

DISCUSSION PAPER SERIES

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Labour Market Dynamics**

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ABSTRACT

Population Growth, Immigration and Labour Market Dynamics*

This paper examines the role of population flows on labour market dynamics across immigrant and native-born populations in the United Kingdom. Population flows are large, and cyclical, driven first by the maturation of baby boom cohorts in the 1980s, and latterly by immigration in the 2000s. New measures of labour market flows by migrant status uncover both the flow origins of disparities in the levels and cyclicalities of immigrant and native labour market outcomes, as well as their more recent convergence. A novel dynamic accounting framework reveals that population flows have played a nontrivial role in the volatility of labour markets among both the UK-born and, especially, immigrants.

JEL Classification: E24, J6

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1. Introduction

This paper examines the role of population flows on the labour market dynamics of immigrant and native-born populations in the United Kingdom. We present novel stylised facts on population flows, driven both by immigration and by changes in birth rates among UK-born population cohorts, alongside new evidence on the patterns of labour market flows by migrant status over time. We then introduce an analytical framework that accommodates the interaction between population flows with labour market dynamics. This facilitates and enhances understanding of the forces that underlie changes in immigrant and native-born labour market stocks over time and, thereby, changes in aggregate labour market outcomes.

We begin in section 2 by documenting some motivating stylised facts on the evolution of the working-age population in the United Kingdom, and the underlying flows that shape its path. UK population growth has been considerable, averaging 0.5 percent a year over the sample period. Two forces underlie this trend: First, the maturation of the UK baby boom cohorts, and the simultaneous decline in the number of retirees in the late 1970s and early 1980s—legacies of the two world wars and their aftermath. Second, rises in UK immigration, especially from the 2000s onward. The share of immigrants in the working-age population peaked at 19 percent in 2016, up from 10 percent in 2000. Yearly net growth in the immigrant population reached a maximum of 7 percent in 2006.

The UK gross population flows are much larger than net population growth. In any given year, movements in and out of the working-age population comprise 5 to 6 percent of the population stock. Population flows are therefore comparable in magnitude to the stock of unemployed workers, for example. Importantly for the results of this paper, these population flows are often aligned with cyclical fluctuations in the labour market. The large UK-born baby boom cohorts suffered the misfortune of attaining working age at the onset of the 1980s recession. Latterly, the large rises in immigration in the mid-2000s, associated with the accession of the A8 countries to the European Union (EU) in 2004, coincided with the onset of the Great Recession, the imprint of which can be seen in cyclical movements of migrant flows.¹

¹ Migration and birth rate variation contribute to population change in many other industrialised countries. The US working-age population has grown by around 0.4 percent a year, largely as a result of immigration.

Population change can also influence labour market flows. It has long been known that changes in labour market stocks are shaped by changes in inflows and outflows of workers between labour market states (Perry 1972; Marston 1976). The literature on labour market dynamics has, however, largely ignored the influence of population flows and by extension the roles of immigration, or birth and death rates, in shaping labour market stocks. To understand labour market behaviour in periods of rapid change in cohort size, or immigration, in section 3 we introduce a novel labour market flows methodology that accommodates population dynamics, disaggregated by migrant status. This facilitates a study of labour market flows, population flows, their interaction, their origins by migrant status, and their role in shaping labour market outcome, that informs all the remaining analyses of the paper.

Crucial inputs into this framework are requisite measures of labour market stocks and flows by migrant status, as well as population flows by labour force and migrant status. Such measures are not straightforward to compile, since there exists no standard source that provides such data at the required level of disaggregation. A key contribution of the paper, then, is to provide such measures. We do so in section 4 by combining micro-data from the UK Labour Force Survey (LFS) with official data on population flows published by the UK Office for National Statistics (ONS).

What emerges, is a novel set of time series on UK labour market and population flows. In the remainder of section 4, we document the empirical properties of these series, and use them to paint a qualitative picture of the role of gross worker and population flows in the evolution of UK labour market stocks.

Starting with these stocks, we identify three broad trends. First, there are clear levels differences in the labour market experiences of immigrant and UK-born individuals: Immigrants are more likely to be unemployed, much less likely to be employed, and much more likely to be out of the labour force. Second, immigrants historically have faced greater

Conversely, many eastern European countries have experienced working-age population declines of a similar order of magnitude, largely because of emigration to western Europe. Recessions also affect immigrant flows elsewhere. Spain experienced a 0.5 percent yearly fall in its population in the years following its 2009-11 downturn, driven by the return migration of many immigrants. Allied to this, domestic birth rates across much of the OECD have been falling continuously over the last 40 years and this has begun to affect working age populations in the last decade. See OECD Population Statistics (2019).

cyclicalities in their labour market outcomes. These two results echo the conclusions of earlier work for the UK by Bell (1997), and Dustmann, Glitz and Vogel (2010).

A third message of our analysis of stocks is more novel, however: We find that more recent data point to *convergence* in the labour market experiences of immigrant and native populations. Moreover, it appears to have been pervasive, with both the levels and cyclicalities of immigrant stocks aligning more closely to those of the UK-born over time.

An important contribution of section 4, is that it provides a sense of the flow origins of these outcomes. A first impression is provided by our measures of labour market flows by migrant status. These reveal that immigrants historically have faced both higher rates of job loss, as well as lower rates of job finding—both from unemployment and from nonparticipation. Moreover, historically all of these differences have tended to be exacerbated in times of recession. Together, then, these features of the labour market flows provide an account for the flow origins of the differences in the levels and cyclicalities of immigrant labour market stocks.

In addition, the measures in section 4 provide new insights into the interaction of population flows and the labour market. First, echoing the aggregate population flows that motivated our analysis, we find that gross population flows by labour force state are substantial. Up to one third of all observed flows among the UK-born, and up to one half of all observed flows among immigrants, are accounted for by gross population flows. Second, gross population flows are cyclical. The counterpart of the UK-born working-age population growth around the early 1980s recession are large rises in population inflows to unemployment and, to a lesser extent, nonparticipation. The burst of immigration in the mid-2000s was followed by large declines in immigrant population inflows to employment, and rises in their population inflows to unemployment, at the onset of the Great Recession. Third, a final message of the population flows is that the later burst of immigration in the 2000s was accompanied by increased population churn—that is, rises in both population inflows and outflows among the immigrant population. This, in turn, is consistent with the greater incidence of return migration among new EU migrants.

The upshot of these new measures, then, is that there is a *prima facie* case for examining the potential role of population flows in accounting for UK labour market variation around

two cycles: First, among the UK-born in the late 1970s and early 1980s; and, second, among immigrants around the Great Recession.

We take up this task in section 5, which develops an accounting framework for decomposing the variance of changes in labour market stocks into contributions accounted for by labour market flows and, importantly, gross population flows. The framework accommodates labour market and population flows across all three labour market states, and allows for the out-of-steady-state transmission of current flows into future stocks. The existing literature, by contrast, typically has adopted a steady-state framework in which the population is assumed constant, or non-steady-state frameworks that implicitly assume population flows are neutral with respect to labour market dynamics.²

Application of our decomposition reveals that these assumptions are unduly restrictive, especially for immigrants. We present decompositions of changes in the unemployment, employment and nonparticipation rates by migrant status into parts accounted for by changes in rates of job loss, job finding, participation *and* working-age population entry/exit.

The results reinforce the qualitative impression from informal inspection of the time series in section 4. Starting with the earlier episode, between 1978 and 1988, we find that the population inflows that accompanied the maturation of the baby boom cohorts accounted for 8 percent of the large rise and fall in unemployment, and 16 percent of the variation in the nonparticipation rate, among the UK-born. Thus, part of the labour market volatility witnessed during the late 1970s and 1980s can be traced to an accident of history.

Most starkly, our key finding is that population flows had a profound impact on the labour market outcomes of immigrants during the Great Recession and its aftermath. Cyclical declines in immigrant population inflows to employment explain over one third of the cyclical path of employment, and over 40 percent of the variation in the nonparticipation rate among immigrants. Along with a cyclical rise in immigrant population inflows to unemployment, the combined contribution to immigrant unemployment was 14 percent. Population flows have thus played a key role in the labour market dynamics of immigrant populations in the United Kingdom.

² Examples of such steady-state frameworks include Elsby, Michaels and Solon (2009), Fujita and Ramey (2009), Shimer (2012), Gomes (2012), Petrongolo and Pissarides (2008), and Elsby, Smith and Wadsworth (2011). Non-steady-state frameworks include Smith (2012), and Elsby, Hobijn and Sahin (2013, 2015).

In the remainder of the paper, we investigate two further dimensions to our analysis. First, we juxtapose the role of labour market flows between UK-born and immigrant populations. This reveals that although, like the UK-born, job loss is the main contributor to immigrant unemployment variation, rates of job finding contribute relatively little to explaining unemployment among immigrants. Furthermore, the participation margin between unemployment and nonparticipation looms larger for immigrants, accounting for more than 20 percent of the variation in unemployment, consistent with labour market attachment tending to be weaker among immigrants.

Second, and finally, we extend our methodology to accommodate a subgroup-to-aggregate decomposition that measures contributions of flows by migrant status to each *aggregate* labour market stock rate over time. This reveals that immigrant contributions have become increasingly under-represented in the contribution of job-finding flows, a corollary of job-finding rates among immigrants falling less during the Great Recession, and rising more in the subsequent recovery, itself a contributory factor to the convergence in the levels of immigrant and native job-finding rates noted earlier.

In conclusion, we propose several contributions to our understanding of labour and population flows, and their particular role in labour market dynamics by migrant status. UK population flows have been considerable, dominated first by the entry of the baby boom generations in the early 1980s, and then by accelerated immigration, particularly in the 2000s. We provide new measures of labour market and population flows that uncover both the flow origins of disparities in the levels and cyclicalities of immigrant and UK-born labour market outcomes, as well as their more recent convergence. A novel accounting framework formalises these effects, and reveals that population flows have played nontrivial roles in the volatility of labour markets among the UK-born and, especially immigrants.

2. Immigration and working-age population dynamics

We begin by documenting the evolution over time of the working-age population in the United Kingdom, and the underlying flows that shape its path. We define the working-age population as men aged 16 to 64 and women aged 16 to 59.³ Changes in this stock are then driven by population flows. Specifically, the change over time of the working-age population can be disaggregated according to

$$\Delta \text{Pop}_t = \text{Immigrants}_t + 16\text{-year olds}_t - \text{Emigrants}_t - 65\text{-year olds}_t - \text{Deaths}_t. \quad (1)$$

The first two components comprise gross population inflows. The last three comprise all gross population outflows.

Aggregate measures for each of these gross population flows can be derived for the years 1975 to 2016 using micro-data from the UK Labour Force Survey (LFS), combined with UK Office for National Statistics (ONS) official data on population and deaths by age, and on immigration and emigration for the entire UK population.⁴ We combine these measures with LFS micro-data to infer gross population flows for the working-age population into and out of the states of employment, unemployment and nonparticipation.⁵ Derivations of the variables used in what follows are provided in Appendix A.

The resulting time series for the populations studied are illustrated in the two panels of Figure 1. Figure 1A presents the OECD/ILO-based stocks of employment, unemployment and nonparticipation and their composition by immigrant status. Viewed over the whole sample period, the UK working-age population grew by some 6.5 million over 40 years, an average annual growth rate close to 0.5 percent. The majority of this growth can be traced to two broad eras of accelerated population advance. The first period contains the maturation of the UK baby boom cohorts over the course of the late 1970s and 1980s. By contrast, the

³ Although now moving toward equality at age 66 in 2021, for the majority of the sample period these upper age bounds defined state pensionable age in the United Kingdom.

⁴ See in particular the ONS datasets: *Population Estimates for UK, England and Wales, Scotland and Northern Ireland*; *Deaths by single year of age tables – UK*; and *Migration Statistics Quarterly Report*.

⁵ Our sample period ends in 2016 for two reasons. First, it aligns with a full cycle of unemployment; second, Brexit and the Coronavirus pandemic have had a major adverse effect on sampling and the population grossing weights in the LFS. This induces much greater uncertainty in the estimates of the immigrant and native-born populations after 2016. More details on the construction of these population flows and weighting are given in Appendix A.

second era has its origins in rising immigration, especially in the 2000s, associated in part with the accession of the A8 countries to the European Union in 2004. The scale of this rise is such that the immigrant share in the working-age population grew from around 8 percent to more than 18 percent over the sample period.

Figure 1B plots the gross population flows, expressed as shares of working-age population, that underlie these episodes of population growth. This underscores the role of the baby boom cohorts in driving population growth initially, as indicated by the large spike in the inflow of 16-year-olds early in the sample period. Likewise, the large rise in the immigrant population is reflected in a growing gap between immigration and emigration later in the sample.

Figure 1B also provides further perspectives on the flow origins of population growth. First, the gross population flows are large: The number of UK-born individuals recorded as entrants and exits from the working age population in these annual flows comprises 4 to 5 percent of the aggregate UK working-age population. The equivalent flows for migrants rises over time, equal to 2.5 percent of the aggregate UK working-age population, and 50 percent of the immigrant population by the end of the sample. The magnitudes of aggregate gross population flows are thus comparable, for example, to the total stock of unemployed workers in the UK. We will see later that these population flows are consequently nontrivial relative to the gross flows across labour market states.

Second, a lot of churn underlies net population growth. A prominent example is the rise in net immigration in latter part of the sample which, Figure 1B reveals, was accompanied by large rises in both immigration *and* emigration flows. This suggests that the growth of the immigrant population increasingly was driven by rises in short-term (return) migration spells.

Third, the population flows are correlated with the business cycle, and in a way that offers a potential explanation for cyclical movements in labour market outcomes. While it has long been known that birth rates fall in recessions, (Yule 1906), it does not follow that the arrival of 16-year-olds into the working-age population also should coincide with a cyclical shock. Yet, as Figure 1B shows, this misfortune befell the large baby-boom birth cohorts that turned 16 during the downturn at the start of the 1980s. In turn, this was accompanied by a near-contemporaneous fall in the number of 60/65 year olds leaving the

working-age population, a combination of the low birth rates during the First World War, and increased deaths among the same birth cohorts who were then of fighting age in the Second World War.⁶

Figure 1 also identifies cyclical components to migration flows. These are especially apparent in the latter part of the sample, as the scale of immigration grows. In particular, both immigration and emigration flows can be seen to ramp up with the economic expansion of the early 2000s, subside with the onset of the Great Recession, and then recover in its wake.⁷ Among the population flows, only deaths display a lack of cyclicity. Deaths among the working-age population have been in secular decline over the 40 years covered by our data.

3. Labour market dynamics with population flows

We begin with a set of accounting identities for the evolution over time of labour market stocks, and the flows that underlie them. Motivated by the preceding evidence, our goal is to provide an accounting framework that can accommodate population dynamics arising from two potential sources: flows driven by changes in UK-born cohort size associated with the baby boom; and immigrant flows.

Labour market stocks evolve as current members of the working-age population transition across states and as individuals enter and exit the working-age population to and from each state. Denoting the measured stocks of the employed, unemployed and nonparticipants respectively by E , U and N , and indexing “migrant” and “native” status respectively by a subscript $k \in \{m, n\}$, we can formalize this process in the following Markov chain,

$$\begin{bmatrix} E \\ U \\ N \end{bmatrix}_{kt} = \begin{bmatrix} 1 - p_{EU} - p_{EN} & p_{UE} & p_{NE} \\ p_{EU} & 1 - p_{UE} - p_{UN} & p_{NU} \\ p_{EN} & p_{UN} & 1 - p_{NE} - p_{NU} \end{bmatrix}_{kt} \begin{bmatrix} E \\ U \\ N \end{bmatrix}_{kt-1} + \begin{bmatrix} Q_E \\ Q_U \\ Q_N \end{bmatrix}_{kt} \quad (2)$$

for $k \in \{m, n\}$.

⁶ A ripple of this can also be seen in the fall in the number of 65-year-olds from 2010 to 2012 in Figure 1.

⁷ Emigration appears to have reversed polarity, being countercyclical in the 1980s and pro-cyclical in the 2000s. This is in part a reflection of the rise in the stock of immigrants. The magnitude of UK emigration by non-British citizens is now greater than emigration by British citizens.

Here, p_{ijkt} denotes the probability that a working-age individual of migrant status k in labour market state i transitions to state j from year $t - 1$ to year t . Thus, the transition probabilities p_{ijkt} summarize labour market dynamics among current members of the working-age population. Adding to the existing modelling framework, we then augment these transitions by net flows into the working-age population. Specifically, the Q_{ikt} in equation (2) denote the *net* population inflow into labour market state i between year $t - 1$ and year t by migrant status k . These net inflows can in turn be disaggregated into gross inflows and outflows, along the lines of Figure 1B and equation (1). It is in this way that population flows alter the dynamics of employment, unemployment and nonparticipation.

A consequence of the sustained growth in the UK working-age population over the sample is that the labour market stocks inherit this trend. To focus on shorter-run variation in these stocks, then, we normalize the dynamic system in equation (2) by the working-age population. Using lower-case letters to denote upper-case variables as a share of the working-age population, $x \equiv X/Pop$ for all $X \in \{E, U, N, Q\}$, we can write

$$\underbrace{\begin{bmatrix} e \\ u \\ n \end{bmatrix}_{kt}}_{\mathbf{s}_{kt}} = \frac{1}{1 + q_{kt}} \underbrace{\begin{bmatrix} 1 - p_{EU} - p_{EN} & p_{UE} & p_{NE} \\ p_{EU} & 1 - p_{UE} - p_{UN} & p_{NU} \\ p_{EN} & p_{UN} & 1 - p_{NE} - p_{NU} \end{bmatrix}_{kt}}_{\mathbf{P}_{kt}} \underbrace{\begin{bmatrix} e \\ u \\ n \end{bmatrix}_{kt-1}}_{\mathbf{s}_{kt-1}} + \underbrace{\begin{bmatrix} q_E \\ q_U \\ q_N \end{bmatrix}_{kt}}_{\mathbf{q}_{kt}} \quad (3)$$

where $q_{kt} \equiv q_{Ekt} + q_{Ukt} + q_{Nkt}$ is the total net rate of growth of the working-age population of group k .

The direct effects of net new population inflows on employment, unemployment and nonparticipation rates are captured by the final term in (3). Each state is boosted by direct net inflows. In addition, the scalar $1/(1 + q_{kt})$ reflects a dampening effect of total population growth q_{kt} on labour market dynamics. Intuitively, using the example of employment, population flows into unemployment and nonparticipation increase the working age population without raising employment, and so tend to reduce (growth in) employment as a share of the working-age population. As indicated by the annotation of equation (3), for future reference we will express this dynamic system in matrix notation as

$$\mathbf{s}_{kt} = \mathbf{P}_{kt} \mathbf{s}_{kt-1} + \mathbf{q}_{kt} \quad (4)$$

4. Measurement of stocks and flows

It is not straightforward to measure all the constituent components of the laws of motion summarized by the accounting framework in equation (3). Neither labour market, nor population flows are published officially at the required levels of disaggregation. For this reason, we use individual micro-data underlying the LFS and combine it with official sources to infer the requisite components of equation (3). The frequency of the estimates that we study in what follows thus mirrors the frequency of the LFS micro-data files, which are available for every other year from 1975 to 1983, and on an annual basis thereafter.

4.1 Labour market flows

Our starting point is to infer labour market transition rates by migrant status—the p_{ijkt} in equation (3). The LFS asks all individuals interviewed in the second quarter of each year about their labour force status one year prior to the interview date. This information on *recalled status* may be combined with the individual's reported current status to infer measures of annual worker flows, and thereby the accompanying transition rates.

A key advantage of this approach is that it provides measures of transition rates over a long sample period covering three distinct economic cycles, 1975-1986, 1987-1995 and 2006-2016. No other UK data source allows measurement of worker flows over such a long period. In addition, the information in the LFS on recalled status has the benefit of being asked of all individuals, not just those who remain at the same address, unlike the quarterly LFS longitudinal data, which are available only from 1992. It is also straightforward to calculate flows, since the current and recalled status of a particular individual are contained in the same data set.

The use of recalled data does raise issues about the accuracy of remembered status, however. Studies investigating recall accuracy indicate that over short periods—up to about three years—recall bias is not severe (Paull 2002; Elias 1996). If individuals are asked to recall over longer periods, unemployment tends to be underreported; this does not appear to be simply short spells being forgotten, but is rather a general tendency to underreport. The one-year recall required of respondents in this paper falls well within the horizon where results should not be adversely affected by recall bias. However, Bell and Smith (2002) find recalled stocks accurate, and transitions between employment and unemployment correctly

recalled, but where spells are short, transitions between unemployment and nonparticipation estimated from recalled data tally less well with contemporaneous reports. As Akerlof and Yellen (1985) suggest, this might be because individuals tend to remember better the most personally-important or salient events. Moves between the two non-employment states are unlikely to be as psychologically “painful” or “enjoyable” as losing or gaining a job.⁸

4.2 Population flows

We then augment these LFS measures of labour market transitions with yearly measures of net population flows by labour market state and migrant status—the q_{ikt} in equation (3), and their disaggregation into gross population inflows and outflows. We denote the out of the population state by A (for “abroad”), and so we can relate net and gross population flows as follows

$$q_{ikt} = (A \text{ to } i)_{kt} - (i \text{ to } A)_{kt}, \quad \text{for all } i \in \{E, U, N\}. \quad (5)$$

We derive these flows partly from LFS data, and partly from official sources from the UK Office for National Statistics (ONS).

We begin by estimating immigration and emigration flows. While the ONS publishes aggregate measures of these flows for the entire UK population, it does not provide disaggregations for the working-age population, or across labour market states, or of emigration by migrant (country of birth) status. For these reasons, we harness additional information from the LFS micro-data.

The LFS elicits data on the month and year of entry into the United Kingdom for foreign-born individuals. Using this information, we define a new immigrant as a foreign-born individual of working age who reports having been in the United Kingdom for less than one year at the date of interview. We can then split such new immigrants in the LFS between

⁸ Retrospective and current labour market status definitions in the LFS differ with regard to students. The recall data record students as a separate inactive category. The current data classify students as either employed, unemployed or inactive depending on whether report that they have been in work or have actively searched for a job. We choose to classify all students as nonparticipants, in both retrospective and contemporaneous reports, when calculating transition rates.

employment, unemployment and nonparticipation, according to their reported current labour market status.⁹

By contrast, the LFS offers no simple way of estimating emigration outflows. For this reason, we proceed by taking official ONS measures of emigration outflows for the *entire* population, and use it to impute their counterpart for the *working-age* population. Specifically, for each year of data, we assume that the ratio of emigrant outflows to immigrant inflows in the working-age population mirrors its analogue for the total population in the official ONS figures. We then apply this ratio to our estimated working-age immigrant inflows to compute the working-age emigrant outflow. We then apportion these aggregate emigrant outflows to labour market states by immigrant status according to the shares of the latter in the working-age population.

The assumption, then, is that emigration is random with respect to labour force and immigrant status. There is some sporadic evidence to suggest that this may not hold. Accordingly, we examine the robustness of our results to variation in this assumption in Appendix B. We find that reasonable perturbations do not alter our results significantly.

We then turn to estimation of the gross population flows associated with 16-year-old entrants to, and 60/65-year-old exits from the working-age population. For both of these flows, we begin with measures of these age cohort sizes from the LFS micro-data, which in turn can be split into employment, unemployment and nonparticipation by immigrant status according to reported current and retrospective labour market states, and reported immigrant status. These numbers are then grossed up using official Office for National Statistics (ONS) weights to produce estimates for the respective populations of 16-year-olds and 60/65-year-olds.

Finally, we estimate gross population outflows due to deaths. Official statistics for age-specific death rates by year are published by the ONS. However, no information is published on labour market state and country of birth at time of death. The literature on mortality by immigrant and labour market status is rather sparse. In what follows, we augment the ONS age-specific death rates according to the range of estimates given in Dunlavy, Juarez and

⁹ The LFS derived inflow series and the ONS inflows derived from the International Passenger Survey are broadly in line until 2017, when the two series begin to diverge. See Wadsworth (2019).

Rostila (2018), who provide mortality estimates by country of birth and labour market status, albeit for Sweden.¹⁰ The size of these estimated working-age deaths are much smaller than the immigration or birth cohort population flows and so are unlikely to influence greatly the calibrations we present below, though we again test our sensitivity of the results to variants in these assumptions in Appendix B.¹¹ More detail of these calculations and data sources are provided in Appendix A.

4.3 Results

We now integrate this picture of the evolution of population dynamics into the parallel evolution of labour market dynamics over the sample period.

Labour market stocks. Figure 2 begins by contrasting the performance of labour market stocks among immigrant and UK-born populations. This reveals three broad messages.

First, there are clear levels differences between the labour market outcomes of immigrant and native individuals in the United Kingdom. In particular, averaged over the full sample window, immigrants are more likely to be unemployed, much less likely to be employed, and much more likely to be out of the labour force.

Second, immigrants historically have faced much more variation in their labour market outcomes, both at cyclical and medium-run frequencies. Their unemployment rates have historically risen more in recessions, mirrored by larger cyclical declines in employment. This is consistent with the findings of Dustmann, Glitz and Vogel (2010). Over the medium-run, immigrant nonparticipation rates rose as a share of their sub-population in the first half of the sample, and have fallen considerably in the latter half. These trends can in turn be seen in mirror image in the immigrant employment rate series.

A third message of Figure 2, however, is that more recent data paint a picture of *convergence* between migrant and native individuals. Remarkably, this can be seen across the board: the levels of unemployment, employment and nonparticipation as a fraction of

¹⁰ The authors find an approximate 50 percent excess mortality rate for the unemployed in the population of 25- to 64-year-olds, that appears to be offset by foreign-born status. Immigrants in employment have significantly lower mortality rates than Swedish-born employed.

¹¹ Average yearly deaths among the working-age population are around 80,000. Average yearly emigrant flows are around 300,000, and average yearly 65/60-year-old cohort outflows are around 700,000.

the working-age population have converged, and the cyclicalities of immigrant unemployment was comparable to that of natives in the recent Great Recession.

Labour market and population flows. Figures 3, 4, and 5 then track the transition rates that underlie these changes in labour market stocks by migrant status, providing a qualitative sense of the flow origins of these outcomes, and of the role of population flows in this mix. The six panels in Figure 3 plot the evolution of the transition probabilities between the usual labour force states of employment (E), unemployment (U), and nonparticipation (N) in the aggregate, and separately for immigrants and the UK-born population. Figure 4 summarises the overall magnitude of population flows. The six panels in Figure 5 then plot the inflows to, and outflows from, each labour force state due to population change, expressed as a fraction of the respective working-age sub-populations. The interaction between the labour market dynamics in Figure 3, and the population dynamics in Figures 4 and 5, is rich and complex and will be the subject of formal analysis in the ensuing sections. Transition rates across labour market states vary by age, but this matters less for an analysis of changes over time, if the age shares in population, or the transition rates at each age, are stable.¹² However if the population changes then we would expect aggregate flows to change in relative importance according to where in age distribution population is changing.¹³

Labour market flows. We begin by returning to the picture of labour market stocks in Figure 2, which highlighted three key differences between the labour market experiences of immigrants and the UK-born: in their *levels*, in their historical *cyclicalities*, and in the recent *convergence* of their labour market stocks. A first contribution of the flows depicted in Figure

¹² For example, the transition from nonparticipation to employment was dominated by 16 year olds in the 1970s, when most people left school and entered work and the baby boom cohorts entered the labour market. As tertiary education rates have risen over time and the baby boom cohorts aged, the dominance of 16 year olds in the N to E transition has fallen. Results showing the changing age shares of each transition rate over time are available on request.

¹³ Perry (1970), Shimer (1998), and Barnichon and Figura (2013) use shift-share approaches to study the related question of the effects of changes in the age composition of the labour force on trend and cyclical changes in the US unemployment rate. Immigration-driven population change is likely different, occurring along the age distribution at any given point in time, rather than rippling through the age distribution over time from age 16, (see Appendix D). However, by explicitly modelling population entry and exit, our flows framework captures to some extent the effects of age composition. Further disaggregation by age is precluded by the sample sizes of available data.

3, then, is to provide an account of the flow origins of these differences. And the qualitative account turns out to be a relatively simple one.

Consider first the *levels* differences between immigrants and the UK-born. Figure 3 identifies two clear sources for the higher rate of unemployment among immigrants: they typically face both higher rates of job loss to unemployment (E to U), and, for much of the sample, lower rates of job finding from unemployment (U to E). In turn, both of these differences additionally help to explain why employment rates are so much lower among immigrants in Figure 2. But the flows in Figure 3 also reveal another crucial difference: immigrants typically have been much less likely to transition from nonparticipation to employment (N to E). In addition to contributing to lower immigrant employment rates, this helps to explain why nonparticipation rates are so much higher among immigrants in Figure 2.

Differences in the patterns of these transition rates over time also help to give a sense of the origins of the historical differences in the *cyclicalities* of labour market stocks between the two groups. In particular, all of the aforementioned differences in flows are exacerbated in times of recession. Rates of job loss have risen more, and rates of job finding fallen more, among immigrants in each of the recessions in the sample, especially in the early 1980s and early 1990s. Interestingly, this differential cyclicity in job loss and job finding is apparent both in flows between unemployment and employment, as well between nonparticipation and employment.

Similar forces also provide a simple account for the recent *convergence* of the labour market experience of immigrants and natives. The last fifteen years of the sample period were accompanied by a broad-based convergence of labour market flows between the two groups. Although some differences remain—immigrants still face higher rates of entry into unemployment, for example—these differences have narrowed since the turn of the century. Indeed, some differences have reversed. Most strikingly, average job finding rates are now *higher* among immigrants than the UK-born. This is true not only for unemployed immigrants, but also for those out of the labour force, among whom the rate of job finding has exhibited a remarkable upward surge since the early 2000s. These profound trends have contributed to convergence in both the levels of unemployment, employment and nonparticipation between the two groups, as well as their cyclicity.

In addition to elucidating these forces, Figure 3 yields further novel insights into the labour market experience of immigrants. One such example is that immigrants have exhibited greater rates of transition between unemployment and nonparticipation—both the N-to-U and U-to-N flows tend to be higher among immigrants in Figure 3. This finding is consistent with the intuitive notion that immigrants exhibit weaker attachment to the labour force relative to the UK-born.

Population flows. Figures 4 and 5 provide a first snapshot of a key contribution of this paper, namely documenting the behaviour of the population flow rates for each group. Together, they provide new insights into the *levels* of gross population flows, their *cyclicalities* (and, thereby, their co-movement with the cyclical labour market dynamics documented above), and the recent emergence of increased population *churn* among immigrants. We now document each of these three key features of the data.

Figure 4 begins by providing a sense for the *levels* of gross population flows. It reports the shares of annual gross population and labour market flows among all transitions, averaged over the 1975 to 2016 sample period. Gross population flows are large—they account for a significant fraction of total labour reallocation in the United Kingdom. Together, population entry and exits account for nearly one half of all transitions among immigrants, and nearly one third of all transitions among the UK-born. The flow of population entrants into nonparticipation (A to N) is the third largest of all annual flows among the UK-born, and is also, typically, the modal flow among the immigrant population. Strikingly, almost half of all population inflows into employment (A to E) originate from immigrants. By contrast, population entrants into nonparticipation (A to N) are dominated by UK-born inflows (that is, 16-year-olds who turn working age while still in full-time education). Importantly for what follows, observe that gross population inflows generally are larger than gross population outflows over the sample period, especially among immigrants, reflecting the net population growth highlighted earlier. As a result, we will see that population inflows will tend to loom larger in labour market dynamics.

Figure 5 then plots the time series each of the gross population flows by migrant status over the sample period. In addition to reiterating the magnitude of population flows, a key first impression of Figure 5 is that there is also considerable time variation in these flows. We highlight two dimensions to this time variation. The first is that the population flows

exhibit a degree of *cyclicality*. This is most apparent in two episodes, corresponding to the cycles surrounding the 1980s recession, and the Great Recession. Moreover, both episodes suggest a narrative whereby the evolution of population flows contributed to the cyclicality of labour market stocks of both UK-born and immigrant populations.

An especially simple narrative emerges for the 1980s episode. The most prominent feature of the population flows during this period is a large spike in population inflows into unemployment (A to U) in Figure 5. This is especially apparent among the UK-born. For them, the maturation of the large baby-boom cohorts coincided with a deep recession, frustrating the job-finding prospects of school leavers. Similar trends also are apparent among immigrant population flows. These too exhibit a spike in new immigrant inflows to unemployment (A to U). But also apparent is a gradual fall in immigrant inflows into employment (A to E). These features provide a first glimpse of the interaction between population flows and the cyclicality of labour market stocks: We will see that population flows contributed to the rises in unemployment, and falls in employment, seen among both groups during the 1980s recession.

The later episode surrounding the Great Recession was not accompanied by large shifts in population flows among the UK-born. However, the large growth in immigration in this later period was accompanied by cyclical gross population flows among immigrants. Figure 5 suggests two cyclical features around the Great Recession. Most prominent is a large decline in population inflows into employment (A to E) among immigrants that followed in the wake of the first wave of A8 migrants who were allowed access to the United Kingdom from 2004. In addition, immigrant population inflows into unemployment (A to U) rose after the onset of the Great Recession in 2007. Both of these forces naturally contribute to decline in employment, and rise in unemployment, among immigrants during the Great Recession, a point we will confirm more formally later.

The recent evolution of the population flows in Figure 5 also conveys a final message—that recent immigration was accompanied by increased *churn* of immigrant populations. Immigrant population flows reached their peak in the immediate aftermath of the accession of the A8 countries to the European Union in 2004. A distinctive characteristic of this wave of immigration, however, was its association with elevated rates of *both* population inflows

and population outflows—that is, greater churn—consistent with a greater incidence of return migration among these new EU migrants.

In summary, UK population growth over the past forty years has been substantial, driven first by rises in the UK-born population and then rises in the immigrant population. The population flows that underlie this growth are large in magnitude, and have co-varied with the UK business cycle. Most recently, growth in immigration has further been associated with increased population churn. These insights underscore the value of the measurement of gross population flows.

A host of questions emerge from these observations. How do population dynamics interact with labour market dynamics? To what extent does variation in population flows contribute to variation in labour market outcomes? And how does this vary across UK-born and immigrant subpopulations? These are the questions we take up in what follows.

5. Accounting for the role of population flows

The preceding section made a qualitative case for the role of movements in population flows in shaping labour market stocks. In this section, we seek to provide a sense of how to gauge the quantitative magnitudes of this role.

5.1 Decompositions by migrant status

To do so, we return to the normalized Markov chain in equation (4). To recap, changes over time t in the vector of labour market states \mathbf{s} by migrant status k , \mathbf{s}_{kt} , are shaped by transition rates across labour market states among the existing working-age population, summarized in the probability matrix \mathbf{P}_{kt} , and net inflows due to population growth into each state, summarized by the vector \mathbf{q}_{kt} . We then apply the stock-flow variance decomposition developed in Elsby, Hobijn and Sahin (2015) to the data documented above. Specifically, the Markov chain described in equation (4) converges to a flow steady state

$$\bar{\mathbf{s}}_{kt} = (\mathbf{I} - \mathbf{P}_{kt})^{-1} \mathbf{q}_{kt}. \quad (6)$$

Dynamic convergence to this flow steady state can in turn be expressed in the form of a partial adjustment mechanism,

$$\Delta \mathbf{s}_{kt} = \mathbf{A}_{kt} \Delta \bar{\mathbf{s}}_{kt} + \mathbf{B}_{kt} \Delta \mathbf{s}_{kt-1}, \quad (7)$$

where $\mathbf{A}_{kt} \equiv (\mathbf{I} - \mathbf{P}_{kt})$ and $\mathbf{B}_{kt} \equiv (\mathbf{I} - \mathbf{P}_{kt})\mathbf{P}_{kt-1}(\mathbf{I} - \mathbf{P}_{kt-1})^{-1}$. The first term on the right-hand side of (7) reflects contemporaneous changes in discrete-time transition probabilities – the elements p_{ijkt} of the transition matrix \mathbf{P}_{kt} – which shift the flow steady state $\bar{\mathbf{s}}_{kt}$. The second term summarises the impact of past changes in transition probabilities on the current labour market state.

To understand how the contribution of transition probabilities to labour market dynamics can be estimated, it is helpful to reformulate (7) as a distributed lag of growth in steady-state labour market stocks. By repeated substitution (7) becomes

$$\Delta \mathbf{s}_{kt} = \sum_{\ell=0}^{t-1} \mathbf{C}_{k\ell t} \Delta \bar{\mathbf{s}}_{kt-\ell} + \mathbf{D}_{kt} \Delta \mathbf{s}_{k0}, \quad (8)$$

where $\mathbf{C}_{k\ell t} \equiv (\prod_{n=0}^{\ell-1} \mathbf{B}_{kt-n})\mathbf{A}_{kt-\ell}$, $\mathbf{D}_{kt} \equiv \prod_{\ell=0}^{t-1} \mathbf{B}_{kt-\ell}$ and $\Delta \mathbf{s}_{k0}$ is the change in labour market stocks in the first period of available data.

The flow steady states in this distributed lag adjust immediately to any change in the transition probabilities p_{ijkt} and the population flows q_{ikt} . The $\mathbf{C}_{k\ell t}$ coefficients then embody the effect of all the histories of transition probabilities. The influence of changes in each flow on labour market dynamics can then be approximated by taking a first-order expansion to the change in the flow steady state labour market stocks,¹⁴

$$\Delta \bar{\mathbf{s}}_{kt} \approx \sum_{i \neq j} \frac{\partial \bar{\mathbf{s}}_{kt}}{\partial p_{ijkt}} \Delta p_{ijkt} + \sum_i \frac{\partial \bar{\mathbf{s}}_{kt}}{\partial q_{ikt}} \Delta q_{ikt}. \quad (9)$$

The derivatives $\partial \bar{\mathbf{s}}_{kt} / \partial p_{ijkt}$ and $\partial \bar{\mathbf{s}}_{kt} / \partial q_{ikt}$ can be calculated analytically using (6) and standard rules for matrix derivatives. Inserting the approximation (9) into the distributed lag (8) allows one to express changes in labour market stocks by migrant status linearly in changes in the flows, and thereby allows a decomposition of the variance over time in the stocks into contributions accounted by each flow p_{ijkt} and q_{ikt} .¹⁵

5.2 Aggregate decomposition

In addition to delivering stock-flow decompositions for each subgroup—in this case, immigrants and the UK-born—the analytical framework can be extended to assess how each

¹⁴ Note that the steady-state stocks, unlike the actual stocks, are functions solely of the current transition rates. This fact lends itself to the decomposition above.

¹⁵ The decomposition is a straightforward extension of that suggested by Fujita and Ramey (2009).

of the subgroup flows contributes to the variation in *aggregate* labour market stocks. Intuitively, aggregate labour market dynamics are shaped by aggregate labour market and population flows which, in turn, are weighted sums of subgroup flows. The aggregate decomposition formalises this logic.

Although rising, immigrants comprise a modest share—averaging around 12 percent over the sample window —of labour force stocks, ¹⁶ limiting their impact on aggregate labour market dynamics. Nevertheless, we present the aggregate decomposition for two reasons. First, it quantifies how immigrant flows percolate to the aggregate, and how changes in both the extent of immigration, and its composition across labour force states, can amplify, or moderate, the contributions of immigrant flows relative to their base shares of labour force stocks. Second, from a methodological perspective, the method we develop can be applied to study the role of composition across any given set of subgroups, beyond our focus on migrant status – for example, by age, gender, ethnicity, skill or region – a feature we hope will be useful for future work.

The individual contributions of the subgroup transitions reflect both the (time-varying) relative share of immigrants in the aggregate, as well as the differing importance of each transition path across the two groups. For example, the aggregate job loss transition probability p_{EU} can be rewritten as the weighted sum of migrant and native job loss transition probabilities,

$$p_{EUt} = \varepsilon_t p_{EUmt} + (1 - \varepsilon_t) p_{EUnt}, \quad (10)$$

where $\varepsilon_t \equiv E_{mt-1}/E_{t-1}$ is the share of immigrants in employment at time $t - 1$. Analogous expressions hold for each of the remaining gross labour market and population flows.

If each group's transition probabilities were identical, equation (10) and its analogues would be degenerate, and each group's contribution to the aggregate would be proportional to their share in the respective labour market stocks. If not, equation (10) and its analogues provide a mechanism for gauging the extent to which the contributions of subgroup flows deviate from their shares in the stocks.

¹⁶ The respective sample average shares of immigrants among the unemployed, employed, and those out of the labour force over the full sample period are, respectively, 12 percent, 9 percent, and 12 percent.

The law of motion for the aggregate stock of employment, for example, can be expanded as follows (noting that $p_{EEKt} = 1 - p_{EUkt} - p_{ENkt}$),

$$E_t = [\varepsilon_t p_{EEmt} + (1 - \varepsilon_t) p_{EEnt}] E_{t-1} + [v_t p_{UEmt} + (1 - v_t) p_{UEnt}] U_{t-1} \\ + [\eta_t p_{NEmt} + (1 - \eta_t) p_{NEnt}] N_{t-1} + [\omega_t a_{Emt} + (1 - \omega_t) a_{Ent}] Pop_{t-1}, \quad (11)$$

where, mirroring the definition of ε_t above, $v_t \equiv U_{mt-1}/U_{t-1}$, $\eta_t \equiv N_{mt-1}/N_{t-1}$, and $\omega_t \equiv Pop_{mt-1}/Pop_{t-1}$ are the shares of migrants in the respective stocks of employment, unemployment, nonparticipation, and the working-age population.¹⁷

Since the aggregate equivalent of (6) is

$$\bar{\mathbf{s}}_t = (\mathbf{I} - \mathbf{P}_t)^{-1} \mathbf{q}_t, \quad (12)$$

all terms in \mathbf{P}_t and \mathbf{q}_t can be similarly expressed as weighted averages of subgroup probabilities and flow transition rates. This allows the total differential of the steady state stocks $\bar{\mathbf{s}}_t$ with respect to a given transition probability p_{ijt} to be disaggregated as

$$\sum_{i \neq j} \frac{\partial \bar{\mathbf{s}}_t}{\partial p_{ijt}} \Delta p_{ijt} = \sum_{i \neq j} \frac{\partial \bar{\mathbf{s}}_t}{\partial p_{ijmt}} \Delta p_{ijmt} + \sum_{i \neq j} \frac{\partial \bar{\mathbf{s}}_t}{\partial p_{ijnt}} \Delta p_{ijnt}. \quad (13)$$

The transition rate contributions of the subgroups thus sum to the total contribution of each transition rate. Analytical expressions for subgroup derivatives with respect to the overall steady state $\partial \bar{\mathbf{s}}_t / \partial p_{ijkt}$ based on (9) and (13) can be obtained, and hence their contribution to overall labour market dynamics calculated, for both migrants and natives.¹⁸

5.3 Results

The data identify two distinct episodes of accelerated population growth, coincident with two large recessions—in the late 1970s and 1980s, and the recent Great Recession. Population growth was driven by two sources. First, the entry of UK-born baby boom birth-year cohorts then, latterly, immigration. This motivates two cuts of the data that we pursue: by population growth episode (1978-88 and 2007-16),¹⁹ and by immigrant status.

Figure 2 confirms that these episodes are ripe for a study of the interaction of labour and population flows, with considerable cyclical variation in unemployment, employment, and nonparticipation. Accordingly, we apply the decompositions developed above to each of

¹⁷ Since the shares are measured at time $t - 1$, they are predetermined in the decomposition calculations.

¹⁸ See appendix C for derivations.

¹⁹ These episodes also correspond to full cycles of the unemployment rate, from trough to trough.

these episodes. Mirroring the qualitative insights in Figures 3 and 5, we will see that changes in population flows contributed importantly to UK-born labour market volatility in the 1980s. Most strikingly, however, we will show that population flows are fundamental to the evolution of immigrant labour market stocks, and increasingly so over time.

The decomposition results for immigrants are reported in Table 1, with analogous results for the UK-born in Table 2. These summarise the contribution of each gross flow, based on equation (9), in accounting for the change in each of the three labour market states over the two episodes. The variance contributions of labour market flows are grouped in the upper panels, and those of population flows are in the lower panels.

Population flows. We begin by summarising the contributions of gross population flows. As we have foreshadowed, these loom especially large in the labour market dynamics of immigrants, and so we describe the population flows results for immigrants in Table 1 first, and then turn to those for the UK-born in Table 2.

Immigrant population flows. Table 1 delivers a central contribution of the analysis. It reveals that gross population flows play a substantial role in explaining variation in all three immigrant labour market states, and more so in periods when immigration has accelerated.

A first message of Table 1 is that the contribution of gross population flows to immigrant labour market volatility is dominated by the variation in population inflows—primarily to employment, but also to unemployment. This provides some quantitative support to the qualitative impression of Figure 5 that A to E and A to U flows are both larger in magnitude, and exhibit more considerable time variation, during these episodes.

Table 1 also echoes a further impression of Figure 5—that population inflows, in particular those from A to E and A to U, contributed in a nontrivial way to the substantial variation in immigrant labour market stocks in the 1980s seen in Figure 2. 10 percent of the large fall and then rise in the immigrant employment rate, and 20 percent of the countercyclical movement in the immigrant nonparticipation rate, can be traced to the variation in population inflows during this early episode.

But the key message of Table 1 is that population flows have played a dominant role in immigrant labour market dynamics in the later 2007-2016 episode surrounding the Great Recession and its aftermath. Again, as foreshadowed by Figure 5, population inflows, especially those into employment, account for the lion's share of the contribution. Population

inflows account for over 14 percent of the rise and subsequent fall in immigrant unemployment in the 2007-2016 cycle. But, most notably, they account for over *one third* of the variation in immigrant employment rates, and over *40 percent* of the variation in immigrant nonparticipation rates in the Great Recession and its wake.

The upshot, then, is that population flows play a key role in the labour market dynamics of immigrant populations in the United Kingdom. Moreover, these flows are a force that *exacerbates* the considerable labour market volatility faced by migrants, amplifying the cyclical responses of immigrant unemployment, employment and nonparticipation. An integrated study of labour market and population flows thus adds considerable insight into the labour market outcomes experienced by UK immigrants.

UK-born population flows. Consider now the population flows results for the UK-born reported in the lower panels of Table 2. These again confirm the qualitative picture conveyed in Figure 5, in several senses.

First, the effects of population flows are concentrated in the earlier 1978-1988 economic cycle among the UK-born, when the downturn coincided with the entry of large baby boom cohorts into the working-age population. This left a particular imprint on the paths of unemployment and nonparticipation among the UK-born during this period.

Second, as with immigrants, the contributions of gross population flows among the UK-born are again dominated by variation in inflows. Population inflows account for 8 percent of the large rise and fall of UK unemployment in the 1980s, driven entirely by the large spike in population inflows to unemployment (A to U) during that period apparent in Figure 5. An echo of this trend can be seen in the evolution of nonparticipation: Population inflows, in particular A to N flows, account for 16 percent of the variation in the nonparticipation rate, which rose and then fell by around 3 percentage points between 1978 and 1988.

The baby boomers thus entered a labour market at the onset of a large recession; rather than find work, many were channelled into unemployment and nonparticipation. The rise in nonparticipation was mitigated in part by a rise in participation outflows among the cohorts of older workers reaching retirement age in this same period. There is a much lower role for population flows in the latter period. Fluctuations in UK-born birth cohorts did not coincide with the timing of the Great Recession.

Labour market flows. A further contribution of Tables 1 and 2 is to provide additional perspective on the labour market flows of immigrant and UK-born populations.

Since the UK-born account for the bulk—around 90 percent—of UK labour market stocks, the labour market flow contributions in Table 2 largely confirm the results of earlier studies of labour market flows among the UK population as a whole, albeit without accommodating population flows (Petrongolo and Pissarides 2008; Elsby, Smith and Wadsworth 2011; Smith 2012).²⁰ Consistent with those studies, Table 2 finds that the bulk of variation in unemployment, employment, and nonparticipation is traced to variation in rates of job loss and job finding. And, as in earlier studies, of these, job loss is the larger (and growing) contributor to unemployment variation. The contribution of the participation margin in explaining unemployment variation among the UK-born is smaller, at around 8 to 13 percent. This, too, is consistent with aggregate UK results reported in Elsby, Smith and Wadsworth (2011).²¹

The novel contribution of Tables 1 and 2 to our understanding of labour market flows lies instead in the *juxtaposition* of their behaviour among immigrants and the UK-born. Contrasting Tables 1 and 2 reveals that the labour market experience of immigrants has a distinctive character.

First, as for the UK-born, job loss is the main contributor to immigrant unemployment variation, particularly via E to U flows. Nearly 60 percent of the variation in immigrant unemployment in the Great Recession episode is accounted for by variation in this job loss flow. In contrast to the UK-born, however, rates of job finding contribute relatively little to explaining unemployment among immigrants. Moreover, the contributions of the job finding flows have fallen noticeably over time, to as little as 3 percent in the 2007-16 period. This confirms the impression from Figure 3 that the job finding rates for immigrants did not alter

²⁰ Elsby, Smith and Wadsworth's (2011) decomposition of the variation in aggregate *steady-state* unemployment for the 1975-2010 period estimates that 70 percent is accounted for by job loss. Similarly, Smith (2012) estimates the job loss contribution to variation in aggregate *actual* unemployment over the period 1992 to 2007 to be approximately 70 percent. In Petrongolo and Pissarides (2008), the relative contributions of job loss in explaining aggregate steady-state unemployment variation vary from 30 to 50 percent, depending on the time period studied.

²¹ The participation margin appears to be more prominent in the United States: Elsby, Hobijn and Sahin (2015) report US variance contributions around twice those reported here for the United Kingdom.

much during the Great Recession, and so do not explain much of the rise in immigrant unemployment during this time.

Second, it is also apparent that the participation margin between unemployment and nonparticipation looms large for immigrants, accounting for more than 20 percent of the variation in unemployment. Immigrants appear to be more fluid across the participation margin, and in a cyclical fashion. The suggestion, then, is that labour market attachment among immigrants tends to be weaker, such that the participation margin binds more.

Aggregate decomposition. In our final set of results, we turn to the subgroup-to-aggregate decomposition derived in section 5.2. Recall that this provides a sense of the extent to which subgroups—in our case, immigrants and the UK-born—play a disproportionate role in contributing to variation in aggregate labour market stocks. Specifically, as we noted earlier, if worker flows were homogeneous across subgroups, their contributions to the aggregate will be in proportion to their shares in the respective stocks. For reference, the respective sample average shares of immigrants among the unemployed, employed and nonparticipants were 7 percent, 8 percent, and 9 percent between 1977 and 1988, and 14 percent, 18 percent, and 17 percent between 2007 and 2016. Immigrant flow contributions to aggregate labour market variation will deviate from these shares when their flows exhibit more, or less, cyclicity.

Table 3 reports the results of this exercise. It reports the percentage contribution of each aggregate labour market flow to the variation in each aggregate labour market state, and the share of that contribution accounted for by immigrant flows.

Table 3 reaffirms the impression of Table 2 of the importance of variation in job finding and job loss flows in shaping variation in aggregate unemployment over the sample windows. Job loss rather than job finding dominates the changes over the 2007-2016 cycle. Immigrant contributions in job loss are broadly proportional to their labour force shares. Importantly, though, they are under-represented in the contribution of job-finding flows. This echoes the earlier impression of Figures 2 and 3 that underscored *convergence* in unemployment rates of immigrants and UK-born workers. Table 3 suggests that part of the reason for such convergence is that job-finding rates among immigrants fell less during the Great Recession, and rose more in the subsequent recovery.

Turning to the variation in aggregate employment and nonparticipation, Table 3 indicates a more prominent role for job finding variation. Immigrants are again under-represented in the contribution of U to E flows, but contribute broadly in line with their share of the stocks in the job finding flows from N to E. The latter, in turn, are shown to be important in explaining aggregate variation in employment and nonparticipation.

Finally, the results for population flows in Table 3 indicate more modest contributions to changes in aggregate stocks relative to explaining changes in subgroup stocks. This is not so surprising given the results of Table 2, and that fact that the UK-born comprise the vast majority of labour force stocks over the sample. As in Table 2, we see that population inflows into unemployment (A to U) and nonparticipation (A to N) contributed respectively around 8 percent of the rise in unemployment, and 13 percent of the rise in nonparticipation, in the 1978-88 cycle. And, consistent with the narrative of the maturation of the baby boom cohorts, this contribution is dominated by the UK-born, with immigrants under-represented. However, in the later 2007-16 cycle, Table 3 reveals prominent contributions of population inflows into employment (A to E) and nonparticipation (A to N) to aggregate variation in the nonparticipation rate and, to a lesser extent, the employment rate. Consistent with the shifting influences of population change over the sample window, these later contributions are dominated by immigrants.

6. Summary and discussion

Economists have long realised that a better understanding of what drives changes in the labour market states can be gleaned from an examination of the associated worker flows. The existing literature has however largely overlooked the role of population change. Yet for any economy, like the UK, subject to large population change, it would seem pertinent to examine the role of gross population flows in shaping labour market stocks. This paper is the first attempt to accommodate the contribution of gross population flows, and their interaction with labour market flows, into a dynamic accounting framework, and to model the individual contributions of subgroups, in this case immigrant and native-born workers, to aggregate variation in labour market states.

Our results have yielded several contributions. They have underscored that UK population flows have been considerable, dominated first by the entry of the baby boom generations in the early 1980s, and then by accelerated immigration, particularly in the 2000s. This in turn helps explain cyclical variation in unemployment and employment rates over two different cycles that would be otherwise obscured by a dynamic analysis that ignored population flows. We have also documented new measures of labour market and population flows, uncovering both the flow origins of disparities in the levels and cyclicalities of immigrant and UK-born labour market outcomes, as well as their more recent convergence. And, we have devised a novel accounting framework that formalises these effects, and reveals that population flows have been an important contributor to UK labour market dynamics among the UK-born and, especially immigrants.

Our analysis suggests that aggregate explanations of the main drivers of change in a labour market stock may not always be appropriate for different sub-groups of the population. In addition to their very different population flow contributions, immigrants are under-represented in the contribution of job-finding transitions in explaining variations in unemployment over the cycle. This helps our understanding of the recent convergence in unemployment rates of immigrants and UK-born workers.

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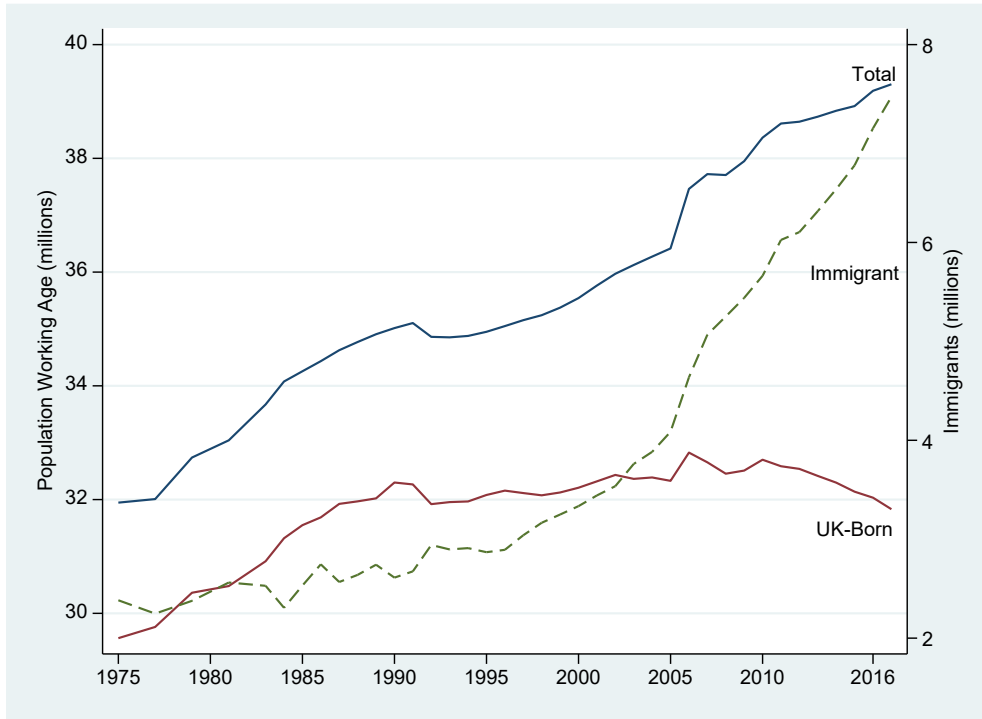
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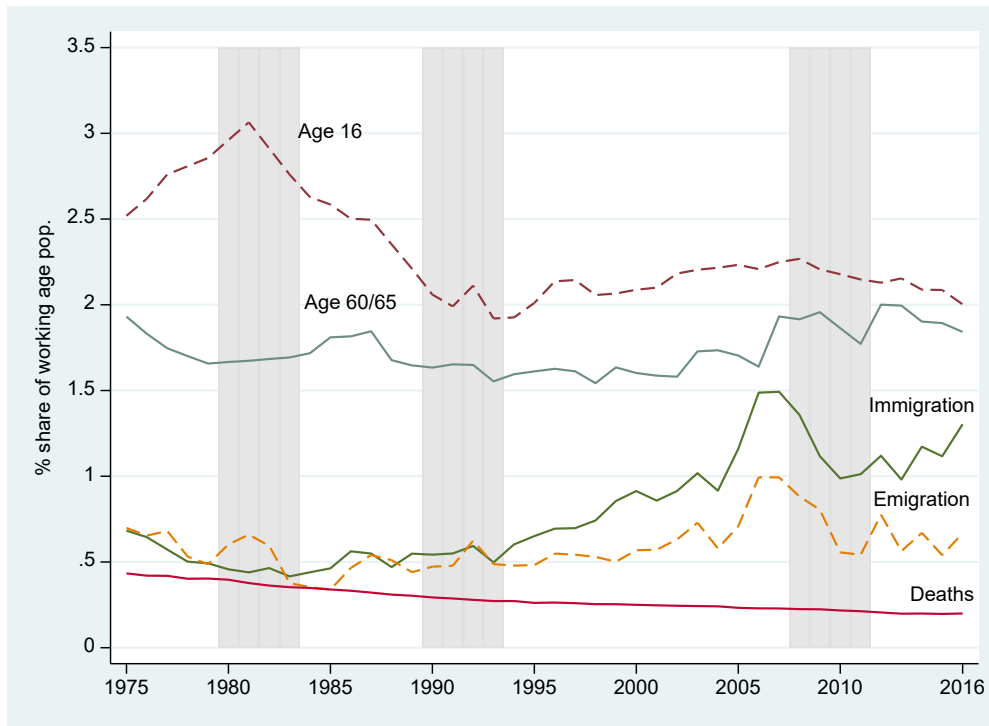
Yule, U. 1906. "On the Changes of Marriage and Birth Rates in England and Wales During the Past Half Century: With an Inquiry as to Their Probable Causes." *Journal of the Royal Statistical Society* LXIX: 88-147.

Figure 1. Working-age population dynamics, 1975 to 2016

A. Working-age population by immigrant status



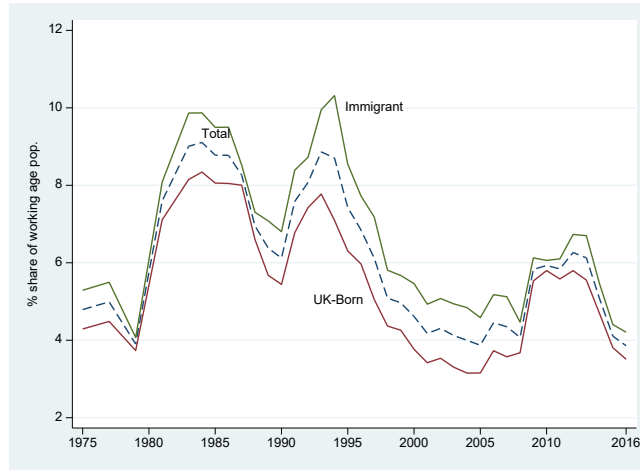
B. Annual population flows as a share of working-age population



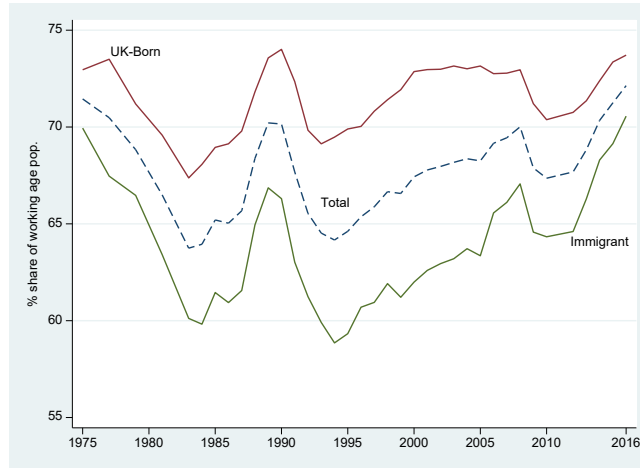
Source: LFS and ONS. Authors' calculations

Figure 2. Labour market stocks by immigrant status

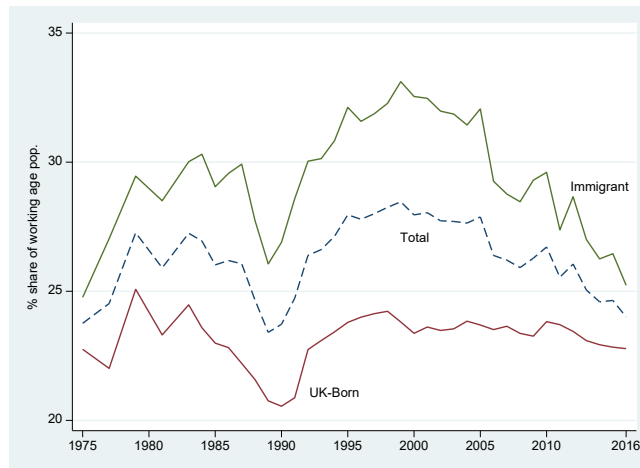
A. Unemployment-population ratio



B. Employment-population ratio



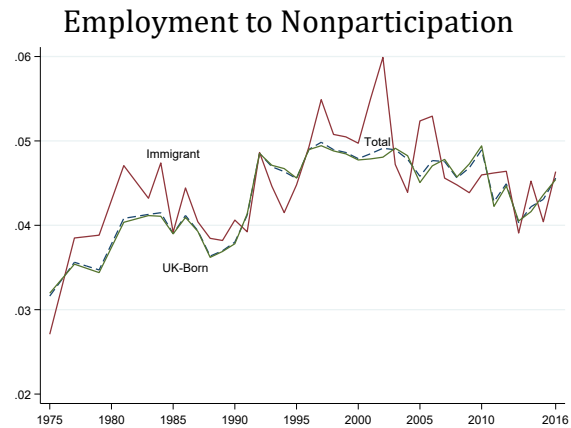
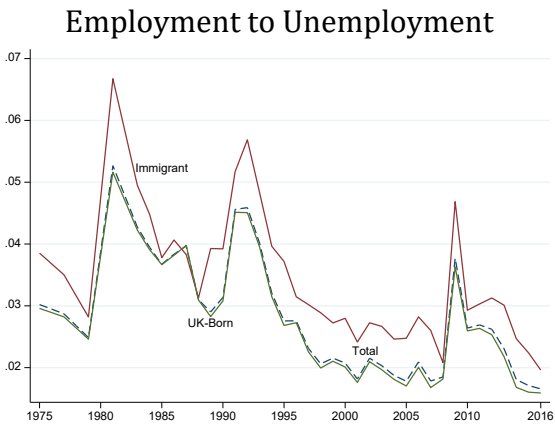
C. Nonparticipation-population ratio



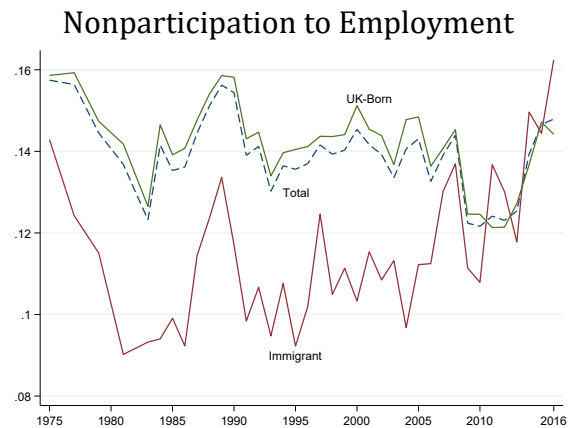
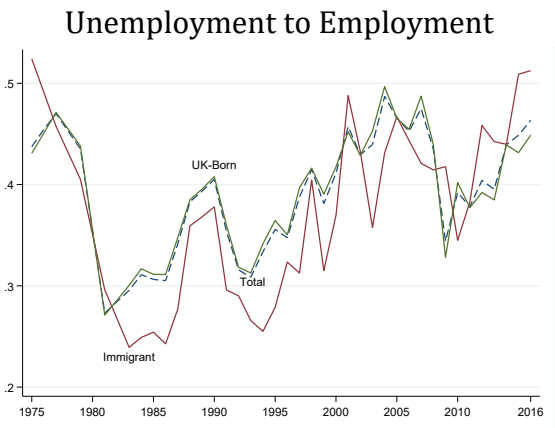
Source: LFS. Authors' calculations

Figure 3. Labour market transition probabilities by immigrant status, 1975 to 2016

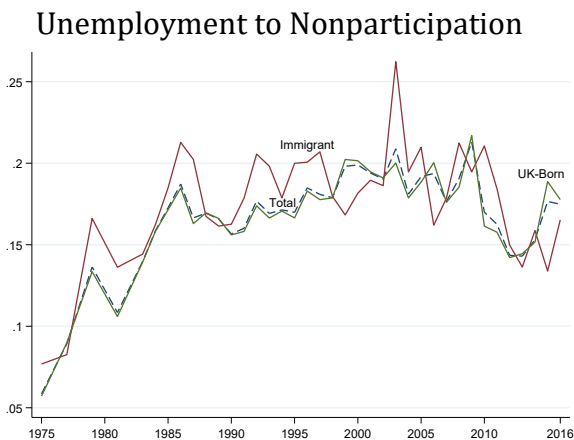
(i) Job loss



(ii) Job finding

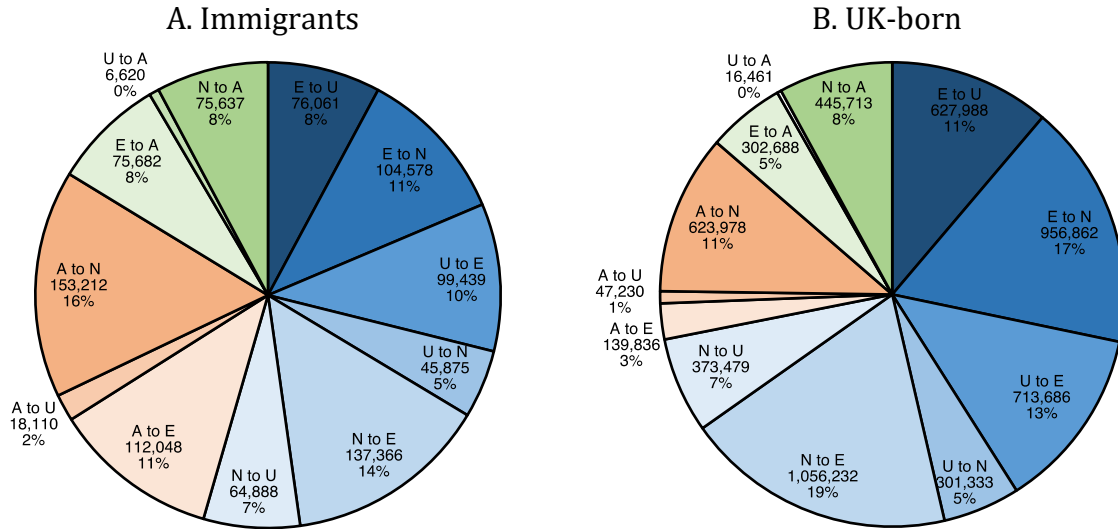


(iii) Participation margin



Source: LFS. Note: Transition rates are reported as a fraction of the respective origin stock of each group.

Figure 4. Gross population and labour market flows as a share of all flows



Source: LFS. Note: The annotations report both the annual levels, and percentages of total flows by migrant status, accounted for by each of the gross population and labour market flows, averaged over the years 1975-2016.

Figure 5. Gross population flows by immigrant status, 1975 to 2016

(i) Population inflows

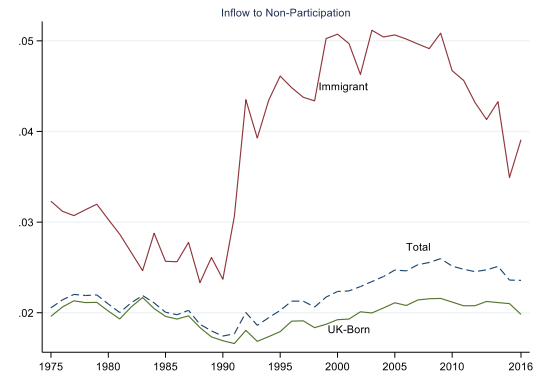
Inflow to employment (A to E)



Inflow to unemployment (A to U)

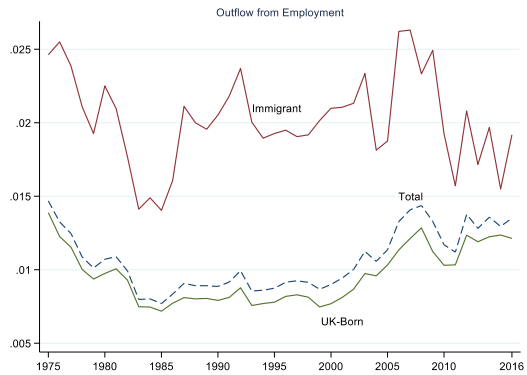


Inflow to nonparticipation (A to N)

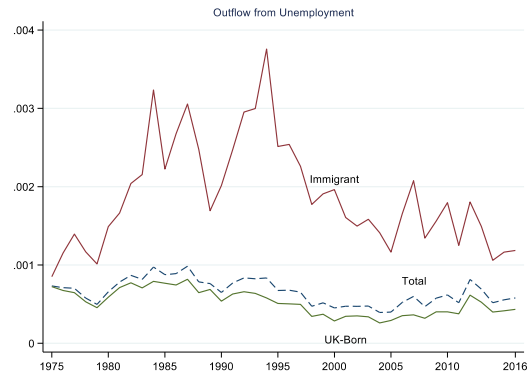


(ii) Population outflows

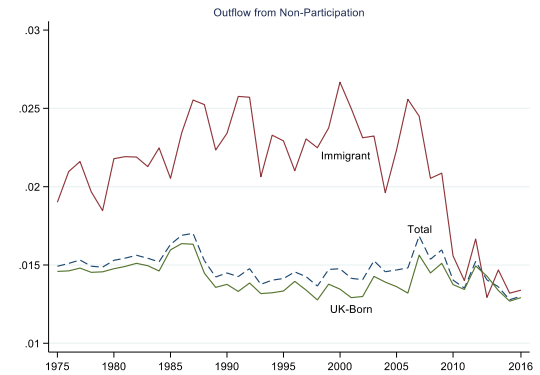
Outflow from employment (E to A)



Outflow from unemployment (U to A)



Outflow from nonparticipation (N to A)



Source: LFS. Note: Population flows by migrant status are reported as fractions of the respective working-age populations of each group.

Table 1. Flow variance contributions (percent) for two eras: Immigrants

	Unemployment		Employment		Nonparticipation	
	1978-88	2007-16	1978-88	2007-16	1978-88	2007-16
<i>Job loss</i>						
E to U	45.8	58.8	31.7	26.0	3.6	4.6
E to N	0.5	-0.1	14.4	-0.2	28.8	7.8
Total	46.3	58.7	46.1	25.8	32.4	14.4
<i>Job finding</i>						
U to E	25.4	4.6	18.3	7.6	-4.5	1.2
N to E	0.5	-1.4	19.7	26.1	32.8	41.1
Total	25.9	3.2	38.0	33.7	28.3	42.3
<i>Participation</i>						
U to N	8.4	7.2	0.2	2.3	1.6	5.2
N to U	14.3	17.8	6.4	0.3	13.9	-14.4
Total	22.7	25.0	6.6	2.6	15.5	-9.2
<i>Pop. inflows</i>						
A to E	2.5	5.7	7.4	26.0	11.7	37.5
A to U	3.0	8.5	1.1	3.1	1.0	-2.6
A to N	0.9	0.1	1.5	6.9	7.7	8.7
Total	6.4	14.3	10.0	36.0	20.4	43.6
<i>Pop. outflows</i>						
E to A	-0.8	0.9	-1.7	0.3	-3.5	-2.4
U to A	-0.9	-1.3	-0.3	0.2	0.3	0.2
N to A	0.6	-0.1	1.3	1.6	6.6	8.7
Total	-1.1	-0.5	-0.7	2.1	3.4	6.5
Pop. flows total	5.3	13.8	9.3	38.1	23.8	50.1

Notes. Author calculations using LFS. E=Employment, U=Unemployment, N=Nonparticipation, A=Abroad (out of population).

Table 2. Flow variance contributions (percent) for two eras: UK-born

	Unemployment		Employment		Nonparticipation	
	1978-88	2007-16	1978-88	2007-16	1978-88	2007-16
<i>Job loss</i>						
E to U	41.1	54.5	36.6	42.6	5.8	13.8
E to N	0.1	0.7	10.9	6.5	26.3	33.2
Total	41.2	55.2	47.7	49.1	32.1	47.0
<i>Job finding</i>						
U to E	37.5	36.1	35.2	26.7	3.3	8.7
N to E	0.3	-0.4	9.7	24.1	37.3	65.4
Total	37.8	35.7	46.9	50.8	40.6	74.1
<i>Participation</i>						
U to N	8.3	-6.2	-0.1	2.2	9.2	4.5
N to U	5.0	14.5	3.2	-0.7	5.8	-23.2
Total	13.3	8.3	3.1	1.5	14.0	-18.7
<i>Pop. inflows</i>						
A to E	-0.1	0.3	-0.2	0.9	0.9	2.4
A to U	8.0	0.6	5.1	0.2	1.3	-0.4
A to N	0.2	1.2	0.7	1.9	14.0	1.4
Total	8.1	1.2	5.6	3.0	16.2	3.4
<i>Pop. outflows</i>						
E to A	-0.1	-0.3	-0.3	-1.3	-3.6	-5.8
U to A	-0.5	-0.5	-0.4	-0.3	0.1	0.2
N to A	0.2	0.1	-0.4	-2.9	-0.3	-0.3
Total	-0.4	-0.7	-1.1	-4.4	-3.8	-5.9
Pop. flows total	7.7	0.5	4.5	-1.4	12.4	-2.5

Notes. Author calculations using LFS. E=Employment, U=Unemployment, N=Nonparticipation, A=Abroad (out of population).

Table 3. Aggregate variance contributions (percent) for two eras

	Unemployment		Employment		Nonparticipation	
	1978-88	2007-16	1978-88	2007-16	1978-88	2007-16
<i>Job loss</i>						
E to U	41.8	56.0	36.4	40.5	5.8	12.1
<i>Immigrant %</i>	<i>9.8</i>	<i>16.6</i>	<i>9.9</i>	<i>15.8</i>	<i>15.5</i>	<i>14.0</i>
E to N	0.2	0.6	11.1	4.3	27.7	22.3
<i>Immigrant %</i>	<i>50.0</i>	<i>16.6</i>	<i>11.7</i>	<i>-4.2</i>	<i>11.9</i>	<i>9.4</i>
<i>Job finding</i>						
U to E	36.6	31.8	33.7	23.1	2.3	7.0
<i>Immigrant %</i>	<i>5.7</i>	<i>2.8</i>	<i>6.5</i>	<i>7.4</i>	<i>-8.0</i>	<i>7.1</i>
N to E	0.4	-0.5	10.7	25.3	38.1	64.9
<i>Immigrant %</i>	<i>25.0</i>	<i>40.0</i>	<i>17.8</i>	<i>20.6</i>	<i>12.9</i>	<i>22.0</i>
<i>Participation</i>						
U to N	8.3	-4.7	-0.1	2.5	7.2	3.5
<i>Immigrant %</i>	<i>8.4</i>	<i>10.2</i>	<i>50.0</i>	<i>32.0</i>	<i>2.8</i>	<i>65.7</i>
N to U	5.6	14.9	3.5	-0.6	7.0	-21.9
<i>Immigrant %</i>	<i>14.3</i>	<i>18.1</i>	<i>14.3</i>	<i>20.0</i>	<i>-1.4</i>	<i>-16.9</i>
<i>Pop. inflows</i>						
A to E	0.1	1.1	0.3	4.5	1.9	11.1
<i>Immigrant %</i>	<i>200.0</i>	<i>45.5</i>	<i>71.4</i>	<i>75.0</i>	<i>73.7</i>	<i>82.0</i>
A to U	7.6	1.7	4.7	1.0	0.8	-0.7
<i>Immigrant %</i>	<i>2.6</i>	<i>58.8</i>	<i>2.1</i>	<i>60.0</i>	<i>12.5</i>	<i>114.3</i>
A to N	0.3	-0.3	0.8	3.3	12.5	5.1
<i>Immigrant %</i>	<i>33.3</i>	<i>133.3</i>	<i>12.5</i>	<i>54.5</i>	<i>0.1</i>	<i>76.4</i>
<i>Pop. outflows</i>						
E to A	-0.1	-0.2	-0.4	-1.7	-4.1	-4.4
<i>Immigrant %</i>	<i>100.0</i>	<i>100.0</i>	<i>25.0</i>	<i>23.5</i>	<i>-22.0</i>	<i>2.2</i>
U to A	-0.6	-0.5	-0.2	-0.3	0.2	0.2
<i>Immigrant %</i>	<i>16.6</i>	<i>20.0</i>	<i>50.0</i>	<i>33.3</i>	<i>50.0</i>	<i>50.0</i>
N to A	0.3	-0.1	-0.2	-2.5	0.9	0.8
<i>Immigrant %</i>	<i>33.3</i>	<i>100.0</i>	<i>50.0</i>	<i>4.0</i>	<i>122.2</i>	<i>175.0</i>

Appendix

A. Data sources

The labour market gross flow data used in this analysis are derived from the 2nd quarter of the UK Labour Force Survey (LFS). In this quarter, individuals are asked to recall their labour market status one year earlier. Comparison with current labour market status forms the basis of the gross transitions. We define working age as 16-64 for men and 16-59 for women. The population flows are also partly derived from the LFS and partly from external sources.

Immigration. The LFS elicits month and year of entry into the UK (for both UK nationals and non-nationals). We define a new arrival as anyone of working age in the UK for less than 1 year from the date of interview. Immigrants are split into employment, unemployment and nonparticipation according to self-reported current labour market state.

16-year-old entrants. New entrants to the working age population are defined by self-reported age in the LFS grossed up to population by the LFS population weights. Entrants are split into employment, unemployment and nonparticipation according to self-reported current labour market state. The totals are split by self-reported immigrant status. These estimates are then balanced against official Office for National Statistics (ONS) estimates for 16-year-olds found [here](#).

Emigration. The ONS publishes yearly estimates of emigration outflows (and immigration inflows) for *all* British and Non-British citizens (not working age) [here](#). Our working-age estimates for emigration are obtained by assuming the ratio of emigrants to immigrants is the same for the working-age and total populations and then using our estimated working age immigrant inflows to calculate the working age emigrant number.

65-year-old leavers. These are estimated directly from the LFS by self-reported age, immigrant status and retrospective labour market state. These estimates are then balanced against official Office for National Statistics (ONS) estimates for 60/65-year-olds found [here](#).

Deaths. Yearly age-specific death rates can be found at the ONS weblink [here](#). No information is given on labour market state at time of death. We therefore apportion deaths according to the estimates given in Dunlavy, Juarez and Rostila (2018).

Population grossing weights. Each individual surveyed in the LFS is given a specific weight which when multiplied across the sample aggregates to the UK population. The weight can be thought of as the number of people the individual represents. This weighting factor takes account of differential non-response to the survey (not specific questions in the survey)

among different subgroups in the population, based on age, gender and region, comparing the survey response of a particular age-gender-region cell with the expected response based on the Census. The grossing weights are benchmarked to population projections for each year based on the decennial Census. However, there is no disaggregation of the sample weights by migrant status. If the sample population suddenly falls – as the migrant sample population did in 2017 after Brexit and in 2020 during the pandemic – there is no correction to the underlying Census-based population projections. Instead, the weights change for *other* groups in the sample to equate the grossed survey population with the population projection for that year. See [here](#) for more details. This makes the grossed-up population subgroup estimates in the LFS after 2016 very uncertain. We therefore exclude these years from the analysis until this issue is resolved.

B. Robustness checks

Emigration weights. Recall that our baseline results proceeded under the assumption, that emigration is random with respect to labour force and immigrant status. There is some (albeit limited) evidence to suggest that this may not hold.

Bijwaard, Schluter and Wahba (2014) use Dutch register data to show that those unemployed comprise around between 40 and 50 percent of all emigrants from *immigrant* populations, and that exit rates rise with unemployment duration. No information is supplied on non-immigrant emigration patterns, however.

Recently-published UK Home Office exit checks for non-EEA nationals (Home Office 2017) suggest that student visas comprise two-thirds of all non-EEA migrant exits. Non-EEA nationals cannot be unemployed when their visa expires unless they already have UK citizenship. These patterns may not be representative of the exit patterns for EEA nationals, however, who were free to move in and out of the UK prior to 2021. Such EEA nationals comprised around one third of all immigrants in our sample in 2016.

Accordingly, in this appendix we investigate the sensitivity of these baseline results to alternative assumptions made about the (unknown) shares of workers by labour force state in emigration outflows. Table B.1 presents decomposition results (for the immigrant population) implied by using different weights on the share of employed, unemployed and nonparticipants in emigration, based on the hints from the literature cited in section 4.2. We allow for the possibility that the short duration of many student immigrant stays, and unsuccessful job search among unemployed EEA workers, may change the weights from the baseline assumption that emigration is random with respect to labour force status. The new weights double the share of nonparticipants in emigration outflows among the immigrant population from 0.26 to 0.52, and raise the share of unemployment from 0.02 (the

unemployment share in the immigrant working-age population over the sample window) to 0.12 (the maximum unemployment population share over the sample window).

Table B.1 shows that allowing unemployment to adapt over the cycle through more emigration of unemployed workers raises the overall contribution of population outflows to immigrant unemployment by around 10 percentage points in the 2007-2016 cycle, accompanied—as it must in the accounting identity—by small adjustments in all the other population inflows and labour market outflows. The increase in the population outflow contribution stems from small rises in each of the three constituent population outflows. The role of population outflows in explaining nonparticipation is also raised by a similar amount. However, population inflows into employment (A to E) and the job finding flow from N to E however, remain the dominant flows explaining employment and nonparticipation volatility among immigrants. Job loss remains the dominant flow in explaining unemployment volatility among immigrants. The weighting changes only matter when there is notable cyclical movement in population outflows. In the 1978-88 period, with less volatility in population outflows, the weighting changes matter less.

Ignoring population flows. To get a sense of the implications of ignoring population flows, as is customary in the existing literature, or more accurately subsuming population flows into the other six transition probabilities, Table B.2 documents the transition contribution estimates for immigrants based on a conventional six-flow transition matrix.²² The main effect of this re-organisation is to raise the contribution of job finding in the accounting decomposition. In the earlier 1978-88 episode, when immigration was stable, the omission of population flows does not make much difference to the decomposition. In the later 2007-16 episode, however, the omission of population flows appears to ascribe a much larger role for job finding; specifically, to U to E flows in explaining changes in unemployment, and to N to E flows in explaining changes in employment and nonparticipation. Table 1 suggests that this is because much of this N-to-E effect stems from direct population movements into employment from outside the country. Many more of the flows classified as from nonparticipation to employment were from migrant nonparticipants in their origin country. Both job loss and job finding roles in explaining immigrant unemployment variation are also inflated in a similar way.

²² The retrospective LFS data suggest that around 50 percent of all immigrant inflows were nonparticipants one year earlier—i.e., prior to migration—with a further 40 percent employed. Most population inflows would thus be counted among the employed or nonparticipants in the denominators of the transition rates.

Table B.1. Flow variance contributions (percent) for two eras: Immigrants

Robustness check: Different emigrant weights (E=0.36, U=0.12, N=0.52)

	Unemployment		Employment		Nonparticipation	
	1978-88	2007-16	1978-88	2007-16	1978-88	2007-16
<i>Job loss</i>						
E to U	48.9	55.1	31.6	25.4	4.3	4.7
E to N	0.6	-0.2	14.9	-0.5	29.1	8.6
Total	49.5	54.9	46.5	24.9	33.4	13.3
<i>Job finding</i>						
U to E	22.0	2.8	14.6	6.0	-3.5	1.1
N to E	0.5	-1.2	18.6	24.3	31.3	37.1
Total	22.5	1.6	33.2	30.3	27.8	38.2
<i>Participation</i>						
U to N	7.4	5.5	0.1	1.7	0.1	2.8
N to U	13.4	16.0	6.0	0.3	10.4	-13.0
Total	20.8	21.5	6.1	2.0	10.5	-10.2
<i>Pop. Inflows</i>						
A to E	2.4	4.0	6.9	25.1	10.9	35.7
A to U	3.5	10.3	1.8	4.1	1.6	-1.4
A to N	0.4	-2.0	0.9	5.4	6.8	8.1
Total	6.3	12.3	9.6	34.6	19.3	42.4
<i>Pop. Outflows</i>						
E to A	-0.1	3.3	0.1	0.8	-1.2	0.2
U to A	0.6	5.3	2.4	2.0	1.1	1.0
N to A	0.5	0.9	2.3	5.3	9.1	15.1
Total	1.0	9.5	4.8	8.1	9.0	16.3
Pop. flows total	7.3	21.8	14.4	43.7	28.3	58.7

Table B.2. Flow variance contributions (percent) for two eras: Immigrants

Robustness check: No population flows

	Unemployment		Employment		Nonparticipation	
	1978-88	2007-16	1978-88	2007-16	1978-88	2007-16
<i>Job loss</i>						
E to U	48.6	66.5	34.7	40.3	5.3	4.2
E to N	0.5	0.2	16.0	5.3	41.2	23.0
Total	49.1	66.7	50.7	45.6	46.5	27.2
<i>Job finding</i>						
U to E	30.0	18.5	22.2	13.9	-4.6	3.5
N to E	0.5	-1.5	21.4	36.8	46.2	82.7
Total	30.5	16.0	43.6	50.7	41.6	86.2
<i>Participation</i>						
U to N	6.5	0.1	0.4	2.4	2.9	4.6
N to U	14.0	16.1	5.2	1.3	9.1	-18.0
Total	20.5	16.2	5.6	3.7	12.0	-13.4

Appendix C1: Derivation of equation (7)

Equation(6):
$$\Delta \mathbf{s}_t = \mathbf{A}_t \Delta \bar{\mathbf{s}}_t + \mathbf{B}_t \Delta \mathbf{s}_{t-1}$$

Equation (7):
$$\Delta \mathbf{s}_t = \sum_{k=0}^{t-1} \mathbf{C}_{k,t} \Delta \bar{\mathbf{s}}_{t-k} + \mathbf{D}_t \Delta \mathbf{s}_0, \quad \text{where} \quad \mathbf{C}_{k,t} = \left(\prod_{n=0}^{k-1} \mathbf{B}_{t-n} \right) \mathbf{A}_{t-k}, \quad ,$$

$$\mathbf{D}_{k,t} = \prod_{k=0}^{t-1} \mathbf{B}_{t-k} .$$

At $t=0$

$$\Delta \mathbf{s}_0 = \Delta \bar{\mathbf{s}}_0$$

At $t=1$

$$\Delta \mathbf{s}_1 = \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_1 \Delta \bar{\mathbf{s}}_0$$

In general:
$$\Delta \mathbf{s}_t = \sum_{k=0}^{t-1} \left(\prod_{n=0}^{k-1} \mathbf{B}_{t-n} \right) \mathbf{A}_{t-k} \Delta \bar{\mathbf{s}}_{t-k} + \prod_{k=0}^{t-1} \mathbf{B}_{t-k} \Delta \mathbf{s}_0$$

$$\begin{aligned} \Delta \mathbf{s}_1 &= \sum_{k=0}^{1-1} \left(\prod_{n=0}^{k-1} \mathbf{B}_{1-n} \right) \mathbf{A}_{1-k} \Delta \bar{\mathbf{s}}_{1-k} + \prod_{k=0}^{1-1} \mathbf{B}_{1-k} \Delta \mathbf{s}_0 \quad \text{since } t=1 \\ &= \left(\prod_{n=0}^{-1} \mathbf{B}_{1-n} \right) \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_1 \Delta \mathbf{s}_0 \quad \text{since } k=0 \\ &= \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_1 \Delta \mathbf{s}_0 \quad \text{since } \prod_0^{-1}(\cdot) \text{ is the empty product with value 1} \end{aligned}$$

At $t=2$

$$\Delta \mathbf{s}_2 = \mathbf{A}_2 \Delta \bar{\mathbf{s}}_2 + \mathbf{B}_2 \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_2 \mathbf{B}_1 \Delta \bar{\mathbf{s}}_0$$

$$\begin{aligned} \Delta \mathbf{s}_2 &= \sum_{k=0}^{2-1} \left(\prod_{n=0}^{k-1} \mathbf{B}_{2-n} \right) \mathbf{A}_{2-k} \Delta \bar{\mathbf{s}}_{2-k} + \prod_{k=0}^{2-1} \mathbf{B}_{2-k} \Delta \mathbf{s}_0 \quad \text{since } t=2 \\ &= \underbrace{\left(\prod_{n=0}^{-1} \mathbf{B}_{2-n} \right) \mathbf{A}_2 \Delta \bar{\mathbf{s}}_2}_{k=0} + \underbrace{\left(\prod_{n=0}^0 \mathbf{B}_{2-n} \right) \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_2 \mathbf{B}_1 \Delta \mathbf{s}_0}_{k=1} \quad \text{since } k = \{0,1\} \\ &= \underbrace{\mathbf{A}_2 \Delta \bar{\mathbf{s}}_2}_{k=0} + \underbrace{\mathbf{B}_{2-0} \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1}_{k=1, n=0} + \mathbf{B}_2 \mathbf{B}_1 \Delta \mathbf{s}_0 \\ &= \mathbf{A}_2 \Delta \bar{\mathbf{s}}_2 + \mathbf{B}_2 \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_2 \mathbf{B}_1 \Delta \mathbf{s}_0 \end{aligned}$$

$$\text{At } t=3 \quad \Delta \mathbf{s}_3 = \mathbf{A}_3 \Delta \bar{\mathbf{s}}_3 + \mathbf{B}_3 \mathbf{A}_2 \Delta \bar{\mathbf{s}}_2 + \mathbf{B}_3 \mathbf{B}_2 \mathbf{A}_1 \Delta \bar{\mathbf{s}}_1 + \mathbf{B}_3 \mathbf{B}_2 \mathbf{B}_1 \mathbf{A}_0 \Delta \bar{\mathbf{s}}_0$$

Appendix C2: Derivation of the Sub-Group decomposition and Evaluation of Partial Derivatives

For any aggregate transition probability , for example

$$p_{eu} = \varepsilon_t p_{m,t}^{eu} + (1 - \varepsilon_t) p_{n,t}^{eu}$$

the total differential is

$$dp_{eu} = \varepsilon_t dp_m^{eu} + (1 - \varepsilon_t) dp_n^{eu}$$

The aggregate steady state stocks, \bar{s}_t , then become a function of 18 rather than 9 transition probabilities. The approximation of the steady-state changes using a first-order Taylor series becomes,

$$\Delta \bar{s}_t \approx \sum_{i \neq j} \frac{\partial \bar{s}_t}{\partial p_{ijkt}} \Delta p_{ijkt} + \sum_i \frac{\partial \bar{s}_t}{\partial a_{ikt}} \Delta a_{ikt},$$

The portion of the total differentiation of the steady state with respect to the transition rates p_{eu}^M and p_{eu}^{UK} is

$$d\bar{s} = \frac{\delta \bar{s}}{\delta p_m^{eu}} dp_m^{eu} + \frac{\delta \bar{s}}{\delta p_n^{eu}} dp_n^{eu}$$

which must by construction be equal to $d\bar{s} = \frac{\delta \bar{s}}{\delta p_{eu}} dp_{eu}$

- since $\bar{s} = f(x)$ and $x = x(x_1, x_2)$ implies $d\bar{s} = f_x \left(\frac{\delta x}{\delta x_1} dx_1 + \frac{\delta x}{\delta x_2} dx_2 \right)$ so that

$$d\bar{s} = \frac{\delta \bar{s}}{\delta p_{eu}} (\varepsilon_t dp_{eu}^M + (1 - \varepsilon_t) dp_{eu}^{UK}) = \frac{\delta \bar{s}}{\delta p_{eu}} dp_{eu}$$

Generalising to any transition p_{ij} the total differential with respect to a given transition probability can be disaggregated as

$$\left(\sum_{i \neq j} \frac{\delta \bar{s}}{\delta p_{ij}} \Delta p_{ij} \right)_t = \left(\sum_{i \neq j} \frac{\delta \bar{s}}{\delta p_{ij}^m} \Delta p_{ij}^m \right)_t + \left(\sum_{i \neq j} \frac{\delta \bar{s}}{\delta p_{ij}^n} \Delta p_{ij}^n \right)_t$$

ie the transition rate contributions of the sub-groups add to the total contribution of each transition rate.

The partial derivatives in each bracket on the right hand side can be evaluated as follows

Given $\bar{s}_{kt} = (\mathbf{I} - \mathbf{P}_{kt})^{-1} \mathbf{q}_{kt} \equiv \mathbf{A}_{kt}^{-1} \mathbf{q}_{kt}$

Using the product rule for matrix derivatives

$$\frac{\partial \bar{\mathbf{s}}_{kt}}{\partial x} = \mathbf{A}_{kt}^{-1} \frac{\partial \mathbf{q}_{kt}}{\partial x} + \mathbf{q}_{kt} \frac{\delta \mathbf{A}_{kt}^{-1}}{\delta x}$$

and the derivative of a matrix inverse

$$\frac{\delta A^{-1}}{\delta x} = -A^{-1} \frac{\delta A}{\delta x} A^{-1}$$

Gives

$$\frac{\partial \bar{\mathbf{s}}_{kt}}{\partial x} = \mathbf{A}_{kt}^{-1} \frac{\partial \mathbf{q}_{kt}}{\partial x} - \mathbf{A}_{kt}^{-1} \frac{\partial \mathbf{A}_{kt}}{\partial x} \mathbf{A}_{kt}^{-1} \mathbf{q}_{kt} = \mathbf{A}_{kt}^{-1} \left(\frac{\partial \mathbf{q}_{kt}}{\partial x} - \frac{\partial \mathbf{A}_{kt}}{\partial x} \bar{\mathbf{s}}_{kt} \right).$$

Given the group specific steady state decomposition $\bar{\mathbf{s}}_{kt} = (\mathbf{I} - \mathbf{P}_{kt})^{-1} \mathbf{q}_{kt}$

$$\text{with } \mathbf{q}_{kt} = \begin{bmatrix} a_{Ek} \\ a_{Uk} \\ a_{Nk} \end{bmatrix}, \quad \text{then } \frac{\partial \mathbf{q}_{kt}}{\partial a_{Ekt}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{q}_{kt}}{\partial a_{Ukt}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{q}_{kt}}{\partial a_{Nkt}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The aggregate equivalent of \mathbf{q}_{kt} is

$$\mathbf{q}_t = \begin{bmatrix} a_E^m + a_E^n \\ a_U^m + a_U^n \\ a_N^m + a_N^n \end{bmatrix}_t$$

So that

$$\frac{\partial \mathbf{q}_t}{\partial a_{E,m,t}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \mathbf{q}_t}{\partial a_{E,n,t}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Similarly the group specific version of the P matrix is

$$\mathbf{P}_{kt} = \frac{1}{1+a_{kt}} \begin{bmatrix} 1 - p_{kt}^{EU} - p_{kt}^{EN} & p_{kt}^{UE} & p_{kt}^{NE} \\ p_{kt}^{EU} & 1 - p_{kt}^{UE} - p_{kt}^{UN} & p_{kt}^{NU} \\ p_{kt}^{EN} & p_{kt}^{UN} & 1 - p_{kt}^{NE} - p_{kt}^{NU} \end{bmatrix}$$

and

$$\mathbf{A}_{kt} = \mathbf{I} - \mathbf{P}_{kt}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{1+a_{kt}} \begin{bmatrix} 1 - p_{kt}^{EU} - p_{kt}^{EN} & p_{kt}^{UE} & p_{kt}^{NE} \\ p_{kt}^{EU} & 1 - p_{kt}^{UE} - p_{kt}^{UN} & p_{kt}^{NU} \\ p_{kt}^{EN} & p_{kt}^{UN} & 1 - p_{kt}^{NE} - p_{kt}^{NU} \end{bmatrix}$$

$$\mathbf{A}_{kt} = \begin{bmatrix} 1 - \frac{1 - p_{kt}^{EU} - p_{kt}^{EN}}{1 + a_{kt}} & \frac{-p_{kt}^{UE}}{1 + a_{kt}} & \frac{-p_{kt}^{NE}}{1 + a_{kt}} \\ \frac{-p_{kt}^{EU}}{1 + a_{kt}} & 1 - \frac{1 - p_{kt}^{UE} - p_{kt}^{UN}}{1 + a_{kt}} & \frac{-p_{kt}^{NU}}{1 + a_{kt}} \\ \frac{-p_{kt}^{EN}}{1 + a_{kt}} & \frac{-p_{kt}^{UN}}{1 + a_{kt}} & 1 - \frac{1 - p_{kt}^{NE} - p_{kt}^{NU}}{1 + a_{kt}} \end{bmatrix}$$

So that the first partial derivative is

$$\frac{\partial \mathbf{A}_{kt}}{\partial p_{kt}^{EU}} = -\frac{1}{1 + a_{kt}} \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The other partials follow accordingly.

The aggregate A matrix, \mathbf{A}_t can be written as a function of all the sub-group transition probabilities

$$A_t = \frac{1}{1 + a_E^M + a_E^{UK} a_E^m + a_E^n + a_U^m + a_U^n + a_N^m + a_N^n}_t^*$$

$$\left[\begin{array}{ccc} 1 - (\varepsilon_t p_{eu}^M + (1 - \varepsilon_t) p_{eu}^{UK}) - (\varepsilon_t p_{en}^M + (1 - \varepsilon_t) p_{en}^{UK}) & \omega_t p_{ue}^M + (1 - \omega_t) p_{ue}^{UK} & \gamma_t p_{ne}^M + (1 - \gamma_t) p_{ne}^{UK} \\ \varepsilon_t p_{eu}^M + (1 - \varepsilon_t) p_{eu}^{UK} & 1 - (\omega_t p_{ue}^M + (1 - \omega_t) p_{ue}^{UK}) - (\omega_t p_{un}^M + (1 - \omega_t) p_{un}^{UK}) & \gamma_t p_{nu}^M + (1 - \gamma_t) p_{nu}^{UK} \\ \varepsilon_t p_{en}^M + (1 - \varepsilon_t) p_{en}^{UK} & \omega_t p_{un}^M + (1 - \omega_t) p_{un}^{UK} & 1 - (\gamma_t p_{ne}^M + (1 - \gamma_t) p_{ne}^{UK}) - (\gamma_t p_{nu}^M + (1 - \gamma_t) p_{nu}^{UK}) \end{array} \right]_{kt}$$

The sub-group partial derivatives of this matrix are now

$$\frac{\partial \mathbf{A}_t}{\partial p_{mt}^{EU}} = -\frac{1}{1 + a_t} \begin{bmatrix} -\varepsilon_t & 0 & 0 \\ \varepsilon_t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \frac{\partial \mathbf{A}_t}{\partial p_{nt}^{EU}} = -\frac{1}{1 + a_t} \begin{bmatrix} -(1 - \varepsilon_t) & 0 & 0 \\ (1 - \varepsilon_t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $a_t = a_E^m + a_E^n + a_U^m + a_U^n + a_N^m + a_N^n$

Similarly the sub-group components of p_{en} are now

$$\frac{\partial \mathbf{A}_t}{\partial p_{mt}^{EN}} = -\frac{1}{1 + a_t} \begin{bmatrix} -\varepsilon_t & 0 & 0 \\ 0 & 0 & 0 \\ \varepsilon_t & 0 & 0 \end{bmatrix} \quad \text{and} \quad \frac{\partial \mathbf{A}_t}{\partial p_{nt}^{EN}} = -\frac{1}{1 + a_t} \begin{bmatrix} -(1 - \varepsilon_t) & 0 & 0 \\ 0 & 0 & 0 \\ (1 - \varepsilon_t) & 0 & 0 \end{bmatrix}$$

Following in this way

$$\frac{\partial \mathbf{A}_{kt}}{\partial p_{kt}^{UE}} = -\frac{1}{1 + a_{kt}} \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \frac{\partial \mathbf{A}_{kt}}{\partial p_{kt}^{UN}} = -\frac{1}{1 + a_{kt}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and

$$\frac{\partial \mathbf{A}_t}{\partial p_{mt}^{UE}} = -\frac{1}{1+a_{kt}} \begin{bmatrix} 0 & \omega_t & 0 \\ 0 & -\omega_t & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \frac{\partial \mathbf{A}_t}{\partial p_{nt}^{UE}} = -\frac{1}{1+a_{kt}} \begin{bmatrix} 0 & 1-\omega_t & 0 \\ 0 & -(1-\omega_t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{A}_t}{\partial p_{mt}^{UN}} = -\frac{1}{1+a_{kt}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_t & 0 \\ 0 & \omega_t & 0 \end{bmatrix} \quad \frac{\partial \mathbf{A}_t}{\partial p_{nt}^{UN}} = -\frac{1}{1+a_{kt}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(1-\omega_t) & 0 \\ 0 & 1-\omega_t & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{A}_{kt}}{\partial p_{NEkt}} = -\frac{1}{1+a_{kt}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\frac{\partial \mathbf{A}_t}{\partial p_{mt}^{NE}} = -\frac{1}{1+a_t} \begin{bmatrix} 0 & 0 & \gamma_t \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma_t \end{bmatrix} \quad \frac{\partial \mathbf{A}_t}{\partial p_{nt}^{NE}} = -\frac{1}{1+a_t} \begin{bmatrix} 0 & 0 & 1-\gamma_t \\ 0 & 0 & 0 \\ 0 & 0 & -(1-\gamma_t) \end{bmatrix}$$

$$\frac{\partial \mathbf{A}_{kt}}{\partial p_{NUkt}} = -\frac{1}{1+a_{kt}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\frac{\partial \mathbf{A}_t}{\partial p_{mt}^{NU}} = -\frac{1}{1+a_t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma_t \\ 0 & 0 & -\gamma_t \end{bmatrix} \quad \frac{\partial \mathbf{A}_t}{\partial p_{nt}^{NU}} = -\frac{1}{1+a_t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1-\gamma_t \\ 0 & 0 & -(1-\gamma_t) \end{bmatrix}$$

Since $\mathbf{A}_{kt} \equiv (\mathbf{I} - \mathbf{P}_{kt})$ can be written as

$$\frac{1}{1+a_E^M + a_E^{UK} + a_U^M + a_U^{UK} + a_N^M + a_N^{UK}} \begin{bmatrix} 1-p_{EE} & -p_{UE} & -p_{NE} \\ -p_{EU} & 1-p_{UU} & -p_{NU} \\ -p_{EN} & -p_{UN} & 1-p_{NN} \end{bmatrix}_{kt}$$

Then the derivatives of \mathbf{A}_{kt} with respect to the net population flows are

$$\frac{\partial \mathbf{A}_{kt}}{\partial a_{ikt}} = \frac{\partial (\mathbf{I} - \mathbf{P}_{kt})}{\partial a_{ikt}} = \frac{\partial (\mathbf{I})}{\partial a_{ikt}} - \frac{\partial (\mathbf{P}_{kt})}{\partial a_{ikt}} = -\frac{\partial (\mathbf{P}_{kt})}{\partial a_{ikt}} = -\left(\frac{1}{1+a_{ikt}}\right)^2 \begin{bmatrix} p_{EE} & p_{UE} & p_{NE} \\ p_{EU} & p_{UU} & p_{NU} \\ p_{EN} & p_{UN} & p_{NN} \end{bmatrix}$$

$$\frac{\partial \mathbf{A}_{kt}}{\partial a_{ikt}} = - \left(\frac{1}{1 + a_{ikt}} \right)^2 \mathbf{P}_{kt}, \quad \forall i \in \{E, U, N\}, k \in \{M, UK\}$$

Finally, note that the a_i s are simply the difference between population inflows and outflows,

$$a_{ikt} = l_{ikt} - o_{ikt}$$

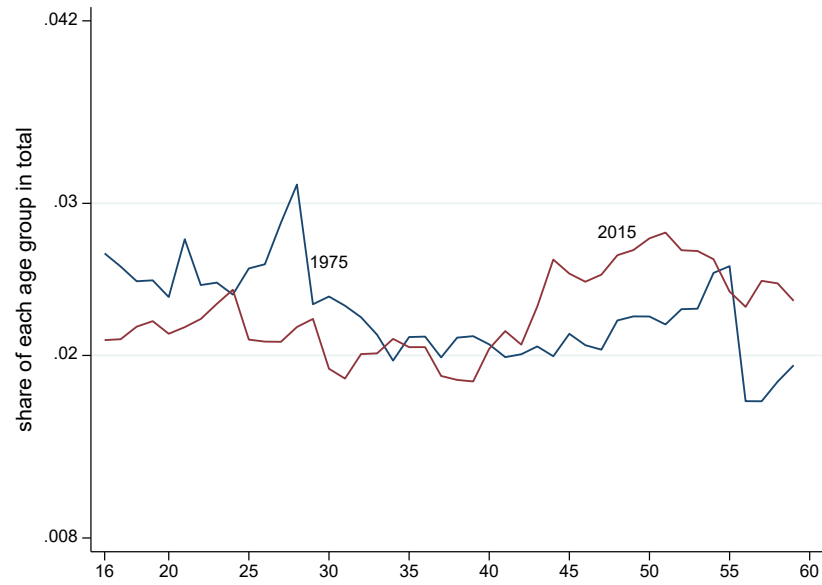
It must therefore be that the *gross* population flow partial derivatives are given by

$$\frac{\partial \mathbf{q}_{kt}}{\partial l_{ikt}} = - \frac{\partial \mathbf{q}_{kt}}{\partial o_{ikt}}, \quad \frac{\partial \mathbf{A}_{kt}}{\partial l_{ikt}} = - \frac{\partial \mathbf{A}_{kt}}{\partial o_{ikt}}.$$

Thus, the “factor loadings” in the Taylor series must be symmetric. The only difference in the contributions of the gross population flows them comes from any differences in the change in the gross flows in the Taylor series approximation, using $da_{ikt} = dl_{ikt} - do_{ikt}$

Appendix D. Changing Age Shares in the UK-Born and Immigrant Working Age Populations

UK-Born



Immigrant

