I Z A Institute
of Labor Economics
Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 14515

Foundations of Utilitarianism under Risk and Variable Population

Dean Spears
Stéphane Zuber

JUNE 2021

DISCUSSION PAPER SERIES

IZA DP No. 14515

Foundations of Utilitarianism under Risk and Variable Population

Dean Spears
University of Texas at Austin, Indian Statistical Institute, IZA and IFFS

Stéphane Zuber
CNRS

JUNE 2021

ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Foundations of Utilitarianism under Risk and Variable Population

Utilitarianism is the most prominent family of social welfare functions. We present three new axiomatic characterizations of utilitarian (that is, additively separable) social welfare functions in a setting where there is risk over both population size and the welfares of individuals. First, we show that, given uncontroversial basic axioms, Blackorby et al.'s (1998) Expected Critical-Level Generalized Utilitarianism (ECLGU) is equivalent to a new axiom holding that it is better to allocate higher utility-conditional-on-existence to possible people who have a higher probability of existence. The other two novel characterizations extend classic axiomatizations of utilitarianism from settings with either social risk or variable-population, considered alone. By considering both social risk and variable population together, we clarify the fundamental normative considerations underlying utilitarian policy evaluation.

JEL Classification: D63, D81, J10

Keywords: social risk, population ethics, utilitarianism, expected critical-level generalized utilitarianism, prioritarianism

Corresponding author:
Dean Spears
Department of Economics
The University of Texas at Austin
2225 Speedway
BRB 1.116, C3100
Austin, Texas 78712
USA

E-mail: dspears@utexas.edu

1 Introduction

Utilitarianism is the most prominent family of social welfare functions in welfare economics and economic policy. So additively separable social evaluation — the distinguishing feature of utilitarianism — is at once a core tool of public economics and an enduring focus of debate in welfare economics. In the prior literature, there are two major axiomatic paths to characterizing utilitarianism, each with known criticisms. One, in risky settings, begins with respecting *ex ante* Pareto improvements for individuals (Harsanyi, 1955), but this is criticized by egalitarians and others for ignoring the distribution of utility (*e.g.* Fleurbaey, 2010). Another, in variable-population settings, begins with independence of the existence of unaffected lives (Blackorby and Donaldson, 1984; Blackorby, Bossert and Donaldson, 1995), but this too is criticized in the population literature. We identify a new path to utilitarianism, by studying risky, variable-population social settings, where the number and lifetime utilities of possible people vary across probabilistic states. We clarify the additional properties of utilitarianism in our more general setting.

We present three new characterizations of Expected Critical-Level Generalized Utilitarianism (ECLGU) for evaluation of variable-population risky social prospects. To do this, we introduce novel axioms about the *combination* of risk and variable population. For example, we propose an axiom that the *probability* of a person existing and that person's *utility*-conditional-on-existence are *complements*, so it is better to allocate higher chances of existence to lives that would have more wellbeing. This axiom allows us to characterize generalized utilitarianism without directly assuming either *ex ante* Pareto or independence of the existence of unaffected lives, which are the classic axioms for additive separability when either risk or population, respectively, are considered in isolation (Blackorby, Bossert and Donaldson, 1998, 2005) — although, of course, both of these principles are entailed by ECLGU. Our other axioms allow other novel characterizations.

The ECLGU family is an important set of social welfare functions that includes leading candidates in the population ethics literature. In particular, classical total utilitarianism is a special case of ECLGU. Because ECLGU allows utilities to be transformed before they are aggregated, it also permits sensitivity to the distribution of utility: a concave transformation yields social evaluation that prefers an equal distribution with the same total utility. We emphasize that such transformation may be, but need not be, individuals' own von Neumann Morgenstern transformations for risky choice. Such generality is a benefit of our avoidance of an explicit axiom respecting individuals' *ex ante* Pareto improvements in expected utility.

Because ECLGU offers an approach to giving priority to lower-utility individuals while retaining additive separability, we contrast ECLGU with a leading alternative approach to inequality: the expected equally-distributed equivalent (EEDE), described by Fleurbaey (2010).[1] Because EEDE, as we make it precise, cares about the *pattern* of inequality across persons, whether to add additional people depends, in part, on their consequences for inequality.

In the next section, we present examples where ECLGU and EEDE disagree or notably agree, in order to motivate our novel axioms. Subsequently, we present our three characterizations. The first theorem uses a new axiom

[1] Many of Fleurbaey's results are general enough to include non-egalitarian approaches such as classical utilitarianism and other families; we focus on a particular subset, detailed below, that is separable in same-number, risk-free cases but not otherwise.

that is sufficient to characterize ECLGU in the presence of a set of uncontroversial basic axioms that are used throughout the population economics literature. This axiom requires complementarity between the probability of a person existing and utility-conditional-on-existence. We thus provide an axiomatization of utilitarianism that relies neither on *ex ante* Pareto, nor on independence of the existence of unaffected lives, which is an alternative to the existing literature.

We then explore other axiomatizations of ECLGU in our setting. We show that independence of the existence of unaffected lives and (a variant of) *ex ante* Pareto have to be supplemented with other properties. Our second characterization adds to the basic axioms two axioms that make use of both social risk and variable population, namely Existence independence of the sure and Social risk neutrality in population size. This characterization is important in part because risk-neutrality in the size of perfectly equal populations is an axiom that would be widely accepted among candidate approaches to population ethics, including the EEDE approach. Moreover, independence of risk-free individuals has been noted in the prior literature to be attractive in same-number evaluations of social risk and inequality, even to egalitarians. This is because the presence of risk-free individuals does not change the social evaluator's information and therefore undermines the normative rationale for disregarding individuals' own risk preferences (*cf.* Fleurbaey, 2018).

Our third theorem is most conceptually similar to prior characterizations, particularly the Harsanyi approach. It uses an "Individual dominance" axiom that respects *social* risk preferences over *single-person* populations. This permits us to characterize ECLGU in a way that generalizes and weakens *ex ante* Pareto beyond expected utility and even beyond individuals' von Neumann-Morgenstern transformations. Again, this axiom has to be supplemented with other properties to obtain ECLGU. With Theorem 2 and 3, we thus improve our understanding of the normative underpinning of the ECLGU family.

We view our contribution as new avenues to understand ECLGU. This is significant because this family of social welfare functions is of leading practical importance in public economics and contemporary empirical policy analysis. Despite this importance, theorists recognize that prior characterizations have been subject to important criticisms. Theorem 1 provides a new approach that avoids using these controversial axioms. Together with Theorems 2 and 3, these new characterizations offer a better understanding of leading tools for social evaluation that are additively separable, that allow us to evaluate populations of different size, and that allow either utilitarian aggregation or transformed aggregation that is sensitive to inequality in utility.

2 Motivating examples

Harsanyi (1955) and Blackorby, Bossert and Donaldson (1995) have influentially characterized additively separable utilitarianism. Here, we provide three more characterizations. These are of course equivalent to one another and to others in the literature. Our further characterizations advance the literature because they clarify what is fundamental to expected utilitarianism; because they highlight new, potentially unintuitive consequences of rejecting expected utilitarianism; and because, in different ways and to different degrees, they sidestep familiar debates in social risk or population ethics. The importance of these contributions, therefore, depends in part on our intuitive understanding of them.

The examples in this section provide that motivating intuition for our new axioms. We focus in this section on the motivation for our Theorems 1 and 2. These examples distinguish expected critical-level generalized utilitarianism (ECLGU), as proposed by Blackorby, Bossert and Donaldson (1998), from expected equally-distributed-equivalent (EEDE) criteria, as proposed by Fleurbaey (2010). We define ECLGU as the expectation of a value function $\sum_i \left[g(u_i) - g(c)\right]$ where u_i is the utility of individual i existing in a particular population, g is increasing, and c is a critical level of utility. We define EEDE as the expectation of a value function $g^{-1}\left(\frac{1}{n}\sum_i g(u_i)\right)$, where n is the number of people who exist.[2] Both ECLGU and EEDE are sensitive to unequal distribution if g is concave, but ECLGU is additively separable in all cases. EEDE is additively separable only in same-population, risk-free cases.[3]

In these examples, rows are individual people (or potential people) and columns are risky social states. Numbers are lifetime utilities. Let $*$ denote a person not existing in a state. We use $\succsim_{\mathrm{E}}$ for EEDE. We use $\succsim_{\mathrm{U}}$ for ECLGU. Example 1, previously discussed by Fleurbaey (2010) and Broome (2015), illustrates why EEDE rejects separability across persons in risky cases.

[2] For formal definitions of the criteria, see Definitions 1 and 2.

[3] Fleurbaey's (2010) approach is more general and is compatible with other non-additive social criteria. Fleurbaey also considers non-continuous social orderings. Note also that EEDE, as we define it, is related to but distinct from expected average generalized utilitarianism that uses a value function $\frac{1}{n}\sum_i g(u_i)$; like EEDE, average generalized utilitarianism is insensitive to population size.

Example 1 (EEDE rejects separability) *Assume that social states are equally likely:*

$$\overbrace{\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}}^{u} \succ_E \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}^{v},$$

but $u \sim_U v$.

This is a fixed-population example. Both people have an equal chance of receiving 1 or 0. These utilities summarize everything relevant about lifetime wellbeing, including any experience of inequality. For the top person, the mapping of states to outcomes is the same in v as in u. The bottom person faces the same individual prospect in v as in u. But v will have an unequal outcome and u will have an equal outcome. EEDE favors ex-post egalitarian outcomes and so rejects separability across persons and prefers u.

2.1 Leveling down for probabilistic people

Our principal novel axiom holds that, in allocating probabilities of existence and utilities conditional-on-existence among probabilistically possible people, a higher probability of existence and a higher utility-conditional-on-existence are complements. ECLGU holds that it is better if the people who are more likely to exist are also the people who would have better lives, if they existed. In contrast, because EEDE cares about distributional properties, whether EEDE prefers to increase the probability that a person will exist also depends on the chances for inequality among other possible people. Strikingly, EEDE can therefore prefer to "level down" probabilistic people and reject the complementarity of wellbeing and the chances of existence.

Example 2 (Leveling down for probabilistic people) *Assume that all four states are equally probable. Assume that* $g(u) = \sqrt{u}$.

$$\overbrace{\begin{pmatrix} 2 & \mathbf{2} & * & * \\ * & * & * & 1 \\ * & 1 & * & * \end{pmatrix}}^{w} \prec_E \overbrace{\begin{pmatrix} 2 & * & * & * \\ * & * & \mathbf{1} & 1 \\ * & 1 & * & * \end{pmatrix}}^{z}, \text{ but } w \succ_U z.$$

In Example 2, both options have an expected population size of one. The person in the top row has higher utility-conditional-on-existence than the person in the middle row. Whether the top or middle person has a $\frac{1}{2}$ or $\frac{1}{4}$ chance of existence depends on the choice. The person in the bottom row is unaffected by the choice.

The decision between w and z allocates the bolded utilities-conditional-on-existence of 2 and 1 between the top two people in the middle two equally-likely states. Everything else is the same between w and z. Under ECLGU, $w \succ_U z$, because the higher utility level is then more likely to exist. But under EEDE, $w \prec_E z$ because of the inequality if the top and bottom people existed together at 2 and 1. Choosing z would be a form of leveling down to avoid inequality. In section 4, we characterize ECLGU with the principle behind Example 2.

2.2 Existence independence of the sure and social risk neutrality in population size

One well-studied path towards ECLGU in risk-free variable population settings is an existence independence property (Blackorby, Bossert and Donaldson, 2005). Such a property holds that social evaluations do not depend on the existence of unaffected people, such as the long dead or those in the distant future. EEDE does not typically satisfy such a property. This is because, in a variable-population setting, adding an extra person has two consequences to an EEDE evaluation: the person's own utility matters and the extra person may change distributional properties, such as inequality. To ECLGU, such distributional effects are irrelevant. But to EEDE, adding a sure person can reverse a choice. Example 3 shows this by expanding Example 1:

Example 3 (EEDE rejects existence independence of the sure) *Let social states have equal probability and let* $g(u) = -e^{-1.065u}$.

$$\overbrace{\begin{pmatrix} 1 & 0 & * \\ 1 & 0 & * \\ * & * & 0.5 \end{pmatrix}}^{x} \succ_E \overbrace{\begin{pmatrix} 1 & 0 & * \\ 0 & 1 & * \\ * & * & 0.75 \end{pmatrix}}^{y}.$$

$$\overbrace{\begin{pmatrix} 1 & 0 & * \\ 1 & 0 & * \\ * & * & 0.5 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}}^{x'} \prec_E \overbrace{\begin{pmatrix} 1 & 0 & * \\ 0 & 1 & * \\ * & * & 0.75 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}}^{y'}.$$

Notice that, under EEDE, x has the advantage of *ex post* equality, but y has the advantage of 0.75 being a better lifetime utility for one possible person than 0.5. Under some quantitative parameterizations, x is preferred. But adding the sure person in x' changes this advantage, as ex-post inequality appears, so y' is preferred. Such a preference reversal violates existence independence. To ECLGU, $y \succ_U x$ and $y' \succ_U x'$ because only the fact that $0.75 > 0.5$ matters.

In section 5, we present a characterization of ECLGU that builds upon an axiom that rules out choosing the way EEDE does in Example 3. However, we also show that Existence independence is not sufficient for ECLGU. This is because criteria that maximize the expectation of the multiplicative value function $\prod_i g(u_i)$ also satisfy Existence independence.

In section 5, we therefore use an additional property: Social risk neutrality in population size for perfectly equal populations. This principle holds that—in the hypothetical case of no risk or inequality in utility-conditional-on-existence, because utility is always one fixed level for everyone—social evaluation is risk neutral in the number of people who live. Example 4 informally expresses this property.

Example 4 (Social risk neutrality in population size) *Having 10 billion lives, each at a utility of u, is just as good as a fifty-fifty chance of 5 or 15 billion lives, each at that same utility level u.*

Such risk neutrality will be an implication of any approach that maximizes the expectation of an additive social welfare function. Importantly, however, social risk neutrality in population size is *also* a property of EEDE, of average utilitarianism, and broadly of other non-separable approaches in the population literature. What singles out ECLGU is the combination of the two properties, our Theorem 2 shows: along with our basic axioms, existence independence of the sure and social risk neutrality in population size characterize ECLGU.

ECLGU and EEDE are also distinguished by the following stronger case of social risk in population size.

Example 5 (Strong social risk neutrality in population size) *Assume that social states are equally likely:*

$$\overbrace{\begin{pmatrix} 1 & * \\ 1 & * \end{pmatrix}}^{u} \succ_E \overbrace{\begin{pmatrix} 1 & * \\ * & 1 \end{pmatrix}}^{v},$$

but $u \sim_U v$*.*

Notice that Example 5 is formally similar to Example 1; the only difference is that lifetime utilities of 0 have been replaced with nonexistence. The implication of EEDE in that case is striking because there is no *ex post* inequality among the people who exist in any outcome. Average utilitarianism would agree with EEDE in this case. The example reveals a tension for EEDE: on the one hand, EEDE does not directly value population expansions in risk-free cases; but on the other hand, it is risk averse over population size in the particular case of non-existence. Our Social risk neutrality in population size axiom is weak enough to be compatible with EEDE's choice in Example 5 because it makes no claim about populations of size zero; it only applies to positive-size populations. However, this illustrates a difference between ECLGU and other approaches that emerges in the context of all of the axioms.

To a reader who agrees with EEDE's egalitarian evaluations in Examples 2, 3 or 5, our paper provides new arguments against generalized utilitarianism. Alternatively, to a reader who agrees with generalized utilitarianism's evaluations in these cases, our paper shows that, in the context of some widely accepted basic axioms, these principles are sufficient to characterize ECLGU. Either way, these examples clarify the foundations of utilitarianism for economic policy analysis in the real-world case of both social risk and variable population.

3 Framework and background

3.1 The framework

The set of positive integers is denoted by $\mathbb{N}$. The set of all real numbers is denoted by $\mathbb{R}$. The set of non-negative (resp. positive) real numbers is denoted by $\mathbb{R}_+$ ($\mathbb{R}_{++}$).

The set of possible individuals is $\mathbb{N}$. $\mathcal{N}$ is the set of all possible finite subsets of $\mathbb{N}$, which are possible populations with typical element N. We consider a welfarist framework where the only information necessary for social decisions is the utility levels of people alive in a certain state of affairs provided some people exist.[4] If no people exist (that is, $N = \emptyset$), we let $\star$ denote the welfare information. If some people exist, the welfare information is given by $u = (u_i)_{i \in N} \in \mathbb{R}^N$ where $N \in \mathcal{N}$ is the non-empty population, and u_i is for each existing individual i the lifetime utility experienced by i. Although it is

[4]Blackorby, Bossert and Donaldson (1998) explicitly state Welfarism as an axiom. Here, to simplify the analysis, we only implicitly make the assumption in order to avoid introducing states of affairs, lotteries over states of affairs, and the corresponding welfarist information. But we could as well use the framework of Blackorby, Bossert and Donaldson (1998). Welfarism itself is often derived from more basic principles, typically Pareto indifference and Anonymity. See Blackorby, Bossert and Donaldson (1999, 2005) for an example in a variable-population framework assuming a multi-profile setting; and Blackorby, Bossert and Donaldson (2006) in a fixed-population framework assuming a single profile of utility.

common in the population ethics literature to interpret the sign of lifetime utility, we need not do so because we make no assumptions about it; ECLGU can use any critical level.

The set of all possible alternatives is $U = \left(\cup_{N\in\mathcal{N}}\mathbb{R}^N\right) \cup \{\star\}$. For each $u \in U$, we denote $N(u)$ the set of individuals alive in u, and $n(u)$ the number of individuals alive in u. For an individual $i \in \mathbb{N}$, we let U_i be the set of alternatives where i exists. Formally, $U_i = \{u \in U | i \in N(u)\}$. Also, for two alternatives u and $v \in U$ such that $N(u) \cap N(v) = \emptyset$, we denote uv the alternative w such that $N(w) = N(u) \cup N(v)$, $w_i = u_i$ for all $i \in N(u)$ and $w_j = u_j$ for all $j \in N(v)$. Similarly, we use uvw when we "merge" three alternatives such that $N(u) \cap N(v) = N(u) \cap N(w) = N(v) \cap N(w) = \emptyset$.

We consider risk over social states of affairs. We do so by introducing lotteries over U. A lottery p over U is a mapping $p : U \to [0, 1]$ such that there exists a finite set $V \subset U$ such that $\sum_{v\in V} p(v) = 1$, $p(v) > 0$ for all $v \in V$ and $p(u) = 0$ for all $u \in (U \setminus V)$. We denote P the set of all such lotteries. For simplicity, we abuse notation and denote u the degenerate lottery such that alternative $u \in U$ occurs with probability 1 (so that all other alternatives have probability 0).[5] For any $p \in P$ and any individual i, we denote $\pi_i(p) = \sum_{u\in U_i} p(u)$ the total probability that individual i exists with lottery p.

For any population $N \in \mathcal{N}$, we denote P_N the set of lotteries concerning only population N: that is, for each $u \in U$ such that $p(u) > 0$, it must be the case that $u \in U_N$. Such same-population lotteries have attracted attention in the discussion about whether we should use an additively separable von

[5]Hence, we implicitly assume the property of Social certainty consistency by Blackorby, Bossert and Donaldson (1998).

Neumann-Morgenstern function in a risky context. For any $i \in N$ and any $p \in P_N$, we denote p_i the lottery on $\mathbb{R}$ (or similarly the lottery in $P_{\{i\}}$) defined as follows: for any $x \in \mathbb{R}$, $p_i(x) = \sum_{u|N(u)=N, u_i=x} p(u)$.

Note that our framework combines risk in two ways: on who exists and on the utility of the existing persons. Concretely, what we have in mind are situations where people may be born or not (because fertility patterns change for instance) and then face risks *during their lifetimes* that affect their overall lifetime utilities. Hence, we do not require an interpretation of risky personal identity beyond ordinary usage: a person may or may not be born, and if so may face risk.[6] Note also that we do not use any explicit time structure in our setting. Our risky variable population framework can be applied naturally to intertemporal issues (for instance, our current actions may change who exist in the future). In that case—which is our preferred interpretation—we assume that a population comprises everyone who ever exists in the world, like in Blackorby, Bossert and Donaldson (2005). In this interpretation, time would not play any significant role in welfare evaluation: one of our basic axioms is Anonymity, ruling out discounting or any other discrimination on the basis of time. But our analysis could also be applied to single generations.[7]

We investigate a social welfare relation $\succsim$ over P (below, when we assume Social expected-utility, we will assume that $\succsim$ is a complete and transitive ordering). We denote $\succ$ the asymmetric part and $\sim$ the symmetric part of $\succsim$.

[6] Individuals here are assumed to be people with given name at specific locations (in space and time) and well-defined values and preferences. We do not use a more elusive notion of identities or souls that could be incarnate in different places and times and have different values, preferences and evaluations of their own lives depending on when and where they are born.

[7] Under such an interpretation, we would leave open the question of how to aggregate welfare across generations, which could involve time discounting.

3.2 Basic principles

We assume the principles in this section throughout our analysis. Our first three axioms apply only to risk-free distributions with fixed population sizes, meaning only to lotteries where a specific population size and allocation occurs in all possible outcomes. Our fourth axiom introduces risk, assuming that risk is treated by maximizing the expectation of a social objective.[8] Our fifth assumption introduces variable population with a minimal assumption that there is a case in which two populations of different sizes can be compared.

Strong Pareto. For all $N \in (\mathcal{N} \setminus \{\emptyset\})$, for all u, $v \in U$ such that $N(u) = N(v) = N$, if $u_i \geq v_i$ for all $i \in N$, then $u \succsim v$; if furthermore $u_j \succ v_j$ for some $j \in N$ then $u \succ v$.

Anonymity. For all u, $v \in U$ such that $n(u) = n(v) > 0$, if there exists a bijection $\varpi : N(u) \to N(v)$ such that $u_i = v_{\varpi(i)}$ for all $i \in N(u)$, then $u \sim v$.

Continuity. For all $N \in (\mathcal{N} \setminus \{\emptyset\})$, the restriction of $\succsim$ to $\{u \in U | N(u) = N\}$ is continuous in lifetime utilities.

Social expected-utility. There exists a function $V : U \to \mathbb{R}$ such that, for all p, $p' \in P$:

$$p \succsim p' \iff \sum_{u \in U} p(u) \times V(u) \geq \sum_{u \in U} p'(u) \times V(u).$$

[8] In contrast with our approach, which preserves Social expected-utility but weakens *ex ante* Pareto in individuals' expected utility, a number of other important contributions to the recent literature have furthered Harsanyi's theorem by weakening Social expected-utility (Fleurbaey, 2009; Mongin and Pivato, 2015; Zuber, 2016; McCarthy, Mikkola and Thomas, 2020). We will discuss the Social expected utility assumption in the conclusion.

Strong Pareto, Anonymity, Continuity, and Social expected-utility all could apply in the same way to a fixed-population setting. The next axiom is our first variable-population axiom. It assumes a minimal degree of comparability across population sizes and rules out an implausible social evaluation that would always regard an additional person as an improvement or as a worsening, irrespective of the utility level.

Minimal existence of a critical level. There exist $u \in U$ and $c \in \mathbb{R}$ such that there exist $i \notin N(u)$ and $v \in U$ defined by $N(v) = N(u) \cup \{i\}$, $v_i = c$, and $v_j = u_j$ for all $j \in N(u)$ for which $u \sim v$.

In the context of Pareto and continuity in a complete and transitive ordering, Minimal existence of a critical level would follow immediately from the apparently weaker assumption that there are $\bar{c}, \underline{c} \in \mathbb{R}$ such that if $\bar{v}_i = \bar{c}$, $\underline{v}_i = \underline{c}$, and definitions are otherwise as in the axiom, then $\bar{v} \not\prec u$ and $\underline{v} \not\succ u$. This weakening is an application of Broome (2005).

3.3 Variable-population social welfare functions

For any $N \in \mathcal{N} \setminus \{\emptyset\}$, a function $\Xi : \mathbb{R}^N \to \mathbb{R}$ is said to be normalized if $\Xi(u) = a$ whenever $u_i = a$ for all $i \in N$.

Proposition 1 *If the social welfare ordering $\succsim$ on U satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, and Minimal existence of a critical level, then there exist a function $W : (\mathbb{N} \cup \{0\}) \times \mathbb{R} \to \mathbb{R}$ increasing and continuous in its second argument, a function $\Xi_0 : \{\star\} \to \{0\}$, and for each $n \in \mathbb{N}$ a continuous, increasing, symmetric and normalized function Ξ_n*

such that, for all p, $p' \in P$:

$$p \succsim p' \Longleftrightarrow \sum_{u \in U} p(u) \times W\Big(n(u), \Xi_{n(u)}(u)\Big) \geq \sum_{u \in U} p'(u) \times W\Big(n(u), \Xi_{n(u)}(u)\Big).$$

Proof. Given that $\succsim$ satisfies Strong Pareto, Anonymity and Continuity, we can use Theorem 1 in Blackorby, Bossert and Donaldson (1998) to prove that there exist a function $\bar{W} : (\mathbb{N} \cup \{0\}) \times \mathbb{R}$ increasing and continuous in its second argument, a function $\Xi_0 : \{\star\} \to \{0\}$, and for each $n \in \mathbb{N}$ a continuous, increasing, symmetric and normalized function Ξ_n such that, for all u, $v \in U$:[9]

$$u \succsim v \Longleftrightarrow \bar{W}\Big(n(u), \Xi_{n(u)}(u)\Big) \geq \bar{W}\Big(n(v), \Xi_{n(v)}(v)\Big).$$

But the principle of Social expected-utility implies that there exists a continuous function $V : U \to \mathbb{R}$ such that, for all u, $v \in U$:

$$u \succsim v \Longleftrightarrow V(u) \geq V(v).$$

Hence there must exist a continuous and increasing function $\Phi : \mathbb{R} \to \mathbb{R}$ such that $V(u) = \Phi \circ \bar{W}\big(n(u), \Xi_{n(u)}(u)\big)$ for all $u \in U$. Denoting $W := \Phi \circ \bar{W}$ and using Social expected-utility, we obtain the result. ■

An example of variable social welfare orderings in the class described in Proposition 1 is EEDE by Fleurbaey (2010).

[9] Note that the proof of Theorem 1 in Blackorby, Bossert and Donaldson (1998) is a previous result by Blackorby and Donaldson (1984) that assumes that $\succsim$ restricted to sure alternatives is representable by a function $V : U \to \mathbb{R}$. Because we assume Social expected-utility, we have this representability assumption built into our axiomatization.

Definition 1 *A social welfare ordering $\succsim$ is an EEDE social welfare ordering if there exists a continuous and increasing function $g : \mathbb{R} \to \mathbb{R}$ such that for all p, $p' \in P$:*

$$p \succsim p' \iff \sum_{u \in U} p(u) \times g^{-1} \left(\tfrac{1}{n(u)} \sum_{i \in N(u)} g(u_i) \right) \geq \sum_{u \in U} p'(u) \times g^{-1} \left(\tfrac{1}{n(u)} \sum_{i \in N(u)} g(u_i) \right).$$

An alternative family has been axiomatized by Blackorby, Bossert and Donaldson (1998), namely ECLGU:

Definition 2 *A social welfare ordering $\succsim$ is an ECLGU social welfare ordering if there exist $c \in \mathbb{R}$ and a continuous and increasing function $g : \mathbb{R} \to \mathbb{R}$ such that $g(0) = 0$ and for all p, $p' \in P$:*

$$p \succsim p' \iff \sum_{u \in U} p(u) \times \left(\sum_{i \in N(u)} \left[g(u_i) - g(c) \right] \right) \geq \sum_{u \in U} p'(u) \times \left(\sum_{i \in N(u)} \left[g(u_i) - g(c) \right] \right).$$

If g is the identity function, then ECLGU is simply critical-level utilitarianism. The case where g is a concave transformation is known as prioritarianism, because the social evaluation gives more priority to those who are worse off.[10]

4 No leveling down for probabilistic lives

Our first representation theorem builds upon the principle in Example 2. We use a novel axiom that combines features of the *ex ante* Pareto axiom that drives the Harsanyi theorem with the existence independence that drives the

[10] For a discussion of various prioritarian criteria, see Parfit (1995); Adler and Treich (2017).

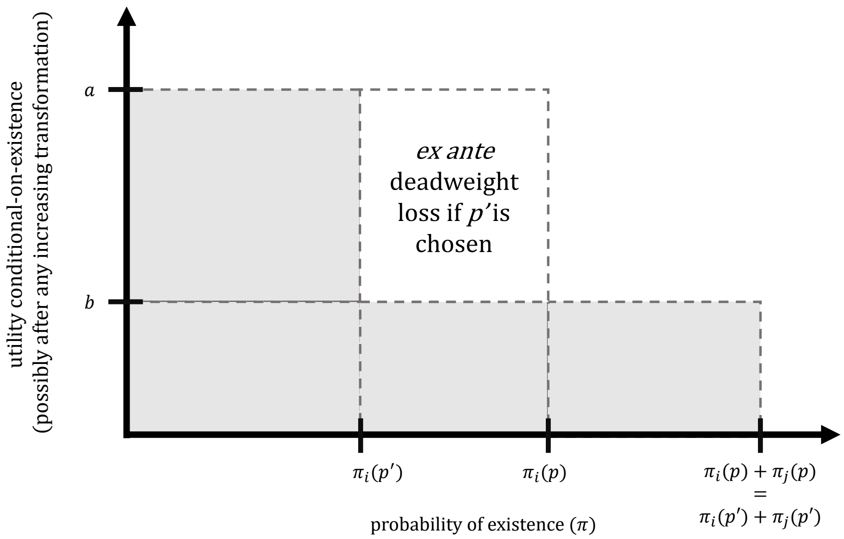

Figure 1: No leveling down for probabilistic lives, illustrated

Blackorby and Donaldson (1984) theorems, without fully assuming either. In a same-number (or, more precisely, a same probabilities of existence) choice with no utility risk conditional on existence, this axiom holds that the *ex ante* probability of existence of an individual is a complement to lifetime utility. In other words, it is better if higher utility-conditional-on-existence is allocated to probabilistic people who are more likely to exist and lower utility-conditional-on-existence is allocated to probabilistic people who are less likely to exist, in any all-else-equal binary choice.[11]

No leveling down for probabilistic lives. For all p, $p' \in P$, if there exist two individuals i, $j \in \mathbb{N}$ and two real numbers $a > b$ such that:

[11] In our context of Anonymity and Social expected-utility, it is equivalent to assume a version where two utility levels are allocated to fixed probabilities or a version where two probabilities are allocated to fixed utility levels.

(i) For all $k \in \mathbb{N} \setminus \{i, j\}$, $p_k = p'_k$ and there exists $c_k \in \mathbb{R}$ such that $u_k = c_k$ for all $u \in U_k$ such that $p(u) > 0$ or $p'(u) > 0$;

(ii) $u_i = a$ for all $u \in U_i$ such that $p(u) > 0$ or $p'(u) > 0$;

(iii) $u_j = b$ for all $u \in U_j$ such that $p(u) > 0$ or $p'(u) > 0$;

(iv) $\pi_i(p) + \pi_j(p) = \pi_i(p') + \pi_j(p')$;

then $p \succ p'$ if and only if $\pi_i(p) > \pi_i(p')$.

Figure 1 illustrates the principle, focusing on the two people i and j. Conditional on existence, i has higher utility a than j's utility b. The empty rectangle represents a deadweight loss of *ex ante* wellbeing if p' is chosen over p. The axiom is named in reference to the well-studied "leveling down" objection to those versions of egalitarianism that sometimes prefer making individuals worse off in order to reduce inequality (Parfit, 1995). The axiom has in common with Harsanyi's approach that it pays *some* restricted amount of attention to *ex ante* properties and has in common with Blackorby and Donaldson's approach that it makes *some* variable-population evaluations without regard to the full distribution of welfare.

This principle is related to the Probability-adjusted Suppes-Sen principle of Asheim and Zuber (2016), who study social evaluations in a space in which utility-conditional-on-existence is always certain. They construct rank-ordered allocations that are probability-weighted cumulative functions of utility based on individuals' probabilities of existence and (sure) utility levels. Probability-adjusted Suppes-Sen implies that a higher rank-ordered allocation is always better: this happens when some probability has been moved towards higher utility levels.

In the context of the basic axioms, No leveling down for probabilistic lives is sufficient to characterize ECLGU:

Theorem 1 *A social welfare ordering* $\succsim$ *on* U *satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level and No leveling down for probabilistic lives if and only if it is an ECLGU social ordering.*

Proof. In the Appendix. ■

5 Existence independence and risk neutrality

5.1 Existence independence

In Example 3, a social ordering was reversed by addition of a person who was certain to exist at a specific, constant utility in any possible state, instead of being certain not to exist. Existence independence of the sure rules out such social evaluations:

Existence independence of the sure. For all p, p', q, $q' \in P$, if there exists an alternative $u \in U$ such that:

- for all $v \in U$ such that $p(v) > 0$, there exists $w \in U$ such that $N(w) \cap N(u) = \emptyset$, $v = uw$ and $p'(w) = p(v)$;
- for all $v \in U$ such that $q(v) > 0$, there exists $w \in U$ such that $N(w) \cap N(u) = \emptyset$, $v = uw$ and $q'(w) = q(v)$;

then $p \succsim q$ if and only if $p' \succsim q'$.

In risky social choice, egalitarians sometimes justify violating *ex ante* Pareto with the observation that the social evaluator may have more relevant information than individuals, particularly where the possible states are anonymous permutations of one another (Fleurbaey, 2010). People whose outcomes are identical in all possible states would seem to represent no missing information — an argument in favor of Existence independence of the sure. And yet, as in Example 3, their existence can transform otherwise equal outcomes into possible instances of inequality.

In population ethics, Blackorby, Bossert and Donaldson (1998, 2005) have argued that independence of the sure is highly plausible because, at a minimum, lifetime utilities are sure for people who are dead. Their "Independence of the utilities of the dead" axiom is equivalent to Independence of the sure in the presence of Anonymity. They introduced existence independence to the population ethics literature as an attractive axiom when initially studying cases with no risk at all — that is to say, comparing sure alternatives. Any intuitive attractiveness of existence independence offers one argument against non-separable approaches such as EEDE: EEDE satisfies independence of the sure in fixed-population, risk-free cases but not in variable-population, risk-free cases. ECLGU satisfies Existence independence of the sure, and therefore also satisfies a weaker property, Existence independence for sure alternatives:

Existence independence for sure alternatives. For any u, v, w in U, if $N(u) \cap N(w) = N(v) \cap N(w) = \emptyset$, then $uw \succsim vw$ if and only if $u \succsim v$.

Blackorby, Bossert and Donaldson (1998) have characterized social welfare orderings satisfying this principle together with our other basic axioms.

Proposition 2 (Blackorby, Bossert, Donaldson, 1998) *If $\succsim$ is a social welfare ordering that satisfies Strong Pareto, Anonymity, Continuity, Existence independence for sure alternatives, Minimal existence of a critical level and Social expected-utility then there exist two continuous and increasing functions $\varphi : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that $\varphi(0) = g(0) = 0$, and for all p, $p' \in P$:*

$$p \succsim p' \iff \sum_{u \in U} p(u) \times \varphi\left(\sum_{i \in N(u)} \left[g(u_i) - g(c)\right]\right) \geq \sum_{u \in U} p'(u) \times \varphi\left(\sum_{i \in N(u)} \left[g(u_i) - g(c)\right]\right).$$

Proof. This is Theorem 3 of Blackorby, Bossert and Donaldson (1998). Recall that our social welfare ordering is welfarist in the sense of Blackorby, Bossert and Donaldson (1998). We note that Minimal existence of a critical level is assumed by Blackorby, Bossert and Donaldson (1998) but not stated as an axiom. ■

Proposition 2 shows that Existence independence for sure alternatives is not enough to characterize ECLGU social orderings in the risky context. Indeed, even the stronger Existence independence of the sure is not enough. This is because multiplicative criteria that take the expectation of $\prod_i g(u_i)$ also satisfy the property (and, all our other properties provided, there exists c such that $g(c) = 1$).

5.2 Risk neutrality in population size

Given the result in Proposition 2 we need an additional property to characterize ECLGU. Our next axiom, building upon Example 4, holds that social evaluation is neither risk-loving nor risk-averse in population size, for cases of equal utility-conditional-on-existence:

Social risk neutrality in population size for perfect equality. For all p, $p' \in P$, for any alternative $w \in U$ such that $n(w) > 0$, if there exist two alternatives u and v in U such that $N(u) \cap N(v) = N(u) \cap N(w) = N(v) \cap N(w) = \emptyset$, $u_i = v_j = w_k = e$ for all $i \in N(u)$, $j \in N(v)$ and $k \in N(w)$, $p(uvw) = 1/2$, $p(w) = 1/2$, $p'(uw) = 1/2$ and $p'(vw) = 1/2$, then $p \sim p'$.

Social risk neutrality in population size for perfect equality imposes that, for cases of equality and given a pre-existing population w at the equal level, society is risk-neutral with respect to the risk on future population size. Bommier and Zuber (2008) discuss a version of this axiom.[12]

Social risk neutrality in population size for perfect equality is consistent with a very large variety of social criteria in population ethics. In particular, it is satisfied by every social criterion that maximizes an expectation of a function of the form $V(u) = \Xi_n(u)$ or $V(u) = n\big(\Xi_n(u) - c\big)$, with Ξ a normalized function like in Proposition 1. This means that it is accepted by average and total versions of utilitarianism, EEDE, each with or without positive critical levels, as well as by maximin. It is also accepted by linear combinations of these criteria, for instance by social criteria using a function of the form $V(u) = \alpha\Xi_n(u) + (1-\alpha)n\big(\Xi_n(u) - c\big)$. Social risk neutrality in population size is however rejected by "variable value" approaches to population ethics, which

[12]More precisely, they explore a principle that denies it, which they call Preference for Catastrophe Avoidance. We note that they use Independence of the utilities of the dead, which is a version of Independence for the sure, but do not explicitly cite Independence for sure acts. Theirs is the first use of this axiom of which we are aware, although they cite a related principle by Keeney (1980) about risk aversion over quantity of deaths. Although Bommier and Zuber are motivated by the risk of human extinction, we note that this possibility would be represented by a positive population size in our framework (not zero), because some human lives have already been lived.

include Rank-Dependent Generalized Utilitarianism (Asheim and Zuber, 2014) and Number-Dampened Generalized Utilitarianism (Ng, 1989), but both of these are already excluded by Existence independence for sure alternatives. Therefore, we interpret Social risk neutrality in population size for perfect equality to be a weak axiom in the context of our other assumptions for Theorem 1, as it allows most approaches to population ethics that are named in the literature.

The next proposition precisely describes the scope of all social orderings that satisfies Social risk neutrality in population size for perfect equality and our other basic axioms.

Proposition 3 *If a social welfare ordering $\succsim$ on U satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level, and Social risk neutrality in population size for perfect equality, then there exist two continuous functions $\Psi : \mathbb{R} \to \mathbb{R}$ and $\Phi : \mathbb{R} \to \mathbb{R}$, and for each $n \in \mathbb{N}$ a continuous, increasing, symmetric and normalized function Ξ_n, such that, for all p, $p' \in P$:*

$$p \succsim p' \iff \sum_{u \in U \setminus \{\star\}} p(u) \times \Big[\Phi\left(\Xi_{n(u)}(u)\right) + n(u) \times \Psi\left(\Xi_{n(u)}(u)\right)\Big] \geq \sum_{u \in U \setminus \{\star\}} p'(u) \times \Big[\Phi\left(\Xi_{n(u)}(u)\right) + n(u) \times \Psi\left(\Xi_{n(u)}(u)\right)\Big].$$

Proof. In the Appendix. ■

Combining Propositions 2 and 3 we obtain our second characterization of ECLGU.

Theorem 2 *A social welfare ordering* $\succsim$ *on* U *satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level, Existence independence of the sure, and Social risk neutrality in population size for perfect equality if and only if it is an ECLGU social welfare ordering.*

Proof. In the Appendix. ■

Because EEDE of the form $V(u) = g^{-1}\left(\frac{1}{n(u)}\sum_i g(u_i)\right)$ satisfies Risk neutrality in population size and satisfies Existence independence for sure alternatives in same-number cases, Theorem 2 implies that the axiomatic gap between ECLGU and this form of EEDE is that ECLGU satisfies Existence independence for sure alternatives in same- or different-number comparisons but EEDE does so only in same-number comparisons. This is despite the fact that the rejection by EEDE of *ex ante* Pareto cannot be justified, in the case of fully risk-free alternatives, by any difference in information between individuals and the social evaluator because the outcome is certain.

6 *Ex ante* Pareto: a generalization

As noted in the introduction, a leading path to obtaining (critical-level) utilitarianism is to use the *Ex ante* Pareto principle. Expected utilitarianism satisfies *Ex ante* Pareto:

***Ex ante* Pareto.** For all $N \in \mathcal{N}$, for all $p, q \in P_N$, if $\sum_{u_i \in \mathbb{R}} p_i(u_i) \times u_i \geq \sum_{u_i \in \mathbb{R}} q_i(u_i) \times u_i$ for all $i \in N$, then $p \succsim q$. If the inequality is strict for at least one $i \in N$ then $p \succ q$.

Clearly, EEDE with concave g rejects *Ex ante* Pareto (to see this, add a small amount of utility to each outcome in v in Example 1). But ECLGU also rejects Pareto when g is concave because g plays a role of increasing risk aversion with respect to lifetime utility. We can however provide the following generalization of *Ex ante* Pareto:

Individual dominance. For all $N \in \mathcal{N}$, for all $p, q \in P_N$, if $p_i \succ q_i$ for all $i \in N$, then $p \succ q$.

Individual dominance only uses social preferences and does not make any reference to individual attitudes under risk and uncertainty.[13] In particular, it allows lifetime utilities not to be measured by individuals' von Neumann Morgenstern (VNM) utility functions (whereas *Ex ante* Pareto makes sense only if individual preferences satisfy the VNM axioms and lifetime utilities are measured by VNM utilities).[14] Individual dominance together with our basic axioms yields the Expected Number-Weighted Generalized Utilitarian (ENWGU) family:

[13]Related principles in the recent literature are the Anteriority and Reduction to Prospects axioms of McCarthy, Mikkola and Thomas (2020). These axioms hold that risky social distributions can be ranked like individual prospects in specific cases (either when individuals face the same prospects in the two social distributions, or when social distributions are egalitarian and individuals have the same preferences). The Anteriority and Reduction to Prospects axioms are key to their proof that the social preorder is generated by the individual preorder. The main difference in terms of interpretation is that we do not have individual preorders but only a social ordering of individual prospects: that is social distributions where only one individual exists. We do not say anything about individual attitudes towards risk.

[14]See related arguments by McCarthy (2017). In McCarthy's terminology, we permit, but do not require, claim (X): "It is a substantive ethical question what the relation is between the individual preorder and the one-person social preorder" (p. 247). If one agrees with McCarthy that—"(Y) It is a conceptual truth that the individual preorder and the one-person social preorder coincide"—then this flexibility is of no theoretical value.

Definition 3 *A social welfare ordering* $\succsim$ *is an ENWGU social ordering if there exist a continuous and increasing function* $g : \mathbb{R} \to \mathbb{R}$*, and two functions* $F : \mathbb{N} \to \mathbb{R}$ *and* $G : \mathbb{N} \to \mathbb{R}$ *such that, for all* p*,* $p' \in P$*:*

$$p \succsim p' \iff \sum_{u \in U} p(u) \times \left(F\big(n(u)\big) \sum_{i \in N(u)} g(u_i) + G\big(n(u)\big) \right) \geq \sum_{u \in U} p'(u) \times \left(F\big(n(u)\big) \sum_{i \in N(u)} g(u_i) + G\big(n(u)\big) \right).$$

Proposition 4 *If a social welfare ordering* $\succsim$ *on* U *satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, and Individual dominance, then it is an ENWGU social ordering.*

Proof. In the Appendix. ■

To obtain our third characterization of ECLGU, we need to add another axiom. In this case, Social risk neutrality in population size for perfect equality is not a sufficient axiom. Indeed, social criteria that maximize the expectation of a function of the form

$$V(u) = \alpha \tfrac{1}{n(u)} \sum_{i \in N(u)} g(u_i) + (1 - \alpha) \sum_{i \in N(u)} \big[g(u_i) - g(c)\big]$$

are ENWGU social orderings and they satisfy Social risk neutrality.[15] These would be convex combinations of average and critical-level generalized utilitarianism and would inherit the disadvantages of each that are documented

[15] Indeed, we can write $V(u) = \alpha g(\Xi_{n(u)}(u)) + n(u)(1 - \alpha)\big(g(\Xi_{n(u)}(u)) - g(c)\big)$, where $\Xi_{n(u)}(u) = g^{-1}\left(\frac{1}{n(u)} \sum_{i \in N(u)} g(u_i)\right)$. Hence, it is compatible with the form described in Proposition 3.

in the population literature.

To complete the third characterization, our next axiom posits that whether adding risk-free lives is good should be independent of the prospects faced by a fixed set of unconcerned individuals.

Independent value of additional lives from the prospects of the unconcerned. For all p, q, p', $q' \in P$, if there exist $i \in \mathbb{N}$ and $x \in \mathbb{R}$ such that $w \in U$ is defined by $N(w) = \{i\}$ and $w_i = x$ and:

- for all $u \in U$, $p(u) > 0$ if and only if $p'(u) > 0$, and $i \notin N(u)$ whenever $p(u) > 0$;
- for all $u \in U$ such that $p(u) > 0$, $q(uw) = p(u)$ and $q'(uw) = p'(u)$;

then $q \succ p$ if and only if $q' \succ p'$.

This axiom is satisfied by social welfare orderings other than ECLGU, such as a version of maximin that only attends to whether a probability is zero or positive: $\min_{\{u:p(u)>0\}}\{\min_{i\in N(u)} u_i\}$. Independent value of additional lives from the prospects of the unconcerned will be sufficient to obtain ECLGU social orderings. As the proof of Theorem 3 shows, however, Independent value of additional lives from the prospects of the unconcerned can be replaced with Constant critical level for risk-free distributions, which it implies in the context of our basic axioms.

Constant critical level for risk-free distributions. There exists $c \in \mathbb{R}$, such that for all $u \in U$ and for all $i \notin N(u)$, if $v \in U$ is such that $N(v) = N(u) \cup \{i\}$, $v_i = c$ and $v_j = u_j$ for all $j \in N(u)$, then $u \sim v$.

Theorem 3 *A social welfare ordering $\succsim$ on U satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level, Individual dominance, and either Independent value of additional lives from the prospects of the unconcerned or Constant critical level for risk-free distributions if and only if it is an ECLGU social welfare ordering.*

Proof. In the Appendix. ■

7 Conclusion

We characterize additively separable generalized utilitarianism in a space that combines variable population and social risk. Because actual policies have uncertain outcomes and may change the size of the population, this setting is the real-world case. Special cases of the ECLGU family include total utilitarianism, critical-level utilitarianism, and versions of these with "prioritarian" transformations that are sensitive to inequality and emphasize changes in the well-being of the worse-off. Because we do not assume *ex ante* Pareto (for example, by using our Individual dominance axiom), in our representations the transformation g may or may not be interpreted as distinct from individuals' own VNM functions for risk aversion.

Although our characterizations result in a familiar representation—and therefore are logically equivalent to prior characterizations—we have used this rich setting to highlight new axioms. These axioms are designed to avoid begging contested questions in welfare economics. To readers who find our novel axiomatizations compelling, our results can be read as new arguments in favor of separable social evaluation. For example, our axiom against probabilistic leveling down notes a new disadvantage, only apparent in variable-population cases, of non-separable egalitarianism.

To other readers, our results can be read to clarify what is fundamental about utilitarianism, for better or worse. Some may interpret our paper to offer new illustrations of the normative costs of ECLGU. One cost may be that ECLGU prevents social evaluations from having risk aversion over large changes in the mere size of the intertemporal human population. Another may be that ECLGU disregards the fact that adding new people to the population can change its degree of inequality and other distributional properties.

A key assumption that we have kept throughout the paper is that the social ordering can be represented by an expected utility. The expected utility assumption has been called into question in the literature, especially when society faces uncertainty (when there are no well-defined probabilities): see Fleurbaey (2009), Mongin and Pivato (2015), Zuber (2016), and McCarthy, Mikkola and Thomas (2020). We have studied a setting where the social ordering is over *lotteries*, in which case expected utility is often seen as more normatively appealing (Fleurbaey, 2018). Moreover, a theme of the recent literature is that social expectation-taking tends to reappear under weak rationality assumptions. It could still be of interest to study whether our results hold under weaker social rationality requirements. We conjecture that some of the findings in Theorem 1, namely that CLGU holds at least for sure alternatives, would still be true because our proof mainly uses (probability) continuity and ideas of stochastic dominance.[16] We also conjecture that our results in Theorems 2 and 3 may also be true for rank-dependent expected

[16]See steps 2 and 3 of the proof. We would thus obtain Existence independence for sure alternatives.

utility models, that are the prominent alternative to expected utility when we consider lotteries.[17] We leave those extensions to future research.

A Appendix: Proofs of the results

A.1 Proof of Theorem 1

Step 1: Proof that ECLGU satisfies all the properties. It can easily be checked that an ECLGU social welfare ordering satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level and No leveling down for probabilistic lives.

Step 2: Proof that $\succsim$ satisfies Property 1. We first show that, if social welfare ordering $\succsim$ on U satisfies Social expected-utility and No leveling down for probabilistic lives then it satisfies the following property:

Property 1. For all p, $p' \in P$, if there exist three alternatives u, v, w in U such that $N(u) \cap N(v) = \emptyset$, $N(w) = \{i\}$ where $i \notin N(u) \cup N(v)$, and $p(uvw) = 1/3$, $p(\star) = 2/3$, $p'(u) = 1/3$, $p'(v) = 1/3$ and $p'(w) = 1/3$, then $p \sim p'$.

To prove that the property must be satisfied, consider p and $p' \in P$ like those described in Property 1 and assume by contradiction that $p \succ p'$. By Social expected-utility (and the definition of p and p'), it must be the case that

$$\tfrac{1}{3}V(uvw) > \tfrac{1}{3}V(u) + \tfrac{1}{3}V(v) + \tfrac{1}{3}V(w).$$

[17] Indeed, our proofs use functional equations that may hold on restricted rank-ordered subsets.

Let w' be an alternative such that $N(w') = \{j\}$, where $j \notin (N(u) \cup N(v) \cup \{i\})$, and $w'_j > w_i$. By Strong Pareto and Anonymity, $V(w') > V(w)$, but we can find $0 < \varepsilon < 1/3$ small enough so that:

$$\tfrac{1}{3}V(uvw) - \tfrac{1}{3}V(u) - \tfrac{1}{3}V(v) - \tfrac{1}{3}V(w) > \varepsilon\big(V(w') - V(w)\big). \tag{A.1}$$

Let $\hat{p}$ be the lottery such that $\hat{p}(u) = 1/3$, $\hat{p}(v) = 1/3$, $\hat{p}(w) = 1/3 - \varepsilon$ and $\hat{p}(w') = \varepsilon$. By Eq. (A.1),

$$\tfrac{1}{3}V(uvw) > \tfrac{1}{3}V(u) + \tfrac{1}{3}V(v) + \left(\tfrac{1}{3} - \varepsilon\right)V(w) + \varepsilon V(w').$$

By Social expected-utility, this implies that $p \succ \hat{p}$. But, by No leveling down for probabilistic lives we should have $p \prec \hat{p}$. Indeed, in $\hat{p}$ we have increased the probability of existence of j and decreased by the same amount that of i, where j has higher utility than i, while maintaining the probability of existence of other people.

The contradiction shows that we cannot have $p \succ p'$. We can similarly prove that we cannot have $p \prec p'$ (now taking l with lower utility than i).

Step 3: Proof that $\succsim$ satisfies Existence independence for sure alternatives. Using Property 1, we can show that $\succsim$ satisfies Existence independence for sure alternatives.

To see why this is the case, consider any u, u', v' like described in this axiom. Let i be any individual in $N(v')$. Denote v the alternative such that $N(v) = N(v') \setminus \{i\}$ and $v_j = v'_j$ for all $j \in N(v') \setminus \{i\}$. Denote w the alternative such that $N(w) = \{i\}$ and $w_i = v'_i$. We also denote p, p', q, $q' \in P$ such that:

- $p(uv') = 1/3$, $p(\star) = 2/3$, $p'(u) = 1/3$, $p'(v) = 1/3$ and $p'(w) = 1/3$;

- $q(u'v') = 1/3$, $q(\star) = 2/3$, $q'(u') = 1/3$, $q'(v) = 1/3$ and $q'(w) = 1/3$.

By Property 1, we know that $p \sim p'$ and $q \sim q'$ so that $p \succsim q$ if and only if $p' \succsim q'$. By Social expected-utility, this means that:

$$\tfrac{1}{3}V(uv') \geq \tfrac{1}{3}V(u'v') \Longleftrightarrow \tfrac{1}{3}V(u)+\tfrac{1}{3}V(v)+\tfrac{1}{3}V(w) \geq \tfrac{1}{3}V(u')+\tfrac{1}{3}V(v)+\tfrac{1}{3}V(w).$$

The equivalence simplifies to $V(uv') \geq V(u'v') \Longleftrightarrow V(u) \geq V(u')$. By Social expected-utility, this implies that $uv' \succsim u'v'$ if and only if $u \succsim u'$.

Step 4: Conclusion. $\succsim$ is a social welfare ordering that satisfies Strong Pareto, Anonymity, Continuity, Existence independence for sure alternatives, Minimal existence of a critical level and Social expected-utility. By Proposition 2, we thus know that there exist two continuous and increasing functions $\varphi : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that $\varphi(0) = g(0) = 0$, and for all p, $p' \in P$:

$$p \succsim p' \Longleftrightarrow \sum_{u\in U} p(u)\times\varphi\left(\sum_{i\in N(u)} \Big[g(u_i) - g(c)\Big]\right) \geq \sum_{u\in U} p'(u)\times\varphi\left(\sum_{i\in N(u)} \Big[g(u_i) - g(c)\Big]\right). \tag{A.2}$$

Consider two populations N, $N' \in \mathcal{N}$ such that $N \cap N' = \emptyset$, an individual $i \notin N \cup N'$ and four alternatives u, u', w, w' in U such that:

- $N(u) = N \cup N' \cup \{i\}$, $N(u') = N$, $N(w) = N'$, $N(w') = i$,

- $u_j = u'_j$ for all $j \in N$, $u_k = w_k$ for all $k \in N'$ and $u_i = w'_i = c$, where c is the level obtained in the formula above.

And consider p and p' such that $p(u) = 1/3$, $p(\star) = 2/3$, $p'(u') = 1/3$, $p'(w) = 1/3$ and $p'(w') = 1/3$, then $p \sim p'$. We obtain, by Property 1 and the formula above, that:

$$\begin{aligned}
& \tfrac{1}{3}\varphi\left(\sum_{i\in N}\big[g(u_i)-g(c)\big]+\sum_{j\in N'}\big[g(u_j)-g(c)\big]\right) \\
= \; & \tfrac{1}{3}\varphi\left(\sum_{i\in N}\big[g(u_i)-g(c)\big]\right)+\tfrac{1}{3}\varphi\left(\sum_{j\in N'}\big[g(u_j)-g(c)\big]\right)
\end{aligned}$$

Denoting a the real number $a=\sum_{i\in N}\big[g(u_i)-g(c)\big]$ and b the real number $b=\sum_{j\in N'}\big[g(u_j)-g(c)\big]$, we thus get the equality

$$\varphi(a+b)=\varphi(a)+\varphi(b).$$

We can actually get this equality for any pair of real numbers $(a,b)\in\mathbb{R}^2$. Indeed, any real number can be reached as a sum of transformed utilities minus the transformed critical level (indeed, we can add the utility of as many people as we want and we can vary their utility level above and below c). We thus get the Cauchy functional equation $\varphi(a+b)=\varphi(a)+\varphi(b)$ for all $(a,b)\in\mathbb{R}^2$. Given that φ is continuous and increasing, we know that there must exist a positive real number α such that $\varphi(a)=\alpha a$ for all $a\in\mathbb{R}$ (Aczél, 1966, Chap. 2).

So we can conclude that there exists a continuous and increasing function $g:\mathbb{R}\to\mathbb{R}$ such that, for all p, $p'\in P$:

$$p\succsim p' \Longleftrightarrow \sum_{u\in U}p(u)\times\left(\sum_{i\in N(u)}\big[g(u_i)-g(c)\big]\right)\geq\sum_{u\in U}p'(u)\times\left(\sum_{i\in N(u)}\big[g(u_i)-g(c)\big]\right).$$

The social welfare ordering $\succsim$ is an ECLGU social welfare ordering.

A.2 Proof of Theorem 2

Step 1: Proof that ECLGU satisfies all the properties. It can easily be checked that an ECLGU social welfare ordering satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level, Existence independence of the sure, and Social risk neutrality in population size for perfect equality.

Step 2 Given that $\succsim$ on U satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, and Existence independence of the sure (and therefore Existence independence of the sure for same-populations), we know by Proposition 2 that there exist two continuous and increasing functions $\varphi : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that $\varphi(0) = g(0) = 0$, and for all p, $p' \in P$:

$$p \succsim p' \Longleftrightarrow \sum_{u \in U} p(u) \times \varphi\left(\sum_{i \in N(u)} \left[g(u_i) - g(c)\right]\right) \geq \sum_{u \in U} p'(u) \times \varphi\left(\sum_{i \in N(u)} \left[g(u_i) - g(c)\right]\right). \tag{A.3}$$

Consider p, p', q, $q' \in P$, such that there exist distinct alternatives u, v, v', w, and w' in U for which:

- $N(u) \cap N(v) = N(u) \cap N(v') = N(u) \cap N(w) = N(u) \cap N(w') = \emptyset$;
- $p(uv) = p(uv') = q(uw) = q(uw') = p'(v) = p'(v') = q'(w) = q'(w') = 1/2$.

By Existence independence of the sure, and using Eq. (A.3), we obtain that:

$$\begin{aligned}
&\tfrac{1}{2}\varphi\left(\sum_{i\in N(u)}[g(u_i)-g(c)]+\sum_{i\in N(v)}[g(v_i)-g(c)]\right)+\tfrac{1}{2}\varphi\left(\sum_{i\in N(u)}[g(u_i)-g(c)]+\sum_{i\in N(v')}[g(v_i')-g(c)]\right)\\
&\geq \tfrac{1}{2}\varphi\left(\sum_{i\in N(u)}[g(u_i)-g(c)]+\sum_{i\in N(w)}[g(w_i)-g(c)]\right)+\tfrac{1}{2}\varphi\left(\sum_{i\in N(u)}[g(u_i)-g(c)]+\sum_{i\in N(w')}[g(w_i')-g(c)]\right)\\
\Longleftrightarrow &\tfrac{1}{2}\varphi\left(\sum_{i\in N(v)}[g(v_i)-g(c)]\right)+\tfrac{1}{2}\varphi\left(\sum_{i\in N(v')}[g(v_i')-g(c)]\right)\\
&\geq \tfrac{1}{2}\varphi\left(\sum_{i\in N(w)}[g(w_i)-g(c)]\right)+\tfrac{1}{2}\varphi\left(\sum_{i\in N(w')}[g(w_i')-g(c)]\right)
\end{aligned}$$

Denote $a = \sum_{i\in N(u)}\big[g(u_i) - g(c)\big]$, $x = \sum_{i\in N(v)}\big[g(v_i) - g(c)\big]$, $x' = \sum_{i\in N(v')}\big[g(v_i') - g(c)\big]$, $y = \sum_{i\in N(v)}\big[g(v_i) - g(c)\big]$ and $y' = \sum_{i\in N(v')}\big[g(v_i') - g(c)\big]$. We get that, for any real numbers a, x, x', x and y':

$$\varphi(a+x)+\varphi(a+x') \geq \varphi(a+y)+\varphi(a+y') \Longleftrightarrow \varphi(x)+\varphi(x') \geq \varphi(y)+\varphi(y').$$

Let $I = \varphi(\mathbb{R})$, which is an open interval in $\mathbb{R}$ because φ is continuous and increasing. First fix a and denote $\psi_a : I \to \mathbb{R}$ the continuous function such that $\psi_a(x) = \varphi\big(a + \varphi^{-1}(x)\big)$. For the above equivalence to hold, there must exist an increasing function $\Psi_a : \mathbb{R} \to \mathbb{R}$ such that, for all z, $z' \in I$: $\psi_a(z) + \psi_a(z') = \Psi_a(z + z')$. This is a Pexider functional equation, and it is known that in that case Ψ_a and ψ_a must be affine (Aczél, 1966). Hence there exist $\alpha_a \in \mathbb{R}_{++}$ and $\beta_a \in \mathbb{R}$ such that $\psi_a(z) = \alpha_a z + \beta_a$.

Define the functions $\alpha : \mathbb{R} \to \mathbb{R}_{++}$ and $\beta : \mathbb{R} \to \mathbb{R}$ by $\alpha(a) = \alpha_a$ and $\beta(a) = \beta_a$ for all $a \in \mathbb{R}$. By definition of function ψ_a, we obtain that, for all $x \in \mathbb{R}$, $\psi_a(\varphi(x)) = \varphi\big(a + \varphi^{-1} \circ \varphi(x)\big) = \varphi(a + x)$. But by our result above, it is also the case that $\psi_a(\varphi(x)) = \alpha(a)\varphi(x) + \beta(a)$. We thus end up with the functional equation: $\varphi(a+x) = \alpha(a)\varphi(x) + \beta(a)$ for all $(a,x) \in \mathbb{R}^2$. By Corollary 1 (pp. 150–151) in Aczél (1966), this equation implies that either φ is affine or that it is a positive affine transformation of the function $x \to \alpha e^{\alpha x}$ for some $\alpha \neq 0$.

Step 3: Conclusion. By Step 2, we know that

$$V(u) = \varphi\left(\sum_{i \in N(u)} \big[g(u_i) - g(c)\big]\right),$$

where φ is affine or that it is a positive affine transformation of the function $x \to \alpha e^{\alpha x}$ for some $\alpha \neq 0$.

Also given that $\succsim$ satisfies Social risk neutrality in population size for perfect equality, we know by Prop. 3 that there exist a positive number a, a number b, two continuous functions $\Psi : \mathbb{R} \to \mathbb{R}$ and $\Phi : \mathbb{R} \to \mathbb{R}$, and for each $n \in \mathbb{N}$ a continuous, increasing, symmetric and normalized function Ξ_n such that, for u such that $n(u) > 0$:

$$V(u) = a\Big[\Phi\left(\Xi_{n(u)}(u)\right) + n(u) \times \Psi\left(\Xi_{n(u)}(u)\right)\Big] + b.$$

Suppose that φ is a positive affine transformation of the function $x \to \alpha e^{\alpha x}$ for some $\alpha \neq 0$. Consider any $n \in \mathbb{N}$ and $x \in \mathbb{R}$, and let u be such that $n(u) = n$ and $u_i = x$ for all $i \in N(u)$. We must have:

$$\alpha e^{\alpha n x} = a\Big[\Phi\left(x\right) + n \times \Psi\left(x\right)\Big] + b.$$

But this cannot be true if $x \neq 0$. Hence only the affine form is possible for φ.

A.3 Proof of Proposition 4

Consider any $N \in \mathcal{N}$ and let $n = |N|$. For any $i \in N$ let us define $\succsim_i$ the ordering on P_N such that, for any p, $q \in P_N$, $p \succsim_i q$ if and only if $p_i \succsim q_i$. By Social expected-utility, defining $W_i(p) = \sum_{u_i \in \mathbb{R}} p_i(u_i)V(u_i)$, we have that $p \succsim_i q$ if and only if $W_i(p) \geq W_i(q)$. Similarly, defining $W_0(p) = \sum_{u \in U_N} p(u)V(u)$, we have that $p \succsim q$ if and only if $W(p) \geq W(q)$.

Consider $F = (W_0, (W_i)_{i \in N}) : P_N \to \mathbb{R}^{n+1}$. By definition $F(P_N)$ is convex because $W_i(\kappa p + (1-\kappa)q) = \kappa W_i(p) + (1-\kappa)W_i(q)$ for each $i \in N$ or $i = 0$. Individual dominance implies that, if $W_i(p) > W_i(q)$ for all $i \in N$, then $W_0(p) > W_0(q)$. By Proposition 2 in De Meyer and Mongin (1995), there must exist non-negative numbers λ_i and a number γ such that, for each $p \in P_N$:

$$W_0(p) = \sum_{i \in N} \lambda_i W_0(p) + \gamma.$$

Focusing on lotteries yielding sure outcomes, we get that, for all u, $v \in U_N$,

$$u \succsim v \Longleftrightarrow \sum_{i \in N} \lambda_i V(u_i) \geq \sum_{i \in N} \lambda_i V(v_i).$$

By Anonymity and Strong Pareto, it must be the case that all the λ_i must be the same positive number λ. Denoting g the function such that $g(u_i) = V(u_i)$ for each $u_i \in \mathbb{R}$, we obtain that for all p, $q \in P_N$,

$$p \succsim q \Longleftrightarrow \sum_{u \in U_N} p(u) \left[\sum_{i \in N} g(u_i)\right] \geq \sum_{u \in U_N} p(u) \left[\sum_{i \in N} g(u_i)\right].$$

Observe that the function g does not depend on N.

By Social expected-utility, and given that a VNM utility function is defined up to an increasing affine transformation, it must be the case that, for each $n \in \mathcal{N}$, for each $u \in U_N$:

$$V(u) = F(N)\left[\sum_{i \in N} g(u_i)\right] + G(N),$$

for some positive $F(N)$ and some number $G(N)$. By Anonymity, $F(N)$ and $G(N)$ only depend on population size.

A.4 Proof of Theorem 3

Step 1: Proof that ECLGU satisfies all the properties. It can easily be checked that an ECLGU social welfare ordering satisfies Strong Pareto, Anonymity, Continuity, Social expected-utility, Minimal existence of a critical level, Individual dominance and Independent value of additional lives from the prospects of the unconcerned.

Step 2: Proof that $\succsim$ has a constant critical level. We show that if $\succsim$ satisfies Strong Pareto, Continuity, Social expected-utility, Minimal existence of a critical level and Independent value of additional lives from the prospects of the unconcerned, then it satisfies the Constant critical level for risk-free distributions property.

By Minimal existence of a critical level, there exist $\bar{u} \in U$ and $c \in \mathbb{R}$ such that there exist $k \notin N(\bar{u})$ and $\bar{v} \in U$ defined by $N(\bar{v}) = N(u) \cup \{k\}$, $\bar{v}_k = c$ and $\bar{v}_l = \bar{u}_l$ for all $l \in N(\bar{u})$ for which $\bar{v} \sim \bar{u}$. Let ε be some positive number. Define $\tilde{v} \in U$ by $N(\tilde{v}) = N(u) \cup \{k\}$, $\tilde{v}_k = c + \varepsilon$ and $\bar{v}_l = \bar{u}_l$ for all $l \in N(\bar{u})$ By Strong Pareto, we must have $\tilde{v} \succ \bar{u}$, which by Social expected-utility implies that $V(\tilde{v}) > V(\bar{u})$.

Consider any $u \in U$ and any $i \notin N(u)$, and define $w \in U$ such that $N(w) = N(u) \cup \{i\}$, $w_i = c + \varepsilon$ and $w_j = u_j$ for all $j \in N(u)$. Let p, $p' \in P$ such that $p(u) = p'(w) = q$ and $p(\bar{u}) = p'(\tilde{v}) = 1 - q$, with $q \in (0, 1)$. Independent value of additional lives from the prospects of the unconcerned, if $p' \succ p$ for some q, this should be true whatever q is. But $p' \succ p$ means that:

$$qV(w) + (1-q)V(\tilde{v}) > qV(u) + (1-q)V(\bar{u})$$

which can be written

$$V(\tilde{v}) - V(\bar{u}) > \tfrac{q}{1-q}\Big[V(u) - V(w)\Big].$$

Given that $V(\tilde{v}) > V(\bar{u})$, this must be true, whatever the values $V(w)$ and $V(u)$, for small enough value of q.

Thus, whatever q is, we have:

$$qV(w) + (1-q)V(\tilde{v}) > qV(u) + (1-q)V(\bar{u})$$

which can be written

$$V(w) - V(u) > \tfrac{1-q}{q}\Big[V(\tilde{v}) - V(\bar{u})\Big].$$

Given that $\frac{1-q}{q}$ can be as low as we want, we need to have $V(w) - V(u) \geq 0$, and therefore $w \succsim u$.

So for every $\varepsilon > 0$, $V(w) - V(u) \geq 0$, where $v \in U$ such that $N(w) = N(u) \cup \{i\}$, $w_i = c + \varepsilon$ and $w_j = u_j$ for all $j \in N(u)$, we have $w \succsim u$. By continuity, if $v \in U$ such that $N(v) = N(u) \cup \{i\}$, $v_i = c + \varepsilon$ and $v_j = u_j$ for all $j \in N(u)$, then $v \succsim u$.

We can prove that $u \succsim v$ in the same way by using a negative $\varepsilon < 0$.[18] Hence $u \sim v$.

Step 3: Conclusion. Given that $\succsim$ satisfies Strong Pareto, Continuity, Social expected-utility, and Individual dominance, we know by Proposition 4 that it is an ENWGU social ordering. Then there exist a continuous and increasing function $g : \mathbb{R} \to \mathbb{R}$, and two functions $F : \mathbb{N} \to \mathbb{R}$ and $G : \mathbb{N} \to \mathbb{R}$ such that for any $u \in U$ the VNM social utility function is

$$V(u) = F\big(n(u)\big) \sum_{i \in N(u)} g(u_i) + G\big(n(u)\big).$$

By Step 3, we also know that $\succsim$ satisfies Constant critical level. Hence, there exists a $c \in \mathbb{R}$ such that, for any $n \in \mathbb{N}$ and any $x \in \mathbb{R}$, if $u, v \in U$ are such that $n(u) = n$, $N(v) = N(u) \cup \{i\}$ (with $i \notin N(u)$), $u_j = v_j = x$ for all $j \in N(u)$ and $v_i = c$, then $V(u) = V(v)$, so that:

$$F(n)ng(x) + G(n) = F(n+1)\big[ng(x) + g(c)\big] + G(n+1).$$

Given that g is increasing in x and that this equality must be true for all $x \in \mathbb{R}$, we must have $F(n+1) = F(n)$. Hence, we also obtain by the equality that $G(n+1) - G(n) = -g(c)$. Hence, by iterating on n (given that these results are true for all $n \in \mathbb{N}$), we have $F(n) = F(0) = a$ and

[18] Note that Independent value of additional lives from the prospects of the unconcerned implies that, if $p \succsim p'$ for some q, then this should be true whatever q.

$G(n) = G(0) - ng(c) = b - ng(c)$, where $a > 0$ and b is a real number. Hence,

$$V(u) = a \sum_{i \in N(u)} \big[g(u_i) - g(c)\big] + b.$$

References

Aczél, J. (1966). *Lecture on Functional Equations and their Applications.* Academic Press.

Adler, M.D., Treich, N. (2017). Utilitarianism, prioritarianism, and intergenerational equity: A cake eating model. *Mathematical Social Sciences* **87**: 94–112.

Asheim, G.B., Zuber, S. (2016). Evaluating intergenerational risks. *Journal of Mathematical Economics* **65**: 104–117.

Asheim G.B., Zuber S. (2014). Escaping the repugnant conclusion: Rank-discounted utilitarianism with variable population. *Theoretical Economics* **9** 629–650.

Blackorby, C., Bossert, W., Donaldson, D. (1995). Intertemporal Population ethics: Critical-level utilitarian principles. *Econometrica.* **63**: 1303–1320.

Blackorby, C., Bossert, W., Donaldson, D. (1998). Uncertainty and critical-level population principles. *Journal of Population Economics* **11**: 1–20.

Blackorby, C., Bossert, W., Donaldson, D. (1999). Information Invariance in Variable-Population Social-Choice Problems. *International Economic Review* **40**: 403–422

Blackorby, C., Bossert, W., Donaldson, D. (2001). Population ethics and the existence of value functions. *Journal of Public Economics* **82**: 301–308.

Blackorby, C., Bossert, W., Donaldson, D. (2005). *Population Issues in Social Choice Theory, Welfare Economics, and Ethics.* Cambridge: Cambridge University Press.

Blackorby, C., Bossert, W., Donaldson, D. (2006). Anonymous single-profile welfarism. *Social Choice and Welfare* **27**: 279–287

Blackorby, C., Donaldson, D. (1984). Social criteria for evaluating population change. *Journal of Public Economics* **25**: 13–33.

Bommier, A., Zuber, S. (2008). Can preferences for catastrophe avoidance reconcile social discounting with intergenerational equity? *Social Choice and Welfare* **31**: 415–434.

Broome, J. (2005). Should we value population? *Journal of Political Philosophy* **13**: 399–413.

Broome, J. (2015). Equality versus priority: A useful distinction. *Economics and Philosophy* **31**: 219–228.

De Meyer, B., Mongin, P. (1995). A note on affine aggregations. *Economics Letters* **47**: 177–183.

Fishburn, P.C., Keeney, R.L. (1975). Generalized utility independence and some implications. *Operations Research* **23**: 928–940.

Fleurbaey, M. (2009). Two variants of Harsanyi's aggregation theorem. *Economics Letters* **105**: 300–302.

Fleurbaey, M. (2010). Assessing risky social situations. *Journal of Political Economy* **118**: 649–680.

Fleurbaey, M. (2018). Welfare economics, risk and uncertainty. *Canadian Journal of Economics* **51**: 5–40.

Harsanyi, J. (1955). Cardinal welfare, individualistic ethics and interpersonal comparisons of utility. *Journal of Political Economy* **63**, 309–321.

Keeney, R. (1980). Equity and public risk. *Operations Research* **28**: 527–534.

McCarthy, D. (2017). The priority view. *Economics and Philosophy* **33**(2): 215–257.

McCarthy, D., Mikkola, K., and Thomas, T. (2020). Utilitarianism with and without expected utility. *Journal of Mathematical Economics* **87**: 77-113.

Mongin, P., and Pivato, M. (2005). Ranking multidimensional alternatives and uncertain prospects. *Journal of Economic Theory* **157**: 146–171.

Ng, Y-K. (1989). What should we do about future generations? Impossibility of Parfit's theory X. *Economics and Philosophy* **5**: 235–253.

Parfit, D. (1995) Equality or Priority? Lawrence, KA: University of Kansas.

Zuber, S. (2016). Harsanyi's theorem without the sure-thing principle: On the consistent aggregation of Monotonic Bernoullian and Archimedean preferences. *Mathematical Social Sciences* **63**: 78–83.

Acknowledgements. Zuber's research has been supported by the Agence nationale de la recherche through the Investissements d'Avenir program (ANR-17-EURE-01) and by the Riksbankens Jubileumsfond (the Swedish Foundation for Humanities and Social Sciences). Spears' research is supported by grant K01HD098313 and by P2CHD042849, Population Research Center, awarded to the Population Research Center at The University of Texas at Austin by the Eunice Kennedy Shriver National Institute of Child Health and Human Development; the content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health. The authors' collaborative research is supported by a Dr. Cécile DeWitt-Morette France-UT Endowed Excellence Grant.