NDB schemes may give rise to the largest underestimation of actual liabilities, because they underestimate not only the liabilities of those already retired, but also those of active contributors. These estimates also rely on using the correct mortality data.

This differs from the liability estimates in NDC schemes, in which liabilities for the active workforce are covered by their accumulated contributions, and assets and liabilities fully match. In the estimation of implicit pension debt, liabilities toward active workers typically amount to two-thirds of overall liabilities (Holzmann et al. 2001). Hence the underestimation of liabilities in NDC schemes applies only for pensions in disbursement and the applied too-low life expectancy when converting individual accumulations into the initial pension benefit. Hence the underestimation amounts to only one-third of the full liability. However, compared to financial defined contribution (FDC) schemes where the assets are – in principle – immune to mortality rate misestimation, the wrong choice of mortality rates/life expectancy estimates should impact assets in NDC schemes. This is due to the PAYG asset, which is calculated as the present value of future contributions minus the liabilities created; the latter depend, of course, on the selected mortality/life expectancy estimates. Applying the too-low period life expectancy estimates underestimates these future liabilities and thus overestimates the PAYG asset.⁹

For FDB and FDC schemes, typically periodic and often annual actuarial assessments take place. While assets are – in principle – immune to the selected mortality estimates, liabilities are affected by the incorrect selection, and the scope of this underestimation has been hinted above. In funded schemes, however, additional underestimation of the liability may take place through the choice of a too-high discount rate. Sometimes the choice of the discount rate is determined by the return of the asset side of the funded scheme, for which highly optimistic assumptions are

⁹ In reality, of all the NDC countries only Sweden has a formal (automatic) balancing mechanism and it approximates the PAYG asset from cross-sectional revenue data. Hence, underestimation of life expectancy will also affect its NDC scheme as the PAYG asset will tend to be overestimated.

assumed. For most pension economists, there is no link to the rate of return as the discount rate is governed by other considerations.

4.3. Adding heterogeneity considerations to estimation of life expectancy

For pension schemes the issue of correct estimation of future life expectancy of a retiree cohort is complicated by the increasing recognition that this mean estimate has a dispersion that is linked to the level of lifetime income and accumulated saving (Ayuso, Bravo, and Holzmann 2017a, 2017b). Hence, not taking account of both issues may lead to multiple and interrelated distortions at both the individual and scheme level.

For example, the wrong mean estimate for life expectancy leads to underestimation of the pension scheme's liability. If the same common life expectancy is applied to all cohort members at retirement, a second underestimation of the liability is introduced, as richer individuals have a higher life expectancy. For the highest income decile, the individual subsidy rate may reach 30 percent for women and 15 percent for men. In contrast, those in the lowest income decile may face a tax rate of 20 percent or more on a much lower pension. The scope of the aggregate underestimation of liabilities will depend on the distributive characteristics and choice of the mean value (average or median), but is hypothesized to be 10–15 percent.

The interaction of both effects on individuals and their labor supply and savings decisions has not yet been worked out. For lower-income groups, the tax rate effect of heterogeneity is counteracted by an incorrect lower mean value. For higher-income groups, the subsidy effect of heterogeneity is fortified by the subsidy effect of a too-low mean estimate. How this affects individuals' decisions will also depend on their perceptions of their own life expectancy. Understanding these mechanisms is important for designing appropriate policy interventions. For correct mean and heterogeneity estimates, the effectiveness of some policy proposals has already been estimated (Holzmann et al. 2017).

4.4. Life expectancy measures in recent pension reforms

In recent decades, most OECD countries responded to continuous growth in life expectancy with pension reforms in which a common feature is to create an automatic link between future pensions and changes in life expectancy. The link between life expectancy and pension benefits has been accomplished in at least six different ways (Whitehouse 2007; OECD 2017):

- a) By introducing FDC plans as a (often partial) substitute for unreformed NDB pensions (e.g., Mexico, Poland, Sweden);
- b) By introducing an automatic link between life expectancy and pension benefits, for example through demographic sustainability factors (e.g., Finland, Portugal, Spain);
- c) By linking the normal retirement age to life expectancy (so far 10 countries including Denmark, Italy, the Netherlands, and Portugal);
- d) By connecting years of contributions needed for a full pension to life expectancy (e.g., France);

- e) By substituting traditional NDB public schemes with NDC schemes that replicate some of the features of FDC plans, namely the way in which pension (annuity) benefits are computed (Sweden, Poland, Latvia, Italy, Norway);
- f) By linking penalties (bonuses) for early (late) retirement to years of contributions and normal retirement age (e.g., Portugal).

These reforms represent a fundamental change in the way longevity risk is shared between contributors and retirees, and between current and future generations, that has become more explicit and, in principle, based on automatic rules rather than ad hoc changes as in the past. However, the legislated automaticity in countries must still stand the test of time. And even if automatically introduced and moving in the right direction, these measures are almost always incomplete and insufficient to assure financial sustainability issues triggered by life expectancy changes. This adds to issues of not selecting the correct life expectancy under these mechanisms and the incentive distortions involved.

In almost all cases and countries, the period life expectancy measure has been used to link longevity and pension benefits, which, as discussed above, results in underestimating remaining lifetime at retirement. This option has consequences on the way longevity risk is shared between generations. The financial consequences of underestimating life expectancy during retirement are ultimately borne by those who fund the pension scheme; i.e., younger cohorts in NDB/NDC schemes and private contributors/sponsors in FDB/FDC schemes.

For instance, demographic sustainability factors introduced in Finland, Portugal, and Spain automatically link initial pension benefits to life expectancy observed at the time of retirement, leading, in most cases, to a reduction in pension entitlements. These sustainability factors are computed as a simple ratio between period life expectancy observed at some reference age (e.g., 65 in Portugal and 67 in Spain) in some (past) reference year (e.g., 2000 in Portugal and 2012 in Spain) and period life expectancy observed at the time of retirement (in Spain according to mortality tables for the pensioner population as they are designed by the social security system). By design, these sustainability factors are not consistent with an actuarially neutral pension scheme since they do not guarantee that by adjusting (reducing) initial pension benefits but paying them for a longer period the scheme is financially neutral. To the extent that trends in period and cohort life expectancies differ between past and current generations, the system could redistribute in favor of older cohorts and have a negative impact on the public pension system's sustainability.

Linking the normal retirement age to period life expectancy instead of cohort life expectancy will extend working lives but will be insufficient to preserve actuarial neutrality between contributions and benefits, thus maintaining major distortions in individual labor supply and saving decisions, wasting the macroeconomic impact of higher levels of employment on investment, gross domestic product, consumption, and public finances. Moreover, the lower-than-consistent-with-actuarial-neutrality increase in the retirement age reduces the size of the

positive effect on pension adequacy resulting from longer contribution careers and higher pension accruals.

Linking penalties (bonuses) for early (late) retirement to years of contributions and normal retirement age using period life expectancy measures does not ensure actuarial neutrality between contributions and benefits. Actuarial neutrality depends on the parameters that determine the annuity factor – survival probabilities, indexation rate, discount rate – that revert to life expectancy when the indexation rate equals the discount rate. Increases in life expectancy require higher penalties for early retirement and lower bonuses for late retirement to keep up with actuarial neutrality. Adopting a period approach in measuring life expectancy to assess the work incentives around retirement ages systematically underestimates the magnitude of the penalties (bonuses) for early (late) retirement needed to ensure actuarial neutrality and a fair share of longevity risk between generations, and is likely to reduce labor supply.

5. Conclusions and next steps

While the general discussion about the change in life expectancy of the population and its past and projected future increases has finally reached policy makers and the public at large, more technical and political attention is needed on the selection of estimates of life expectancy and their application. This paper substantiates why it is so important to get life expectancy estimates right for pension policy, how it can be done, and how to overcome the key obstacles.

At the technical level, few arguments arise for not estimating and applying cohort life expectancies that take account of past and expected future declines in mortality rates. Technical issues remain regarding how best to estimate cohort values, and the best methods depend on high-quality data. But even for countries with excellent data quality, estimates seemingly differ across groups of countries. Some recent methods that take account of the acceleration in the decrease in mortality rates, particularly at higher ages, suggest a noticeable and rising underestimation of cohort life expectancy by conventional methods. Yet the potential estimation errors that may emerge from a commonly applied approach (e.g., across the European Union) are likely to be dwarfed by the magnitude of the differences generated between the cohort and period approaches.

The paper presents estimates of such differences for five countries: three have official estimates of both cohort and period life expectancy (Australia, the UK, and the US); and two have only official period tables that are supplemented by cohort values estimated herein (Portugal and Spain). Significant differences are estimated between cohort and period life expectancies: differences reach 8–15 years at birth but are reduced to 2–4 years at age 65, and all projected differences tend to decrease over time but do not disappear. Differences in estimated life expectancies by gender are also found but are mostly moderate and are not systematic across countries or over time.

Translating these differences in life expectancy at age 65 (around the age of retirement) amounts to an implicit subsidy to the average retiree that can reach 30 percent or more of the pension wealth of the current working generation. That is, using period life expectancy to calculate the initial benefit at retirement offers a too-generous benefit level that is not consistent with actual financial sustainability. To address the implications at a later stage will require additional contributions or budget transfers by future working generations or a partial default for those currently working (i.e., future retirees).

Selection of the correct life expectancy estimate is also increasingly important for day-to-day pension policy. Most countries have undertaken some kind of reform that legally links pension schemes to the development of the officially measured change in life expectancy. This naturally includes NDC countries and calculation of the initial benefit, but also includes the many NDB countries that have linked benefit levels and/or retirement ages to such a life expectancy measure, and the fewer countries that have a financial stability mechanism for their pension scheme. In all cases, inadequate choice of the life expectancy measure leads to incentive distortions and miscalculations of financial sustainability.

The proposed solution is simple as well as effective: convince governments that it is in their interest to apply the best estimates of cohort life expectancies. The estimation can be performed by national statistical offices in close cooperation with academic and partner institutions in other countries to compare, to learn, and to progress.

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