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ABSTRACT

Artificial Intelligence, Income Distribution and Economic Growth

The economic impact of Articial Intelligence (AI) is studied using a (semi) endogenous growth model with two novel features. First, the task approach from labor economics is reformulated and integrated into a growth model. Second, the standard representative household assumption is rejected, so that aggregate demand restrictions can be introduced. With these novel features it is shown that (i) AI automation can decrease the share of labor income no matter the size of the elasticity of substitution between AI and labor, and (ii) when this elasticity is high, AI will unambiguously reduce aggregate demand and slow down GDP growth, even in the face of the positive technology shock that AI entails. If the elasticity of substitution is low, then GDP, productivity and wage growth may however still slow down, because the economy will then fail to benefit from the supply-side driven capacity expansion potential that AI can deliver. The model can thus explain why advanced countries tend to experience, despite much AI hype, the simultaneous existence of rather high employment with stagnating wages, productivity, and GDP.

JEL Classification: O47, O33, J24, E21, E25

Keywords: technology, artificial intelligence, productivity, labor demand, income distribution, growth theory

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1 Introduction

The possible impact of automation technologies such as Artificial Intelligence[1] (AI) and robotics on unemployment has become globally a hot topic. As pointed out by Acemoglu and Restrepo (2018b, p.2) AI is the "most discussed automation technology." Governments and global development organizations have scrambled to respond to the expected new future of work.[2] Concerns include fears that AI would accelerate the automation of jobs, causing mass technological unemployment, and that its economic benefits would accrue to only a few, driving up inequality (Frey and Osborne, 2017; Korinek and Stiglitz, 2017). The COVID-19 pandemic is expected to give a further impetus to the digitization and automation of the global economy (Bloom and Prettner, 2020; Schrage, 2020).

To evaluate these concerns the main conceptual economic model that have been used by economists is the so-called task approach - proposed and elaborated primarily by Autor et al. (2003); Acemoglu and Autor (2011), Autor and Dorn (2013), Acemoglu and Restrepo (2018a) and Acemoglu and Restrepo (2020). It has proven an useful framework from which to evaluate the impact that an automation technology such as AI can have on labor markets. The identification of various tasks, which differ in terms of whether they are susceptible to automation, and the indirect effects of automation on labor demand through *reinstatement effects* are important insights. It has shown that fears of mass technological employment are exaggerated, and that moreover, that the net impact of AI on jobs can in principle be positive.

There is however a shortcoming in the task approach, as applied to automation. In this

[1]There is no single generally accepted definition of AI. One typical definition is that AI is the "simulation of human intelligence processes by machines, especially computer systems. These processes include *learning* (the acquisition of information and the rules for using the information), *reasoning* (using the rules to reach approximate of definite conclusions) and *self-correction*" - see the Tech Dictionary and IT Encyclopedia at `https://whatis.techtarget.com`.

[2]See for instance the ILO's Global Commission on the Future of Work, The WTO's World Trade Report 2017 or the World Bank's World Development Report 2019.

paper we argue that this shortcoming is that the jobs impact from the reinstatement effect is fundamentally uncertain, as also Agrawal et al. (2019) pointed out. This is because the size of the reinstatement effect will depend on (i) the extent of economic growth created by AI, and (ii) the extent to which economic growth stimulates the demand for labor, which in turn depends (iii) on growth in labor productivity, labor wages, and the income share of labor. Because the task approach is not an an economic growth model, it is unable to model these dynamic aspects.

This shortcoming means that an economic growth model is needed to provide better insights on the impact of AI on growth and distributional issues. There have indeed been a number of recent advances in economic growth models focusing on automation and artificial intelligence - amongst other e.g. Benzell et al. (2018); Cords and Prettner (2019); Gries and Naudé (2018); Hémous and Olsen (2018) and Prettner and Strulik (2017). While these provide valuable insights, these models tend to suffer from two shortcomings of their own. First, they lack the insights of the task approach into the distinction between tasks and jobs, and typically do not allow for substitution between tasks to be taken into consideration. And second, endogenous growth models are supply-driven, thus ignoring the role of aggregate demand (Dutt, 2006).

The upshot is that the task approach is a valuable contribution to model the impact of AI. However, it is not a growth model and cannot be used to analyze the dynamics of growth and labor demand that results from the consequences of AI on income distribution. Ideally, one would require an endogenous growth model with the task approach incorporated. The contribution of this paper is to provide such an (semi) endogenous growth model with the task approach incorporated, and moreover that can deal with demand constraints. The model is used to explore the impact of AI on jobs, inequality, wages, labor productivity and long-run GDP growth.

The rest of the paper is structured as follows. In section 2 the relevant literature related to the impacts of AI and the task approach is discussed. In section 3 an (semi) endogenous growth model is introduced that includes constraints from the demand-side, and is also consistent with the task approach. In section 4 the dynamics of the model in terms of impact of AI on jobs, inequality, wages, labor productivity and long-run GDP growth are traced. Section 5 concludes.

2 Literature Review

The task approach is a theoretical framework in labor economics wherein a distinction is made between the skills that production factors have and the tasks that they perform. Key references are Acemoglu and Autor (2011), Acemoglu2018a, Acemoglu2018b, Acemoglu2019, Acemoglu2020, Autor et al. (2003); Autor and Dorn (2013) and Autor and Salomons (2018).

A task is "a unit of work activity that produces output" (Autor and Dorn, 2013, p.186). A product or service is the result of skills applied to various tasks. Various categories of tasks have been described in the literature. Autor et al. (2003) makes a distinction between routine cognitive and manual tasks, abstract analytical and managerial tasks and non-routine manual tasks. The manner in which various tasks results in an output of a final good can be described by a Constant Elasticity of Substitution (CES) production function, as in Autor and Dorn (2013, p.187):

$$Y = \left[\int_0^1 y(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \tag{1}$$

Where Y is the output of a final good, $y(i)$ the different tasks needed to produce the output Y, and η the elasticity of substitution between tasks. It is often assumed in this literature that $\eta = 1$ in which case the production function has a Cobb-Douglas specification: $Y = \int_0^1 y(i)di$

(Acemoglu and Autor, 2011). Because a task can be produced or performed by either low (L), medium (M), high-skilled (H)labor or capital (K), the production function for a task can be written, following Autor (2013), as:

$$y(i) = A_L \alpha_L l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i) + A_K \alpha_K(i) k(i) \quad (2)$$

Where $l(i)$, $m(i)$, $h(i)$ are respectively the number of low, medium and high-skilled laborers doing task (i), and $k(i)$ the capital used for task (i). The productivity of labor and capital in a task (i) are expressed by α_L, α_M, α_H and α_K. The A represents a factor-augmenting technology in the carrying out of tasks.

According to Autor and Dorn (2013, pp.188-189) "the most important innovation offered by this task-based framework is that it can be used to investigate the implications of capital (embodied in machines) directly displacing workers from tasks that they previously performed". For example, if improvements in algorithms (AI) occur (reflected in A_K) the α_K would improve, not for all tasks, but say for a specific range $(i) \subset [I', I]$ of tasks - maybe those than can be more easily codified, such as routine tasks, for example. Then, if some of these tasks are done by medium-skilled workers, then some of the $m(i)$ will be displaced with capital (machines, robots and computers) taking over the performance of their tasks - i.e. their tasks are automated.

When automation lowers the demand for labor performing routine tasks, as in this example, it is described as *routine replacing technical change* (RRTC). It is often assumed that medium-skilled labor is more subject to RRTC (Autor and Salomons, 2018; Gregory et al., 2019). RRTC has been used to explain labor market trends including occupational polarization (Autor et al., 2006), growing wage premia (Dastory, 2019), the declining share of labor in total income (vom Lehn, 2018) and and offshoring (Goos et al., 2014).

The task approach has also become the model of choice in economics to study the impacts of artificial intelligence (AI) on labor markets. Earlier versions of the task approach implied a significant displacement effect of automation, through the mechanisms as described above.[3] In a more recent elaboration of the task approach, Acemoglu and Restrepo (2018b) introduces and model a "reinstatement effect which can counter the typical displacement" effect of automation. With a reinstatement effect, the automation of tasks by AI should not be assumed to automatically lead to net job losses over the longer-term - technological unemployment should be due to short-term labor market frictions.

The essence of the reinstatement effect is that the automation technology (AI) can raise labor productivity, wages, and through this aggregate demand - which then indirectly raises the demand for labor. Autor and Salomons (2018, pp.12-13) describes three effects through which this indirect demand for labor may increase, namely an *Uber effect*, a *Walmart effect*, and a *Costco effect*. For more on these effects, see Autor and Salomons (2018) or the discussion in Gries and Naudé (2018).

The net impact of automation on jobs will therefore depend on the relative strengths of the displacement effect ***and*** the countervailing reinstatement effect. An underlying assumption is that AI-driven innovation will be different from other types of automation (e.g. robots) in being more likely to generate countervailing reinstatement effects. Agrawal et al. (2019) differs however from this assumption. They explain that AI is essentially a prediction technology and that it will predominantly displace labor from tasks that requires prediction, and that it will create (reinstate) jobs elsewhere only in tasks that can be done better with support of superior prediction technology, and tasks that will benefit upstream or downstream from these new tasks. They provide the example of "an artificial intelligence that automates translation on an online trading platform significantly enhances international trade" and

[3] Earlier estimates of the potential displacement of labor were alarming, for instance suggesting that up to 47 % in the USA and 54 % of jobs in the EU could be automated in the near future, see e.g. Frey and Osborne (2013, 2017) and Bowles (2017). Later estimates were much lower, see e.g. Arntz et al. (2016) and Arntz et al. (2017).

thereby allow new jobs to be created in trade (Agrawal et al., 2019, p.3). However they also point out that due to the nature of AI as prediction tool, it is very difficult in advance to conclude whether or not net job creation will be positive, recognising that “The net effect is an empirical question and will vary across applications and industries”[Ibid, p.4].

The task approach is fundamentally, as the above discussion implies, an extremely useful framework from which to evaluate and think about the impact that an automation technology such as AI can have on labor markets. The identification of various tasks that will differ in terms of whether they are susceptible to automation (codification) and the indirect effects of automation on labor demand through reinstatement effects are important insights. It has shown that fears of mass technological employment are exaggerated, and that the impact of AI on net jobs can in principle be positive.

There is however, a shortcoming in the task approach. This is that the impact of the reinstatement effect is fundamentally uncertain - the point also made by Agrawal et al. (2019). This is because the extent of the reinstatement effect will depend on (i) the extent of economic growth created by AI and (ii) the extent to which this economic growth leads to a rise in the demand for labor, which in turn depends (iii) on growth in labor productivity, labor wages, and the income share of labor. Because the task approach is not an an economic growth model, it is unable to model these dynamic aspects.

That the reinstatement effect will depend on the income share of labor and its dynamics are recognised. For instance Acemoglu and Restrepo (2018b, p.33) are concerned about rising income inequality if the growth generated by automation and AI is not inclusive, as it could mean that “the rise in real incomes resulting from automation ends up in the hands of a narrow segment of the population with much lower marginal propensity to consume than those losing incomes and their jobs.” Such a redistribution towards households with a lower marginal propensity to consume would cause aggregate demand to grow much slower and to constrain GDP growth. With lower GDP growth there would in turn be less incentives for entrepreneurs to innovate, and less growth in the productivity of labor and in the demand for it (Gries and Naudé, 2018).

This shortcoming in the task approach to model dynamic aspects related to income distribution and demand, suggests that an economic growth model is needed to provide better insights on the impact of AI on growth and distributional issues. There have indeed been a number of recent advances in economic growth models focusing on automation and artificial intelligence. Both endogenous growth models and overlapping generations (OLG) models of AI have been proposed, see e.g. Benzell et al. (2018); Cords and Prettner (2019); Gries and Naudé (2018); Hémous and Olsen (2018) and Prettner and Strulik (2017). While these provide interesting insights, and generally confirm that the growth and distributional dynamics of automation would matter for the size of any reinstatement effects, these models tend to suffer from two shortcomings of their own.

First, they lack the insights of the task approach into the distinction between tasks and jobs, and does not allow for substitution between tasks to be taken into consideration. And second, most endogenous growth models are supply-driven, ignoring the role of aggregate demand (Dutt, 2006). In standard endogenous growth models, aggregate demand is typically modelled assuming representative intertemporal choices based on a representative household's Euler equation. The representative household assumption in standard endogenous growth models assumes away differences in intertemporal decisions of rich and poor households and their respective effects on aggregate consumption and savings. This, is not adequate when asymmetries in factor rewards and potential changes in income distribution are key features of interest - as is the case when considering an automation technology (Gries and Naudé, 2018).

The upshot is that the task approach is a valuable contribution to model the impact of AI. However, it is not a growth model and cannot be used to analyze the dynamics of growth and labor demand that results from the consequences of AI on income distribution. Ideally, one would require an endogenous growth model with the task approach incorporated.

The contribution of this paper is to rectify these shortcomings. Therefore, in the next section a (semi) endogenous growth model that incorporates both the task approach as well as demand constraints are set out.

3 A New Theoretical Model: Labor Tasks, Demand, and Growth

This section proposes an endogenous growth model that incorporates an automation technology such as AI, changes in tasks, as well as aggregate demand constraints. The model presented here builds on, extends, and refines our earlier work in Gries and Naudé (2018), Gries (2019) and Gries (2020b).

We start off (in 3.1) by describing the production of final consumption goods by sales-maximising firms who use labor, intermediate goods, as well as AI. In section 3.2 the nature of the relationship between AI and labor is set out, in 3.3 intermediate goods production is specified, and in section 3.4 the aggregate budget constraints, income and its distribution is derived. Having dealt with aggregate supply, the paper turns to aggregate demand in section 3.5. It introduces a particular novelty of this paper, namely the rejection of the typical assumption in endogenous growth models of a representative household. In section 4 the model is solved and a stationary equilibrium solution provided.

3.1 Final goods-producing firms

Final goods for consumption are produced by firms using labor, AI services, and intermediate inputs. Actual sales of output may fall short of potential sales due to market frictions and shocks in final goods markets. To maximize sales, firms will incur marketing and R&D expenses, buy labor and AI services in a competitive market, and purchase intermediate goods. The following sub-sections elaborate this maximization problem.

3.1.1 Output of Final Goods

First we show how the output of a firm is produced. Let firm $i \in \mathcal{F}$ be a representative firm that produces final goods using labor, AI services and intermediate inputs. Given that AI services can substitute for or complement labor, we will refer to labor and AI together as ***human service*** inputs. We lump labor and AI together as human services, because AI is a software and information technology that human-related. The term "intelligence" already suggests that non-physical, productive labor abilities are provided by this technology. We denote these human service inputs by H_{Qi}.

In addition to human services, the firm sources $N_i(t)$ differentiated intermediate inputs $x_{ji}(t)$ which are offered by $N(t)$ intermediate input-producing firms. Given this, we can write $Q_i(t)$, the potential output of final goods by firm i, as

$$Q_i(t) = H_{Qi}^{1-\alpha} \sum_{j=1}^{N_i} x_{ij}^{\alpha}(t) = N_i(t) H_{Qi}^{1-\alpha} x_i^{\alpha}(t) \tag{3}$$

3.1.2 Market frictions, sales promotion, and expected sales

Next, we derive[4] the implications for firms who realize that, due to stochastic market frictions, not all of their potential output will necessarily be sold. Assume that firm i can only sell Φ so that its effective sales ratio is $\phi_i(t) = \frac{\Phi_i(t)}{Q_i(t)} \leq 1$. The firm's subjective interpretation of $\phi_i(t) \leq 1$ is that this shortfall in sales is due to the fact that customers are insufficiently informed about products, prices, qualities, and general market conditions. The extent of

[4]The modelling of mismatch and frictions here is close to the ideas developed in Gries (2020a).

this mismatch between potential and actual sales, $\delta_i(t)$ determines the effective sales ratio, i.e.:

$$\phi_i(t) = 1 - \delta_i(t) \tag{4}$$

As a response to a sub-optimal effective sales ratio, firms allocate human services $H_{\phi i}$ to promote sales so as to counter this mismatch (δ_i) and improve the likelihood of selling all potential output in the market. The match-improving mechanism can thus be represented as $m_i = m_i(H_{\phi i})$, with $\frac{\partial m_i(t)}{\partial H_{\phi i}(t)} > 0$. Given that δ_i' denotes the stochastic market frictions which the firm perceives as exogenous, the total mismatch of potential and actual sales is

$$\delta_i(t) = \delta_i'(t) - m_i(H_{\phi i})$$

Each individual firm i observes that the expected effective sales ratio $E[\phi_i]$ is monotonically increasing with $H_{\phi i}$, and decreasing with δ_i',such that

$$E[\phi_i] = E[\phi_i(\delta_i', H_{\phi i})] \quad \text{with} \quad \frac{\partial E[\phi_i]}{\partial H_{\phi i}} > 0, \quad \frac{\partial E[\phi_i]}{\partial \delta_i'} > 0. \tag{5}$$

3.1.3 Factor demands

Having described the firm's output and expected effective sales ratio in the previous two sub-sections, we can now derive the firm's factor demands from its profit maximization. We start by denoting the price of the human services factor as p_H, and the price of intermediate inputs x as p_x. Now we can specify the firm's profit maximization function as:

$$\max_{H_{Qi}, H_{\phi i}, x_i} : E[\Pi_i(t)] = E[\phi_i(t)] Q_i(t) - p_H'(t)(H_{\phi i}(t) + H_{Qi}(t)) - N_i(t) p_x(t) x_i(t). \tag{6}$$

For all firms, the procedure to maximize profits has two components: first, firms need to organize an efficient sales process, and secondly, they need to determine optimal production.

First, the organization of an *efficient sales process.* Firm i allocates $H_{\phi i}$ to the search and information process and improves its effective sales. In order to sell all potential output, the firm increases $H_{\phi i}$ until all goods that have been produced and supplied can be expected to be absorbed by the market. The firm's total revenues $E[\phi_i] Q_i$ are determined by the expected success rate of selling the produced output $E[\phi_i]$ and the production of even more goods Q_i. As each element depends on the respective human service input, we assume that placing an already existing (but not yet demanded) output in the market is more effective than producing a new unit of output. That is, until the point when all production in fact finds a customer, the marginal revenue generated by human services in the matching process is greater than the marginal revenue of human service in production, and zero otherwise

$$\frac{\partial E[\phi_i]}{\partial H_{\phi i}} Q_i > E[\phi_i](1-\alpha)\frac{Q_i}{H_{Qi}} \quad for \quad E[\phi_i] \leq 1.$$

As a result, the firm will increase $H_{\phi i}$ until the expected sales ratio becomes

$$E[\phi_i] = 1, \tag{7}$$

and thus no unsold output remains. The firm will be in a sales equilibrium. Any time when conditions (7) holds, (5) defines a function for the allocation of labor to each firm i's sales activities[5]

$$H^*_{\phi i} = H_{\phi i}(E[\delta'_i]), \quad \frac{\partial H_{\phi i}}{\partial E[\delta'_i]} > 0. \tag{8}$$

Secondly, the firm needs to determine *optimal factor inputs.* Under the condition that $E[\phi_i] = 1$, a firm's profit (6) is

$$E[\Pi_{Qi}] = Q_i - N_i p_x x_i - p'_H \left(H^*_{\phi i} + H_{Qi}\right),$$

[5]See appendix A for the implicit function theorem.

As p'_H is the price payable to human services (also in production), the first-order condition for the efficient use of labor in production gives

$$H_{Qi}\left(t\right) = \left(1-\alpha\right) Q_i\left(t\right) p'_H\left(t\right)^{-1}, \tag{9}$$

and the demand for intermediate goods can be derived as

$$x_i\left(t\right) = \left(\frac{\alpha}{p_x\left(t\right)}\right)^{\frac{1}{1-\alpha}} H_{Qi}. \tag{10}$$

3.2 Human services: Labor and AI

Having explained the production of optimal final output, given market frictions, and the resulting demand for human services and intermediate inputs in section 3.1, in this section we elaborate the human services input, and clarify the relationship between labor and AI.

3.2.1 The "production" of human services

Human services H_i, as already indicated consisting of labor and AI, are produced following the task-based approach. As such, $H = H(L_L, A_L, A_{IT}, L_{IT})$, where L_L is the number of workers each providing one unit of "human intelligent activity", A_L is an index of general knowledge, A_{IT} is the total number of AI technology services (e.g., software programs) in the economy, and L_{IT} the IT-labor providing IT services. Further, L_L and L_{IT} are different groups of labor, allow us to have two separated segments in the labor market. The general function $H = H(...)$ can be specified as

$$H = \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, \tag{11}$$

where z denotes each task in a unit interval $[N-1, N]$, and $h(z)$ is the output of task z. As tasks range between $N-1$ and N, the total number of tasks is constant. While formally following the task-based approach, the slightly different interpretations of the variables as specified here better reflects the nature of IT and AI technologies.

Each task can either be produced only with labor, $l(z)$, or only with AI-labor services, $l_{IT}(z)$, if the task can be automated. Therefore, there are two sets of tasks. Tasks $z \in [N-1, N_{IT}]$ can be produced by both labor and AI services, and tasks $z \in (N_{IT}, N]$ can only be produced by labor. Thus, the output of a task can be generated in two ways, namely

$$h(z) = \begin{cases} A_L \gamma_L(z) l(z) + A_{IT} \gamma_{IT}(z) l_{IT}(z) & \text{if } z \in [N-1, N_{IT}] \\ A_L \gamma_L(z) l(z) & \text{if } z \in (N_{IT}, N] \end{cases} . \qquad (12)$$

Here $\gamma_L(x)$ is the classic productivity of labor of task z and A_L generally available knowledge, which is usable without rivalry and labor augmenting.

The AI service consists of three elements which reflects the fact that modern AI is the result of combinations of software / algorithms and software expertise. Thus the first element is $l_{IT}(z)$ which is IT-specific labor- in other words so-called IT specialists. The task-related productivity or skills and expertise of these specialists is the second element and is given by $\gamma_{IT}(z)$. The third element or ingredient in AI services is AI technology, denoted A_{IT}. A_{IT} could, for instance, indicate the number or quality of software programs/algorithms available in the economy. As each software program has no rivalry in use the same program can be applied in each task. Therefore, the property of a software technology is contained in A_{IT} .

For an existing stock of AI technology the number and kind of tasks which are used and which fully substitute for labor (automation) will be endogenous. The relative factor prices and efficiency of these services will determine the extent of the use of automation technologies. Thus the degree of automation in this model is endogenous. In the next subsection this process is described in detail.

For now, it can be noted that if a task z with prize $p_h(z)$ is produced with pure labor $h(z) = A_L\gamma_L(z)l(z)$, and labor rewards are calculated according to marginal productivity, then $p_h(z)A_L\gamma_L(z) = w_L$. Symmetrically, the same task could be produced with an AI technology so that $p_h(z)A_{IT}\gamma_{IT}(z) = w_{IT}$. Given these two conditions, and given wages in the market, for any particular task the firm will choose choose the kind of production (automation or not) that results in the lowest unit labor costs. Thus, if the following condition holds, the task will be automated:

$$\frac{w_{IT}}{p_h(z)A_{IT}\gamma_{IT}(z)} < \frac{w_L}{p_h(z)A_L\gamma_L(z)}$$

This rule leads to condition (13) which identifies the switching point between automated (AI) tasks and labor tasks. If tasks are ordered in such a way that $\frac{A_L\gamma_L(z)}{A_{IT}\gamma_{IT}(z)}$ is increasing in z and the tasks with lower numbers $z \in [N-1, N_{IT}]$ are the automated tasks, task N_{IT} is the switching point from an automation task to a labor task. N_{IT} is the highest number in this order for which

$$\frac{A_L\gamma_L(N_{IT})}{A_{IT}\gamma_{IT}(N_{IT})} < \frac{w_L}{w_{IT}} \tag{13}$$

holds. Apart from these automated (AI) tasks $[N-1, N_{IT}]$, all other tasks $(N_{IT}, N]$ are produced with standard labor. Thus, the costs and respectively the price $p_h(z)$ for any task z is

$$p_h(z) = \begin{cases} \frac{w_{IT}}{A_{IT}\gamma_{IT}(z)} & \text{if } z \in [N-1, N_{IT}] \\ \frac{w_L}{A_L\gamma_L(z)} & \text{if } z \in (N_{IT}, N]. \end{cases} . \tag{14}$$

3.2.2 Human service firm's optimization

The next step in modelling human services is to specify how and by whom the human service is "produced". Human services are produced by small firms who take the price for human services, the price for each task, and wages for various labor inputs as given.

To do this, it is assumed that human services are provided by service firms in competitive markets. These human-service firms will aim to maximize profits for a given price p_H subject to the production process in (11), such that

$$\pi_H = p_H H - p_h(z)h(z) = p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} - p_h(z)h(z).$$

From which the demand for task z can be derived to be:

$$h(z) = \frac{p_H^\sigma H}{p_h(z)^\sigma}. \tag{15}$$

Combining (14) and (15) we can derive the demand for automation and labor tasks z as follows[6]

$$h(z) = \begin{cases} p_H^\sigma H \left(\frac{A_{IT}}{w_{IT}}\right)^\sigma \gamma_{IT}(z)^\sigma & \text{if } z \in [N-1, N_{IT}] \\ p_H^\sigma H \left(\frac{A_L}{w_L}\right)^\sigma \gamma_L(z)^\sigma & \text{if } z \in [N_{IT}, N] \end{cases} \tag{16}$$

Further, from (16) and (12) we can obtain the optimal demand for IT labor:

$$l_{IT}(z) = \begin{cases} \frac{p_H^\sigma H}{(w_{IT})^\sigma} (A_{IT})^{\sigma-1} \gamma_{IT}(z)^{\sigma-1} & \text{if } z \in [N-1, N_{IT}] \\ 0 & \text{if } z \in [N_{IT}, N] \end{cases} \tag{17}$$

and standard labor:

$$l_L(z) = \begin{cases} 0 & \text{if } z \in [N-1, N_{IT}] \\ \frac{p_H^\sigma H}{(w_L)^\sigma} (A_L)^{\sigma-1} \gamma_L(z)^{\sigma-1} & \text{if } z \in (N_{IT}, N] \end{cases} \tag{18}$$

Relative labor productivity can be determined from factor abundance and technology- and productivity-related parameters. Assuming that all types of labor are fully used in the

[6]For details see Appendix B.

various tasks, labor in all tasks add up to given total labor in each labor market segment

$$L_{IT} = \int_{N-1}^{N_{IT}} l_{IT}(z)dz, \text{ and} \tag{19}$$

$$L_{L} = \int_{N_{IT}}^{N} l_{L}(z)dz. \tag{20}$$

By using (17), (18), (19) and 20) relative labor productivity is then obtained as:

$$\frac{w_L}{w_{IT}} = \left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}} \left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \tag{21}$$

3.2.3 Optimal number of automated tasks

Combining relative marginal productivity (21) with condition (13) and applying the *implicit function theorem* gives an expression for calculating the optimal number of automated tasks in the economy, which are endogenous:[7].

$$N_{IT} = N_{IT}\left(L_{IT}, L_L, A_{IT}, ...\right), \text{ with } \frac{dN_{IT}}{dL_{IT}} > 0, \quad \frac{dN_{IT}}{dL_L} < 0, \quad \frac{dN_{IT}}{dA_{IT}} > 0. \tag{22}$$

This result indicates that the number of automated tasks crucially depends on the relative availability of the production factors as well as the availability of AI technologies. If IT labor is broadly available and hence its relative wage is low, more tasks will be automated.

Similarly, if IT knowledge and AI algorithms are readily available, relative wages $\frac{w_L}{w_{IT}}$ increase and make standard labor tasks relatively more expensive. This results in a higher share of automated tasks. The clear implication is that if an economy is advanced in terms of IT technologies and IT labor, this economy will be more automated.

[7]For details see the Appendix B

3.2.4 Optimal human service supply

From the demands for the various tasks total human service production can be derived. Aggregating automated tasks and labor, equation (11) leads to

$$H = \left(\int_{N-1}^{N_{IT}} h(z)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} .$$

Using (16), (84) and 85), respectively, and re-arranging gives the expression for total production of human services as:[8]

$$H = \left(\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_{IT} L_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\int_{N_{IT}}^{N} \gamma_{L}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_{L} L_{L})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

In order to simplify this expression Acemoglu and Restrepo (2018a,b) and Acemoglu and Restrepo (2019) propose two definitions that allow for a more compact expression. With the definitions

$$\Gamma(N_{IT}, N) = \frac{\int_{N_{IT}}^{N} \gamma_{L}(z)^{\sigma-1} dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz + \int_{N_{IT}}^{N} \gamma_{L}(z)^{\sigma-1} dz} \tag{23}$$

and

$$\Pi(N_{IT}, N) = \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz + \int_{N_{IT}}^{N} \gamma_{L}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma-1}} \tag{24}$$

one may substitute $\int_{N_{IT}}^{N} \gamma_{L}(z)^{\sigma-1} dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$ and $\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz = (1 - \Gamma(N_{IT}, N))\Pi(N_{IT}, N)^{\sigma-1}$ and thus rewrite the aggregate optimal human service production as

$$H = \Pi\left(N_{IT}, N\right) \left[\left(1 - \Gamma\left(N_{IT}, N\right)\right)^{\frac{1}{\sigma}} \left(A_{IT} L_{IT}\right)^{\frac{\sigma-1}{\sigma}} + \Gamma\left(N_{IT}, N\right)^{\frac{1}{\sigma}} \left(A_{L} L_{L}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} . \tag{25}$$

This expression is similar to the familiar Constant Elasticity of Supply (CES) production function.

[8] For details see Appendix B.

3.2.5 Earning shares of laborers

From equation (25) the earning share of each group of L_L and L_{IT} can be deduced. After rearranging these, the earning share of standard labor from revenues earned by human services can be written as:[9]

$$\begin{aligned} \phi_L &= \frac{w_L L_L}{p_H H} = \frac{1}{1+\left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}} \quad \text{(non-IT labor)}, \qquad (26) \\ \phi_{IT} &= \frac{w_{IT} L_{IT}}{p_H H} = 1-\phi_L \quad \text{(IT labor)}. \end{aligned}$$

3.3 Intermediate goods-producing firms

In our model we have final-good producing firms (whose final goods production under market frictions was set out above in section 3.1), as well as small firms that produce human services, described in the previous section (3.2). The third group of firms consists of firms producing intermediate goods that are used by final goods-producing firms. In this sub-section we describe these firms in greater detail.

3.3.1 Market entry of intermediate goods-producing firms

The intermediate goods-supplying firms in our model are monopolists because they each sell an unique product which is the outcome of entrepreneurial (product) innovation. The costs for the typical firm (denominated in units of final output) to produce one unit of x is c_x, and the profits this result in is $\pi_x = (p_x - c_x)\, x$.

[9]For details see Appendix B.

Using the demand function (10) and plugging in $p_x = \alpha H_Q^{1-\alpha} x^{-(1-\alpha)}$ results in:

$$\pi_x(t) = \alpha H_Q^{1-\alpha} x(t)^{-(1-\alpha)} x(t) - c_x x(t) \tag{27}$$

From the first-order condition[10] and using (10) and (29), the optimal price p_x and optimal production of intermediate goods $x(t)$ are, respectively:

$$p_x = \frac{c_x}{\alpha}. \tag{28}$$

and

$$x(t) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q, \tag{29}$$

Given (29) and (28), maximum profits $\pi_x(t)$ are:

$$\pi_x(t) = \left(\frac{1}{\alpha} - 1\right)(c_x)^{\frac{-\alpha}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} H_Q. \tag{30}$$

The present value of this future profit flow, discounted at the steady-state interest rate r, is:

$$V_x(t) = \frac{1}{r}\pi_x(t) = \int_t^{\infty} \pi_x(t) e^{-r(v,t)(v-t)} dv. \tag{31}$$

Here, $\frac{1}{r}\pi_x$ is the present value of profits per innovation and $\frac{1}{r}\pi_x \dot{N}$ are the total profits of the intermediate goods producing firm (which is essentially a new firm) of introducing $\dot{N}(t)$ new goods. In addition to the cost of innovation, the new firm also has to cover the costs of market entry (e.g., commercialization costs) for the new intermediate good, which is ν.

[10]The first-order condition is $\frac{\partial \pi_x}{\partial x} = \alpha^2 (1-\theta_i) H^{1-\alpha} x^{\alpha-1} - c_x = 0$, thus $c_x = \alpha^2 (1-\theta_i) H^{1-\alpha} x^{\alpha-1} \Leftrightarrow x^{1-\alpha} = (c_x)^{-1} \alpha^2 (1-\theta_i) L^{1-\alpha}$.

Thus, the total entry cost of the start-up with innovation rate $\dot{N}$ and thus total investment is

$$\dot{N}\nu = I. \tag{32}$$

With competitive market entry, the net rents of a new firm turn to zero and the net present value of the new firm just about covers its total start-up costs:

$$\frac{1}{r}\pi_x(t)\dot{N}(t) - I(t) = 0. \tag{33}$$

With $\dot{N}\nu = I$ the steady-state interest rate is:

$$r = \frac{\pi_x(t)}{\nu} \tag{34}$$

3.3.2 Supply of innovative intermediate products

Innovation in the intermediate goods market is exogenously given as $\dot{A}(t) = \frac{dA(t)}{dt}$, which is the number of innovative intermediate products invented at t. These innovative intermediate products are not automatically successful in the market. The success or failure to find a buyer can be modelled as an *aggregate matching* process.[11]

In such a matching process, the number of new intermediate products successfully entering the market $\dot{N}$ is a function of two elements: (i) the given number of new, innovative intermediate products $\dot{A}(t)$ potentially ready for market entry, and (ii) the number of opportunities for market entry that entrepreneurs (start-ups) discover. These opportunities are determined

[11] For a micro-foundation of this process see Gries and Naudé (2011).

by the capacity of the market. Absorption capacity for intermediate goods is a function of total effective demand for intermediate goods in the economy $X^{eD}(t)$.

Through an aggregate matching function, these two elements can be combined and the resulting process of market entry can be described as $\dot{N} = f(\dot{A}, X^{eD})$. For simplicity, it is assumed here that the matching technology is subject to constant economies of scale, so that the number of new products in the market will be given by

$$\dot{N}(t) = \left(X^{eD}(t)\right)^{\varphi} (\dot{A}(t))^{1-\varphi} \tag{35}$$

where φ is the contribution of market opportunities. Although the assumption of a macro-matching process is basic, it represents the main idea behind the mechanism. Given (35), the growth of new products in the economy is a *semi*-endogenous process because the number of new products $\dot{A}$ is fixed but the number of new technologies implemented to establish intermediate products $\dot{N}$ is endogenous.

3.4 Aggregate production and income distribution

Having specified final goods and intermediate goods production, and in having showed how the task approach can be used to account for human service production in the preceding sections, this sub-section is concerned with the aggregate budget constraint and the distribution of income to the various agents in the economy, starting with labor income (3.4.1).

3.4.1 Labor income

As we discussed in the preceding sections, human services H are allocated to two activities, namely production H_Q and sales promotion H_ϕ, $H = H_Q + H_\phi^*$. For the representative firm

$H^*_{\phi i}$ - i.e. use in sales promotion - has already been determined by condition (7) and (8). Thus, the allocation of human services to production must be

$$H_Q = H - H_\phi. \tag{36}$$

From (9) we know that human service in production is paid according to its marginal productivity with the price $p'_H(t)$. However, not only do firms have to pay human services used in physical production H_Q, they also need to pay human services used in sales promotion H_ϕ. As factor rewards are paid in physical output goods at an amount $(1-\alpha)Q$, all human services needs to be paid out of this amount. Also, with a homogeneous H in a perfectly integrated human service market, only one price is paid to to H, irrespective of whether used in production or sales promotion.

Finally, because total payment for H cannot exceed the contribution of H to effective production (9), we obtain an average income that is paid to all human services. Thus, total or aggregate human service income is $p_H H = p'_H H_Q = (1-\alpha)Q$[12] and the price for H is

$$p_H(t) = (1-\alpha)\frac{Q(t)}{H}. \tag{37}$$

3.4.2 Wealth holders' income

$N(t)\,\pi_x(t)$ denotes total debt issued in the economy. All new products, results of innovation (R&D), are financed by issuing new debt, $\dot{N}(t)\nu = \dot{F}(t)$. As wealth holders, profits accrue to the owners of this debt - the financiers:

$$N(t)\,\pi_x(t) = r(t)\,F(t). \tag{38}$$

[12] $p'_H H_Q = p_H(H_Q + H_\Phi) \Leftrightarrow p_H = p'_H \frac{H_Q}{(H_Q+H_\Phi)} = (1-\alpha)\frac{Q}{H_Q}\frac{H_Q}{(H_Q+H_\Phi)} = (1-\alpha)\frac{Q}{H}$. Note that Q depends on H_Q.

3.4.3 Production and income constraints

Effective output in the economy has to be divided amongst intermediate goods x, standard labor L_L, and the IT technology service provider L_{IT}. The budget constraint for effective output is therefore

$$Q(t) = N(t)H_Q^{1-\alpha}x^{\alpha}(t) = N(t)p_x(t)x(t) + w_L(t)L_L + w_{IT}(t)L_{IT} \tag{39}$$

Note that effective output is not the same as GDP or aggregate income. As x is produced by using c_x units of final goods, net final output and thus *income* is

$$Y(t) = Q(t) - N(t)x(t)c_x. \tag{40}$$

Further, (39) and (40) imply that $Q - Nxc_x = Np_x x - Nxc_x + w_L L_L + w_{IT}L_{IT}$. With the definition of profits in the intermediate goods sector (27), the income constraint then becomes:

$$Y(t) = N(t)\pi_x(t) + w_L(t)L_L + w_{IT}(t)L_{IT} \tag{41}$$

According to (41) total or aggregate income in the economy consists of profits, labour, and technology income. Given equation (34) this means that $Y = rN\nu_x + w_L(t)L_L + w_{IT}(t)L_{IT}$. Value added generated by innovative intermediate firms therefore turns into the income of financial asset owners $r(t)F(t)$. The growth process is thus essentially a process of financial wealth accumulation through the financing of new products and (intermediate-good producing) new ventures, which we label a Silicon Valley model of growth.

Finally, using (38) results in the familiar income decomposition of GDP:

$$Y(t) = r(t)F(t) + w_L(t)L_L + w_{IT}(t)L_{IT}. \tag{42}$$

In addition to income of financial wealth owners, value added generated by the human service input is distributed to labour ($w_L(t)L_L$) and the providers of the AI technologies and services ($w_{IT}(t)L_{IT}$).

3.4.4 Income distribution

To allow us to eventually trace the distributional consequences of progress in artificial intelligence (AI), the income shares of the three input and resource providing agents in the model need to be derived. These are the income of standard labor ($w_L(t)L_L$), the AI service providers ($w_{IT}(t)L_{IT}$), and the financial investors ($r(t)F(t)$).

Wages and income share of labor: Using the expression for factor demand (37) and (26), wages can be related to total income as follows:

$$w_L = \phi_L \frac{p_H H}{L_L} = \frac{\phi_L}{1+\alpha}\frac{Y(t)}{L_L}. \tag{43}$$

Because ϕ_L is constant, w_L is the standard wage rate in the economy. The income share of standard labour can now be derived by using (43), (29) and (40) as:[13]

$$\frac{w_L(t)L_L}{Y(t)} = \frac{\phi_L}{(1+\alpha)} < 1 \tag{44}$$

Wages and income share of the AI provider: The factor reward, or wage rate, of the economic agent that provides the AI at amount A_{IT} can be derived in a symmetrical manner as in (43) and can thus be specified as:

$$w_{IT} = (1-\phi_L)\frac{p_H H}{L_{IT}} = \frac{1-\phi_L}{1+\alpha}\frac{Y(t)}{L_{IT}} \tag{45}$$

[13]Note that $\frac{1-\alpha}{1-\alpha^2} = \frac{1}{(1+\alpha)}$. See also Appendix C.

The income share of providers of the AI service is accordingly:

$$\frac{w_{IT}(t)\, L_{IT}}{Y(t)} = \frac{1-\phi_L}{1+\alpha} < 1. \tag{46}$$

Income share of financial investors: The income share of financial investors can be calculated using (40), (38), and (27) as [14]

$$\frac{N(t)\, \pi_x(t)}{Y(t)} = \frac{\alpha}{1+\alpha}. \tag{47}$$

3.5 Aggregate expenditure and income

To understand and analyze the role of aggregate demand it is necessary to specify the consumption and savings behaviour of the agents in the economy.

In standard endogenous growth models, aggregate demand is typically modelled assuming *representative* intertemporal choices based on a *representative* household's Euler equation.[15] This, however, is not adequate when asymmetries in factor rewards and potential changes in income distribution are key features of interest - as is the case when considering automation technology. The representative household assumption in standard endogenous growth models assumes away differences in intertemporal decisions of rich and poor households and their respective effects on aggregate consumption and savings. In Appendix D examples are provided for specific intertemporal choices at individual or group level. Moreover, if group preferences are heterogeneous, they may lead to heterogeneous consumption and savings behaviour which needs to be taken into consideration given that it specifies that effective aggregate supply and demand for intermediate inputs depends on aggregate demand.

[14] Details of the calculation are contained in Appendix C.

[15] $\frac{\dot{C}}{C} = \frac{r_D - \rho}{\eta_U}$ with ρ denoting the representative agent's time preference rate and η_U the intertemporal elasticity of substitution.

The novel model proposed here does not assume away the idea of rational intertemporal choices, as is usually the case in endogenous growth models. However, what it does reject is the idea of a simple aggregation rule like a representative household (Gries, 2019). Instead, for present purposes the Keynesian tradition is followed by assuming that some households only earn wage income $w_L L_L$ and another group of households earn only financial income from assets rF. A third group, providers of AI-services, is also regarded as its own group. Each group has its own consumption preferences and patterns. Labor income accrues to poorer households while financial wealth holders and AI service providers accrue income for richer households.

3.5.1 Consumption and investment expenditure

From (44) we know that the share of labor income. We define group-specific intertemporal choice models and assume plausible group-specific parameters for the choice problem, then suggest that total wage income is fully consumed and that labor income is the only source of consumption expenditure. This is a traditional assumption in Keynesian growth models (Gries, 2019). Yet in Appendix D we show that motivating this assumption by suggesting group-specific optimal intertemporal choices is not difficult. The important assumption is that groups are different and have different expenditure behavior.

According to (44) the share of labor income is $\frac{w_L L_L}{Y} = \frac{\phi_L}{(1+\alpha)}$. As labor belongs to poorer households, it is assumed here that total wage income will be fully consumed and that total wage income is the only source of consumption expenditure.[16] Further, in an economy with non-perfect matching, consumers also devote income to search and matching activities whenever their desired consumption cannot find a suitable output. Searching for appropriate consumption goods leads to the experience that investing fraction θ_j of their income

[16] In Appendix D we show that once we depart from the representative household approach, motivating this assumption by group-specific optimal intertemporal choices is not difficult.

in the search and matching procedure would reduce the mismatch.[17] Therefore aggregate consumption is:

$$C(t) = w_L(t) L_L (1-\theta) + \varepsilon = c(1-\theta) Y(t) + \varepsilon \tag{48}$$

$$\text{with} \quad c = \frac{\phi_L}{(1+\alpha)} \tag{49}$$

Note here that c is the economy's marginal (and average) rate of consumption. ε denotes a randomness in consumption demand with an expected value $E[\varepsilon] = 0$.

As far as investment expenditure is concerned, in our model the innovation by intermediate-goods producing start-up ventures requires investment. It is assumed that such investment ν is identical for each innovation. Thus total start-up investments $I(t)$ are described by

$$I(t) = \nu \dot{N}(t). \tag{50}$$

3.5.2 The Keynesian income-expenditure equilibrium

Income Y can be used for consumption C and investment I. Thus demand for GDP is $Y^D \equiv C + I$. While the consumption rate is determined by (48) and a constant fraction of total effective income, investments are driven only by the market entry of new goods (i.e.,innovation), $\dot{N}$. With the consumption rate (49) a constant, the Keynesian income-expenditure mechanism can be applied to determine effective total demand, Y^D. Therefore, in income-expenditure equilibrium, aggregate effective demand equals effective income

$$Y(t) \stackrel{!}{=} Y^D(t) \equiv C(t) + I(t), \tag{51}$$

[17] In section 4.2 when we introduce the aggregate *match-improvement function* (58) we will see, how θ affects the matching process.

and we obtain the well-known Keynesian income-expenditure multiplier for the effective expected demand in aggregate goods market

$$Y^D(t) = \frac{I(t) + \varepsilon\,(t)}{1 - c\,(1-\theta)} = \frac{\nu \dot{N}(t) + \varepsilon_I}{1 - c\,(1-\theta)}. \tag{52}$$

3.5.3 Expected aggregate demand for total production

To determine the total or aggregate demand for final output Q, we begin with the demand for GDP, $Y^D(t) \equiv C\,(t) + I(t)$. We also need to add the demand for input goods taken from final goods sector $N(t)x(t)c_x$. The Keynesian income-expenditure mechanism tells us that effective aggregate demand for GDP is $\frac{\nu \dot{N}+\varepsilon}{1-c(1-\theta)}$, adding $N(t)x(t)c_x$ gives the effective demand for total output Q, namely

$$Q^D = \frac{\nu \dot{N}(t) + E\left[\varepsilon\,(t)\right]}{1 - c\,(1-\theta)} + N(t)x(t)c_x.$$

Demand is hence an endogenous value in which investment expenditures are independent from households' savings decisions. Further, to determine the expected excess demand ratio under current demand conditions, we need to divide by $Q(t)$. As a result, the aggregate effective demand ratio $\lambda(t)$ describes the ratio of effective aggregate demand to current output

$$\lambda(t) = \frac{Q^D(t)}{Q(t)},$$

and in expected values we obtain the *ratio of expected aggregate demand* [18]

$$E\left[\lambda(t)\right] = \frac{\nu}{1 - c\,(1-\theta)} \frac{1}{L_Q^* \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} g_N + \alpha^2. \tag{53}$$

[18] $\frac{E[Q^D]}{Q} = \frac{\nu}{1-c(1-\theta)} \frac{\dot{N}(t)}{Q(t)} + \alpha^2$ and using (3) and (29) we obtain (53).

4 Solving the model

In this section, we depart from the perspective of individual firms and consumers who counter their perceived frictions and move to that of an omniscient observer of the mechanisms. From this angle, the driver of perceived friction is an aggregate market mismatch. This provokes an adjustment that leads to a reallocation of resources. While this reallocation can neutralize the mismatch, it also reduces human services used in production and thus output. The mismatch is closely related to the demand side, so the demand side restricts the current output and growth rate. The growth path is demand-restricted. Further, we also show that this growth path is stationary.

4.1 Solving for technology growth

We start to solve the model by determining the semi-endogenous growth rate of new products that successfully enter and remain in the market $g_N(t) = \frac{\dot{N}(t)}{N(t)}$. From Equation (35) we know that the growth rate of implemented technologies depends on effective demand for intermediate goods and thus depends on labor in effective production H_Q, and is

$$g_N(t) = \frac{\dot{N}(t)}{N(t)} = \left(\left(\frac{\alpha^2}{c_x} \right)^{\frac{1}{1-\alpha}} H_Q \right)^{\varphi} (g_A)^{1-\varphi} \tag{54}$$

This process is semi-endogenous, as the exogenous g_A is an essential driver of g_N. However, the extent to which the exogenous innovative process g_A becomes usable and implemented in the economy is endogenous.

4.2 From perceived individual frictions to aggregate market mismatch

In section 3.1 we introduced the notion of a firm facing market friction in selling its potential output.[19] From the perspective of an individual firm i, we have discussed firm i's perception of market mismatch δ_i which they relate to their individual market conditions and their counter-activities. They use human services $H_{\phi i}$ for placement and reduce their individual sales problems accordingly. Furthermore, in the preceding sections we explained that it is not only firms that are affected by a market mismatch. In their search for the desired consumption goods, consumers also face a mismatch and hence spend a fraction θ_j of their income on this search.

Further, in this section we suggest that individual (idiosyncratic) problems in the market are not the only reason for firms' sales and customers' purchase problems. These problems are in fact, also due to aggregate market conditions, even if individual decision-makers are not aware of this fact.

What are the reasons for firms' sales problems? From the perspective of firms, effective sales are determined by stochastic market mismatch δ_i, [$\phi_i(t) = 1 - \delta_i(t)$ see 4]. Thus, to answer this question we need to find out more about the random variable $\delta_i(t)$. Furthermore, what is behind the firm's perceived market frictions $\delta_i'(t)$?

We assume that the mismatch is determined by two components, (i) aggregate market conditions and (ii) an idiosyncratic component for each individual firm. (i) The first component, the aggregate component, is a current shortage of aggregate demand $\delta^D(t)$ as being the difference between total supply and effective aggregate demand $Q^D(t)$

$$\delta^D(t) = \frac{Q(t) - Q^D(t)}{Q(t)} = 1 - \lambda(t). \tag{55}$$

[19]This section is closely related to the modelling in Gries (2020a).

(ii) Second, and in addition, while $\delta^D(t)$ is the aggregate market component, we have to add the idiosyncratic component for each firm. Sales problems are also firm-specific obstacles and are described by the random variable ε_{Fi}, with $1 > E[\varepsilon_{Fi}] > 0$. For given aggregate market conditions $\delta^D(t)$, ε_{Fi} is the element of the mismatch that is due to individual firm conditions. Therefore, total individual friction perceived by each firm i combines the aggregate market and idiosyncratic component and can be described as

$$\delta_i'(t) = \delta^D(t)\,\varepsilon_{Fi}. \tag{56}$$

However, individual firms or consumer do not have this insight into the breakdown of the friction. An individual firm only perceives an expected sales ratio $E[\phi_i(t)] = 1 - E[\delta_i(t)]$, interpreting it as being caused by a friction that can be addressed by allocating more labor towards the matching process $[\frac{\partial E[\phi_i(t)]}{\partial E\left[\delta_i'\right]} < 0, \quad \frac{\partial E[\phi_i(t)]}{\partial H_{\phi i}(t)} > 0$ see (5) section 3.1].

We have to aggregate to connect these individual activities with total and current market conditions to determine aggregate market equilibrium. Assuming that ε_{Fi} are i.i.d for $i \in \mathcal{I}$, we can aggregate ($\varepsilon_{Fi} = \varepsilon_F$,) and obtain as general or representative perceived friction $\delta'(t)$; and in expectations

$$E[\delta'(t)] = (1 - E[\lambda])\,E[\varepsilon_F] - cov(\varepsilon, \varepsilon_F), \quad \text{with} \quad cov(\varepsilon, \varepsilon_F) < 0. \tag{57}$$

This shows the full mechanism that leads to mismatches. However, we have not specified how counter-measures by firms and customers affect the mismatch. To do this, we define the aggregate match-improvement function $m(t)$ for the aggregate market. We assume that matching of the two market sides is determined by the firms' allocation of human services to combat mismatch $H_{\phi i}(t)$ and of the fraction $\theta(t)$ of consumers' income spent to find the desired consumption good

$$m = L_{\phi}(t)(1-\theta(t))^{-1}, \quad \text{with } \frac{dm}{dH_{\phi}} > 0, \quad \frac{dm}{d\theta} > 0. \tag{58}$$

Thus, the rate of expected effective aggregate mismatch -after implementing counter-measures - is

$$E[\delta(t)] = E[\delta'(t)] - m. \tag{59}$$

When the mismatch is completely eliminated, such that the aggregate expected mismatch becomes zero, we obtain a perfect matching

$$E[\delta(t)] = 0. \tag{60}$$

Further, and most importantly, equation (60) implies that firms are in sales equilibrium, and also the aggregate market is in equilibrium. Specifically, (i) firms are in sales equilibrium as the expected effective sales ratio turns to one,

$$1 = E[\phi(t)]. \tag{61}$$

(ii) For the aggregate market we know that effective sales require the respective effective demand, $E[\Phi] = E\left[Q^D\right]$. As the effective sales ratio is $1 - E[\delta(t)]$, we obtain that the mismatch $E[\delta(t)]$ also describes the gap between effective demand and production $1 - E[\delta(t)] = E[\lambda(t)] = \frac{E\left[Q^D(t)\right]}{Q(t)}$. Thus, whenever (60) holds and the mismatch is removed the aggregate goods market, too, is in equilibrium, since

$$1 = E[\lambda(t)]. \tag{62}$$

4.3 The aggregate model in two equations

Using (54) reduces the system to the following two simultaneous equations, namely (61a) and (62a).

4.3.1 Firms' sales equilibrium

From (7) in section 3.1 we know that a firm allocates human services in the market placement process until all output is sold. On aggregate (59) and (61) tell us that producers and customers allocate resources to improving aggregate matching until all production is sold. Using the constraint for human service allocation (36) we can now state firms' sales equilibrium for the representative producer as[20]

$$H_Q = cov\left(\varepsilon, \varepsilon_F\right)\left(1-\theta\right) + H. \qquad (61a).$$

4.3.2 Aggregate market equilibrium

In section 3.5 we determined aggregate demand and the aggregate effective demand ratio (see 53). Aggregate goods market equilibrium requires that demand equals production and supply, such that the effective demand ratio turns to one and plugging in (62) and (54) gives[21]

$$H_Q = \left(\frac{\nu}{\left(1-\alpha^2\right)\left(1-c\left(1-\theta\right)\right)}\right)^{\frac{1}{(1-\gamma)}} \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{(1-\alpha)(1-\gamma)}} g_A. \qquad (62a)$$

Two equations (61a) and (62a) hence remain to solve for the two endogenous variables, namely human services used in production H_Q and consumers' spending on search and

[20]For details see Appendix E.
[21]For details see Appendix E.

matching θ.

4.4 Current market equilibrium

We can now solve for equilibrium. Combining (61a) with (62a) we are left with only equation (63) and one variable, H_Q

$$0 = F = H_Q^{(1-\gamma)} - \frac{\nu}{(1-\alpha^2)\left(1 - c\frac{H-H_Q}{-cov(\varepsilon,\varepsilon_F)}\right)} \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}} (g_A)^{1-\gamma}. \tag{63}$$

As we cannot explicitly solve for H_Q, we apply the *Implicit Function Theorem* to determine the equilibrium $\tilde{H}_Q$, and other interesting variables.

Proposition 1 ***Current market equilibrium:*** *Equation (63) implicitly defines a function for*

(i) the equilibrium value of $\tilde{H}_Q$

$$\tilde{H}_Q = H_Q\left(\nu, g_A, c_x, A_{IT}, ..., cov\left(\varepsilon, \varepsilon_F\right)\right), \ with \ \frac{d\tilde{H}_Q}{dg_A} > 0, \qquad \frac{d\tilde{H}_Q}{d\nu} > 0. \tag{64}$$

Further, (64) leads to

(ii) the rate of consumers' spending on improving the matching process

$$\tilde{\theta} = 1 - \frac{H - \tilde{H}_Q}{-cov\left(\varepsilon, \varepsilon_F\right)}, \quad with \quad cov\left(\varepsilon, \varepsilon_F\right) < 0 \tag{65}$$

(iii) total production of the final good

$$\tilde{Q}(t) = N\left(t\right) \tilde{H}_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}, \tag{66}$$

(iv) total income and hence the level of the growth path

$$\tilde{Y}(t) = N(t)\left(1-\alpha^2\right)\tilde{H}_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}, \tag{67}$$

(v) the growth rate of income (GDP) gives

$$\tilde{g}_Y = \frac{\dot{Y}(t)}{Y(t)} = g_N = \frac{\dot{N}(t)}{N(t)} = \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}}\left(\tilde{H}_Q\right)^{\gamma}(g_A)^{1-\gamma}, \tag{68}$$

and (vi) the real rate of return on financial investment

$$\tilde{r} = \tilde{g}_Y. \tag{69}$$

For a proof, see Appendix F.

With $\tilde{H}_Q$ and $\tilde{H}_\phi$ we have determined a current market equilibrium at a level below potential output $\tilde{H}_Q < H$. Further, as $\tilde{H}_Q$ depends on demand side parameters, e.g. ν, the level of the income path and the growth rate is restricted by the demand side. It is also interesting to note that in this kind of economy, the return on investment is equal to the growth rate. While (69) is a result that is also found in other mainstream models, causality is different. In this model g_N is the driver of r. As more products enter the market, profits improve and the return on investments increases. In endogenous growth theory, r is the result of an intertemporal choice and drives both the growth rate and the savings rate.

4.5 Stationarity of Equilibrium

Although market equilibrium for each period is described in section 4.4, two important questions remain. First, how can the equilibrium output steadily remain below potential output and represent a long-term stationary equilibrium? Second, how can aggregate demand become central and determine both the stationary level and the speed of the growth path? The next two subsections provide the answers.

Why are these questions worth asking? Mainstream dynamic macroeconomics is based on the idea that the path of potential growth - often regarded as the outcome of some kind (variety) of neoclassical or endogenous growth model - is the only relevant process for economic growth. After a temporary deviation from this path, the economy normally returns to it and continues to grow as described in the fundamental growth model. There is no permanent deviation. We would hence suggest that the equilibrium we just derived can indeed become a permanent, stationary process. In other words, such a demand-restricted growth path (path level and growth rate) is a stationary path. The economy will not necessarily return to the path of potential growth.

4.5.1 The stationary no-expectation-error equilibrium

In this approach we suggest a different concept for a stationary equilibrium[22] that is directly related to stochastic modeling. We describe stationary behavior from the perspective of individual decision-makers.

We assume that a systematic difference of expectations and planning with the average outcome of a stochastic variable is perceived as an inconsistency of one's own behavior and reality that leads to an adjustment in behavior towards an experience of less inconsistency. E.g if in a stochastic environment an individual plans and organizes a specific outcome - according to their subjective expectations - and their plans and outcome do not coincide with observed expected values, we refer to this difference as an expectation error. As a consequence the individual learns from this error and changes their behavior by adjusting their plans. Individual behavior becomes stationary if the planned and realized outcome is

[22]The equilibrium concept is close to the ideas introduced in Gries (2020b).

indeed the observed expected outcome. This condition defines a behavioral equilibrium such that it implies no (need) for a change in behavior. Thus, we refer to this condition as the ***no-expectation-error equilibrium*** (n-e-ee). It is an equilibrium in terms of a stationary behavior.

In this approach, the general concept of a no-expectation-error equilibrium can be illustrated by looking at the matching procedure. The mismatch $E[\delta]$ defines the gap between planned production $Q_i(t)$ and the mean of effective sales $E[\Phi_i(t)] = (1 - E[\delta(t)])\, Q_i(t)$. Thus, as long as firms and customers do not allocate sufficient resources to counter the mismatch they cannot expect the mismatch to disappear, and $E[\delta] = E[\delta'] - m(L_\phi, \theta) > 0$. Thus, individuals face an expectation error as their actions do not coincide with the observed expected values. In other words, there is an error in their planning as their subjective expectations are false. Thus, they continue to adjust their plans until they correctly expect and plan their counter-activities, such that the expected mismatch is on average fully eliminated $0 = E[\delta] = E[\delta'] - m(L_\phi, \theta)$. As a result, there is no expectation error with respect to the final goods matching mechanism. Firms are in sales equilibrium ($E[\phi] = 1$) and we also obtain equilibrium in the aggregate goods market $E[\lambda(t)] = 1$.

Definition 1: No-expectation-error equilibrium. *Firms and customers are in no-expectation-error equilibrium (n-e-ee) if (i) the expected mismatch is correctly predicted, such that respective planned counter-activities fully eliminate the expected mismatch*

$$E[\delta'] = m(L_\phi, \theta). \tag{70}$$

and (ii) furthermore, firms and customers exhibit stationary behavior (no change in behavior is necessary) as they expect what they plan and realize, such that firms remain in sales *equilibrium and the aggregate market continues to remain in market equilibrium,*

$$E[\phi] = 1, \text{ and}$$
$$E[\lambda] = 1.$$

Using Definition 1 above we see that the equilibrium which is determined in proposition 1 is indeed a stationary equilibrium. Thus, we can state the following proposition.

Proposition 2 ***Steady state equilibrium:*** *The market equilibrium derived in proposition 1 is a no-expectation-error equilibrium and thus a stationary equilibrium.*[23]

This outcome, a demand-restricted stationary growth path below the level of potential growth, is the most significant difference in our model from mainstream endogenous growth models, and a fundamental contribution of this paper. Such as demand-restricted stationary growth path could not occur under the perfect market conditions of typical endogenous growth models. There are two reasons why and how can this happen here. First, firms observe a market mismatch which provides incentives for firms and customers to act. Both can respond to this perceived mismatch by allocating resources to reduce perceived frictions and improve the match between demand and supply. In response, labor potentially available for production is allocated to improve the matching process and expenditure that is potentially usable for consumption demand is spent on the search. This resource reallocation leaves the economy below the potential production level.

Having proposed an endogenous growth model with aggregate demand constraints and that incorporates the task approach, the next section uses this model to analyze the dynamic,

[23]Proof: As (64) and (65) in proposition 1 satisfies condition (60), (61) and (62), respectively, conditions of Definition 1 are satisfied. Thus, firms and customers are in a no-expectation-error equilibrium and exhibit stationary behavior.

long-run impacts of artificial intelligence.

5 The Dynamic, Long-Run Impacts of AI

The previous section of this paper proposed an appropriate endogenous growth model to identify labor market and growth consequences of technological progress in AI. In this section the model is used to analyze the long-run impacts of AI. For simplicity we assume a once and for all push in the availability of AI technologies indicated by the parameter A_{IT}.

5.1 Impact on human service production

Progress in AI ($\frac{dA_{IT}}{A_{IT}}$) will increase the supply of the human service input H

$$\frac{dH}{H} = \eta_{H,A_{IT}} \frac{dA_{IT}}{A_{IT}} > 0, \quad \text{with} \quad \eta_{H,A_{IT}} > 0, \quad \text{for} \quad \sigma > 1, \tag{71}$$

given that[24]

$$\eta_{H,A_{IT}} = \frac{\left[1 - \left(\frac{A_L}{A_{IT}} \frac{L_L B_1(N_{IT})}{L_{IT} B_2(N_{IT})} \left(\frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}}\right] (\gamma_{IT}(N_{IT}))^{\sigma-1} \left(B_1\left(N_{IT}\right)\right)^{-\frac{1}{\sigma}} \eta_{N_{IT},A_{IT}} + (\sigma - 1)}{(\sigma - 1)\left[1 + \left(\frac{A_L L_L}{A_{IT} L_{IT}} \left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}\right]}. \tag{72}$$

This is an interesting, and perhaps counter-intuitive result result. In a standard constant elasticity of substitution (CES) approach, as in Gries and Naudé (2018), H would grow at rate $\frac{dA_{IT}}{A_{IT}}$ for large values of A_{IT}. The task-based approach incorporated in this model however leads to the result that the growth rate of H depends on the elasticity of substitution between AI and human labor. If the elasticity of substitution is high, i.e. $\sigma > 1$, more AI

[24] For calculations see Appendix H.

will lead to an even stronger expansion in human service production and availability. From the supply side a high substitution rate between AI and labor is favorable, as it means that labor supply constraints will have less inhibiting effect on production. Easy substitution of labor by the AI technology thus facilitates the extra expansion of tasks. On the other hand, if AI is a complement to labor, i.e. ($\sigma > 1$), this effect will be limited, and progress in AI can lead to a reduction in the growth rate of human services.

5.2 Impact on inequality

To analyze the impact of AI ($\frac{dA_{IT}}{A_{IT}}$) on inequality, the changes it brings about in the income share of labor, the income share of the technology providers, and the income share of financial wealth holders will be determined.

Income share of labor: The income share of labor was described in equation (44). The derivative of (26) shows how the income share of labor income changes as a result of new AI technologies:[25]

$$\frac{d\left(w_L L_L/Y\right)}{w_L L_L/Y} = \frac{d\phi_L}{\phi_L} = \eta_{\phi_L,A_{IT}} \frac{dA_{IT}}{A_{IT}} < 0, \text{ for } 1 < \sigma, \tag{73}$$

$$\text{with } \eta_{\phi_L,A_{IT}} = \frac{1}{\sigma} \frac{\frac{\Gamma \frac{d\Gamma}{dN_{IT}} \frac{N_{IT}}{\Gamma} \frac{dN_{IT}}{dA_{IT}} \frac{A_{IT}}{N_{IT}}}{(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1)\Gamma(N_{IT}(A_{IT}),N)^2} + (1-\sigma)}{\left(\Gamma\left(N_{IT}\left(A_{IT}\right),N\right)^{-1}-1\right)^{-\frac{1}{\sigma}} \left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{-\frac{\sigma-1}{\sigma}} + 1} < 0. \tag{74}$$

and with $\frac{d\Gamma}{dN_{IT}}\frac{N_{IT}}{\Gamma} < 0$. For $1 < \sigma$ (high elasticity of substitution) the income share of labor will decline since $\Gamma \frac{d\Gamma}{dN_{IT}} \frac{N_{IT}}{\Gamma} \frac{dN_{IT}}{dA_{IT}} \frac{A_{IT}}{N_{IT}} < 0$. However, if $1 > \sigma$ the income share of labor will not necessarily increase, in contrast to what would be the case in a standard CES approach. This shows that the integration of the task-based approach in the model means

[25] For calculations see Appendix H.

that automation will decrease the share of labor income no matter the size of σ. This is because $\Gamma \frac{d\Gamma}{dN_{IT}} \frac{N_{IT}}{\Gamma} \frac{dN_{IT}}{dA_{IT}} \frac{A_{IT}}{N_{IT}}$ is always negative. This effect is the same as that identified by Acemoglu and Restrepo (2019, p.9).

It can also be noted that A_{IT} progress will not only depress the income share that labor receives from providing human service inputs (for $1 < \sigma$), but that the share of labor income in the *total economy* will also decline.

Income share of technology providers: Departing from (46) and taking the derivatives shows that the income share of the technology providers increases when there is high elasticity of substitution between AI and labour, as can be seen from:

$$\frac{d\left(w_{IT}L_{IT}/Y\right)}{dA_{IT}} \frac{1}{w_{IT}L_{IT}/Y} = -\frac{\eta_{\phi_L, A_{IT}}}{\left(\frac{1}{\phi_L} - 1\right)} \frac{dA_{IT}}{A_{IT}} > 0 \quad \text{with } \eta_{\phi_L, A_{IT}} < 0 \text{ for } 1 < \sigma. \tag{75}$$

Income share of financial wealth holders: According to (47), the income share of financial wealth holders is $\frac{\alpha}{(1+\alpha)}$. Hence this income share will not change with the technology shock of AI progress, $dA_{IT} > 0$.

5.3 Impact on demand and absorption

In the previous section we learned that a high elasticity of substitution leads to a shift in income distribution in favor of technology providers and financial wealth holders. Further, as argued in section 3.5 consumption demand is determined by standard labor. Thus, the demand side - specifically the consumption rate c - is affected by this asymmetric participation in income benefits from AI technologies. The impact of progress in AI technologies

(dA_{IT}) on the consumption ratio and thus the ratio of market absorption is[26]

$$\frac{dc}{c} = \overset{(-)}{\eta}_{c,A_{IT}} \frac{dA_{IT}}{A_{IT}} < 0, \tag{76}$$

with $\eta_{c,A_{IT}} = \eta_{\phi_L,A_{IT}} < 0$, for $\sigma > 1$. This shows that AI unambiguously tightens the demand constraint when the elasticity of substitution between AI technologies and labor is high, i.e. $\sigma > 1$.

5.4 Impact on long-term efficiency

How close is an economy to potential output? How high is the current equilibrium output compared to potential output? We measure this kind of foregone output by the utilization or deployment rate which we define as $\omega = \tilde{Y}/\tilde{Y}^P$. Thus, $1-\omega$ is a of measure of hidden inefficiency. The economy is below its potentials but is not aware of it. With current equilibrium output (67) and potential output [maximum possible output ($\tilde{H}_Q = H$), $\tilde{Y}^P(t) = N(t)(1-\alpha^2)H\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}$] the *deployment rate*

$$\omega(t) = \frac{\tilde{Y}(t)}{\tilde{Y}^P(t)} = \frac{\tilde{H}_Q(t)}{H}. \tag{77}$$

As a reminder, $\tilde{\omega}$ is the result of optimal individual behavior in steady state. Thus, our question is how does AI technologies affect this path of utilization or deployment of resources?

As we know from (71), changes in AI technology dA_{IT} increase total availability of human services H. This can improve potential and current production through an increase in H_Q (see 64). As effects on H and H_Q are different, the effect on the deployment rate is not clear, $\frac{d\omega(t)}{\omega(t)} = \frac{dH_Q}{H_Q} - \frac{dH}{H}$. While $\frac{dH}{H}$ is known from (71) we need to determine the equilibrium change

[26] For a proof and the respective conditions see Appendix H.

of human service in production as result of AI improvements

$$\frac{d\tilde{H}_Q}{H_Q} = \eta_{H_Q,A_{IT}} \frac{dA_{IT}}{A_{IT}} \gtrless 0 \tag{78}$$

$$\eta_{H_Q,A_{IT}} = \frac{\overset{(ii)\quad <0}{\eta_{\phi_L,A_{IT}}} + \frac{H}{H_\phi}\overset{(i)\quad >0}{\eta_{H,A_{IT}}}}{(1-\gamma)\left(\frac{1}{(1-\theta)c} - 1\right) + \frac{H_Q}{H_\phi}} \gtrless 0 \tag{79}$$

While the total effect $\frac{d\tilde{H}_Q}{H_Q}$ seems generally ambiguous, the first intuitive idea is that it should be positive. However, there are two opposing effects. First, on the supply side, an increase in technology which is quasi factor-augmenting should lead to more factors available for production. However, in our demand constraint growth model, there are further effects. Secondly, the term $H_\phi \eta_{\phi_L,A_{IT}} < 0$ in (78) shows that the potential increase on the supply side is countered by a negative effect through income distribution and a reduction in absorption on the demand side. More inequality and a declining consumption rate restricts the total effect of AI on factor utilization for production which otherwise had been solely $H\eta_{H,A_{IT}}$ in (78). We can even identify the conditions when $\eta_{H_Q,A_{IT}}$ turns negative. Broadly speaking, $\eta_{H_Q,A_{IT}}$ turns negative if A_{IT} and σ are sufficiently large, such that the effect on inequality becomes so dominant that the declining demand suppresses the opportunity for more production.[27]

Now we can turn back to the deployment rate. If we plug in and rearrange, for the change of the deployment rate we obtain

$$\frac{d\omega(t)}{\omega(t)} = \eta_{u,A_{IT}} \frac{dA_{IT}}{A_{IT}} < 0 \quad \text{for } (1-\theta)c < \frac{(1-\gamma)}{2-\gamma} \tag{80}$$

$$\eta_{u,A_{IT}} = \left[\frac{\overset{<0}{\eta_{\phi_L,A_{IT}}} + \frac{H}{H_\phi}\overset{>0}{\eta_{H,A_{IT}}}}{(1-\gamma)\left(\frac{1}{(1-\theta)c} - 1\right) + \frac{H_Q}{H_\phi}} - \overset{>0}{\eta_{H,A_{IT}}}\right] < 0, \tag{81}$$

$\eta_{u,A_{IT}}$ again is ambiguous in general. However, for $\eta_{u,A_{IT}}$ we find a simple (sufficient) con-

[27]See Appendix H.

dition, $(1-\theta)\, c < \frac{(1-\gamma)}{2-\gamma}$ for a an increasing gap between current equilibrium steady state and potential steady state output.[28] This condition also allows for further interesting interpretations. In this comparative static analysis we look at a once-and-for-all increase in AI technologies and obtain as result, that this increase may lead to a positive or negative effect on deployment rate $\omega\,(t)$. However, from (74) and (76) we know that inequality increases and absorption declines if AI continues to change. As a consequence, with more inequality and a declining consumption rate the economy will move towards a declining deployment rate and increasing *structural inefficiency,* which may result in a long-term *stagnating path of growth.*

Unlike in mainstream growth modelling, our model allows us to study long term stationary growth below the potential growth path. One reason for this is the fact that income distribution matters for the demand side and the demand side is able to restrict economic expansion. This hold true not only as a short-term disequilibrium phenomenon. Due to the equilibrium concept for stationary behavior, it sustains. Therefore, in this model a path of long-term stagnating growth has a simple narrative. If AI technologies lead to asymmetric benefits mostly for financial wealth holders and technology owners, such that more inequality reduces labor share of income and the consumption rate, the demand side will grow less than the supply side. Since this approach has no assumption such as Say's law and market adjustments are done through search and sales promotion decisions, the demand side can become a constraint for the supply side. Resources that could be used for more production and absorption are indeed used more and more to achieve a match between supply and demand. The economy could produce more, but has to deploy increasing resources to find the equilibrium.

[28] See Appendix H.

5.5 Impact on wages and labor productivity

Changes in the expected ratio of market absorption $\frac{dc}{c}$ have implications for wages, labor productivity, and GDP growth. Because wages are equal to marginal labor productivity, the effects of AI on wage and labor productivity growth are:

$$\frac{dw_L(t)}{w_L(t)} = \left[\overset{(ii)}{\eta_{c,A_{IT}}}^{<0} + \overset{>0}{\eta_{H_Q,H}} \overset{(i)}{\eta_{H,A_{IT}}}^{>0} \right] \frac{dA_{IT}}{A_{IT}} \lesseqgtr 0. \tag{82}$$

The result in (82) shows that there are two effects of AI on productivity and wages. First, growth in human service input is driven by IT and AI growth and is described by (71).Thus, this effect happens through the higher total human service input $\eta_{H,A_{IT}}$ which may improve productivity and wages [see (i) in 82] and also increases human service in the production sector $\eta_{H_Q,H}$. Second, if the elasticity of substitution is high ($\sigma > 1$) the ratio of market absorption declines $\eta_{c,A_{IT}} < 0$. This is a demand-constraint effect, due to the fact that aggregate demand is not growing sufficiently to absorb additional supply [see (ii) in 82].

If the elasticity of substitution is high, i.e. ($\sigma > 1$), wage income growth will be slower and labor will not equally share in the benefits of $\frac{dH}{H}$ as its share of income will decline. Even if IT and AI technologies have both positive and negative impacts on labor productivity growth, the on total marginal labor productivity and wage growth *stagnate* (are reduced). Also, labor participation through wage growth is reduced. By how much is an empirical question. The important message is that there will be an effect from the demand side.

5.6 Impact on long-term GDP growth

The implementation of IT and AI technologies affect not only wages and labor, but also the GDP growth rate. The same mechanisms that were discussed in section 5.4 are also responsible for a negative net impact on GDP growth. These mechanisms are again (i) a positive productivity effect and (ii) a negative effect of a tightening demand constraint. Overall, however, the net impact is ambiguous. From (68) we know that the GDP growth rate is $\tilde{g}_Y = \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}} \left(\tilde{H}_Q\right)^{\gamma} (g_A)^{1-\gamma}$. Taking the derivative

$$\begin{aligned}
\frac{d\tilde{g}_Y}{\tilde{g}_Y} &= \eta_{H_Q,A_{IT}} \frac{dA_{IT}}{A_{IT}} \lesseqgtr 0 \quad \text{with } \eta_{\phi_L,A_{IT}} < 0 \text{ for } 1 < \sigma, \qquad (83) \\
\eta_{H_Q,A_{IT}} &= \frac{\overset{(ii)\quad <0}{\eta_{\phi_L,A_{IT}}} + \frac{H}{H_\phi}\overset{(i)\quad >0}{\eta_{H,A_{IT}}}}{(1-\gamma)\left(\frac{1}{(1-\theta)c} - 1\right) + \frac{H_Q}{H_\phi}} \gtreqless 0 \quad \text{see (79)},
\end{aligned}$$

we see that the direction of this effect directly depends on the sign of $\eta_{H_Q,A_{IT}}$. As section 5.4 provides an extensive discussion, we can conclude: If A_{IT} is large and σ is high enough, the effect of AI growth on inequality with a declining demand becomes dominant, the GDP growth rate is likely to be reduced even if the shock is a positive technology shock.

6 Summary and Conclusion

Artificial intelligence (AI) has become a significant automation technology, raising concerns about rising technological unemployment and inequality. To understand these labor market impacts of automation better, the task approach has been a valuable addition to the theoretical toolkit of economists. The task approach's central contribution has been to make a distinction between jobs and tasks, and moreover to distinguish various tasks which differ in terms of whether they are susceptible to automation. For instance, a distinction is often made between routine cognitive and manual tasks, abstract analytical and managerial tasks, and non-routine manual tasks. If AI technologies, embodied in capital (machines, robots and computers) do not automate all tasks, but instead only those that can be more easily codified, such as routine tasks, and if some of these tasks are done by medium-skilled workers, then these workers may be displaced by AI – leading to higher unemployment and inequality at the same time.

This, however, is not the end of the story. In more recent elaborations of the task approach, the possibility is recognised that, because AI can raise labor productivity and the wages of some, that aggregate demand would increase and indirectly raise the demand for labor. This has been labelled the reinstatement effect of AI. The implication is that the net impact of AI (automation) on jobs and inequality will therefore depend on the relative strengths of the displacement effect and the countervailing reinstatement effect.

Important as these insights are, the task approach as applied to AI has a shortcoming. In this paper we argued that its shortcoming is that any jobs impact from the reinstatement effect is fundamentally uncertain in the task approach. This is because the reinstatement effect depends on the extent of economic growth created by AI and the extent to which economic growth stimulates the demand for labor. This in turn depends on growth in labor productivity, labor wages, and the income share of labor. Because the task approach is not an economic growth model, it is unable to model these dynamic aspects.

While there have been a number of proposals for economic growth models focusing on automation and artificial intelligence, these have shortcomings of their own. First, they lack the insights of the task approach into the distinction between tasks and jobs, and hence does not allow for substitution between tasks to be taken into consideration. And second, endogenous growth models are supply-driven, ignoring the role of aggregate demand. In standard endogenous growth models, aggregate demand is typically modelled assuming representative intertemporal choices based on a representative household's Euler equation. The representative household assumption in standard endogenous growth models assumes away differences in intertemporal decisions of rich and poor households and their respective effects on aggregate consumption and savings. This is not adequate when asymmetries in factor rewards and potential changes in income distribution are key features of interest - as is the case when considering an automation technology.

Thus, both the task approach and endogenous growth models are currently limited in theoretically modelling key economic impacts of AI and automation. As such, the contribution of this paper was to address these shortcomings of the task approach and endogenous models. We provided a (semi) endogenous growth model with a reformulated task approach incorporated, and moreover that can deal with demand constraints. In our model, we derived a demand-restricted stationary growth path below the level of potential growth. In this our model departs the furthest from typical endogenous growth models. With such a demand-restricted stationary growth path possible, our model suggests two novel impacts of AI, which are not find in current models.

For one, by integrating the task-based approach in our growth model we showed that AI automation can decrease the share of labor income no matter the size of the elasticity of substitution between AI and labor – and increase the income share of financial wealth owners and the owners of the technology. And two, when the elasticity of substitution between AI technologies and labor is high, AI will unambiguously reduce aggregate consumption and thus aggregate demand. As a consequence, with more inequality and a declining consumption rate, the economy will move towards a declining utilization (deployment) rate and increasing structural inefficiency.

The core contribution is to show that, if AI technologies lead to asymmetric benefits mostly for financial wealth holders and technology owners, such that more inequality reduces the labor share of income and the consumption rate, the demand side will grow less than the supply side. Since our model has jettisoned the typical assumption of Say's Law, and instead model market adjustments through search and sales promotion decisions, the demand side can become a binding constraint on the supply side. Resources that could be used for more production and absorption are indeed used more and more to achieve a match between supply and demand. The economy could produce more but has to deploy increasing resources to find the equilibrium. If the progress in AI is significant and the elasticity of substitution between labor and AI high enough, the effect of AI to increase inequality and reduce demand becomes dominant, so that GDP growth is likely to slow down, even if the progress in AI amounts to a positive technology shock.

With a slow (-er) diffusion of AI (and slowing down of innovation in AI), there will be rather slower growth in GDP and productivity, because the economy does not benefit much from the supply-side driven capacity expansion potential that AI can deliver. Wages can however stagnate in line with slower GDP and productivity growth in order to maintain employment levels. Thus, in the model presented here we can explain why contemporary advanced countries experience the simultaneous existence of high employment with stagnating wages, productivity and GDP, all despite much AI hype.

Appendices

A. Final-goods-producing firm

Implicit Function Theorem for optimal $H_{\phi i}$ Condition for applying the implicit function theorem hold: $0 = F = E\left[\phi_i\left(\delta_i, H_{\phi i}\right)\right] - 1$, and $\frac{dF}{dH_{\phi i}} = \frac{\partial E[\phi_i(t)]}{\partial H_{\phi i}(t)} > 0$. For the effect of $E\left[\delta_i\right]$ we use $\frac{dF}{dE[\delta_i]} = \frac{\partial E[\phi_i(t)]}{\partial E[\delta_i]} < 0$.

B. The task-based approach

B1. The optimal allocation of tasks, and task production

Demand for tasks: Human service firms

$$\max : \pi_H = p_H H - p_h(z)h(z) = p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} - p_h(z)h(z).$$

F.O.C.

$$\begin{aligned} p_H \frac{\sigma}{\sigma-1} \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} h(z)^{\frac{\sigma-1}{\sigma}-1} - p_h(z) = 0 \\ p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}-1} h(z)^{\frac{\sigma-1}{\sigma}-1} = p_h(z) \\ p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{1}{\sigma-1}} h(z)^{-\frac{1}{\sigma}} = p_h(z) \\ p_H H^{\frac{1}{\sigma}} h(z)^{-\frac{1}{\sigma}} = p_h(z) \end{aligned}$$

arriving at $h(z) = \frac{H}{p_h(z)^{\sigma}} p_H^{\sigma}$, see (15).

Demand for task z : Using marginal production and productivity rules

$h(z_{IT}) = A_{IT}\gamma_{IT}(z)l_{IT}(z)$	production (12)	$h(z_L) = A_L\gamma_L(z)l_L(z)$
$p_h A_{IT}\gamma_{IT}(z)l_{IT}(z) = l_{IT}(z)w_{IT}$	marginal productivity and factor reward	$p_h A_L\gamma_L(z)l_L(z) = l_L(z)w_L$
$p_h\left(z_{IT}\right) = \frac{w_{IT}}{A_{IT}\gamma_{IT}(z_{IT})}$	price = unit labor costs	$p_h\left(z_L\right) = \frac{w_L}{A_L\gamma_L(z_L)}$

and plugging in gives (16) as being the optimal demand for $h(z)$,

$$h(z) = \frac{H}{\left(\frac{w_{IT}}{A_{IT}\gamma_{IT}(z)}\right)^{\sigma}} p_H^{\sigma}, \qquad h(z) = \frac{H}{\left(\frac{w_L}{A_L\gamma_L(z)}\right)^{\sigma}} p_H^{\sigma},$$
$$h(z) = p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_{IT}(z)^{\sigma}, \quad h(z) = p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma} \gamma_L(z)^{\sigma}.$$

Demand for each kind of labor In order to determine the marginal productivity for each total labor input, the productivity for each kind of labor is derived from (16) and (12), and we can obtain the optimal demand for IT labor :

$$\begin{aligned} h(z) &= p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_{IT}(z)^{\sigma} \\ A_{IT}\gamma_{IT}(z)l_{IT}(z) &= p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_{IT}(z)^{\sigma} \\ l_{IT}(z) &= p_H^{\sigma} H \left(A_{IT}\right)^{\sigma-1} w_{IT}^{-\sigma}\gamma_{IT}(z)^{\sigma-1}, \text{ see (17)}, \end{aligned}$$

and standard labor:

$$l_L(z) = p_H^{\sigma} H \left(A_L\right)^{\sigma-1} w_L^{-\sigma}\gamma_L(z)^{\sigma-1}, \text{ see (18)}.$$

To determine wages for each kind of labor we have to rearrange;further, as the next calculations are symmetric for each kind of labor, we look at details only for L_{IT}

$$l_{IT}(z) = p_H^{\sigma} H \left(A_{IT}\right)^{\sigma-1} w_{IT}^{-\sigma}\gamma_{IT}(z)^{\sigma-1}$$

Total IT labor is fully employed and allocates to all tasks using IT labor.

$$L_{IT} = \int_I^N l_{IT}(z)dz.$$

With the integral in (17) $\left[l_{IT}(z)=\frac{p_H^\sigma}{w_{IT}^\sigma}H\gamma_{IT}(z)^{\sigma-1}\left(A_{IT}\right)^{\sigma-1}\right]$ we obtain

$$\begin{aligned}\int_I^N l_{IT}(z)dz &= \int_I^N \frac{p_H^\sigma}{w_{IT}^\sigma}H\gamma_{IT}(z)^{\sigma-1}\left(A_{IT}\right)^{\sigma-1}dz\\ L_{IT} &= \frac{p_H^\sigma}{w_{IT}^\sigma}H\left(A_{IT}\right)^{\sigma-1}\int_I^N \gamma_{IT}(z)^{\sigma-1}dz\\ w_{IT}^\sigma &= p_H^\sigma\frac{H}{L_{IT}}\left(A_{IT}\right)^{\sigma-1}\int_I^N \gamma_{IT}(z)^{\sigma-1}dz\end{aligned}$$

such that with full employed It labor we can determine the wages of IT labor as

$$w_{IT}=p_H\left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}}\left(A_{IT}\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}, \tag{84}$$

and symmetrically for standard labor

$$w_L=p_H\left(\frac{H}{L_L}\right)^{\frac{1}{\sigma}}\left(A_L\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}. \tag{85}$$

The resulting internal relative factor productivity for labor is:

$$\begin{aligned}\frac{w_L}{w_{IT}} &= \frac{\left(\frac{p_H H}{L_L}\right)^{\frac{1}{\sigma}}\left(A_L\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}}{\left(\frac{p_H H}{L_{IT}}\right)^{\frac{1}{\sigma}}\left(A_{IT}\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}}\\ \frac{w_L}{w_{IT}} &= \left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}}\end{aligned}$$

Endogenously automated tasks N_{IT} **:** From the discussion of (13) it is known that tasks are ordered such that $\gamma(z)=\frac{\gamma_L(z)}{\gamma_{IT}(z)}$, and $\frac{\partial\gamma(z)}{\partial z}>0$. If it is assumed that task N_{IT} is the task that exactly separates the production mode, and if tasks are continued, the condition can be rewritten (13) as follows:

$$\begin{aligned}\frac{A_L\gamma_L(N_{IT})}{A_{IT}\gamma_{IT}(N_{IT})} &< \frac{w_L}{w_{IT}}=\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}}\\ 0 &= G=\gamma\left(N_{IT}\right)-\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{-\frac{1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}}\end{aligned} \tag{86}$$

If $\frac{dG}{dN_{IT}} \neq 0$, G implicitly defines a function $N_{IT} = N_{IT}(L_{IT}, L_L, A_{IT}, ...)$.Thus, we need to calculate the respective interesting derivatives.

$$\frac{dG}{dN_{IT}} = \frac{\partial \gamma(N_{IT})}{\partial N_{IT}} + \left[\begin{array}{c} \frac{1}{\sigma}\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{-\frac{1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{I}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \\ \left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{\int_{N_{IT}}^{N}\gamma_L(N_{IT})^{\sigma-1}dz} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{\int_{N-1}^{N_{IT}}\gamma_{IT}(N_{IT})^{\sigma-1}dz}\right] \end{array} \right] > 0$$

B2. Total supply of human service inputs

From (16) it is known that $h(z) = p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_{IT}(z)^{\sigma}$ for $z \in [N-1, N_{IT}.$, and $h(z) = p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma} \gamma_L(z)^{\sigma}$ for $z \in [N_{IT}, N]$. Plugging this in (11) generates an expression for the total value of H:

$$\begin{aligned} H &= \left(\int_{N-1}^{N_{IT}} h(z)^{\frac{\sigma-1}{\sigma}}dz + \int_{N_{IT}}^{N} h(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_{N-1}^{N_{IT}} \left(p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_I(z)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} \left(p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma} \gamma_L(z)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}. \end{aligned}$$

Using (84) and (85) results in: $w_{IT} = p_H \left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}} (A_{IT})^{\frac{\sigma-1}{\sigma}} \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}$

$$\begin{aligned} H &= \left(\int_{N-1}^{N_{IT}} (\gamma_{IT}(z)^{\sigma})^{\frac{\sigma-1}{\sigma}} dz \left(p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} + \int_{N_{IT}}^{N} (\gamma_L(z)^{\sigma})^{\frac{\sigma-1}{\sigma}} dz \left(p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad (87) \\ &= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{w_L}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\begin{array}{c} \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{p_H\left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}}(A_{IT})^{\frac{\sigma-1}{\sigma}}\left(\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \\ + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{p_H\left(\frac{H}{L_L}\right)^{\frac{1}{\sigma}}(A_L)^{\frac{\sigma-1}{\sigma}}\left(\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \end{array} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

$$= \left(\begin{array}{c} \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{p_H^{-1} H^{-\frac{1}{\sigma}} L_{IT}^{\frac{1}{\sigma}} A_{IT}^{\frac{1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}} \right)^{\sigma-1} \\ + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{p_H^{-1} H^{-\frac{1}{\sigma}} L_L^{\frac{1}{\sigma}} A_L^{\frac{1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}} \right)^{\sigma-1} \end{array} \right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\begin{array}{c} \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz H^{\frac{\sigma-1}{\sigma}} \frac{p_H^{-(\sigma-1)} H^{-\frac{\sigma-1}{\sigma}} (L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} \\ + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \frac{p_H^{-(\sigma-1)} H^{-\frac{\sigma-1}{\sigma}} (L_L A_L)^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} \end{array} \right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \frac{(L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} d \frac{(L_L A_L)^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}}$$

$$H = \left(\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (L_L A_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

B2.1 Earning shares

To determine the contribution of standard labor to total service production one can start off from (87)

$$H = \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{w_{IT}} \right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{w_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}}$$

$$1 = \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \left(\frac{A_{IT}}{w_{IT}} \right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \left(\frac{A_L}{w_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} p_H^{\sigma}$$

$$1 = \left((1 - \Gamma(N_{IT}, N)) \Pi(N_{IT}, N)^{\sigma-1} \left(\frac{A_{IT}}{w_{IT}} \right)^{\sigma-1} + \Gamma(N_{IT}, N) \Pi(N_{IT}, N)^{\sigma-1} \left(\frac{A_L}{w_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} p_H^{\sigma}$$

Plugging in definitions (24) and (23), $\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$, $\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz = (1-\Gamma(N_{IT}, N))\Pi(N_{IT}, N)^{\sigma-1}$ this turns into

$$1 = \left(1 + \frac{(1-\Gamma(N_{IT}, N))}{\Gamma(N_{IT}, N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \Gamma(N_{IT}, N)^{\frac{\sigma}{\sigma-1}} \Pi(N_{IT}, N)^{\sigma} \left(\frac{A_L}{w_L}\right)^{\sigma} p_H^{\sigma}$$

we can further rearrange this equation:

$$\begin{aligned}
\Gamma(N_{IT}, N)^{-\frac{\sigma}{\sigma-1}} \Pi(N_{IT}, N)^{-\sigma} &= \left(1 + \frac{(1-\Gamma(N_{IT}, N))}{\Gamma(N_{IT}, N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{A_L}{w_L}\right)^{\sigma} p_H^{\sigma} \\
\Pi(N_{IT}, N)^{\sigma-1} \Gamma(N_{IT}, N) &= \left(1 + \frac{(1-\Gamma(N_{IT}, N))}{\Gamma(N_{IT}, N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{-1} (A_L)^{-(\sigma-1)} \left(\frac{p_H}{w_L}\right)^{-(\sigma-1)} \quad (88) \\
(A_L)^{(\sigma-1)} \left(\frac{p_H}{w_L}\right)^{(\sigma-1)} &= \frac{1}{\left(1 + \frac{(1-\Gamma(N_{IT}, N))}{\Gamma(N_{IT}, N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right) \Pi(N_{IT}, N)^{\sigma-1} \Gamma(N_{IT}, N)}
\end{aligned}$$

Further, from definition (20) and (18) the following expression can be derived:

$$L_L = \int_{N_{IT}}^{N} \frac{p_H^{\sigma} H}{(w_L)^{\sigma}} (A_L)^{\sigma-1} \gamma_L(z)^{\sigma-1} dz = \frac{p_H^{\sigma} H}{(w_L)^{\sigma}} (A_L)^{\sigma-1} \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz$$

using the definition of labor share of income $\phi_L = \frac{w_L L}{p_H H}$ and using the definitions (24) and (23), $\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$ leads to:

$$\begin{aligned}
\frac{L_L w_L}{p_H H} &= \frac{w_L}{p_H} \frac{1}{H} \frac{p_H^{\sigma} H}{(w_L)^{\sigma}} (A_L)^{\sigma-1} \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \\
\frac{L_L w_L}{p_H H} &= \left(\frac{w_L}{p_H}\right)^{1-\sigma} (A_L)^{\sigma-1} \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}
\end{aligned}$$

and combining with (88) gives labor's share of income fully depending on relative labor rewards $\frac{w_L}{w_{IT}}$

$$\begin{aligned}
\phi_L &= \frac{\Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}}{\left(1 + \frac{(1-\Gamma(N_{IT}, N))}{\Gamma(N_{IT}, N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right) \Pi(N_{IT}, N)^{\sigma-1} \Gamma(N_{IT}, N)} \\
\phi_L &= \left(1 + \frac{(1-\Gamma(N_{IT}, N))}{\Gamma(N_{IT}, N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{-1}
\end{aligned}$$

Labor's share of income in the human service sector is determined by relative factor abundance and productivity parameters. Thus, plugging in the relative factor rewards (21) finally results in:

$$\begin{aligned}
\phi_L &= \left(1+\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\left(\left[\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\Gamma(N_{IT},N)}{(1-\Gamma(N_{IT},N))}\right)^{\frac{1}{\sigma}}\right]\frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{-1} \\
&= \left(1+\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\left(\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_{IT}}{A_L}\right)^{\frac{1}{\sigma}}\left(\frac{(1-\Gamma(N_{IT},N)}{\Gamma(N_{IT},N)}\right)^{-\frac{1}{\sigma}}\right)^{\sigma-1}\right)^{-1} \\
&= \left(1+\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\left(\frac{A_{IT}}{A_L}\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{-\frac{\sigma-1}{\sigma}}\right)^{-1} \\
&= \left(1+\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{-\frac{\sigma-1}{\sigma}}\left(\frac{A_{IT}}{A_L}\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}
\end{aligned}$$

and thus allows one to obtain expression (26).

C. Income distribution

Income Y (GDP) and total production Q. Before determining the income shares it is necessary to determine the relation between income Y (GDP) and total production Q. According to (40)

$$Y(t) = Q(t) - N(t)x(t)c_x = \left(1-\frac{Nxc_x}{Q}\right)Q(t).$$

Applying (29) gives

$$\frac{Nxc_x}{Q} = \frac{Nc_x}{NH_Q^{1-\alpha}\alpha^{\frac{-(1-\alpha)2}{1-\alpha}}c_x^{-\frac{-(1-\alpha)}{1-\alpha}}H_Q^{-(1-\alpha)}} = \frac{c_x}{\alpha^{-2}c_x} = \alpha^2,$$

and for $Y(t)$ the result is:

$$Y(t) = \left(1-\alpha^2\right)Q(t) = \left(1-\alpha^2\right)N(t)\alpha^{\frac{2\alpha}{1-\alpha}}c_x^{-\frac{\alpha}{1-\alpha}}H_Q \tag{89}$$

Income share of labor If A_L is time depending)(i.e. $A_L(t)$) and continuously increasing the long-term position is:

$$\begin{aligned}\frac{w_L L_L}{Y} &= \frac{(1-\alpha)\,\phi_L}{1-\alpha^2} = \frac{\phi_L}{1+\alpha} \\ \lim_{A_{IT}\to\infty}\frac{w_L L_L}{Y} &= \frac{\phi_L}{1+\alpha} = \lim_{A_{IT}\to\infty}\phi_L = \frac{1}{1+\left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}} = 0 \quad \text{for} \quad \sigma > 1.\end{aligned}$$

Wages and income share of IT provider:

$$\frac{w_{IT}(t)\,L_{IT}}{Y(t)} = \frac{(1-\alpha)(1-\phi_L)}{1-\alpha^2} = \frac{1-\phi_L}{1+\alpha}$$

Income share of financial wealth owners: From (27), (29) an (40) it can be seen that:

$$\begin{aligned}\frac{N(t)\,\pi_x(t)}{Y(t)} &= \frac{N\left(\frac{1}{\alpha}-1\right)(c_x)\left[\alpha^{\frac{2}{1-\alpha}}c_x^{-\frac{1}{1-\alpha}}H_Q\right]}{NH_Q^{1-\alpha}x^\alpha - Nxc_x} \\ &= \frac{\left(\frac{1}{\alpha}-1\right)Nc_x x}{NH_Q^{1-\alpha}x^\alpha - Nxc_x} \\ &= \frac{N\left(\frac{1}{\alpha}-1\right)c_x x}{\left(1-\frac{Nxc_x}{Q}\right)NH_Q^{1-\alpha}x^\alpha}\end{aligned}$$

Using $\frac{Nxc_x}{Q} = \alpha^2$ results in:

$$\frac{N(t)\,\pi_x(t)}{Y(t)} = \frac{(1-\alpha)\frac{1}{\alpha}\alpha^2}{(1-\alpha^2)} = \frac{(\alpha-\alpha^2)}{(1-\alpha^2)} = \frac{\alpha}{(1+\alpha)}.$$

D. Intertemporal choices for labor and capital owners

In standard endogenous growth models, aggregate consumption expenditure and savings are determined by a representative household conducting an optimal intertemporal choice according to the Euler equation

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\eta_U}.$$

However, this assumption of a *representative household* is rather restrictive and is introduced more for the sake of simplification. Therefore, in the new model proposed in this paper, this assumption is replaced by assuming two groups of households differing with respect to intertemporal choice behaviour.

(i) It is assumed in this paper that workers with wage income represent the "low per-capita income" group. The second group, the owners of financial assets F, represents the "high per-capita income" group. For these households returns r are the only source of income. (ii) Households in each group make their own intertemporal choices. Both ρ and η_U vary across low- and high-income households. a) *Low-income, wage-earning households*: If it is assumed that the time preference rate of low-income households is high, e.g. $\rho_L \geq r$, and if household debt is not allowed, then the Euler equation $\frac{\dot{C}_L}{C_L} = \frac{r-\rho_L}{\eta_{U_L}}$ implies that these households do not intend to shift intertemporal consumption and simply consume what they earn from wage income. b) *High-income households*: High-income households obtain their total income from returns on financial assets $F(t)\, r$.

Thus, the budget constraint of high-income, financial asset owners is $S_F(t) \leq F(t)\, r - C_F(t)$. As savings are used to purchase newly issued financial assets $\dot{F}(t)$ and these assets finance investments, we obtain $\dot{F}(t) = F(t)\, r - C_F(t)$, and

$$\frac{\dot{F}(t)}{F(t)} = \frac{F(t)}{F(t)} r - \frac{C_F(t)}{F(t)} \tag{A1}$$

Applying the Euler equation for financial investors $\frac{\dot{C}_F}{C_F} = \frac{r-\rho_F}{\eta_{U_F}}$ and using the fact that in long-term steady-state growth of consumption cannot exceed the economy's growth rate g_N, we obtain $g_N \geq \frac{\dot{C}_F}{C_F} = \frac{r-\rho_F}{\eta_{U_F}}$. However, this group's consumption growth remains below GDP growth if $g_N \geq \frac{r-\rho_F}{\eta_{U_F}}$, and this holds for values of η_{U_F} and ρ_F that satisfy $\eta_{U_F} \geq \frac{r-\rho_F}{g_N}$. Further, if $F(t)$ grows equal to the economy's rate of growth g_N [which will be shown in appendix 6]we obtain for $\frac{C_F(t)}{F(t)} = \frac{C_F(t)}{N(t)} = \frac{C_F(0)e^{g_{C_F}t}}{N(0)e^{g_N t}}$ and thus $\lim_{t\to\infty} \frac{C_F(0)}{N(0)} e^{-\left(g_N - g_{C_F}\right)t} = 0$.

In appendix 6 it is shown that this condition fits the requirement of dynamic consistency and allows us to determine the start value $\frac{F(0)}{N(0)}$.

When taking the limit, financial investors' consumption rate turns to zero in steady state, $\lim_{t\to\infty} c_F = \frac{C_F(t)}{Y(t)} = \frac{C_F(t)}{N(t)(\lambda L^{1-\alpha}x^{\alpha} - xc_x)} = \frac{C_F(t)}{N(t)} \left(\tilde{\lambda} L^{1-\alpha}\tilde{x}^{\alpha} - \tilde{x}c_x\right)^{-1} = 0$, for steady state values $\tilde{\lambda}$ and $\tilde{x}$.

This illustrates the assumption that financial investors only save and do not consume - at least in the long term steady state.

E. Calculations to solve the model

Determine the growth rate g_N From (35) and (29) we obtain $X^{eD} = Nx = N\alpha^{\frac{2}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H_Q$. and $\dot{N} = \left(\alpha^{\frac{2}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H_Q\right)^{\gamma} N^{\gamma}(\dot{A}))^{1-\gamma}$. Rearranging gives $\frac{\dot{N}(t)}{N(t)} = \left(\alpha^{\frac{2}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H_Q\right)^{\gamma} (\frac{\dot{A}(t)}{A(t)})^{1-\gamma}$ for $N(t) = A(t)$.

Firm's sales equilibrium Combing (4) for the aggregate economy $\phi(t) = 1 - \delta(t)$ with $\delta(t) = \delta'(t) - m$ (see 59) leads to $\phi(t) = 1 - \delta'(t) + m$, and with the definition of the firms sales frction friction $\delta'(t) = (1-\lambda)\varepsilon_F$, and taking expectations gives

$$= 1 - E[\varepsilon_F] + E[\lambda]E[\varepsilon_F] + cov(\varepsilon, \varepsilon_F) + H_\phi(1-\theta)^{-1}$$

Thus, in sales equilibrium described by condition (61) $E[\phi(t)] \overset{!}{=} 1$, leads to

$$H_\phi(1-\theta)^{-1} = E[\varepsilon_F] - E[\lambda]E[\varepsilon_F] - cov(\varepsilon, \varepsilon_F).$$

Using (36) and (62) we obtain (61a)

Determine current aggregate market equilibrium According to (62) market equilibrium requires $1 = E[\lambda(t)]$. With (53)

$$1 = E[\lambda(t)] = \frac{\nu}{1-c(1-\theta)} \frac{1}{H_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} g_N + \alpha^2,$$

and using (54) leads to

$$1 = \frac{\nu}{1-c(1-\theta)} \frac{1}{H_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} \left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q\right)^{\gamma} (g_A)^{1-\gamma} + \alpha^2.$$

and (62a)

F. Proof of proposition 1

Implicit Function theorem: Function F can be simplified:

$$\begin{aligned}
F &= 0 = H_Q^{(1-\gamma)} - \frac{\nu}{(1-\alpha^2)\left(1 - c\frac{H-H_Q}{-cov(\varepsilon,\varepsilon_F)}\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma} \\
&= \left(1 - c\frac{H-H_Q}{-cov(\varepsilon,\varepsilon_F)}\right)H_Q^{(1-\gamma)} - \frac{\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma} \\
&= \left(-cov(\varepsilon,\varepsilon_F) - c(H-H_Q)\right)H_Q^{(1-\gamma)} + \frac{cov(\varepsilon,\varepsilon_F)\,\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma} \\
&= -cov(\varepsilon,\varepsilon_F)\,H_Q^{(1-\gamma)} - cHH_Q^{(1-\gamma)} + cH_Q^{(2-\gamma)} + \frac{cov(\varepsilon,\varepsilon_F)\,\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}
\end{aligned}$$

To apply the Implicit Function Theorem to $\frac{dF}{dH_Q} \neq 0$:

Derivative: $\frac{dF}{dH_Q}$

$$\begin{aligned}
\frac{dF}{dH_Q} &= H_Q^{-\gamma}\left[-(1-\gamma)\,cov(\varepsilon,\varepsilon_F) - (1-\gamma)\,cH + (2-\gamma)\,cH_Q\right] > 0 \\
0 &< -(1-\gamma)\,cov(\varepsilon,\varepsilon_F) - (1-\gamma)\,cH + (1-\gamma)\,cH_Q + cH_Q \\
0 &< (1-\gamma)\left[-cov(\varepsilon,\varepsilon_F) - cH_\phi + \frac{c}{(1-\gamma)}H_Q\right]
\end{aligned}$$

$$\frac{dF}{dH_Q} = \left[(1-\gamma)\left(-\frac{cov(\varepsilon,\varepsilon_F)}{c} - H_\phi\right) + H_Q\right]cH_Q^{-\gamma}$$

With $cov(\varepsilon,\varepsilon_F) < 0$ and assuming $H_\phi < H_Q$ the derivative $\frac{dF}{dH_Q} > 0$ and the implicit function theorem (requiring $\frac{dF}{dL_Q} \neq 0$) can be applied.

q.e.d.

Other equilibrium values From (61a) and (64) we dictly obtain (65). From Production function (3) and the optimal intermediate goods input (29) ($x(t) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q$) we obtain (66). Using (89) in 6 we obtain the Y. Combining with (66) gives (89). Taking the time derivative of (67) in equilibrium we obtain $\dot{Y}(t) = \dot{N}(t)(1-\alpha^2)H_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}$ and thus $g_Y = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{N}(t)}{N(t)}$, and using (54) we arrive at (68). According to (50) investments are

$I(t) = \dot{N}(t)\nu$ and from (34) we know $r(t) = \frac{\pi_x(t)}{\nu}$. With all profits being saved we obtain $\dot{N}(t)\,\nu = I_x(t) = S(t) = N(t)\,\pi_x(t)$. Plugging in (34) gives (69).

G. Dynamic consistency

Consistent start values of financial and technology stocks:

It can be shown that derived savings can finance the process from the start. According to the discussion in section 3.4 financial wealth income is $rF(t)$. As it is assumed that only labor income consumes, the income of financial asset holders and technology owners only serves for savings and these savings are financing investments for newly introduced goods. Two version of the budget constraint can be derived: one that describes the real investments and innovation which becomes possible ($S = I_x$) and the second that describes the finance mechanism ($S(t) = \dot{F}(t)$)

$$
\begin{aligned}
(i) &\quad : \quad N(t)\,\pi_x(t) + w_{IT}(t)\,L_{IT} = S(t) = I_x(t) = \dot{N}(t)\nu_x \\
(ii) &\quad : \quad rF(t) + w_{IT}(t)\,L_{IT} = S(t) = \dot{F}(t)
\end{aligned}
$$

Equation (45) describes factor rewards for technology owners. $w_{IT}(t) = \frac{1-\phi_L}{1+\alpha}\frac{Y(t)}{L_{IT}}$, with $Y(t) = (1-\alpha^2)\,Q(t) = (1-\alpha^2)\,N(t)\alpha^{\frac{2\alpha}{1-\alpha}}c_x^{-\frac{\alpha}{1-\alpha}}H_Q$ and $(1-\alpha^2) = (\alpha+1)(1-\alpha)$ the implication is that $w_{IT}(t) = N(t)\,(1-\phi_L)\,(1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}c_x^{-\frac{\alpha}{1-\alpha}}H_Q\frac{1}{L_{IT}}$. Defining $Z_{IT} = (1-\phi_L)\,(1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}c_x^{-\frac{\alpha}{1-\alpha}}$ results in $w_{IT}(t) = N(t)\frac{Z_{IT}}{L_{IT}}$, and total income of technology owners is $w_{IT}(t)\,L_{IT} = N(t)\,Z_{IT}$.

Thus from (i) the implication is that:

$$
\begin{aligned}
(i) \quad \dot{N}(t)\,\nu_x &= N(t)\,\pi_x(t) + N(t)\,Z_{IT} \\
\frac{\dot{N}(t)}{N(t)}\nu_x &= \pi_x(t) + zZ_{IT} \\
\frac{\dot{N}(t)}{N(t)} &= r + \frac{1}{\nu_x}Z_{IT}. \quad or \quad r = \frac{\dot{N}(t)}{N(t)} - \frac{Z_{IT}}{\nu_x}
\end{aligned}
$$

From (ii) the implication is that:

$$
\begin{aligned}
(ii) \quad \dot{F}(t) &= rF(t) + N(t)\,Z_{IT} \\
\frac{\dot{F}(t)}{F(t)} &= r + \frac{N(t)}{F(t)}Z_{IT}
\end{aligned}
$$

plugging in from (i) $r = \frac{\dot{N}(t)}{N(t)} - \frac{1}{\nu_x} Z_{IT}$ into (ii) results in:

$$\begin{aligned} \frac{\dot{F}(t)}{F(t)} &= g_N - \frac{1}{\nu_x} Z_{IT} + \frac{N(t)}{F(t)} Z_{IT} \\ g_F &= g_N + \left[\frac{N(0) e^{g_N}}{F(0) e^{g_F}} - \frac{1}{\nu_x} \right] Z_{IT} \end{aligned}$$

and this holds if in steady state growth $g_N = g_F$ and $\frac{N(0)}{F(0)} = \frac{1}{\nu_x}$.

q.e.d.

H. Effects of automation

H1. Factor abundance and comparative advantages of tasks

Due to space limitations, detailed calculations are available from the authors on request.

Effects on human service H: Due to space limitations, detailed calculations are available from the authors on request.

Effects on the income share of labor in the service sector and total economy: $\phi_L, \frac{w_L L_L}{Y(t)}$

Effects on labor share of income in service sector: ϕ_L Departing from (26) with

$$\phi_L = \frac{1}{1 + \left(\Gamma(N_{IT}(A_{IT}), N)^{-1} - 1 \right)^{\frac{1}{\sigma}} \left(\frac{L_{IT}}{L_L} \frac{A_{IT}}{A_L} \right)^{\frac{\sigma - 1}{\sigma}}}$$

$$\frac{d\phi_L}{dA_{IT}} = -\frac{\left[\begin{array}{c}\frac{1}{\sigma}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}-1}(-)\,\Gamma(N_{IT}(A_{IT}),N)^{-2}\frac{d\Gamma}{dN_{IT}}\frac{dN_{IT}}{dA_{IT}} \\ +\frac{\sigma-1}{\sigma}\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}-1}\frac{L_{IT}}{L_L}\frac{1}{A_L}\end{array}\right]}{\left[1+\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\right]^2}$$

$$= \frac{\left[\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}}\frac{1}{\sigma}\left(\frac{\frac{d\Gamma}{dN_{IT}}\frac{dN_{IT}}{dA_{IT}}}{(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1)\Gamma(N_{IT}(A_{IT}),N)^2}-\frac{(\sigma-1)}{A_{IT}}\right)\right]}{\left[1+\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\right]^2}$$

$$\frac{d\phi_L}{\phi_L} = \frac{\left[\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}}\frac{1}{\sigma}\left(\begin{array}{c}\frac{\Gamma\overbrace{\frac{d\Gamma}{dN_{IT}}\frac{N_{IT}}{\Gamma}}^{<0}\frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}}}{(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1)\Gamma(N_{IT}(A_{IT}),N)^2} \\ +(1-\sigma)\end{array}\right)\right]}{1+\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}}\frac{\dot{A}_{IT}}{A_{IT}}$$

$$\frac{d\phi_L}{\phi_L} = \eta_{\phi_L,A_{IT}}\frac{\dot{A}_{IT}}{A_{IT}} < 0$$

$$with \;:\; \eta_{\phi_L,A_{IT}} = \frac{1}{\sigma}\frac{\left(\frac{\Gamma\frac{d\Gamma}{dN_{IT}}\frac{N_{IT}}{\Gamma}\frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}}}{(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1)\Gamma(N_{IT}(A_{IT}),N)^2}+(1-\sigma)\right)}{\left(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1\right)^{-\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{-\frac{\sigma-1}{\sigma}}+1}$$

with $\eta_{\phi_L,A_{IT}}$ for $1 < \sigma$ the share will clearly decline. If $1 > \sigma$ the share will not necessarily increase. Introducing more IT tasks $\Gamma\frac{d\Gamma}{dN_{IT}}\frac{N_{IT}}{\Gamma}\frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}} < 0$ will decrease the share of labor income and overcompensate the potentially positive effect from complementarity, $1 > \sigma$.

Effects on labor share of income in the total economy: $\frac{w_L L_L}{Y}$.

$$\frac{w_L L_L}{Y} = \frac{\phi_L}{(1+\alpha)}, \quad \frac{\frac{dw_L L_L/Y}{dA_{IT}}}{w_L L_L/Y} = \frac{d\phi_L}{\phi_L} = \eta_{\phi_L,A_{IT}}\frac{dA_{IT}}{A_{IT}} < 0$$

Effect on income share of technology providers $\frac{w_{it}l_{it}}{Y}$

$$\begin{aligned}
\frac{w_{IT}(t)\,L_{IT}}{Y(t)} &= \frac{1-\phi_L}{1+\alpha} \\
\frac{d\frac{w_{IT}(t)L_{IT}}{Y(t)}}{\frac{w_{IT}(t)L_{IT}}{Y(t)}} &= \frac{-1}{1+\alpha}\frac{1+\alpha}{1-\phi_L}\frac{\partial\phi_L}{\partial A_{IT}}\frac{dA_{IT}}{A_{IT}}A_{IT} \\
&= \frac{-1}{\frac{1}{\phi_L}-1}\frac{\partial\phi_L}{\partial A_{IT}}\frac{A_{IT}}{\phi_L}\frac{dA_{IT}}{A_{IT}}
\end{aligned}$$

If A_{IT} is repeatedly increasing, the limit is:

$$\lim_{A_{IT}\to\infty}\frac{w_{IT}(t)\,L_{IT}}{Y(t)} = \frac{1-\phi_L}{1+\alpha} = \frac{1}{1+\alpha} - 0.$$

Effects on consumption rate: c

$$\begin{aligned}
c &= \frac{\phi_L}{(1+\alpha)}, \quad dc = \frac{\phi_L}{(1+\alpha)}\frac{\partial\phi_L}{\partial A_{IT}}\frac{dA_{IT}}{A_{IT}}\frac{A_{IT}}{\phi_L} \\
\frac{dc}{c}\frac{A_{IT}}{dA_{IT}} &= \frac{\partial\phi_L}{\partial A_{IT}}\frac{A_{IT}}{\phi_L} = \eta_{c,A_{IT}} = \eta_{\phi_L,A_{IT}}
\end{aligned}$$

Effects of human service in production: H_Q We extend the discussion of the derivative of the implicit function $\tilde{H}_Q$ (section 6) and derive the effect of a change of A_{IT} :

$$\begin{aligned}
\frac{dF}{dA_{IT}} &= -\frac{1}{(1+\alpha)}H_\phi H_Q^{(1-\gamma)}\frac{d\phi_L}{dA_{IT}} - \frac{\phi_L}{(1+\alpha)}H_Q^{(1-\gamma)}\frac{dH}{A_{IT}} \\
&= \left[-\frac{H_\phi}{H}\frac{d\phi_L}{dA_{IT}}\frac{A_{IT}}{\phi_L} - \frac{dH}{A_{IT}}\frac{A_{IT}}{H}\right]\frac{\phi_L}{(1+\alpha)}\frac{H}{A_{IT}}H_Q^{(1-\gamma)} \\
&= -\left[H_\phi\eta_{\phi_L,A_{IT}} + H\eta_{H,A_{IT}}\right]c\frac{1}{A_{IT}}H_Q^{(1-\gamma)}
\end{aligned}$$

$$\frac{dF}{dH_Q} = \left[(1-\gamma)\left(-\frac{cov\left(\varepsilon,\varepsilon_F\right)}{c} - H_\phi\right) + H_Q\right]cH_Q^{-\gamma}$$

$$\frac{dH_Q}{dA_{IT}} = -\frac{\frac{\partial F}{\partial A_{IT}}}{\frac{\partial F}{\partial \tilde{H}_Q}} = -\frac{-\left[H_\phi\eta_{\phi_L,A_{IT}} + H\eta_{H,A_{IT}}\right]\frac{1}{A_{IT}}cH_Q^{(1-\gamma)}}{\left[(1-\gamma)\left(-\frac{cov(\varepsilon,\varepsilon_F)}{c} - H_\phi\right) + H_Q\right]cH_Q^{-\gamma}}$$

using $-cov\left(\varepsilon,\varepsilon_F\right)=\frac{H_\phi}{(1-\theta)}$ see (61a) gives

$$\begin{aligned}\frac{dH_Q}{H_Q} &= \eta_{H_Q,A_{IT}}\frac{dA_{IT}}{A_{IT}}\gtrless 0\\ \eta_{H_Q,A_{IT}} &= \frac{\left[\eta_{\phi_L,A_{IT}}+\frac{H}{H_\phi}\eta_{H,A_{IT}}\right]}{\left[(1-\gamma)\left(\frac{1}{(1-\theta)c}-1\right)+\frac{H_Q}{H_\phi}\right]}\gtrless 0\end{aligned}$$

We want to show that $\left[\eta_{\phi_L,A_{IT}}+\frac{H}{H_\phi}\eta_{H,A_{IT}}\right]$ turns negative, if A_{IT} and σ is sufficiently large: We start with the general identification of $\eta_{H,A_{IT}}$ and $\eta_{\phi_L,A_{IT}}$: According to (72) and (74)

$$\begin{aligned}\eta_{H,A_{IT}} &= \frac{\left[1-\left(\frac{A_L}{A_{IT}}\frac{L_LB_1(N_{IT})}{L_{IT}B_2(N_{IT})}\left(\frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}}\right]\left(\gamma_{IT}(N_{IT})\right)^{\sigma-1}\left(B_1\left(N_{IT}\right)\right)^{-\frac{1}{\sigma}}\eta_{N_{IT},A_{IT}}+(\sigma-1)}{(\sigma-1)\left[1+\left(\frac{A_LL_L}{A_{IT}L_{IT}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}\right]}\\ \text{and } \eta_{\phi_L,A_{IT}} &= \frac{1}{\sigma}\frac{\frac{\Gamma\frac{d\Gamma}{dN_{IT}}\frac{N_{IT}}{\Gamma}\frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}}}{(\Gamma(N_{IT}(A_{IT}),N)^{-1}-1)\Gamma(N_{IT}(A_{IT}),N)^2}+(1-\sigma)}{\left(\Gamma\left(N_{IT}\left(A_{IT}\right),N\right)^{-1}-1\right)^{-\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{-\frac{\sigma-1}{\sigma}}+1}.\end{aligned}$$

As these large terms cannot be easily analyzed we assume large values of A_{IT}, and in order to obtain manageable terms we look at these expression at the limit for $A_{IT}\to\infty$,

$$\begin{aligned}\lim_{A_{IT}\to\infty}\eta_{H,A_{IT}} &= \left(\frac{\left(\gamma_{IT}(N_{IT})\right)^{\sigma-1}\left(B_1\left(N_{IT}\right)\right)^{-\frac{1}{\sigma}}\eta_{N_{IT},A_{IT}}}{(\sigma-1)}+1\right)>0.\\ \lim_{A_{IT}\to\infty}\eta_{\phi_L,A_{IT}} &= \frac{1}{\sigma}\frac{\eta_{\Gamma,A_{IT}}\eta_{N_{IT},A_{IT}}}{\left(1-\Gamma\left(N_{IT}\left(A_{IT}\right),N\right)\right)}+\frac{(1-\sigma)}{\sigma}\end{aligned}$$

Now we need to show that sufficient large values of σ turn $\eta_{\phi_L,A_{IT}}+\frac{H}{H_\phi}\eta_{H,A_{IT}}$ negative.

$$
\begin{aligned}
0 &> \frac{(\gamma_{IT}(N_{IT}))^{\sigma-1}(B_1(N_{IT}))^{-\frac{1}{\sigma}}\eta_{N_{IT},A_{IT}}}{(\sigma-1)}+1+\frac{H}{H_\phi}\frac{1}{\sigma}\frac{\eta_{\Gamma,A_{IT}}\eta_{N_{IT},A_{IT}}}{(1-\Gamma(N_{IT}(A_{IT}),N))}+\frac{H}{H_\phi}\frac{(1-\sigma)}{\sigma} \\
&> (\gamma_{IT}(N_{IT}))^{\sigma-1}(B_1(N_{IT}))^{-\frac{1}{\sigma}}\eta_{N_{IT},A_{IT}}+(\sigma-1) \\
&\quad +\frac{H}{H_\phi}\frac{(\sigma-1)}{\sigma}\frac{\eta_{\Gamma,A_{IT}}\eta_{N_{IT},A_{IT}}}{(1-\Gamma(N_{IT}(A_{IT}),N))}+\frac{H}{H_\phi}\frac{(1-\sigma)(\sigma-1)}{\sigma} \\
0 &> (1-\Gamma(N_{IT}(A_{IT}),N))\,\sigma\,(\gamma_{IT}(N_{IT}))^{\sigma-1}(B_1(N_{IT}))^{-\frac{1}{\sigma}}\eta_{N_{IT},A_{IT}}+(1-\Gamma(N_{IT}(A_{IT}),N))\,\sigma\,(\sigma-1) \\
&\quad +\frac{H}{H_\phi}(\sigma-1)\,\eta_{\Gamma,A_{IT}}\eta_{N_{IT},A_{IT}}+\frac{H}{H_\phi}(1-\Gamma(N_{IT}(A_{IT}),N))(1-\sigma)(\sigma-1)
\end{aligned}
$$

$$
\begin{aligned}
0 &> \sigma(1-\Gamma(N_{IT}(A_{IT}),N))\left[\sigma(\gamma_{IT}(N_{IT}))^{\sigma-1}(B_1(N_{IT}))^{-\frac{1}{\sigma}}\eta_{N_{IT},A_{IT}}+\sigma(\sigma-1)+\frac{H}{H_\phi}(1-\sigma)(\sigma-1)\right] \\
&\quad +\frac{H}{H_\phi}(\sigma-1)\,\eta_{\Gamma,A_{IT}}\eta_{N_{IT},A_{IT}}
\end{aligned}
$$

$$
\begin{aligned}
0 &> \sigma A+\sigma(\sigma-1)+\frac{H}{H_\phi}(1-\sigma)(\sigma-1) \\
0 &> \left[A-1+2\frac{H}{H_\phi}+\sigma\left(1-\frac{H}{H_\phi}\right)-\frac{1}{\sigma}\frac{H}{H_\phi}\right] \\
\lim_{\sigma\to\infty}[...] &= A-1+2\frac{H}{H_\phi}+\infty\left(1-\frac{H}{H_\phi}\right)-0=-\infty
\end{aligned}
$$

q.e.d.

Effects on deployment rate : ω Due to space limitations, detailed calculations are available from the authors on request.

Effects on wages: $w_L(t)$ Due to space limitations, detailed calculations are available from the authors on request.

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