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ABSTRACT

Local Labor Markets with Non-homothetic Preferences

We study the effects on employment, costs of living, and income inequality of local shocks in the housing market or in the productivity of a tradable good. We construct a two-region search and matching model in which housing is considered a necessity good. Mobility of labor implies that any change in one region propagates into the other. The model is analytically tractable and provides some intuitive comparative statics results. We then calibrate the model on the basis of German data. Our simulations indicate that both types of shock produce limited employment gains but have a significant impact on housing prices and real income inequality: poorer, unemployed workers experience a larger increase in their cost of living index. This depends on the assumption of a non-homothetic utility function that generates a specific nominal wage to housing price positive relationship, partially safeguarding employed individuals against the rising cost of living.

JEL Classification: R23, R21, R31, J31, J61, J64, D31

Keywords: local labor markets, income inequality, costs of living, housing expenditures, housing prices

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1 Introduction

Since the end of the financial crisis of 2007-09, house prices and rents have risen rapidly in much of the rich world (see OECD (2023b)[1]. For most households, income growth has not kept pace, with obvious consequences on housing affordability[2]. Recent empirical research has also highlighted how a booming property market may exacerbate income inequality, as poorer families devote a larger fraction of their income on housing expenditures (Dustmann et al. (2021) for Germany and Albouy et al. (2016) for the US). Supply constraints in the housing market are not just one of the main factors behind the steep increase in prices. They also stymie the efficient allocation of labor across regions, by raising the cost of migration (Hsieh and Moretti (2019)). So the most productive areas enjoy higher nominal wages and housing prices but modest employment gains[3].

The aim of this paper is to address both the misallocation and the inequality issues in a unified general equilibrium framework. We construct a two-region search and matching model, in which two goods are produced and sold in the market: a tradable consumption good and a non-tradable one, housing services. The crucial assumption of the model is that all individuals have a non-homothetic constant elasticity of substitution utility function, as in Comin et al. (2021). This allows to consider housing as

[1] In the pandemic period residential property prices have accelerated. In Europe, they started to (slowly) decline in the second half of 2022. See latest available data on https://ec.europa.eu/eurostat/web/products-euro-indicators.

[2] Since 2015, house prices to income ratio of median income household has increased in 33 of the 40 countries studied by the OECD (2023b). In the Eurozone, the same ratio has increased in 16 out of 20 countries. In seven of them the increase has been superior to 20%. See Frayne et al. (2022).

[3] These misallocation losses may be alleviated by reducing commuting costs. See Monte et al. (2018).

a necessity good, whose share on total expenditures decreases with income[4]. The two regions differ only in terms of labor productivity and the decision to migrate depends on job opportunities, the costs of living, and idiosyncratic preferences for a specific location. The model is analytically tractable and we provide some comparative statics results. We then turn to a quantitative analysis, calibrating and simulating the model on the basis of German data.

We find that the non-homotheticity assumption plays a crucial role in amplifying differences in prices, income and costs of living (both across regions and among employed and unemployed workers) following a productivity or a housing supply local shock.

As concerns the increased inequality between workers with a job and those looking for it, the reason is twofold. The first one is straightforward. Any shock that raises housing prices has a stronger impact on poorer, unemployed, workers that spend a larger a fraction of their income on that good. Their cost of living index (i.e. the amount of money they have to pay to reach a given utility level) will increase more than that of the employed workers. The second reason comes directly from the fact that, under non-homothetic preferences, the bargained nominal wage becomes positively influenced by housing prices. This creates a specific positive feedback loop. Any given increase in housing prices leads to higher nominal wages that in turn will increase the demand for housing, raising prices even more. So, when both nominal wages and the relative price in the property market go up, employees with non-homothetic preferences get more expensive housing services but also spend a smaller fraction of their income on them. This second factor, that has a moderating effect on the cost of living index for the employed workers, is not present in the index of the unemployed ones, whose income may not increase as much (or does not increase at all) as to reduce their housing

[4] Several empirical works confirm this feature. See for instance Quigley and Raphael (2004), Albouy et al. (2016), Larrimore and Schuetz (2017) for the United States,Dustmann et al. (2021), for Germany, and Belfield et al. (2015) for the UK.

expenditures share[5].

We find for instance that 1% increase in TFP in the tradable sector in the Western states of Germany raises the cost of living index for the unemployed workers living there by 0.6%, while the the same indicator barely changes for the employees. The resulting employed/unemployed real income gap is 50% larger than it would be obtained simulating the same shock with homothetic preferences.

In terms of welfare, this produces interesting results. If we just look at the current (instantaneous) level of utility, employees are better off and unemployed workers worse off. But the expected discounted lifetime utility goes up for all, as a higher job finding rate and more generous future earnings outweigh the present loss in real income for the unemployed.

Mobility of labor implies a local shock propagates into other areas of the country, and this has an impact on inequality across regions. If a shock attracts more workers in one area, a shrinking labor force in the other regions of the country will depress their housing market. Declining housing prices will lower the cost of living there (and nominal earnings too, for the mechanism explained above). So, in these regions the unemployed workers will experience a greater reduction in their cost of living index, compared to the employed ones. Our simulations suggest that a 1% positive TFP shock in Western Germany reduces the cost of living index by 1% for the unemployed workers and by 0.7% for the employed ones living in the Eastern states. Real incomes increase by roughly the same amount. Under homothetic preferences, the magnitude of variation is 70% lower. If we consider at the entire country, this regional shock on productivity raises the ratio of housing prices between Western and Eastern states by 3% and the variance of the (natural) logarithm of real income by 1.4%, against a 1% and $0,9\%$ change (respectively) in case of homothetic preferences.

[5] In a specific simulation, in which we assume that TFP in the tradable sector goes up in both regions, the cost of living index of employees even decreases while that of the unemployed ones goes up, widening real income inequality even more.

We also look at a change in the housing supply (perhaps stemming from stricter regulations) that raises the equilibrium price in one single region. As dwellings become more expensive there, we expect a larger out-flows migration that in principle could cool the market down, partially offsetting the initial price increase. However, in our model more expensive housing drives nominal wages up, sustaining the demand. We find that a 5% positive increase in the marginal cost in the property market in the Western states of Germany raises housing prices by almost 4%, 25% more of what we obtain shutting the non-homotheticity assumption off. Less affordable housing will increase migration towards the states not hit by the shock. In turn, this will also raise the demand for housing there. In terms of real income and cost of living, unemployed workers are worst hit, both because they are more dependent on housing and because the surge in prices is not partially offset by a nominal pay increase as for the employed workers.

The reason why considering housing a necessity good makes nominal wages dependent on housing prices goes as follows. Under a standard Nash bargaining solution, the equilibrium wage must be such that the firm's marginal costs are equal to workers' marginal utility. When preferences are homothetic, housing prices do not affect this equation. On the contrary, if the housing expenditure share is decreasing with income, higher housing prices raise workers' marginal utility because becoming employed would have the additional advantage of making them less reliant on a good that is relatively more expensive. A higher marginal utility drives the negotiated wage up, that in turn will boost demand for housing, increasing prices even more.

While non-homothetic preferences have an impact on inequality, we find their effect on employment is negligible. In our model, the changes in the unemployment rate following a shock in the tradable sector or in the housing supply are quite limited and do not differ if we drop the non-homotheticity assumption. Unlike Hsieh and Moretti (2019), in our model the culprit does not appear to be an excessively inelastic housing supply either. Rather, we believe they are the results of the specific characteristics of standard search and matching models, that imply a small elasticity of unemployment with respect to productivity or other exogenous shocks[6].

Germany is a significant setting. Unlike other Continental Europe countries, the specific bargaining structure allow nominal wages in Germany to be more dependent on local labor market conditions (Boeri et al. (2021)) and this chimes well with the main mechanism of our model. Moreover, in recent years it experienced a boom in the real estate market (a more than 50 % cumulative growth in residential property prices form 2015 to 2022)[7] and robust growth in nominal earnings (2 - 3 % average annual change from 2008 to 2019). Income inequality increased in the 1990s and 2000s, and recent research has shed light on the role of housing expenditures on that growth (Dustmann et al. (2021)). Data show that this trend stopped after mid 2000s (Biewen et al. (2019)). However, Germans have become increasingly concerned about income disparities[8]. This disconnection between objective indicators and people's feelings may perhaps be explained by knowing that the the former are computed using an identical price index for all categories of individuals, ignoring how income influences tastes.

The present paper connects to a first strand of research that investigates the role of the housing market on income inequality. By estimating a non-homothetic CES utility function, Albouy et al. (2016) find that the increase in rents is the main reason housing shares have increased substantially in the last decades in US, exacerbating the so-called affordability problem for poorer households. In the same vein, Dustmann et al. (2021) document how the increase in housing expenditures have amplified real income inequality in Germany, as poorer households at the bottom quintile of the

[6]This point was first made by Shimer (2005) in the context of business cycle fluctuations. A different parametrization may overcome this problem: see Hagedorn and Manovskii (2008).

[7]The percentage increase was even bigger (about 80%) for the metropolises (Berlin, Hamburg, Munich, Cologne, Frankfurt am Main, Stuttgart, and Düsseldorf). See data from the Federal Statistical Office at https://www.destatis.de.

[8]In 2020 over 50 % of individuals in Germany strongly concurred that income inequalities were excessively high, up from one-third recorded in 2017 and surpassing the OECD average (OECD (2021).

income distribution spend a larger share income on housing and they are also more likely to pay rents, the relative cost of which have increased over time[9]. These are essentially empirical works, while our paper aims to set up a theoretical framework order to consider the interplay between labor and housing markets.

Our results also are also related to a recent literature that studies the relationship between productivity, employment, wages, and housing prices at local level. For instance, Hsieh and Moretti (2019) document how productivity gaps across US states translate into large housing price and nominal wage dispersion, while employment differences are more limited. For them, the reason lies on the small elasticity of the housing supply, so that an increase in the demand caused by stronger migration towards more productive areas drive housing prices and nominal salaries up, and has negligible effects on the supply of new housing units. In our model, on the contrary, the crucial ingredient is the assumption that housing is a necessity good, and how this translates into the positive nominal wage-housing price loop explained above.

The ups and downs in housing prices in the last twenty years (especially in US) and their impact on the business cycle has pushed the economists to look at the relationship between housing and labor markets with a renewed attention[10]. Most papers have put demand effects in the spotlight, showing how a decrease in housing prices may depress spending. For example, Mian and Sufi (2014) find that, during the Great Recession, US counties where the house value depreciated more also exhibited a larger decline in employment in non-tradable sectors, that are more vulnerable to local economic conditions. Branch et al. (2016) focus on liquidity constraints. They obtain that an increase in the eligibility of homes as collateral reduces aggregate unemployment, increases house prices, and drives workers away from the construction sector. The present paper points to a labor supply-side channel by which the housing market may

[9] Shrinking household sizes (especially at the bottom of the income distribution) and the consequent loss in economies of scale is another key factor.

[10] More in general, a thorough exposition on the impact of housing prices on business cycle fluctuations is in Davis and Van Nieuwerburgh (2015).

affect employment. In our model more expensive dwellings has the same impact of a upward shift in the labor supply, that raises nominal pays and reduces employment. It must also be stressed that our focus is on tradable employment[11].

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates the properties of the equilibrium and the comparative statics results. Section 4 shows the quantitative results. Section 5 concludes.

2 The Basic Framework

2.1 Matching Technology

Time is continuous and the model is developed in steady-state. We consider a country composed by two regions, say a and b. Regions differ only in terms of labor productivity, while all the other product and labor market parameters are assumed to be the same. Besides the gain in simplicity, this also allows to isolate more starkly the effects of demand and supply shocks in one region on the entire economy.

In each region, two goods are sold in the market: a consumption good, that can be traded across the regions at a competitive price, and housing services, that are not tradable. To produce the tradable consumption good, firms need to hire workers in the labor market. Conversely, following most of the literature (see Moretti (2011)), we assume that housing services are supplied in the market by landlords that live abroad.

In the entire country there is a measure normalized to L of workers that are infinitely-lived and risk-neutral. Workers can either be employed in the sector that produces the tradabale consumption good or unemployed. In our setting, workers must take two decisions: (i) they have to choose the region to live in, according to a maximization rule it will be presented in the next section; (ii) they must select the

[11] Mian and Sufi (2014) recognize that at theoretical level the correlation between tradable employment and housing price could be negative but they do no find any significant effect in the data.

optimal amounts of the consumption good and housing services.

Before focusing on these actions, we explain the functioning of the frictional labor markets. Following a standard search and matching approach (Pissarides (2000), chapter 1), the flow of hires in the tradable sector of region $i \in \{a, b\}$, M_i depends the number of vacancies, V_i and the number of unemployed people living in region i, U_i. There is no on-the-job search. The matching function is written $M_i = m\,(U_i, V_i)$. Following most of the literature (see Petrongolo and Pissarides (2001)), we impose it is homogeneous of degree 1 and increasing and concave in both arguments. Labor market tightness in region $i \in \{a, b\}$ is denoted by $\theta_i \equiv V_i/U_i$. The rate at which vacant jobs become filled is $q(\theta_i) \equiv m\,(U_i, V_i)\,/V_i$, with $q\,'(\theta_i) < 0$. A job-seeker moves into employment at a rate $f(\theta_i) \equiv m\,(U_i, V_i)\,/U_i = \theta_i\, q(\theta_i)$ with $f\,'(\theta_i) > 0$.[12] We also define $\eta \equiv -q'(\theta_i)\,(\theta_i/q(\theta_i))$, the opposite of the elasticity of the job-filling rate, and we assume to be constant[13]. At an exogenous rate δ a job is destroyed. Let L_i designate the labor force in region $i \in \{a, b\}$, with $L_a + L_b = L$. Then one can write $E_i + U_i = L_i$, with E_i being the measure of employed workers in region i, $i \in \{a, b\}$. The equality between flows in and out of workers' status in steady-state leads to the standard Beveridge curve:

$$u_i = \frac{\delta}{\delta + f(\theta_i)} \quad \text{with } i \in \{a, b\}, \tag{1}$$

in which $u_i \equiv U_i/L_i$ is the unemployment rate in region $i \in \{a, b\}$.

2.2 Workers' Preferences

The most crucial assumption of the model is that all individuals have non-homothetic preferences. Under homothetic utility functions, the percentage of consumption expenditures on a given good does not change with income under constant prices[14]. This

[12] We also assume that $\lim_{\theta_i \to 0} q(\theta_i) = +\infty$, $\lim_{\theta_i \to +\infty} q(\theta_i) = 0$, $\lim_{\theta_n \to 0} f(\theta_i) = 0$ and $\lim_{\theta_n \to +\infty} f(\theta_i) = +\infty$.

[13] This is the case under the standard assumption of a Cobb-Douglas matching function.

[14] This is equivalent to saying that the Engel curves, that illustrate how consumption expenditure on a given good varies with income under constant prices, are straight lines to the origin. See for

assumption does not seem empirically grounded if we consider housing services. Several recent empirical works find poor people spend a higher fraction of their income on housing (see references in footnote 4 and data presented in section 4.1).

To model non-homotheticity, we follow the approach of Comin et al. (2021). Their specific formulation (a non-homothetic constant elasticity of substitution (NHCES) function) is analytically tractable. The instantaneous utility function ν_i for all workers living in region $i \in \{a, b\}$ is implicitly defined by the following equation:

$$1 = \left(Q_{nt,i} \cdot \nu_i^{-(1+\epsilon)}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{t,i} \cdot \nu_i^{-1}\right)^{\frac{\sigma-1}{\sigma}} \quad \text{with } i \in \{a, b\} \tag{2}$$

in which $Q_{t,\,i}$ and $Q_{nt,\,i}$ respectively denote the tradable good and the not tradable housing services consumed in region $i \in \{a, b\}$. As we will see precisely in this section, parameter $\epsilon \geq -1$ captures the extent of non-homotheticity for the housing services. Notice indeed that, with $\epsilon = 0$, ν_i can be explicitly derived and it becomes a standard constant elasticity of substitution utility function.

Parameter $\sigma > 0$ stands for the elasticity of substitution between the two goods. It tells us how the relative expenditure on the goods changes in response to a variation in relative prices[15].

Let r be the discount factor in this economy. We consider $Q_{t,i}$ as the *numeraire* for the economy of region i. So its price is normalized to 1 and it is equal across the regions. Conversely, we denote with $p_{nt,i}$ the price for housing services in region i.

instance Deaton and Muellbauer (1980), chapter 5.

[15]With $\sigma > 1$ (respectively, $0 < \sigma < 1$), an increase in the relative price of housing leads to a decrease (resp. increase) in its relative expenditure. The goods are gross substitutes (resp. complements). With $\sigma = 1$, we are in a Cobb Douglas case and relative expenditures are not affected by relative prices.

2.2.1 Unemployed Workers

The expected discounted utility of the unemployed worker j searching for a job in region $i \in \{a, b\}$, $W^U_{j,i}$ verifies the following Bellman equation:

$$
\begin{aligned}
rW^U_{j,i} &= \max_{Q^U_{t,i},\, Q^U_{nt,i}} \; z_{j,i} + \nu_{U,i} + f(\theta_i)\left[W^E_{j,i} - W^U_{j,i}\right] \\
\text{s.t.} \quad & p_{nt,i} \cdot Q^U_{nt,i} + Q^U_{t,i} = b
\end{aligned}
\tag{3}
$$

The instantaneous utility function $\nu_{U,i}$ is implicitly defined in equation (2). Here we simply add the subscript U to recall we are considering the case of unemployed workers in region i. The random term $z_{j,i}$ stands for the idiosyncratic preference for region i and it is the only difference in preferences across workers. A higher $z_{j,i}$ means a stronger attachment to region i for worker j. The term $W^E_{j,i}$ is the discounted present value of being employed in region i.

This and the following Bellman equations have a standard interpretation. Being unemployed is like holding an asset that gives you a dividend $z_{j,i} + \nu_{U,i}$ and a capital gain, occurring at the rate $f(\theta_{n,i})$, equal to the term inside the square brackets.

The second line in (3) presents the budget constraint for the unemployed workers, in which b stands for the exogenous amount of home production of the consumption good and it is assumed to be identical across regions.

Computing the F.O.C.s for this problem, we get the NHCES Hicksian demand function for each good:

$$
\begin{aligned}
Q^U_{nt,i} &= \left(\frac{p_{nt,i}}{b}\right)^{-\sigma} \cdot \nu_{U,i}^{(1+\epsilon)(1-\sigma)} \\
Q^U_{t,i} &= b^{\,\sigma} \cdot \nu_{U,i}^{1-\sigma},
\end{aligned}
\tag{4}
$$

for $i \in \{a, b\}$.

Moreover, the expenditure function is equal to:

$$
b = \left[\nu_{U,i}^{(1+\epsilon)(1-\sigma)} \cdot p_{nt,i}^{1-\sigma} + \nu_{U,i}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{5}
$$

Let $s_{U,i} \equiv p_{nt,i} Q^U_{nt,i}/b$, the share in total expenditures for the non-tradable housing services for the unemployed workers in region $i \in \{a, b\}$. Then, using the two equations in (4) to get rid of the $\nu_{U,i}$ term, we get:

$$s_{U,i} = p_{nt,i}^{1-\sigma} \cdot b^{\epsilon(1-\sigma)} \cdot (1 - s_{U,i})^{1+\epsilon} \quad (6)$$

for $i \in \{a, b\}$. Totally differentiating this equation, we find the elasticity of the housing share with respect to unemployed workers' home production b:

$$\frac{d\, s_{U,i}}{d\, b} \cdot \frac{b}{s_{U,i}} = \frac{\epsilon(1-\sigma)(1-s_{U,i})}{1+\epsilon s_{U,i}} \quad (7)$$

for $i \in \{a, b\}$. Since $\epsilon \geq -1$, the denominator is always positive and we have that the sign of this elasticity depends on the sign of $\epsilon \cdot (1-\sigma)$. The effect of b on the housing share depends on the non-homotheticity parameter ϵ and the elasticity of substitution σ. If $-1 \leq \epsilon < 0$ and $0 < \sigma < 1$ (the two goods are gross complements), an increase in b leads to a reduction in $s_{U,i}$. This means that housing belongs to that specific subset of normal goods called necessity goods, whose relative expenditure decreases when income increases[16]. In the present paper we will study the equilibrium properties of the model imposing such parameter restrictions, since they match the data in section 4.1. As expected, under homothetic preferences ($\epsilon = 0$), workers do not change their expenditures shares as income changes.

The elasticity of the housing share with respect to its (relative) price is equal to:

$$\frac{d\, s_{U,i}}{d\, p_{nt,i}} \cdot \frac{p_{nt,i}}{s_{U,i}} = \frac{(1-\sigma)(1-s_{U,i})}{1+\epsilon s_{U,i}} \quad (8)$$

for $i \in \{a, b\}$. The sign of the price effect is uniquely determined by the elasticity of substitution σ.

[16] Conversely, with $-1 \leq \epsilon < 0$ and $\sigma > 1$ housing would be a luxury good whose relative expenditure increases with income.

2.2.2 Employed Workers

The utility maximization problem for an employed worker in region $i \in \{a, b\}$ is:

$$\begin{aligned} rW^E_{j,i} &= \max_{Q^E_{t,i},\, Q^E_{nt,i}} \; z_{j,i} + \nu_{E,i} + \delta \left[W^U_{j,i} - W^E_{j,i} \right] \\ \text{s.t.} \quad & p_{nt,i} \cdot Q^E_{nt,i} + Q^E_{t,i} = w_i \end{aligned} \tag{9}$$

With w_i we denote the endogenous nominal wage. The problem is identical to the one presented for the unemployed worker in (3). Following the same steps, we get the Hicksian demand and the expenditure functions:

$$\begin{aligned} Q^E_{nt,i} &= \left(\frac{p_{nt,i}}{w_i} \right)^{-\sigma} \cdot \nu_{E,i}^{(1+\epsilon)(1-\sigma)} \\ Q^E_{t,i} &= b^{\sigma} \cdot \nu_{E,i}^{1-\sigma}, \\ w_i &= \left[\nu_{E,i}^{(1+\epsilon)(1-\sigma)} \cdot p_{nt,i}^{1-\sigma} + \nu_{E,i}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{10}$$

for $i \in \{a, b\}$. Similarly, if $s_{E,i} \equiv p_{nt,i}\, Q^E_{nt,i}/w_i$ is the share in total expenditures for the non-tradable housing services for the employed workers in region $i \in \{a, b\}$, we have:

$$s_{E,i} = p_{nt,i}^{1-\sigma} \cdot w_i^{\epsilon(1-\sigma)} \cdot (1 - s_{E,i})^{1+\epsilon} \tag{11}$$

It is easy to see that with $\epsilon = 0$, then equations (6) and (11) are the same: under homothetic preferences, the share of housing services out of total consumption is the same for unemployed and employed workers. The sign of the elasticities of the housing expenditure share with respect to the wage and the relative price depends on the parameters ϵ and σ, as in the unemployed workers' case:

$$\begin{aligned} \frac{d\, s_{E,i}}{d\, w_i} \cdot \frac{w_i}{s_{E,i}} &= \frac{\epsilon(1-\sigma)(1-s_{E,i})}{1+\epsilon s_{E,i}} \\ \frac{d\, s_{E,i}}{d\, p_{nt,i}} \cdot \frac{p_{nt,i}}{s_{E,i}} &= \frac{(1-\sigma)(1-s_{E,i})}{1+\epsilon s_{E,i}} \end{aligned} \tag{12}$$

for $i \in \{a, b\}$.

2.2.3 Price Indexes and Utility Elasticity

Using the Hicksian demand for the consumption good (the second equations in (4) and (10)) and the definition for the housing share of total expenditures, we get:

$$\begin{aligned} \nu_{U,\,i} &= b\,(1 - s_{U,i})^{\frac{1}{1-\sigma}} \\ \nu_{E,\,i} &= w_i\,(1 - s_{E,i})^{\frac{1}{1-\sigma}} \end{aligned} \tag{13}$$

for $i \in \{a, b\}$. Following the discussion in Comin et al. (2021), we denote with $P_{U,i} \equiv (1 - s_{U,i})^{\frac{-1}{1-\sigma}}$ and $P_{E,i} \equiv (1 - s_{e,i})^{\frac{-1}{1-\sigma}}$ the average price indexes for unemployed and employed workers of region i, respectively. $P_{U,i}$ (resp. $P_{E,i}$) is indeed the amount of income a unemployed (resp. employed) worker needs to reach a level of utility $\nu_{U,\,i}$ (resp $\nu_{E,\,i}$) equal to 1. This also implies that $\nu_{U,\,i}$ and $\nu_{E,\,i}$ coincide with the real wage and the real income for (respectively) employed and unemployed workers in region $i \in \{a, b\}$.[17].

Notice that in the case $0 < \sigma < 1$ and $-1 < \epsilon < 0$, equations (6) and (11) tell us that $s_{U,i} > s_{E,i}$, as long as the nominal wage w_i is higher than the unemployed workers' income, b (this turns out to be always verified in equilibrium).

We have therefore that $P_{U,i} > P_{E,i}$, for $i \in \{a, b\}$. In an economy in which housing services are a necessity, the larger their share out of total expenditures the more expensive the cost of living. Poorer, unemployed workers need to spend more money to reach the same level of utility of an employed worker.

It is also useful to compute the elasticity of employees' instantaneous utility with respect to the wage. Using the second equation in (13) and (11), we have:

$$\begin{aligned} &\mu_i \equiv \frac{d\,\nu_{E,\,i}}{d\,w_i} \cdot \frac{w_i}{\nu_{E,\,i}} = \frac{1}{1 + \epsilon \cdot s_{E,i}}, \quad \text{with} \\ &\mu'(p_{nt,i}) > 0 \quad \text{if and only if} \quad -1 < \epsilon < 0 \quad \text{and} \quad 0 < \sigma < 1 \end{aligned} \tag{14}$$

for $i \in \{a, b\}$. As expected, for $\epsilon = 0$, the elasticity μ_i is unitary and the instantaneous utility is linear in income. It is greater than one if $-1 < \epsilon < 0$. This is because, $\nu_{E,\,i}$

[17] The ratio $P_{U,i}/P_{E,i}$ is the true Konüs index that compares differences in the cost of living between unemployed and employed workers. See Diewert (2009).

is positively influenced by w_i not just directly, as in the homothetic case, but also via the change in $s_{E,i}$. In our scenario where housing is a necessity good, higher nominal earnings reduce the expenditure share $s_{E,i}$ and the cost of living index $P_{E,i}$, thereby increasing employees' instantaneous utility.

The sign of the derivative in the second line of (14) is easily obtained looking at the second equation in (12). As long as as $-1 < \epsilon < 0$ and $0 < \sigma < 1$, a given percentage increase in the nominal wage has a positive impact on employees' utility that is smaller in an economy with a cheap housing market. The rationale is quite intuitive. An increase in workers' wage lowers the fraction of their income spent on housing, if it is necessity good. If the (relative) price of housing services $p_{nt,i}$ is low, this means they shift towards a consumption bundle in which they will buy a larger share of the relatively more expensive tradable good. The pay rise has therefore a more modest effect on workers' utility.

2.3 Firms in the consumption good sector

Following a textbook search and matching model (Pissarides (2000), chapter 1), we impose the one firm - one job assumption. Each firm-worker pair in region i produces an amount y_i of the tradable consumption good, with $i \in \{a, b\}$. We also assume that $y_a > y_b$. This is the only exogenous difference between the two regions.

The expected discounted returns for a firm operating in region $i \in \{a, b\}$, J_i^E verifies the following Bellman equation:

$$rJ_i^E = y_i - w_i + \delta \left(J_i^V - J_i^E\right), \quad \text{with } i \in \{a, b\} \tag{15}$$

At the RHS of (15) we have the firm's revenues, namely the amount of the units of the consumption good produced y_i net of the wage bill, and the capital loss occurring at rate δ, with J_i^V being the expected value of a vacancy. It is determined as follows:

$$rJ_i^V = -k + q(\theta_i)\left(J_i^E - J_i^V\right), \quad \text{with } i \in \{a, b\} \tag{16}$$

The expected value of vacancy is given by the vacancy costs k, expressed in terms of the consumption good, and the capital gain that accrues from the match, multiplied by the job filling rate.

As common in search and matching models, a free-entry zero profit condition determines the equilibrium values of tightness θ_i, conditional on the nominal wage. Free-entry of vacancies and zero profits imply that $J_i^V = 0$. Substituting this into (15) and (16), one gets:

$$\frac{y_i - w_i}{r + \delta} = \frac{k}{q(\theta_i)} \quad \text{with } i \in \{a, b\} \tag{17}$$

Firms' expected discounted revenues (the LHS of (17)) are equal to the expected cost of posting a vacancy (the RHS of (17)).

2.4 Wage bargaining

The nominal wage is negotiated between each firm and worker at individual level. We assume an axiomatic Nash solution to split the surplus $W_{j,i}^E - W_{j,i}^U + J_i^E - J_i^V$ originated from the match. The nominal wage w_i solves the following problem:

$$w_i = \text{argmax} \left[W_{j,i}^E - W_{j,i}^U \right]^{\beta} \left[J_i^E - J_i^V \right]^{1-\beta} , \tag{18}$$

with $i \in \{a, b\}$. Parameter β denotes the exogenous bargaining power of a worker ($0 < \beta < 1$). By ordinality, we can consider the $log(.)$ of the function in (18) to find w_i. Knowing that $J_i^V = 0$ and taking $W_{j,i}^U$ as given, the F.O.C. is:

$$\beta \frac{\frac{d W_{j,i}^E}{d w_i}}{W_{j,i}^E - W_{j,i}^U} + (1-\beta) \frac{\frac{d J_i^E}{d w_i}}{J_i^E} = 0 \quad \text{with } i \in \{a, b\} \tag{19}$$

Using (9) and (15) yields

$$\frac{d W_{j,i}^E}{d w_i} = \frac{\frac{d \nu_{E,i}}{d w_i}}{r + \delta}; \quad \text{and} \quad \frac{d J_i^E}{d w_i} = \frac{-1}{r + \delta} \quad \text{with } i \in \{a, b\}$$

So the F.O.C. in (19) becomes:

$$\frac{1-\beta}{J_i^E} = \beta \frac{\frac{d \nu_{E,i}}{d w_i}}{W_{j,i}^E - W_{j,i}^U} \quad \text{with } i \in \{a, b\} \tag{20}$$

At the LHS we have the cost of a marginal increase in the nominal wage, as a higher pay reduces firms' expected revenues. At the RHS there is the marginal gain. It is given by the marginal change in employees' instantaneous utility (the numerator at the RHS) over workers' quasi-rents from the match (the denominator). Using equations (9), (13), (14), and (15), the F.O.C (20) can be written as follows[18]:

$$\frac{1-\beta}{y_i - w_i} = \frac{\beta\,\mu_i \cdot (1-s_{E,i})^{\frac{1}{1-\sigma}}}{w_i \cdot (1-s_{E,i})^{\frac{1}{1-\sigma}} + z_{j,i} - rW^U_{j,i}}, \tag{21}$$

with $i \in \{a, b\}$ and for a generic worker j. Using equations (3), (16), (20), the zero profit condition $J_i^V = 0$, the expression for the elasticity μ_i from equation (14) and rearranging, we have:

$$\begin{aligned} w_i &= \frac{\beta\,(y_i + k\cdot\theta_i) + (1-\beta)\,b\,(1+\epsilon\cdot s_{E,i})\left(\frac{1-s_{U,i}}{1-s_{E,i}}\right)^{\frac{1}{1-\sigma}}}{1+(1-\beta)\epsilon\cdot s_{E,i}} \\ w_i' &\equiv \left.\frac{d\,w_i}{d\,p_{nt,i}}\right|_{\bar{\theta}_i} > 0 \quad \text{if} \quad -1 < \epsilon < 0 \quad \text{and} \quad 0 < \sigma < 1 \end{aligned} \tag{22}$$

with $i \in \{a, b\}$. This complex expression boils down to the standard search and matching wage schedule (see Pissarides (2000), chapter 1), in case of homothetic preferences (that is $\epsilon = 0$ and $s_{U,i} = s_{E,i}$): the nominal pay is a weighted average of labor productivity y_i and the amount of home production b, the weights being represented by workers' bargaining power β, with in addition a fraction of the vacancy costs[19].

Non-homothetic preferences introduce a new, crucial element in the wage equation (22). From eqs. (6) and (11), we know that $s_{U,i}$ and $s_{E,i}$ do depend on the price of the housing services $p_{nt,i}$. So, unlike the homothetic scenario, in which $\epsilon = 0$, $s_{U,i} = s_{E,i}$, and w_i is unaffected by the housing shares, under non-homothetic preferences the nominal wage is influenced by $p_{nt,i}$. More precisely, the derivative in the second line

[18] Computations are in Appendix A.

[19] This can be explained by noting that, at $\epsilon = 0$, workers' instantaneous utility is linear in wage (see the second equation in (13)), exactly as it is assumed in a textbook matching model, in which $u(w) = w$.

of (22) tells us that, for any given level of θ_i, w_i is increasing in $p_{nt,i}$, if $0 < \sigma < 1$ and $-1 < \epsilon < 0$, for $i \in \{a, b\}$. Therefore, if under homothetic preferences more expensive housing costs lower the real wage but leave the nominal wage intact, under non-homothetic preferences the rise in the price level is partially offset by a higher nominal pay.

To understand this point, it is more convenient to inspect equation (21). First, it is easy to see that employers' marginal costs (the term at the LHS of (21)) are increasing in w_i and not affected by $p_{nt,i}$. Conversely, at the RHS, workers' marginal gains are decreasing in w_i and do depend on the housing market. Moreover, when $0 < \sigma < 1$ and $-1 < \epsilon < 0$, an increase in $p_{nt,i}$ raises the expression at the RHS of (21). At the equilibrium a higher $p_{nt,i}$ raises the nominal wage.

Why so? The result crucially hinges on the variable μ_i. We have seen that, if $-1 < \epsilon < 0$ and $0 < \sigma < 1$, such elasticity is larger the higher $p_{nt,i}$ is. Every additional unit of income has a greater value for the employed workers when the housing market is more expensive. This means that, in the wage negotiation, the gain for employees of receiving one unit more of the surplus of the match is larger when housing is less affordable. *Ceteris paribus*, the outcome of the surplus division tilts more in favour of the workers, resulting in higher nominal pays.

2.5 Workers' Location Decision

To determine the measure of workers choosing to live in either region, we introduce a condition, borrowed from Moretti (2011), that states that a generic worker j's relative preference for region a over region b is:

$$z_{j,a} - z_{j,b} \sim g\left[-\lambda, \lambda\right], \tag{23}$$

with $g(.)$ being a probability density function. Parameter λ captures the importance of the preference for location and therefore the degree of labor mobility. If λ is large, people's willingness to move in order to reap the benefits of higher real wages or shorter unemployment spells is limited. Conversely, if λ is small, workers are more willing to migrate in search of better economic conditions. With $\lambda = 0$, nobody is attached to a region compared to the other, and there is perfect worker mobility. One can define the value λ^* that belongs to the marginal worker j^*, the one indifferent between searching for a job in region a or in b:

$$rW^U_{j^*,b} - rW^U_{j^*,a} = 0$$

If $\lambda^* \equiv z_{j^*,a} - z_{j^*,b}$, from equation (3) we get:

$$\lambda^* = \nu_{U,b} + f(\theta_b)\left[W^E_{j^*,b} - W^U_{j^*,b}\right] - \nu_{U,a} - f(\theta_a)\left[W^E_{j^*,a} - W^U_{j^*,a}\right] \qquad (24)$$

This equation can be re-written to make more visible the effect of housing and labor markets on the marginal worker's migration decision. Notice first that the F.O.C. in the Nash bargaining problem (19) implies that

$$W^E_{j,i} - W^U_{j,i} = \frac{\beta}{1-\beta} \cdot \frac{(1-s_{E,i})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,i}} \cdot J^E_i = \frac{\beta}{1-\beta} \cdot \frac{(1-s_{E,i})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,i}} \cdot \frac{k}{q(\theta_i)}$$

The last equality is obtained by imposing $J^V_i = 0$ in equation (16). Using this and the first equation in (13), we have:

$$\lambda^* = b\left[(1-s_{U,b})^{\frac{1}{1-\sigma}} - (1-s_{U,a})^{\frac{1}{1-\sigma}}\right] + \frac{\beta \cdot k}{1-\beta}\left[\frac{\theta_b(1-s_{E,b})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,b}} - \frac{\theta_a(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,a}}\right] \qquad (25)$$

The labor forces in both regions can be written as:

$$\begin{aligned} L_b &= H(\lambda^*)\,L \\ L_a &= (1-H(\lambda^*))\,L\,, \end{aligned} \qquad (26)$$

with $H(.)$ being the cumulative density function. It is easy to show that the RHS of (25) is increasing (resp. decreasing) in θ_b (θ_a) and decreasing (increasing) in $s_{U,b}$ and $s_{E,b}$ ($s_{U,a}$ and $s_{E,a}$). A tighter labor market in region b implies a higher job finding rate. This raises λ^*, augmenting the labor force L_b, as more workers are willing to migrate from a to b. Of course the opposite occurs in case of a higher θ_a. As we will see in the Equilibrium section, if housing and the tradable consumption good are gross complements, $s_{U,i}$ and $s_{E,i}$ are increasing in $p_{nt,i}$ (for $i \in \{a, b\}$)[20]. This means that L_b increases also because $p_{nt,a}$ goes up. If housing gets less affordable in the richer region a, workers find more convenient to re-locate in the poorer, less expensive one.

2.6 The housing market

As common in models studying local labor markets (see Moretti (2011)), we assume that the housing supply is in the hands of landowners that live abroad. While this assumption is clearly not realistic, separating workers from landowners in the model allows to distinguish the welfare effects of different shocks across different type of agents. In detail, we follow Hsieh and Moretti (2019) and consider the following housing supply schedule:

$$Q_{nt,i} = \alpha_i \, p_{nt,i}^{\frac{1}{\gamma}} \tag{27}$$

with $\alpha_i > 0$ a region-specific parameter and $i \in \{a, b\}$. Parameter γ stands for the inverse elasticity of the housing supply. At the equilibrium, the demand must be equal to the supply. This implies:

$$\begin{aligned} \alpha_i \, p_{nt,i}^{\frac{1}{\gamma}} &= U_i \cdot Q^U_{nt,i} + E_i \cdot Q^E_{nt,i} \qquad \Longleftrightarrow \\ \alpha_i \, p_{nt,i}^{\frac{1}{\gamma}} &= L_i \, \frac{u_i \cdot b \cdot s_{U,i} + (1 - u_i) \cdot w_i \cdot s_{E,i}}{p_{nt,i}} \end{aligned} \tag{28}$$

with $i \in \{a, b\}$. The second equation is obtained using the definitions for the housing shares out of total expenditures.

[20]The elasticity in (12) shows that, if $0 < \sigma < 1$, then $s_{E,i}$ is increasing in $p_{nt,i}$, *for any given* w_i. In the Equilibrium section, we will see that the sign of the derivative remains the same even when taking into account the wage schedule (22).

3 Equilibrium

3.1 Partial Equilibrium

For the sake of clarity, we find it convenient to present first a partial equilibrium version of the model. For partial equilibrium we mean with $p_{nt,i}$ and λ^* fixed. This is tantamount to saying that housing labor supply is perfectly elastic ($\gamma = 0$) and no migration takes place across regions ($\lambda \to +\infty$ for any generic worker j). Of course, these are extreme assumptions. We consider this scenario just to single out more starkly some characteristics of the model that hold even in the general equilibrium case.

Once we take $p_{nt,i}$ and λ^* as given, the only endogenous variables of the model remain the housing share for the employed workers $s_{E,i}$, the nominal w_i, and labor market tightness θ_i, in each region $i \in \{a, b\}$ (from equation (6) the housing share for the unemployed workers $s_{U,i}$ is uniquely determined for any given $p_{nt,i}$ and b). To determine these three unknowns we have to consider the system composed by the demand equation (11), the free entry zero profit condition (17) and the wage equation (22). The following Lemma presents the results.

Lemma 1 *A steady-state partial (i.e. for any given $p_{nt,i}$ and λ^*) equilibrium of the model exists and it is unique. Moreover, in the case $0 < \sigma < 1$ and $-1 < \epsilon < 0$, we have that $\frac{dw_i}{dp_{nt,i}} > 0$, $\frac{d\theta_i}{dp_{nt,i}} < 0$, $\frac{ds_{U,i}}{dp_{nt,i}} > 0$, and $\frac{ds_{E,i}}{dp_{nt,i}} > 0$.*

The proof is in Appendix B and it simply consists on the application of the implicit function theorem. Here we want to give an interpretation for the signs of these derivatives. The first one states that an increase in the housing prices leads to higher nominal pays. As we have anticipated in section (2.4) (but under the hypothesis of constant θ_i), this results stems from the assumption of non-homothetic preferences that raises workers marginal gains in the bargaining process and drives nominal earnings up.

Less affordable housing also reduces labor market tightness, via its effect on nominal wages. For the free-entry zero profit condition, higher labor costs will dampen firms' vacancy creation. This explains why $\frac{d\theta_i}{dp_{nt,i}} < 0$.

Finally, consider equations (6) and (11). More expensive housing costs raise the share of total expenditures devoted to buying this service for all employed and unemployed workers. As we have seen by examining the elasticities (8) and (12), if the consumption good and housing are gross complements, $s_{U,i}$ and $s_{E,i}$ are increasing in $p_{nt,i}$. Notice that there is a second, indirect effect of $p_{nt,i}$ on $s_{E,i}$, that goes in the opposite direction. This stems from the positive impact of the housing price on the nominal pay. More expensive housing costs exert an upward pressure on the nominal wage. Under non-homothetic preferences, a more generous nominal retribution reduces the housing share $s_{E,i}$. At the equilibrium, however, this negative effect is less strong and $p_{nt,i}$ always raises $s_{E,i}$ when $0 < \sigma < 1$.

3.2 General Equilibrium

Definition *A steady-state general equilibrium is defined as a vector* $[s_{U,i}, s_{E,i}, w_i, \theta_i, p_{nt,i}]$ *for* $i \in \{a, b\}$, *and a value for* λ^* *satisfying: (i) the demand equations expressed in terms of the housing shares for unemployed and employed workers, respectively* (6) *and* (11)*; (ii) the free entry zero profit conditions for firms producing the tradable consumption good, (17); (iii) the wage equation,* (22)*; (iv) the market clearing condition in the housing sector,* (28)*; (v) the migration decision rule,* (25)*.*

Compared to the partial equilibrium case, we consider a system with two additional endogenous variables, $p_{nt,i}$ and λ^*, and two additional equations, (28) and (25). Once all these variables are determined, all the remaining unknowns of the model (workers' utilities, the unemployment rates, and the labor forces in each region) can be easily found via their corresponding equations.

Proposition 1 *In the case* $0 < \sigma < 1$ *and* $-1 < \epsilon \leq 0$*, a steady-state general equilibrium exists and it is unique if* $\frac{1}{\gamma} > (1 + \epsilon)(1 - \sigma)$*. At the equilibrium, we have the following properties:* $w_a > w_b$, $p_{nt,a} > p_{nt,b}$*, and* $P_{U,a} > P_{U,b}$*.*

The formal proof is presented in Appendix C. The sufficient condition for the existence of an a equilibrium is not particularly demanding. It just requires the elasticity of the housing supply $1/\gamma$ to be sufficiently large[21].

Let us focus on the properties of the model. Note first that all the inequalities present in Proposition 1 hold both if preferences are homothetic ($\epsilon = 0$) and if they are not ($-1 < \epsilon < 0$).

In region a nominal wages are higher. This is intuitive, as productivity y_a is greater than y_b, so workers get a share of a larger surplus from the match (from equation (22), nominal pays positively depend on y_a). The second inequality in Proposition 1 states that the region with a higher productivity in the tradable sector also exhibits a higher price level in the non-tradable good (housing): $p_{nt,a} > p_{nt,b}$. This result is known in the literature as the Harrod-Balassa-Samuelson effect[22] and refers to the well-known fact that more developed countries present higher consumer prices compared to less developed ones. A common theoretical explanation for that lies on free labor mobility across sectors that, by equalizing wages, drives up the price of the non-tradable good in the country with a higher productivity in the tradable industry. The mechanism in our model is different: we have just seen that a higher productivity y_a positively affects nominal wage. In turn, this implies a stronger demand for housing and a higher price, as housing is a normal good.

Notice that this second inequality also reinforces the first one: nominal pays in region a are higher than in region b not just because $y_a > y_b$ but also because $p_{nt,a} > p_{nt,b}$. Housing prices exert an upward pressure on nominal wages for the mechanism explained in section 2.4.

Since $p_{nt,a} > p_{nt,b}$, unemployed workers located there will spend a larger share of

[21]Note that in our setting $(1-\epsilon)(1-\sigma)$ is the product of two positive terms lower than 1. If the sufficient condition is fulfilled, the equilibrium in the housing market exists even with an upward sloping demand function (i.e. the RHS of equation 28 increasing in $p_{nt,i}$). Clearly, this is more likely if the housing supply is more elastic (the schedule is flatter in the $(Q_{nt,i}, p_{nt,i})$ space).

[22]See Obstfeld and Rogoff (1996), 1996, chapter 4, for a detailed exposition.

their income on housing: $s_{U,a} > s_{U,b}$. This is a direct consequence of the sign of the elasticity (8) when goods are gross complements. From the definition of the price index for the unemployed workers in section 2.2.3, we have that $P_{U,a} > P_{U,b}$. Unemployed workers in the more productive region has a higher cost of living index.

We cannot state the same for the housing expenditures shares and the price indices of the employed workers. Those living in region a face a more expensive housing market (that raises $s_{E,a}$ and in turn $P_{E,a}$ when $0 < \sigma < 1$). However, they also receive more generous nominal pays, as both y_a and $p_{nt,a}$ are higher. Under non-homothetic preferences, this tends to reduce $s_{E,a}$ and $P_{E,a}$. The final effect cannot be ascertained at the analytical level.

Finally, we are also not able to determine whether the region with higher productivity in the tradable sector also exhibits a lower unemployment rate. This is because we do not know if θ_a is greater than θ_b. The ambiguity lies on two conflicting effects. On the one hand, from the zero profit condition (17), a higher productivity in region a tends to raise vacancy creation and labor market tightness. On the other hand, region a also exhibits larger labor costs, as nominal pays are more generous for the combined effect of higher y_a and $p_{nt,a}$. We have seen in section 2.4 that under homothetic preferences nominal wages are not affected by housing prices. The second effect is therefore weaker and we always obtain that a higher productivity in the tradable sector implies a tighter labor market and a lower unemployment rate. With $\epsilon < 0$ however we cannot ascertain which effect is stronger.

In the rest of the section, we will consider the implications for the entire economy of two different shocks that hit just one region: one that that changes the housing supply and the other affecting productivity in the tradable sector.

3.3 A housing supply shock in one region

The following Proposition summarizes the result:

Proposition 2 *Consider the model with* $0 < \sigma < 1$ *and* $-1 < \epsilon \leq 0$ *and a negative housing supply shock in region i (with* $i \in \{a, b\}$*):* α_i *is lower. At the new steady-state:*

1. *In region i, the price for housing services increases and the labor force is lower. If* $\epsilon = 0$*, there are no effects on the nominal wage and the unemployment rate. Conversely, if* $\epsilon < 0$*, the nominal wage and the unemployment rate in region i is higher.*

2. *In region j (with* $j \in \{a, b\}, i \neq j$*), the labor force and the price for housing services are higher. If* $\epsilon = 0$*, there are no effects on the nominal wage and the unemployment rate. Conversely, if* $\epsilon < 0$*, the nominal wage and the unemployment rate are higher also in region j.*

The proof is in Appendix C. Suppose a negative shock on the housing supply in one region: parameter α_i is now lower. We can interpret such a change as higher costs for building new houses or more legal restrictions in the supply of new housing services. At the new steady-state, the price $p_{nt,i}$ is obviously higher. Facing a more expensive housing market, more people will decide to migrate towards region j. Recall from the discussion on the migration decision (25) that $p_{nt,i}$ positively affects L_j for $i, j \in \{a, b\}, i \neq j$. In turn, this will raise the demand for housing service even there, so that $p_{nt,j}$ will also be higher.

From section 2.4, we know that with $\epsilon = 0$ nominal pays do not react to variations in housing prices. In turn, if nominal wages do not change, the zero profit condition (17) tells us that labor market tightness and the unemployment rate in both regions are unaffected by the negative shock on the housing supply. Things are different if $\epsilon < 0$. Under non-homothetic preferences workers are able to extract higher rents from the wage negotiation. Earnings in nominal terms go up, dampening vacancy creation. Unemployment soars in both regions. In the quantitative section we will see however that such an increase is quite small in magnitude.

Following a negative housing supply shock in just one region, all the unemployed workers in the entire country will be worse off, not just because their cost of living index has increased ($P_{U,i}$ and $P_{U,j}$ are larger because $p_{nt,i}$ and $p_{nt,j}$ go up) but also for the worse labor market conditions. Conversely, the welfare consequences for the employed workers are not clear-cut, as they experience more expensive housing services but also get more generous nominal pays. The effect on price indexes $P_{E,i}$ and $P_{E,j}$ and real wages are ambiguous.

3.4 A productivity shock in one region

Unlike the case of a housing supply shock, a change in the productivity of the tradable sector in one single region does not deliver unambiguous comparative statics results for most of the endogenous variables of the model. So we will devote more attention to the effects of this type of shock in the quantitative session 4.2.

As expected, a positive shock on y_i (with $i \in \{a, b\}$) has beneficial effects on the region in which it has occurred. Nominal wages go up, as workers' quasi-rents from the match have increased. Such a wage surge raises the demand for housing and the equilibrium price $p_{nt,i}$. Of course, this also raises the cost of living index for the unemployed workers $P_{U,i}$, whereas the signs of the change on $s_{E,i}$ and $P_{E,i}$ are uncertain. This is because employees pay more for any single unit of housing but more generous pays change their preferences, reducing their demand for the non-tradable good.

The effects on labor market tightness and unemployment are also ambiguous for the reasons discussed in Proposition 1. Firms have productivity gains but also face larger labor cost and at the analytical level we cannot claim which effect is stronger.

Comparative statics computations does not allow us to have clearcut conclusions on the change in the labor force in both regions. From equation (25), we know that the unemployed workers' decision to migrate depends on labor market tightness and the cost of living. After the positive shock on y_i, unemployed workers in region i face a higher cost of living $P_{U,i}$, but the impact on $P_{E,i}$ and θ_i are ambiguous. So, we cannot ascertain the impact of y_i on L_i and, consequently, L_j. Since in this model a local

Calibration I: variables			
Variables	Values	Interpretation	Source
r	0.00083	discount rate	1% on annual basis
β	0.5	workers' bargaining power	arbitrary choice
δ	0.005	separation rate	Hartung et al. (2018)
γ	2	inverse elasticity of the housing supply	Cavalleri et al. (2019)
σ	0.56	elasticity of substitution between the two good	Finlay and Williams (2022)
η	0.5	the opposite of the elasticity of the job-filling rate	Hosios condition (Hosios (1990))
L_a	33181641	labor force in a	German Federal Statistical Office (https://www.destatis.de)
L_b	7650723	labor force in b	German Federal Statistical Office (https://www.destatis.de)
u_a	0.053	unemployment rate in a	German Federal Statistical Office (https://www.destatis.de)
u_b	0.076	unemployment rate in b	German Federal Statistical Office (https://www.destatis.de)
$s_{E,a}$	0.30	housing expenditure share for employed in a	German Federal Statistical Office
$s_{U,a}$	0.40	housing expenditure share for unemployed in a	German Federal Statistical Office
$s_{E,b}$	0.28	housing expenditure share for employed in b	German Federal Statistical Office
$s_{U,b}$	0.35	housing expenditure share for unemployed in b	German Federal Statistical Office
α_b	1	housing supply parameter $i = b$	normalization
m_a	1	matching function paramter in $i = a$	normalization

Table 1: Calibration I: variables

shock propagates into the other region via its effects on the labor force, the ambiguous effect on y_i on L_j means we are not able to have analytical results on the housing and labor market variables in region j.

4 Quantitative Results

4.1 Calibration

The model is calibrated on the basis of German data in the period 2013-2018. The numerical values of some variables are taken from data (and presented in Table 1), others are obtained by evaluating the model at the steady-state (see Table 2).

We identify as region b the six re-established states of the former German Democratic Republic (GDR) and as region a the ten "old" states of the Federal Republic. The month is the unit of time. The discount rate r is fixed at 1% on an annual basis. The elasticity of the housing supply in Germany in the period considered is about

Calibration II: results		
Variables	Interpretation	Source
ϵ	non-homotheticity parameter	using the four equations (6) and (11) to get a real wage gap of 1.2
b	unemployed workers' home production	housing market equilibrium equation (28) at $i = b$
α_a	housing supply parameter $i = a$	housing market equilibrium equation (28) at $i = a$
$p_{nt,a}$; $p_{nt,b}$	housing prices	equations (6)
w_a; w_b	nominal wages	equations (6) and (11)
θ_a	labor market tightness in region a	steady state equation (1)
θ_b	labor market tightness in region b	steady state equation (1)
k	cost of keeping a vacancy open	zero profit condition (17) at $i = a$ and wage equation (22) at $i = a$
y_a	labor productivity in region a	zero profit condition (17) at $i = a$ and wage equation (22) at $i = a$
m_b	matching function parameter in region b	zero profit condition (17) at $i = b$ and wage equation (22) at $i = b$
y_b	labor productivity in region b	zero profit condition (17) at $i = b$ and wage equation (22) at $i = b$
λ	degree of labor mobility parameter	data on L_a and L_b
λ^*	degree of labor mobility for the marginal worker	migration condition (25)

Table 2: Calibration procedure. Unit of time: month.

0.5, according to Cavalleri et al. (2019)[23]. So parameter γ is equal to 2. As concerns the elasticity of substitution parameter σ, we follow Finlay and Williams (2022) and fix it equal to 0.56 (we consider different values for σ in section 4.3). For Hartung et al. (2018), in the years immediately before those considered for our calibration, in Germany about 0.5% of workers transited from the employment to the unemployment status each month[24]. So parameter δ is equal to 0.005. We consider a standard value for workers' bargaining power β equal to 0.5. As concerns the matching function, we assume a standard Cobb-Douglas functional form: $M_i = m_i V_i^{1-\eta} U_i^{\eta}$ for $i \in \{a, b\}$. We normalize m_a to 1 and, just for simplicity, we also impose the Hosios (1990) condition $\beta = \eta$ that ensures the efficiency of the matching process[25].

Data on the the labor force (L_a, L_b) and the unemployment rate (u_a, u_b) are taken from German Federal Statistical Office, that also provided us the figures on housing[26]

[23]Beze (2023) estimates an elasticity of about 0.25, while Lerbs (2012) obtained a value of 0.4 for the period 2004-2010. We perform a sensitivity analysis on parameter γ in section 4.3.

[24]Carrillo-Tudela et al. (2021) also get similar results.

[25]A sensitivity analysis on β is presented in section 4.3.

[26]For the housing expenditure shares we consider the entry "wohnungsmieten" in the data, that

expenditure shares for different income groups and for the two different regions of the country[27]. We have therefore to make assumptions on how to relate these data on the expenditure shares for employed and unemployed workers in our model. We attribute to all the employed workers the same income group of the net median wage in Germany in those years (about 2500 euros per month). This corresponds to the class $[2000, 2600]$ for both East and West Germany workers. We get for the period $2013-2018$ an average expenditure shares for housing of 30% and 28% for the employed workers in region a and region b, respectively. As concerns the unemployed workers, in absence of data on their median income, we assume that they belong to the second income group from the bottom: $[900, 1300]$.[28] This implies a value for their housing expenditure shares of 40% and 35% for the unemployed workers in region a and region b, respectively.

As concerns the variables obtained using the equilibrium conditions of the model, we start with the four housing expenditure shares equations. It is easy to see that using equations (6) and (11) for $i = a$ and $i = b$, one can get rid of $p_{nt,i}$ and b and obtain an expression for the real wage gap $\left[w_a\ (1 - s_{E,a})^{\frac{1}{1-\sigma}}\right] / \left[w_b\ (1 - s_{E,b})^{\frac{1}{1-\sigma}}\right]$ as a function of the housing expenditure share and the two utility function parameters,

includes both effective and the figurative rents. For a detailed discussion on the pros and cons of considering figurative rents, see Dustmann et al. (2021).

[27]More precisely, we have information on the expenditure shares in East and West Germany for each year in the interval $(2010, 2020)$, for people with a monthly income belonging to the following groups: under 900 euros, $[900, 1300]$, $[1300, 1500]$, $[1500, 2000]$, $[2000, 2600]$, $[2600, 3600]$, and $[3600, 5000]$.

[28]In Germany the primary form of unemployment benefits foresees a payment of 60% of prior average pay (or 67% in case you have children). So 60% of the median wage belong to the upper bound of the $[900, 1300]$ interval. Of course, this is just an approximation, as it means neglecting other sources of income production that are included in our definition of the variable b. It must be also noted that not all registered unemployed workers receive benefits: the duration of the insurance scheme is limited, ranging from 6 to 24 months, on the basis of the recipient's age and time spent on employment, contributing to unemployment insurance. Finally, as documented by Carrillo-Tudela et al. (2021), after the so-called Harz (2005) reform, a not negligible number of unemployed workers ceased to register as such. So they are officially considered as non participants, with no unemployment insurance.

ϵ and σ. Since σ, $s_{E,i}$, and $s_{U,i}$ for $i \in \{a, b\}$ have been determined, one can use information on the real wage gap to pin down the value of ϵ. Recent estimates on this gap range from 1.35 (Heise and Porzio (2019)) to 1.17 (Boeri et al. (2021))[29]. We set it equal to 1.22, a number inside this interval, obtaining a value for ϵ equal to -0.8.

Once ϵ has been determined, we can use equations (6) to write $p_{nt,i}$ (for $i \in \{a, b\}$) as a function of b. We plug this expression for $p_{nt,b}$ in the equilibrium equation in the housing market in region b (28). Imposing the housing supply parameter in region b, α_b, equal to 1, this equation has only one unknown, the value of home production b. Once b has been found, the equilibrium values of $p_{nt,a}$, $p_{nt,b}$, w_a, and w_b are easily obtained via the housing expenditure shares equations (6) and (11). We can then use the equilibrium equation in the housing market in region a, (28), to find α_a, the housing supply parameter in region a.

Since $m_a = 1$, using the figure for the unemployment rate in region a in the steady-state equation (1) we get the value for θ_a. We then re-arrange the zero profit conditions (17) to write y_a as a function of k and y_b as a function of k and m_b. Inserting the expression for y_a in the wage equation (22) at $i = a$, we obtain the equilibrium value for k. We then plug the expression for y_b in the wage equation (22) at $i = b$ and get the matching parameter m_b.

The only two remaining unknowns are λ, the parameter that captures the degree of mobility in equation (23), and λ^*, the threshold value that splits the labor force L into L_a and L_b. We can easily obtain λ^* via equation (25), as all the variables at the RHS have been already determined. We finally assume that the function $g(.)$ in equation (23) is normally distributed with 0 mean and standard deviation λ. Then λ is computed knowing that the integral of the normal distribution in the interval $[\lambda^*, +\infty)$ must be equal to L_a/L.

We check the empirical validity of our calibration by looking at three figures found

[29] Dickey and Widmaier (2021) obtain a real wage gap of about 1.2 even after having accounted for different human capital endowments, location effects, and human capital depreciation.

in data and not used in our procedure. First, the nominal wage gap w_a/w_b. According to Boeri et al. (2021), it is equal to 1.28. In our quantitative exercise it is equal to 1.286. Recall that in the calibration we used information on the real wage gap. That targeting relative real pays we also get realistic figures for the relative nominal values suggests our costs of living differences between West and East Germany are empirically plausible.

We also look at the productivity ratio y_a/y_b. According to our elaborations on OECD (2023a) data, the gap in gross real value added per employee between West and East Germany was about 1.25 in 2013 and declined in the subsequent years. In 2018 it was 1.2. These are approximately the same figures that can be found in Boeri et al. (2021) and Mertens and Mueller (2022). Our calibrated result for y_a/y_b is 1.28, a value not far from these findings.

Finally, we also look at some validations for the labor market numerical values. In our model $1/f(\theta_i)$ is the expected duration in unemployment, the reciprocal of the job finding rate. For Hartung et al. (2018) in the period (2004 - 2014) less than 6% of the unemployed workers found a job each month, with a resulting average expected duration of about 16 months. Data collected by Carrillo-Tudela et al. (2021) for the same period imply similar (if slightly lower) transition rates: about 4%, with an expected duration of about two years. Our calibration, that is based on a subsequent interval of years, delivers an expected duration of 11 months in region a and 16 months in region b. These seem plausible values.

4.2 Simulations

Since comparative statics offers more clearcut results if we consider a change in the housing supply than in the tradable sector productivity, we first simulate a shock on the latter. Numerical exercises also allow us to have insights on some variables not considered in the previous section, such as income inequality and expected utilities.

Productivity shocks

We simulate two different scenarios. In the first one, a positive shock in region a (West Germany) of 1% magnitude raises productivity in the tradable sector y_a. In the second exercise, we look at a 1% change in y_a and 1.5% increase in in y_b. This aims to mirror the trend of the economies in West and East Germany, as data show a slow but interrupted convergence[30].

The main results of the first scenario are summarized in Table 3. To provide a clear picture of the impact of the non-homotheticity assumption on our findings, in the third column we present the values obtained via a sensitivity analysis in which ϵ is equal to 0 (details of the procedure are in Appendix E).

Notice first that housing prices and nominal wages are quite responsive to such productivity increase. In the states directly affected by this change both variables change by roughly the same extent of y_a. This is the consequence of the positive co-dependence between nominal pays and housing prices implied by non-homothetic preferences. When a positive productivity shock hits one area of the country, the implied larger match surplus drives the negotiated wage up. This raises the demand for housing services, so prices increase. For the reasons discussed in section 2.4, more expensive dwellings in turn exert an upward pressure on nominal salaries, creating a positive feedback loop.

Migration towards West Germany weakens the demand for housing in the East. Housing prices go down by 2%. As a result, West Germany housing prices become 3% more expensive compared to the East (i.e. the ratio $p_{nt,a}/p_{nt,b}$ is 3% larger). Under homothetic preferences the relative change is three times smaller. For the just exposed wage price positive link, we expect to have lower nominal pays in region b. However, the effects on nominal pays appear to be very modest.

Variations in housing prices and nominal pays have an impact on the cost of living.

[30] According to our elaborations based on OECD (2023) estimates, in 2008 the value added per worker West-East ratio was about 1.3. In 2018, it was 1.2. In 2020, it was about 1.17.

Percentage change	$\epsilon < 0$	$\epsilon = 0$
L_a	0.5	0.2
L_b	-2.1	-0.3
u_a	-0.0 pp	-0.0 pp
u_b	-0.0 pp	-0.0 pp
$p_{nt,a}$	1.1	0.9
$p_{nt,b}$	-2.0	-0.3
$P_{E,a}$	0.1	0.6
$P_{E,b}$	-0.7	-0.2
$P_{U,a}$	0.6	0.6
$P_{U,b}$	-0.9	-0.2
w_a (real)	1.0 (0.9)	1.0 (0.4)
w_b (real)	-0.0 (0.7)	-0.0 (0.2)
real b in region a	-0.6	-0.6
real b in region b	1.0	0.2
Variance of log income (real)	2.3 (1.4)	3 (0.9)
W_a^E	0.9	0.4
W_a^U	0.9	0.4
W_b^E	0.7	0.2
W_b^U	0.7	0.2

Table 3: Productivity shock in region a: $\Delta y_a / y_a = 1\%$ (percentage changes; for the unemployment rates variation in percentage points).

Recall first from section 2.2.3 that the model implies four different consumer price indexes for the four categories of individuals in the economy: employed and unemployed workers in region a or region b ($P_{E,a}$, $P_{U,a}$, $P_{E,b}$, and $P_{U,b}$ respectively). A positive productivity shock in the tradable good produces two effects on such indexes. In the

Western states of Germany, where the relative price of housing goes up, everyone faces a more expensive cost of living[31]. But there is also a second mechanism in motion when preferences are non-homothetic. More generous nominal salaries lower the share of total expenditures devoted to housing, that is a necessity good (in equation 11, $s_{E,i}$ decreases with w_i). Employed workers want to spend a lower fraction of their income on the good that has become relatively dearer. This tends to reduce their cost of living. The second effect is not present for the unemployed workers, as their income b does not change with productivity. As Table 3 illustrates, the price index for the employed workers barely moves. Conversely, unemployed workers experience a not negligible increase in their cost of living. The opposite occurs in the Eastern states of Germany, where housing prices are lower. Unemployed workers benefit more from such a decrease, as housing is a necessity good. Their price index goes down by almost 1%, compared to −0.7% for the employed workers. Of course such heterogeneity in the variation of cost of living coefficients disappears once we consider a homothetic utility function. From the third column in Table 3 we see that both $P_{E,a}$ and $P_{U,a}$ (resp. $P_{E,b}$ and $P_{U,b}$) go up (resp. go down) by the same 0.6% (resp. 0.2%).

If housing is a necessity good, in the states hit by the shock employed workers experience both a pay rise in nominal terms and a (almost nil) change in their cost of living index. Unemployed workers in the West do not get any increase in nominal terms and face a greater surge in their cost of living. Compared to the homothetic scenario, the variation in their real income gap is more than 50% larger. Conversely, since the reduction in $P_{U,b}$ is larger than in $P_{E,b}$, the income divide in the East is narrower. In the end we get that overall inequality in the country (measured by variance of the natural logarithm of real income) goes up.

It is worthwhile to notice that these results do not imply that unemployed workers

[31]From equations (6) and (11), we have that if the tradable and the non-tradable goods are gross complements (i.e $0 < \sigma < 1$), a higher $p_{nt,i}$ raises both $s_{U,i}$ and $s_{E,i}$, thereby increasing $P_{E,i}$ and $P_{U,i}$ (see their definition in section 2.2.3).

are negatively affected by a positive productivity shock. True, the instantaneous utility in unemployment $z_{j,i} + \nu_{U,i}$ goes down for $i \in \{a, b\}$ and any worker j. But our simulations suggest that the expected lifetime utility $W^U_{j,i}$ (presented in equation 3) does increase in all the country. The positive effect of y_i on both the job finding rate $f(\theta_i)$ and the value of being employed $W^E_{j,i}$ outweighs the current loss in real income.

Finally, we find that increasing productivity has negligible effects in terms of employment. Table 3 shows that the unemployment rate goes down by 0.1 percentage points at most. In the sensitivity analysis presented in section 4.3, we also find that a rigid housing market does not seem to be the culprit. A much larger elasticity $1/\gamma$ produces similar employment effects. In general, standard search and matching models are not able to mimic the observed large fluctuations in unemployment in response to an exogenous productivity shock (see Shimer (2005)). The same occurs in this model.

Our simulations indicate that a 1% increases in y_a raises labor market tightness θ_a by 1.6%. The effect on unemployment however is about half that magnitude[32]. Such a low elasticity translates into a very small change in percentage points.

Table 4 presents the results of the second scenario. Compared to the previous exercise the most interesting findings concern housing prices and the cost of living indexes. Since in this scenario a similar positive productivity shock hits both areas, we find a small impact on inter-regional migration (i.e. small changes in L_a and L_b). In turn, this implies that housing prices do not change as much as in the previous scenario where a larger (resp. lower) labor force raises (resp. decreases) the demand for housing in region a (resp. b). The income effect is therefore stronger than the price effect. In this scenario, under non-homothetic preferences, employed workers in West Germany get a small reduction in their price index.

[32]From equation (1), it is easy to see that the elasticity of the unemployment rate u_i with respect to y_i, μ_{u_i,y_i} is equal to $(1 - u_i)(1 - \eta)\mu_{\theta_i,y_i}$ with μ_{θ_i,y_i} being the elasticity with respect to tightness. Recall that η is imposed equal to 0.5.

Percentage change	$\epsilon < 0$	$\epsilon = 0$
L_a	-0.1	0.02
L_b	0.4	-0.04
u_a	-0.0 pp	-0.0 pp
u_b	-0.0 pp	-0.1 pp
$p_{nt,a}$	0.5	0.8
$p_{nt,b}$	1.3	1.0
$P_{E,a}$	-0.1	0.5
$P_{E,b}$	0.0	0.7
$P_{U,a}$	0.3	0.5
$P_{U,b}$	0.6	0.7
w_a (real)	1.0 (1.1)	1.0 (0.5)
w_b (real)	1.5 (1.5)	1.5 (0.8)
real b in region a	-0.3	-0.5
real b in region b	-0.6	-0.8
Variance of log income (real)	3.2 (1.4)	3.2 (0.2)
W_a^E	1.1	0.5
W_a^U	1.1	0.5
W_b^E	1.4	0.8
W_b^U	1.4	0.8

Table 4: Productivity shock in regions a and b: $\Delta y_a/y_a = 1\%$ and $\Delta y_b/y_b = 1.5\%$ (percentage changes; for the unemployment rates variation in percentage points).

Housing supply shock

We simulate a supply shock that raises of the marginal cost of housing in West Germany (region a). More specifically, we decrease parameter α_a in the supply function (27) by 5%. We can interpret it as higher costs for building houses or stricter legal restrictions

in the supply of new housing services. Table 5 summarizes the results.
As expected, housing prices increase in both areas (4.0%, +3.4%), as more expensive dwellings in the Western states push more workers to find a job in the East (L_b goes up by 3.7%), raising the demand even there.
The cost of living indexes go up for all workers in the economy. Of course, the increase is larger in for workers in the Western states, where the shock has occurred. Notice also that the change in $P_{U,i}$ is $0.5 - 0.6$ percentage points larger than $P_{E,i}$ for $i \in \{a, b\}$. Unemployed workers suffer more from the shock, as housing is a necessity good that makes up a larger share of their total expenditures. Again this differential impact cannot be captured if $\epsilon = 0$. As the third column illustrates, $P_{U,i}$ and $P_{E,i}$ change by the same amount for $i \in \{a, b\}$. Real income decreases for all the workers in the economy and this has straightforward consequences on their expected lifetime utilities.

As concerns income inequality, the variance of the logarithm of real incomes increases by almost 1%. Notice that in the $\epsilon = 0$ case we see a reduction of the same variable. This is because the adoption of a unique price index for employed and unemployed workers alike (implied by the homotheticity assumption) accentuates the real income loss of the richest people (employed workers in the West) and underestimates the loss of the poorest ones (unemployed workers in the East)[33].

4.3 Sensitivity analysis

We focus on three main parameters that could in principle alter the main findings of our simulation results: the inverse of the elasticity of the housing supply γ, workers' bargaining power β, and the elasticity of substitution σ.

Let us consider first γ. According to Hsieh and Moretti (2019), the key reason for the employment misallocation across US regions is an excessively rigid housing supply, that, implying large increases in housing prices for any given change in the demand,

[33]From Table 5, if $\epsilon = 0$, real wages in the West go down by 2.2% (compared to a -1.2% with $\epsilon < 0$) whereas unemployed workers in the East suffer from a 0.9% decrease (-1.6% with $\epsilon < 0$).

Percentage change	$\epsilon < 0$	$\epsilon = 0$
L_a	-0.9	-0.9
L_b	3.7	1.6
u_a	0.0 pp	0.0 pp
u_b	0.0 pp	0.0 pp
$p_{nt,a}$	4.0	3.2
$p_{nt,b}$	3.4	1.2
$P_{E,a}$	1.6	2.2
$P_{E,b}$	1.2	0.9
$P_{U,a}$	2.3	2.2
$P_{U,b}$	1.7	0.9
w_a (real)	0.0 (-1.5)	0.0 (-2.2)
w_b (real)	0.0 (-1.2)	0.05 (-0.9)
real b in region a	-2.3	-2.2
real b in region b	-1.6	-0.9
Variance of log income (real)	0.5 (0.9)	0.1 (-2.7)
W_a^E	-1.6	-2.2
W_a^U	-1.6	-2.2
W_b^E	-1.2	-0.9
W_b^U	-1.2	-0.9

Table 5: Housing supply shock in regions a : $\Delta\alpha_a/\alpha_a = -5\%$ (percentage changes - for the unemployment rates variation in percentage points).

makes migration towards more productive areas too expensive. Employment changes are limited.

In our model productivity shocks produce small effects on employment. Several empirical works indicate that even Germany has a quite inelastic housing supply too.

Cavalleri et al. (2019), Beze (2023), and Lerbs (2012) all estimate an elasticity ranging from 0.25 to 0.5. So in our basic setup, γ is fixed equal to 2. Tables 6 in Appendix E shows the results of a productivity shock and a housing supply shock in case γ is fixed equal to 0.5, implying a counterfactual housing supply elasticity of 2. The central findings of our baseline simulations are unaffected even under an elastic housing supply. Both shocks have a greater impact on the cost of living of unemployed workers. However the welfare effects are not different across individuals: a regional positive productivity shock makes all workers in the economy better off, whereas an increase in the marginal cost of housing reduces the expected lifetime utility of everyone. The sign of the variations in the log income variances are also the same of the baseline simulations. More importantly, employment effects remain very small, despite the larger elasticity $1/\gamma$. In the $\Delta y_a/y_a = 1\%$ scenario, a more elastic housing supply does indeed imply larger migration towards region a: the labor force in a (resp. in b) goes up (resp. down) by almost 1% (resp. 4%). The percentage changes are respectively 0.5 and -2.1% in the baseline simulation. However, such a greater inter-regional workers' relocation does not translate into greater changes in housing prices, precisely because the supply is much more elastic. More importantly, the employment effects remain very small.
We then change β, from 0.5 to 0.4 and 0.6. As we can see from Table 7 in Appendix E, in both cases, and for all simulations, the results are remarkably similar (identical to the first decimal place) to the baseline case with $\beta = 0.5$. So workers' bargaining power appears to be quite uninfluential in our setting, at least if we consider values not too close to its upper and lower bound.

Finally, we consider variations in the elasticity of substitution of goods σ. As Table 8 illustrates, changes are minimal compared to the baseline scenario. We just point out that, with a lower elasticity of substitution ($\sigma = 0.3$) the effects of a housing supply shock on the housing prices get larger in magnitude, as consumers are even less willing to switch from one good to the other. In turn this implies a larger increase in the cost of living index for all workers.

5 Conclusions

Four in five individuals in the OECD believe that income inequalities are excessive in their respective countries (OECD (2021)). This perception seems to be present even where income disparities are low and do not appear to be growing. Germans perceive larger-than-average earnings inequality, even though objective measures of income disparities are below the OECD average and they have remained quite stable in the last decade (Biewen et al. (2019) and Drechsel-Grau et al. (2022)).

One possible explanation for this discrepancy between data and perceptions lies on the fact that most indicators of inequality (computed using either administrative data or household surveys) rely on a unique price or cost-of-living index for all categories of individuals, although it is well known that tastes vary with income[34].

The present paper addresses this point. Positive productivity shocks in the tradable sector or stricter regulations in housing supply at local level have a different impact on employed and unemployed workers, the latter experiencing a larger variation in their cost-of-living index. Inequality is larger than it would be implied using an identical price index for all individuals.

It is important not to infer welfare results from our inequality effects. Our simulations indicate that, if a positive shock in the TFP in the tradable sector lowers unemployed workers' instantaneous utility as living gets more expensive, their expected lifetime utility increases. Better employment opportunities (i.e. a tighter labor market) and the increased value of being employed overcome the present instantaneous loss. So, higher real income dispersion does not entail a welfare loss, at least if we consider a utility function where inequality is not negative *per se*. Of course, our conclusions change with myopic individuals whose current welfare loss matters more than future

[34] The importance of non-homotheticity has been recently emphasized by Handbury (2021). Across US locations, the variety of products and prices offered in stores depends on local income levels, with stores in wealthy cities favouring (both in terms of product variety and relative prices) the consumption bundle of high income households.

expected gains.

Employment changes appear to be limited. In our model we focus just on the labor markets in the tradable sectors. Further analysis could also look at the reallocation effects of housing and productivity shocks when both tradable and non-tradable labor markets are considered. Recent papers have pointed out that the negative consequences of the decline in the manufacturing industries have been concealed by the boom in housing sector employment (Kerwin Kofi et al. (2019)).

Our model does not account for different skill levels. By introducing heterogeneity in workers' abilities we could explore how the property market affects the real skill premium. For Moretti (2013), once cross-regional price differences are properly computed, real income disparities between skills groups in US are less pronounced than nominal ones, as highly educated workers migrate towards more productive areas raising housing prices. Dustmann et al. (2021) do not find evidence for that in Germany, where the share of people at the bottom income quintile located in more expensive regions and cities has increased over the last two decades.

Both the tradable/non-tradable employment inter-linkage and the effects of the property market on the real skill premium are left for future research.

References

Albouy, D., G. Ehrlich, and Y. Liu (2016). Housing Demand, Cost-of-Living Inequality, and the Affordability Crisis. *Discussion Paper, University of Illinois*.

Belfield, C., D. Chandler, and R. Joyce (2015). Housing: Trends in Prices, Costs and Tenure. *Election Briefing Note BN161, Institute for Fiscal Studies*.

Beze, E. (2023). Geographic Constraints and the Housing Supply Elasticity in Germany. Ruhr Economic Papers 1003, RWI - Leibniz-Institut für Wirtschaftsforschung, Ruhr-University Bochum, TU Dortmund University, University of Duisburg-Essen.

Biewen, M., M. Ungerer, and M. Löffler (2019). Why Did Income Inequality in Germany Not Increase Further After 2005? *German Economic Review 20*, 471–504.

Boeri, T., A. Ichino, E. Moretti, and J. Posch (2021). Wage Equalization and Regional Misallocation: Evidence from Italian and German Provinces . *Journal of the European Economic Association 19*(6), 3249–3292.

Branch, W. A., N. Petrosky-Nadeau, and G. Rocheteau (2016). Financial Frictions, the Housing Market, and Unemployment. *Journal of Economic Theory 164*, 101–135.

Carrillo-Tudela, C., A. Launov, and J. Robin (2021). The Fall in German Unemployment: A Flow Analysis. *European Economic Review 132*.

Cavalleri, M. C., B. Cournède, and E. Özsöğüt (2019). How Responsive are Housing Markets in the OECD? National Level Estimates. OECD Economics Department Working Papers 1589, OECD Publishing.

Comin, D., D. Lashkari, and M. Mestieri (2021). Structural Change With Long Run Income and Price Effects. *Econometrica 89 (1)*, 311–374.

Davis, M. and S. Van Nieuwerburgh (2015). Housing, Finance and Macroeconomy. *Handbook of Regional and Urban Economics, Chapter 12, 5A*, 813–886.

Deaton, A. and J. Muellbauer (1980). *Economics and Consumer Behaviour*. Cambridge: Cambridge University Press.

Dickey, H. and A. M. Widmaier (2021). The Persistent Pay Gap between Easterners and Westerners in Germany: A Quarter-Century After Reunification. *Papers in Regional Science 100*(3), 605–631.

Diewert, W. (2009). Cost of Living Indexes and Exact Index Numbers. *UBC Department of Economics, UBC Departmental Archives 288*.

Drechsel-Grau, M., A. Peichl, K. D. Schmid, J. F. Schmieder, H. Walz, and S. Wolter (2022). Inequality and Income Dynamics in Germany. *Quantitative Economics 13*(4), 1593–1635.

Dustmann, C., B. Fitzenberger, and M. Zimmermann (2021). Housing Expenditures and Income Inequality. *The Economic Journal 11953*.

Finlay, J. and T. Williams (2022). Housing Demand, Inequality, and Spatial Sorting. *Available at SSRN: https://ssrn.com/abstract=3635035 or http://dx.doi.org/10.2139/ssrn.3635035*.

Frayne, C., A. Szczypińska, B. Vašíček, and S. Zeugner (2022). Housing Market Developments in the Euro Area: Focus on Housing Affordability. European Economy - Discussion Papers 171, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Hagedorn, M. and I. Manovskii (2008). The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *The American Economic Review 98*, 1692 – 1706.

Handbury, J. (2021). Are Poor Cities Cheap for Everyone? Non-Homotheticity and the Cost of Living Across U.S. Cities. *Econometrica 89*(6), 2679–2715.

Hartung, B., P. Jung, and M. Kuhn (2018). What Hides Behind the German Labor Market Miracle? Unemployment Insurance Reforms and Labor Market Dynamics. *IZA Discussion Paper 12001.*

Heise, S. and T. Porzio (2019). Spatial Wage Gaps and Frictional Labor Markets. *FRB of New York Staff Report 898.*

Hosios, A. J. (1990). On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies 57*, 279–298.

Hsieh, C. and E. Moretti (2019). Housing Constraints and Spatial Misallocation. *American Economic Journal: Macroeconomics 11*, 1–39.

Kerwin Kofi, C., E. Hurst, and M. Schwartz (2019). The Transformation of Manufacturing and the Decline in US Employment. *NBER Macroeconomics Annual 33 (1)*, 307–372.

Larrimore, J. and J. Schuetz (2017). Assessing the Severity of Rent Burden on Low-Income Families. *FEDS Notes, Washington: Board of Governors of the Federal Reserve System.*

Lerbs, O. (2012). House Prices, Housing Development Costs, and the Supply of New Single-Family Housing in German Counties and Cities. *Journal of Property Research 31.*

Mertens, M. and S. Mueller (2022). The East-West German Gap in Revenue Productivity: Just a Tale of Output Prices? *Journal of Comparative Economics 50(3)*, 815–831.

Mian, A. and A. Sufi (2014). What Explains the 2007-2009 Drop in Employment? *Econometrica 82*, 2197–2223.

Monte, F., S. J. Redding, and E. Rossi-Hansberg (2018). Commuting, Migration, and Local Employment Elasticities. *American Economic Review 108(12)*, 3855 – 3890.

Moretti, E. (2011). Local Labor Markets. In *Handbook of Labor Economics, Volume 4B*. O. Ashenfelter and D. Card, editors, Elsevier Science Publisher (North-Holland), Amsterdam.

Moretti, E. (2013). Real Wage Inequality. *American Economic Journal: Applied Economics 5(1)*, 65–103.

Obstfeld, M. and K. Rogoff (1996). *Foundations of International Macroeconomics.* Cambridge, Mass: MIT Press.

OECD (2021). *Does Inequality Matter? How People Perceive Economic Disparities and Social Mobility.* OECD Publishing, Paris.

OECD (2023a). Gdp per hour worked (indicator) - available at https://data.oecd.org/lprdty/gdp-per-hour-worked.htm. *OECD, Paris*.

OECD (2023b). Housing prices - available at https://data.oecd.org/price/housing-prices.htm. *OECD, Paris*.

Petrongolo, B. and C. Pissarides (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature 39*, 716–741.

Pissarides, C. (2000). *Equilibrium Unemployment Theory.* 2nd edition, Cambridge, Mass: MIT Press.

Quigley, J. and S. Raphael (2004). Is Housing Unaffordable? Why Isn't It More Affordable? *Journal of Economic Perspectives 18(1)*, 191–214.

Shimer, R. (2005). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *The American Economic Review 95*, 25–49.

Appendix A: The Wage Equation

We derive the equation (21). From equation (15) and the condition $J_i^V = 0$ we easily get that

$$\frac{1-\beta}{J_i^E} = (1-\beta)\,\frac{r+\delta}{y_i - w_i} \tag{29}$$

for $i \in \{a, b\}$.

As concerns the RHS of (21), using the equations (3) and (9) to derive an expression for $W_{j,i}^E - W_{j,i}^U$ and the equation (13) to compute the derivative $d\nu_{E,i}/d\,w_i$ we obtain:

$$\frac{\frac{d\nu_{E,i}}{dw_i}}{W_{j,i}^E - W_{j,i}^U} = \frac{r+\delta}{z_{j,i} + \nu_{E,i} - rW_{j,i}^U} \cdot \frac{(1-s_{E,i})^{\frac{1}{1-\sigma}}}{1+\epsilon s_{E,i}} \tag{30}$$

for $i \in \{a, b\}$. Rearranging equation (16) with $J_i^V = 0$, we get $J_i^E = k/q(\theta_i)$. We use this expression to write the F.O.C (19) as follows:

$$W_{j,i}^E - W_{j,i}^U = \frac{\beta}{1-\beta}\,\frac{k}{q(\theta_i)}\,\frac{(1-s_{E,i})^{\frac{1}{1-\sigma}}}{1+\epsilon s_{E,i}}$$

with $i \in \{a, b\}$. Using this equation and the fact that $\nu_{U,i} = b\,(1-s_{U,i})^{\frac{1}{1-\sigma}}$ (from equation 13), equation (3) becomes:

$$rW_{j,i}^U = z_{j,i} + b\,(1-s_{U,i})^{\frac{1}{1-\sigma}} + \frac{\beta}{1-\beta}\,k\,\theta_i\,\frac{(1-s_{E,i})^{\frac{1}{1-\sigma}}}{1+\epsilon s_{E,i}}$$

We can insert this expression $W_{j,i}^U$ into equation (30). Knowing that $\nu_{E,i} = w_i\,(1-s_{E,i})^{\frac{1}{1-\sigma}}$ (from equation 13) and that $\mu_i = 1/\,(1+\epsilon s_{E,i})$ (from equation 14), we obtain:

$$\frac{\frac{d\nu_{E,i}}{dw_i}}{W_{j,i}^E - W_{j,i}^U} = \frac{(r+\delta)\,\mu_i}{w_i - b\left(\frac{1-s_{U,i}}{1-s_{E,i}}\right)^{\frac{1}{1-\sigma}} - \frac{\beta}{1-\beta}\,\mu_i\,k\cdot\theta_i}$$

for $i \in \{a, b\}$. Putting together this expression with equation (29), we easily obtain equation (21).

We also want to show that the RHS of equation (21) is increasing in $p_{nt,i}$. Computing the derivative, we get:

$$\beta\left[\frac{\frac{d\mu_i}{dp_{nt,i}}\cdot(1-s_{E,i})^{\frac{1}{1-\sigma}}}{w_i\cdot(1-s_{E,i})^{\frac{1}{1-\sigma}}+z_{j,i}-rW^U_{j,i}}-\mu_i\frac{\frac{1}{1-\sigma}(1-s_{E,i})^{\frac{1}{1-\sigma}-1}\cdot\frac{ds_{E,i}}{dp_{nt,i}}\left[z_{j,i}-rW^U_{j,i}\right]}{\left[w_i\cdot(1-s_{E,i})^{\frac{1}{1-\sigma}}+z_{j,i}-rW^U_{j,i}\right]^2}\right]$$

From equations (10) and (14), it is easy to see that both $\frac{d\mu_i}{dp_{nt,i}}$ and $\frac{ds_{E,i}}{dp_{nt,i}}$ are positive in our scenario with $0<\sigma<1$ and $-1<\epsilon<0$. Moreover, from equation (3) we also get that $z_{j,i}-rW^U_{j,i}<0$. So we conclude that the derivative the RHS of equation (21) with respect to $p_{nt,i}$ is unambiguously positive.

Appendix B: Proof of Lemma 1

If $p_{nt,i}$ and λ^* are assumed to be fixed, an equilibrium exists if the system composed by the three equations (11), (17), and (22) admits a solution for the $s_{E,i}$, θ_i, and w_i in the case $0<\sigma<1$ and $-1<\epsilon<0$, for $i\in\{a,b\}$. If the equilibrium values for these unknowns exist, then all the other endogenous variables of the model (the expected lifetime and the instantaneous utilities) are easily obtained using their corresponding equations[35]. We first apply the implicit function theorem to equation (11). It is easy that w_i is a decreasing function of $s_{E,i}$ in the case $0<\sigma<1$ and $-1<\epsilon<0$. Moreover, $w_i\to+\infty$ as $s_{E,i}\to 0$ and $w_i\to+0$ as $s_{E,i}\to 1$. So for any value of $s_{E,i}$, equation (11) allows to identify a corresponding value of w_i. We can write $w_i(s_{E,i})$ with $w_i'(s_{E,i})<0$. We write down the system of equation (17) and (22) in the following way:

$$\begin{cases}\mathbb{ZP}_i\equiv\frac{y_i-w_i(s_{E,i})}{r+\delta}-\frac{k}{q(\theta_i)}=0\\ \mathbb{W}_i\equiv w_i(s_{E,i})-\frac{\beta\,(y_i+k\cdot\theta_i)+(1-\beta)\,b\,(1+\epsilon\cdot s_{E,i})\left(\frac{1-s_{U,i}}{1-s_{E,i}}\right)^{\frac{1}{1-\sigma}}}{1+(1-\beta)\epsilon\cdot s_{E,i}}=0\end{cases}\tag{31}$$

for $i\in\{a,b\}$. Notice that $\frac{d\mathbb{ZP}_i}{ds_{E,i}}=\frac{\partial\mathbb{ZP}_i}{\partial w_i}\cdot w_i'(s_{E,i})$ is positive because the first derivative is negative and we have just seen that w_i is decreasing in $s_{E,i}$. Moreover, since $q(\theta_i)$

[35] With $p_{nt,i}$ fixed, $s_{U,i}$ is obtained via equation (6).

is a decreasing function, then $\frac{d\,\mathbb{ZP}_i}{d\,\theta_i} < 0$, for $i \in \{a, b\}$. So the first equation of the system describes an increasing relationship in the $(s_{E,i}, \theta_i)$ space. In addition, with $s_{E,i} \to 0$ we have $w_i \to +\infty$ and, for the conditions in footnote 12, $\theta_i \to 0$. Conversely, if $s_{E,i} \to 1$ we have $w_i \to 0$ and θ_i is a positive finite number. Under a Cobb-Douglas matching function $\theta_i = \left(\frac{y_i}{(r+\delta)k}\right)^{\frac{1}{\eta}} \equiv \bar{\theta}_i$.

As concerns the second equation of the system, we get:

$$\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} = \frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} + \frac{\partial\,\mathbb{W}_i}{\partial\,w_i} \cdot w_i'(s_{E,i}) \tag{32}$$

with

$$\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} = -\frac{1-\beta}{1+(1-\beta)\epsilon \cdot s_{E,i}} \left[b \left(\frac{1-s_{U,i}}{1-s_{E,i}}\right)^{\frac{1}{1-\sigma}} \left(\frac{1+\epsilon \cdot s_{E,i}}{(1-s_{E,i})\,(1-s)} + \epsilon\right) - \epsilon w_i \right] \tag{33}$$

for $i \in \{a, b\}$. Notice that $\frac{1+\epsilon \cdot s_{E,i}}{(1-s_{E,i})(1-s)} + \epsilon > 0$ as long as $-1 < \epsilon < 0$ and $0 < \sigma < 1$. So we have $\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} < 0$. Since $\frac{\partial\,\mathbb{W}_i}{\partial\,w_i} = 1$, the second term at the RHS of (32) is negative, and we obtain that $\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} < 0$. It is also easy to see that $\frac{d\,\mathbb{W}_i}{d\,\theta_i} < 0$. Therefore $\mathbb{W}_i = 0$ describes a decreasing relationship in the $(s_{E,i}, \theta_i)$ space. In addition, if $s_{E,i} \to 0$ we have $w_i \to +\infty$ and, $\theta_i \to +\infty$. We can also show (details are available on request) that if $\theta_i = \bar{\theta}_i$, $s_{E,i} \in (0, 1)$.

This implies that there exists a unique equilibrium in $(s_{E,i}, \theta_i)$ levels that satisfy system (31). The equilibrium value of $s_{E,i}$ allows to uniquely identify w_i. In turn, all the other endogenous variables are determined via their corresponding equations. As concerns the derivatives presented in Lemma 1, they are just obtained applying the implicit function theorem to the system (31). More specifically, we get:

$$\frac{d\,s_{E,i}}{d\,p_{nt,i}} = -\left[\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,\theta_i} - \frac{d\,\mathbb{W}_i}{d\,\theta_i} \cdot \frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}}\right] \left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,\theta_i} - \frac{d\,\mathbb{W}_i}{d\,\theta_i} \cdot \frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right]^{-1} > 0$$

since we have:

$$\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} = -(r+\delta) \cdot \frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}} > 0\,; \qquad \frac{d\,\mathbb{W}_i}{d\,\theta_i} < 0 \tag{34}$$

for $i \in \{a, b\}$. Moreover, using equations (32) and (33), we have:

$$\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} = \frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} + w_i'(s_{E,i}) < 0; \qquad \frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}} = -\frac{w_i'(s_{E,i})}{r+\delta} > 0; \qquad \frac{d\,\mathbb{ZP}_i}{d\,\theta_i} < 0 \tag{35}$$

for $i \in \{a, b\}$. We also get:

$$\frac{d\,\theta_i}{d\,p_{nt,i}} = -\left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}} - \frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right]\left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,\theta_i} - \frac{d\,\mathbb{W}_i}{d\,\theta_i} \cdot \frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right]^{-1}$$

for $i \in \{a, b\}$. The second difference at the RHS is positive for the inequalities in equations (34) and (35). From equations (34) and (35) we can also write the first difference as follows:

$$\left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}} - \frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} \cdot \frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right] = -\left(\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} + w_i'(s_{E,i})\right)\frac{1}{r+\delta}\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} + \frac{w_i'(s_{E,i})}{r+\delta}\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}}$$

for $i \in \{a, b\}$. This term is positive because the we have proved that $\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} < 0$. In turn, this means that $\frac{d\,\theta_i}{d\,p_{nt,i}} < 0$

Notice that, for the zero profit condition (17), $\frac{d\,\theta_i}{d\,p_{nt,i}} < 0$ implies that $\frac{d\,w_i}{d\,p_{nt,i}} > 0$. The only reason labor market tightness decreases after a positive variation in $p_{nt,i}$ is because w_i has increased, the other variables of the equation (17) being exogenously given.

Appendix C: Proof of Proposition 1

We divide the proof in four steps.

STEP 1.

From Lemma 1, we have seen that the system composed by the housing share equation for employed workers (11), the free entry zero profit condition (17), and the wage equation (22) admits a unique equilibrium in $s_{E,i}$, θ_i, and w_i for any given $p_{nt,i}$, and λ^* ($i \in \{a, b\}$). Now, we focus on the market clearing conditions in the housing sector in both regions, (28). Notice first that using the Hicksian demand functions (4) and (10) and equation (13), we can write

$$u_i \cdot Q^U_{nt,i} + (1-u_i) \cdot Q^E_{nt,i} = p_{nt,i}^{-\sigma}\left[u_i\, b^{1+\epsilon(1-\sigma)}\,(1-s_{U,i})^{1+\epsilon} + (1-u_i)\, w_i^{1+\epsilon(1-\sigma)}\,(1-s_{E,i})^{1+\epsilon}\right]$$

for $i \in \{a, b\}$. We can insert this expression into the equilibrium condition (28). Using equation (26), we get:

$$(1-H(\lambda^*))\,L = \frac{\alpha_a\, p_{nt,a}^{\frac{1}{\gamma}+\sigma}}{u_a\, b^{1+\epsilon(1-\sigma)}\,(1-s_{U,a})^{1+\epsilon} + (1-u_a)\, w_a^{1+\epsilon(1-\sigma)}\,(1-s_{E,a})^{1+\epsilon}} \quad (36)$$

$$H(\lambda^*)L = \frac{\alpha_b\, p_{nt,b}^{\frac{1}{\gamma}+\sigma}}{u_b\, b^{1+\epsilon(1-\sigma)}\,(1-s_{U,b})^{1+\epsilon} + (1-u_b)\, w_b^{1+\epsilon(1-\sigma)}\,(1-s_{E,b})^{1+\epsilon}} \quad (37)$$

We want to show that the RHS of both equations are increasing in $p_{nt,i}$ for $i \in \{a, b\}$. The numerator is of course increasing in $p_{nt,i}$. As concerns the denominator, applying the implicit function theorem to the system composed by equations (11), (17), and (22) (details are available on request), we obtain that $\frac{d\,w_i}{d\,p_{nt,i}} > 0$, $\frac{d\,s_{E,i}}{d\,p_{nt,i}} > 0$, $\frac{d\,s_{U,i}}{d\,p_{nt,i}} > 0$ and $\frac{d\,\theta_i}{d\,p_{nt,i}} < 0$. Since the unemployment rate u_i is decreasing in θ_i for equation (1), we get that the only positive term[36] of the derivative of the denominator of (36) with respect to $p_{nt,i}$ is :

$$(1+\epsilon(1-\sigma))\,(1-u_i)\,(1-s_{E,i})^{1+\epsilon}\; w_i^{\epsilon(1-\sigma)}\,\frac{d\,w_i}{d\,p_{nt,i}}$$

for $i \in \{a, b\}$. Computing the derivative of (36) with respect to $p_{nt,i}$, we then obtain that sufficient condition for the term at the RHS of (36) to be increasing in $p_{nt,i}$ is that

$$p_{nt,i}^{\frac{1}{\gamma}+\sigma}\,[1+\epsilon(1-\sigma)]\,(1-u_i)\,(1-s_{E,i})^{1+\epsilon}\; w_i^{\epsilon(1-\sigma)}\,\frac{d\,w_i}{d\,p_{nt,i}} <$$
$$\left(\frac{1}{\gamma}+\sigma\right) p_{nt,i}^{\frac{1}{\gamma}+\sigma-1}(1-u_i)\,(1-s_{E,i})^{1+\epsilon}\; w_i^{1+\epsilon(1-\sigma)}$$

This is equivalent to prove that

$$\frac{d\,w_i}{d\,p_{nt,i}}\,\frac{p_{nt,i}}{w_i} < \frac{\frac{1}{\gamma}+\sigma}{1+\epsilon(1-\sigma)}$$

[36]This is because the denominator at the RHS of (36) is decreasing in $s_{E,i}$ and $s_{U,i}$, that are increasing in $p_{nt,i}$. Moreover, the same denominator is decreasing in u_i (that is in turn positively affected by $p_{nt,i}$, because $b^{1+\epsilon(1-\sigma)}\,(1-s_{U,i})^{1+\epsilon} < w_i^{1+\epsilon(1-\sigma)}\,(1-s_{E,i})^{1+\epsilon}$ (as $b < w_i$ and $s_{U,i} > s_{U,i}$ if $-1 < \epsilon < 0$).

In a supplementary note (available on request) we prove that this is always verified if the term at the RHS is greater than 1. This in turn is equivalent to imposing $\frac{1}{\gamma} > (1+\epsilon)(1-\sigma)$.

Therefore, if this condition holds, we have that the term at the RHS of (36) is an increasing function of $p_{nt,i}$

STEP 2

We then use both equations in (36) to get rid of the terms with λ^*:

$$\begin{aligned} \mathbb{L} \equiv L & - \frac{\alpha_a \, p_{nt,a}^{\frac{1}{\gamma}+\sigma}}{u_a \, b^{1+\epsilon(1-\sigma)} \, (1-s_{U,a})^{1+\epsilon} + (1-u_a) \, w_a^{1+\epsilon(1-\sigma)} \, (1-s_{E,a})^{1+\epsilon}} + \\ & - \frac{\alpha_b \, p_{nt,b}^{\frac{1}{\gamma}+\sigma}}{u_b \, b^{1+\epsilon(1-\sigma)} \, (1-s_{U,b})^{1+\epsilon} + (1-u_b) \, w_b^{1+\epsilon(1-\sigma)} \, (1-s_{E,b})^{1+\epsilon}} = 0 \end{aligned} \tag{38}$$

For the result obtained in STEP 1, the implicit function $\mathbb{L} = 0$ describes a decreasing relationship in the $(p_{nt,a}, \, p_{nt,b})$ space. Notice also that $\frac{d\,\mathbb{L}}{d\,y_i} > 0$ for $i \in \{a, \, b\}$, as w_i is increasing in y_i (see equation 22) and $\frac{d\,u_i}{d\,y_i} = \frac{\partial\,u_i}{\partial\,\theta_i} \cdot \frac{\partial\,\theta_i}{\partial\,y_i} < 0$ and $\frac{d\,s_{E,i}}{d\,y_i} = \frac{\partial\,s_{E,i}}{\partial\,w_i} \cdot \frac{\partial\,w_i}{\partial\,y_i} < 0$.

STEP 3

For convenience we re-write here the migration equation (25):

$$\Lambda \equiv \lambda^* - b\left[(1-s_{U,b})^{\frac{1}{1-\sigma}} - (1-s_{U,a})^{\frac{1}{1-\sigma}}\right] - \frac{\beta \cdot k}{1-\beta}\left[\frac{\theta_b(1-s_{E,b})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,b}} - \frac{\theta_a(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,a}}\right] = 0$$

Notice that $\frac{d\,\Lambda}{d\,\lambda^*} > 0$. Moreover we get:

$$\frac{d\,\Lambda}{d\,p_{nt,a}} = \frac{\partial\,\Lambda}{\partial\,p_{nt,a}} + \frac{\partial\,\Lambda}{\partial\,p_{nt,b}} \cdot \frac{\partial\,p_{nt,b}}{\partial\,p_{nt,a}}\bigg|_{\mathbb{L}=0}$$

in which $\frac{\partial\,p_{nt,b}}{\partial\,p_{nt,a}}\Big|_{\mathbb{L}=0} < 0$ is obtained by total differentiating the implicit function $\mathbb{L} = 0$ (see STEP 2). Moreover, we have:

$$\begin{aligned} \frac{\partial\,\Lambda}{\partial\,p_{nt,a}} = & -b\,(1-s_{U,a})^{\frac{1}{1-\sigma}-1} \cdot \frac{d\,s_{U,a}}{d\,p_{nt,a}} + \frac{\beta \cdot k}{1-\beta}\,\frac{(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,a}} \cdot \frac{d\,\theta_a}{d\,p_{nt,a}} + \\ & - \frac{\beta \cdot k}{1-\beta} \cdot \frac{\theta_a(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon \cdot s_{E,a}} \cdot \frac{(1+\epsilon \cdot s_{E,a}) + \epsilon(1-\sigma)(1-s_{E,a})}{(1-\sigma)(1-s_{E,a})(1+\epsilon \cdot s_{E,a})} \cdot \frac{d\,s_{E,a}}{d\,p_{nt,a}} \end{aligned}$$

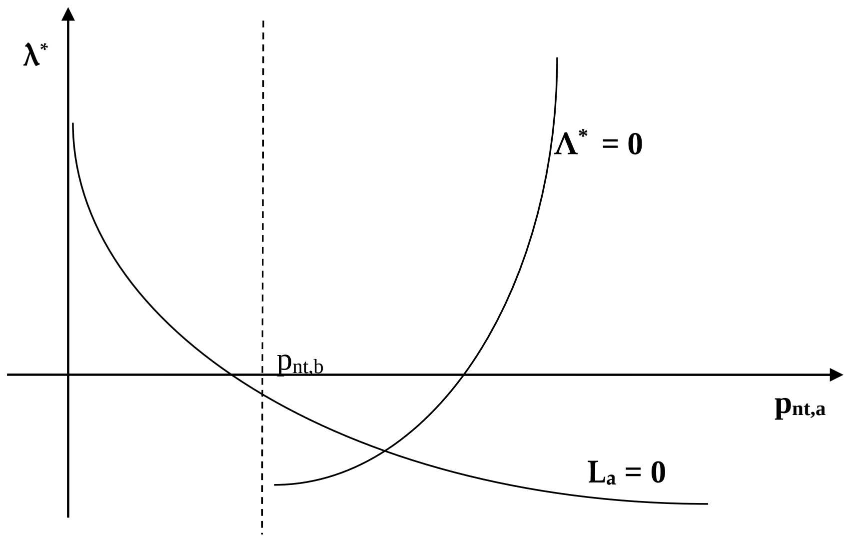

Figure 1: Equilibrium of system (39)

This derivative is negative, since $\frac{d\,s_{U,a}}{d\,p_{nt,a}} > 0$, $\frac{d\,s_{Ea}}{d\,p_{nt,a}} > 0$, $\frac{d\,\theta_a}{d\,p_{nt,a}} < 0$ and $(1 + \epsilon \cdot s_{E,a}) + \epsilon(1-\sigma)(1-s_{E,a}) > 0$. Following the same procedure, it is easy to see that $\frac{\partial\,\Lambda}{\partial\,p_{nt,b}} > 0$. So we have that $\frac{d\,\Lambda}{d\,p_{nt,a}} < 0$. The implicit function $\Lambda = 0$ describes a positive relationship in the $(p_{nt,a},\, \lambda^*)$ space.

STEP 4

We consider the system composed by the migration equation (25) and the market clearing condition in the housing market in region a, the first equation in (36):

$$\begin{cases} \Lambda \; = 0 \\ \mathbb{L}_a \;\equiv\; (1 - H(\lambda^*))\,L \;-\; \dfrac{\alpha_a\,p_{nt,a}^{\frac{1}{\gamma}+\sigma}}{u_a\,b^{1+\epsilon(1-\sigma)}\,\left(1-s_{U,a}\right)^{1+\epsilon} + (1-u_a)\,w_a^{1+\epsilon(1-\sigma)}\,\left(1-s_{E,a}\right)^{1+\epsilon}} = 0 \end{cases} \tag{39}$$

We have proved in STEP 3 that $\Lambda = 0$ a positive relationship in the $(p_{nt,a},\, \lambda^*)$ space. From STEP 1 we also know that the term at the LHS of the second equation of system

(39), is decreasing in $p_{nt,a}$. Moreover, the same term is also decreasing in λ^*, as $H(.)$ is cumulative density function. Therefore, the second equation of system (39) describes a positive relationship in the $(p_{nt,a},\, \lambda^*)$ space. As concerns the limit of the second equation of the system, as $\lambda^* \to \lambda$, there is no labor force in region a and $p_{nt,a} \to 0$. If $\lambda^* \to -\lambda$, $p_{nt,a}$ takes a positive finite number.

We focus now on the limit cases for the the implicit function $\Lambda = 0$, that is obtained using the migration equation (25) and he implicit function $\mathbb{L} = 0$, defined in (38). If $\lambda^* < 0$, $L_a > L_b$ and the first negative term at the LHS of equation (38) must be greater in absolute value the the second one. Since $y_a > y_b$ and $\frac{d\,\mathbb{L}}{d\,y_i} > 0$ for $i \in \{a,\, b\}$ (see STEP 3), this is possible only if $p_{nt,a} > p_{nt,b}$. This implies that the implicit function $\Lambda = 0$ is negative for values of $p_{nt,a}$ close but greater than $p_{nt,a}$. Since it is an increasing function in the $(p_{nt,a},\, \lambda^*)$ space, an equilibrium for the system (39) exists and it is unique. See Figure 1. Once the equilibrium values of $p_{nt,a}$ and λ^* are determined, $p_{nt,b}$ is uniquely obtained via equation (38). From Lemma 1, all the other endogenous variables of the model can be found solving system (31).

Notice that in Figure 1 we have considered an equilibrium with $\lambda^* < 0$ and $L_a > L_b$. We cannot rule out however the possibility that the two curves intersect in the positive hortant.

As concerns the inequalities presented in Proposition 1, we use the implicit function theorem to system (39):

$$\frac{d\,p_{nt,a}}{d\,y_a} = -\left[\frac{d\,\mathbb{L}_a}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,y_a} - \frac{d\,\mathbb{L}_a}{d\,y_a}\cdot\frac{d\,\Lambda}{d\,\lambda^*}\right]\cdot\left[\frac{d\,\mathbb{L}_a}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,p_{nt,a}} - \frac{d\,\mathbb{L}_a}{d\,p_{nt,a}}\cdot\frac{d\,\Lambda}{d\,\lambda^*}\right]^{-1} > 0 \quad (40)$$

since we have:

$$\frac{d\,\mathbb{L}_a}{d\,\lambda^*} < 0\,; \qquad \frac{d\,\mathbb{L}_a}{d\,y_a} > 0\,; \qquad \frac{d\,\mathbb{L}_a}{d\,p_{nt,a}} < 0\,; \qquad \frac{d\,\Lambda}{d\,\lambda^*} > 0$$

$$\frac{d\,\Lambda}{d\,y_a} = \underbrace{\frac{\partial\,\Lambda}{\partial\,y_a}}_{+} + \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+}\cdot\underbrace{\frac{\partial\,p_{nt,b}}{\partial\,y_a}\bigg|_{\mathbb{L}=0}}_{+} > 0 \quad \text{and} \quad \frac{d\,\Lambda}{d\,p_{nt,a}} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-} + \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+}\cdot\underbrace{\frac{\partial\,p_{nt,b}}{\partial\,\partial_{nt,a}}\bigg|_{\mathbb{L}=0}}_{-} < 0$$

All the signs of the derivatives in the first line are easily obtained by differentiating the equations in system (39). The signs of the derivatives in the second line are computed by differentiating the implicit equation $\mathbb{L} = 0$ in (38). The sign of the derivative in equation (40) means that $p_{nt,a} > p_{nt,b}$ as long as $y_a > y_b$. In turn, this implies that $w_a > w_b$ because

$$\frac{d\,w_a}{d\,y_a} = \frac{\partial\,w_i}{\partial\,y_i} + \frac{\partial w_i}{\partial p_{nt,i}} \cdot \frac{\partial p_{nt,i}}{\partial y_i} > 0.$$

The first term at the RHS is positive and it is obtained by applying the the implicit function theorem to system (31). The second term at the RHS is also positive for the results in Lemma 1 ($\frac{\partial w_i}{\partial p_{nt,i}} > 0$) and for what we have obtained by totally differentiating system (39) ($\frac{\partial p_{nt,i}}{\partial y_i} > 0$). The fact that $p_{nt,a} > p_{nt,b}$ also means that $s_{U,a} > s_{U,b}$ for equation (6).

Appendix D: Proof of Proposition 2

The proof consists on applying the implicit function theorem to system (39). We consider a marginal change in α_a but the same procedure applies in case of a variation in α_b. More specifically, we get that

$$\frac{d\,p_{nt,a}}{d\,\alpha_a} = -\left[\frac{d\,\mathbb{L}_a}{d\,\lambda^*} \cdot \frac{d\,\Lambda}{d\,\alpha_a} - \frac{d\,\mathbb{L}_a}{d\,\alpha_a} \cdot \frac{d\,\Lambda}{d\,\lambda^*}\right] \cdot \left[\frac{d\,\mathbb{L}_a}{d\,\lambda^*} \cdot \frac{d\,\Lambda}{d\,p_{nt,a}} - \frac{d\,\mathbb{L}_a}{d\,p_{nt,a}} \cdot \frac{d\,\Lambda}{d\,\lambda^*}\right]^{-1} < 0 \quad (41)$$

since we have:

$$\frac{d\,\mathbb{L}_a}{d\,\lambda^*} < 0\,; \qquad \frac{d\,\mathbb{L}_a}{d\,\alpha_a} < 0\,; \qquad \frac{d\,\mathbb{L}_a}{d\,p_{nt,a}} < 0\,; \qquad \frac{d\,\Lambda}{d\,\lambda^*} > 0$$

$$\frac{d\,\Lambda}{d\,\alpha_a} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+} \cdot \underbrace{\frac{\partial\,p_{nt,b}}{\partial\,\alpha_a}\Big|_{\mathbb{L}=0}}_{-} < 0 \quad \text{and} \quad \frac{d\,\Lambda}{d\,p_{nt,a}} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-} + \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+} \cdot \underbrace{\frac{\partial\,p_{nt,b}}{\partial\,\partial_{nt,a}}\Big|_{\mathbb{L}=0}}_{-} < 0$$

All the signs of the derivatives in the first line are easily obtained by differentiating the equations in system (39). The signs of the derivatives in the second line are computed by differentiating the implicit equation $\mathbb{L} = 0$ in (38).

To evaluate the effects on $p_{nt,b}$ and λ^* of a marginal reduction in α_a we find it easier to focus on an alternative equilibrium system, in which we consider the equilibrium condition in the housing market in region b, the second equation in (36):

$$\begin{cases} \Lambda \; = 0 \\ \mathbb{L}_b \; \equiv \; H(\lambda^*)L - \frac{\alpha_b \, p_{nt,b}^{\frac{1}{\gamma}+\sigma}}{u_b \, b^{1+\epsilon(1-\sigma)} \left(1-s_{U,b}\right)^{1+\epsilon} + (1-u_b) \, w_a^{1+\epsilon(1-\sigma)} \left(1-s_{E,b}\right)^{1+\epsilon}} = 0 \end{cases} \tag{42}$$

The same procedure described in STEP 4 in the previous Appendix allows to find a unique equilibrium in the $(p_{nt,b}, \, \lambda^*)$ space.

Comparative statics is easier if we consider this system as $\frac{d\mathbb{L}_b}{d\,\alpha_a} = 0$. So we get:

$$\frac{d\,p_{nt,b}}{d\,\alpha_a} = -\left[\frac{d\,\mathbb{L}_b}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,\alpha_a}\right]\cdot\left[\frac{d\,\mathbb{L}_b}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,p_{nt,b}} - \frac{d\,\mathbb{L}_b}{d\,p_{nt,b}}\cdot\frac{d\,\Lambda}{d\,\lambda^*}\right]^{-1} < 0 \tag{43}$$

and

$$\frac{d\,\lambda^*}{d\,\alpha_a} = \left[\frac{d\,\mathbb{L}_b}{d\,p_{nt,b}}\cdot\frac{d\,\Lambda}{d\,\alpha_a}\right]\cdot\left[\frac{d\,\mathbb{L}_b}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,p_{nt,b}} - \frac{d\,\mathbb{L}_b}{d\,p_{nt,b}}\cdot\frac{d\,\Lambda}{d\,\lambda^*}\right]^{-1} < 0 \tag{44}$$

since we have:

$$\frac{d\,\mathbb{L}_b}{d\,\lambda^*} > 0\,; \qquad \frac{d\,\mathbb{L}_b}{d\,p_{nt,b}} < 0\,; \qquad \frac{d\,\Lambda}{d\,\lambda^*} > 0$$

$$\frac{d\,\Lambda}{d\,\alpha_a} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-}\cdot\underbrace{\frac{\partial\,p_{nt,a}}{\partial\,\alpha_a}\Big|_{\mathbb{L}=0}}_{-} > 0 \quad \text{and} \quad \frac{d\,\Lambda}{d\,p_{nt,b}} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+} + \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-}\cdot\underbrace{\frac{\partial\,p_{nt,a}}{\partial\,\partial_{nt,b}}\Big|_{\mathbb{L}=0}}_{-} > 0$$

Again, all the signs of the derivatives in the first line are easily obtained by differentiating the equations in system (42). The signs of the derivatives in the second line are computed by differentiating the implicit equation $\mathbb{L} = 0$ in (38).

We get that a marginal reduction in α_a increases $p_{nt,a}$, $p_{nt,b}$, and λ^*. Since the labor force L_a is a decreasing function of λ^*, a lower α_a reduces L_a and raises L_b. For the properties described in Lemma 1, for any $i \in \{a, b\}$ a higher $p_{nt,i}$ raises w_i and $P_{U,i}$, while it reduces θ_i (so u_i goes up for equation 1).

Appendix E: Sensitivity Analysis

We follow two alternative procedures to calibrate our model under homothetic preferences (i.e. $\epsilon = 0$). The first one consists on following the same steps presented in section 4.1. We make only one notable departure. Since with $\epsilon = 0$, the expenditure shares are equal for all workers belonging to the same region, we consider the average values of the expenditure share for the Western states of Germany and the Eastern states, instead of distinguishing between employed workers and poorer, unemployed ones. The resulting calibrated model is then simulated in response to the same shocks considered in section 4.2.

In the second alternative approach, we take the baseline calibrated model and simulate a change on ϵ. Its initial calibrated value is -0.8. We look at the new values for the endogenous variables of the model if $\epsilon = -0.08$, ten times larger in absolute value. We then simulate the same shocks considered in section 4.2 using as starting point the model at $\epsilon = -0.08$.

They are both counterfactual exercises. The first one considers the real average values for the housing expenditures shares, but all the variables that are calibrated conditional on a specific value of ϵ (most of those presented in Table 2) are obtained imposing homotheticity, an assumption that does not match German data, that show housing expenditures shares decrease with income. On the other hand, the second procedure is equivalent to considering a shock on ϵ that changes households' preferences on housing (in the sense it is no longer considered a necessity good) and then look at how the resulting economy would react to the shocks.

Luckily, results are very similar for either option. In Tables 3, 4, and 5 we present the results of the second procedure. The ones obtained via the first approach are available on request.

As concerns the other sensitivity analyses presented in section 4.3, we simply change the values of the parameter of interest in the calibration and then proceed with the simulation.

Percentage change	$\Delta y_a/y_a = 1\%$	$\Delta \alpha_a/\alpha_a = -5\%$
L_a	0.9	-0.6
L_b	-4	2.7
u_a	-0.04 pp	0.002 pp
u_b	-0.002 pp	0.001 pp
$p_{nt,a}$	0.6	1.7
$p_{nt,b}$	-1.6	1.0
$P_{E,a}$	-0.07	0.7
$P_{E,b}$	-0.6	0.4
$P_{U,a}$	0.4	1.0
$P_{U,b}$	-0.7	0.5
w_a (real)	1 (1.1)	0.0 (-0.7)
w_b (real)	0.0 (0.6)	0.0 (-0.4)
real b in region a	-0.4	-1.0
real b in region b	0.7	-0.5
Variance of log income (real)	2.0 (1.5)	0.2 (-1.1)
W_a^E	1.1	-0.7
W_a^U	1.1	-0.7
W_b^E	0.6	-0.4
W_b^U	0.6	-0.4

Table 6: Sensitivity analysis with $\gamma = 0.5$ (percentage changes - for the unemployment rates variation in percentage points)

	$\beta = 0.4$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.6$
Percentage change	$\Delta y_a/y_a = 1\%$	$\Delta\alpha_a/\alpha_a = -5\%$	$\Delta y_a/y_a = 1\%$	$\Delta\alpha_a/\alpha_a = -5\%$
L_a	0.5	-0.8	0.5	-0.8
L_b	-2.1	3.7	-2.1	3.7
u_a	-0.04 pp	0.0 pp	-0.04 pp	0.0 pp
u_b	-0.002 pp	0.0 pp	-0.0 pp	0.0 pp
$p_{nt,a}$	1.1	4.0	1.1	4.0
$p_{nt,b}$	-1.9	3.4	-1.9	3.4
$P_{E,a}$	0.1	1.6	0.1	1.6
$P_{E,b}$	-0.7	1.2	-0.7	1.2
$P_{U,a}$	0.6	2.3	0.6	2.3
$P_{U,b}$	-0.9	1.6	-0.9	1.6
w_a (real)	1 (0.9)	0.0 (-1.5)	1 (0.9)	0.0 (-1.5)
w_b (real)	-0.0 (0.7)	0.0 (-1.2)	-0.0 (0.7)	0.0 (-1.2)
real b in region a	-0.6	-2.3	-0.6	-2.3
real b in region b	0.9	-1.6	0.9	-1.6
Var. log income (real)	2.3 (1.4)	0.5 (0.9)	2.3 (1.4)	0.5 (0.9)
W_a^E	0.9	-1.5	0.9	-1.5
W_a^U	0.9	-1.5	0.9	-1.5
W_b^E	0.7	-1.2	0.7	-1.2
W_b^U	0.7	-1.2	0.7	-1.2

Table 7: Sensitivity analysis with $\beta = 0.4$ and $\beta = 0.6$ (percentage changes - for the unemployment rates variation in percentage points)

	$\sigma = 0.3$	$\sigma = 0.3$	$\sigma = 0.7$	$\sigma = 0.7$
Percentage change	$\Delta y_a/y_a = 1\%$	$\Delta\alpha_a/\alpha_a = -5\%$	$\Delta y_a/y_a = 1\%$	$\Delta\alpha_a/\alpha_a = -5\%$
L_a	0.4	-0.9	0.6	-0.9
L_b	-1.7	3.8	-2.4	3.7
u_a	-0.04 pp	0.0 pp	-0.03 pp	0.0 pp
u_b	-0.002 pp	0.0 pp	-0.0 pp	0.0 pp
$p_{nt,a}$	1.1	4.8	1.1	3.6
$p_{nt,b}$	-1.9	4.3	-2.0	3.1
$P_{E,a}$	0.2	1.8	0.04	1.5
$P_{E,b}$	-0.7	1.5	-0.8	1.2
$P_{U,a}$	0.6	2.6	0.7	2.3
$P_{U,b}$	-0.9	1.9	-1.0	1.6
w_a (real)	1 (0.8)	0.0 (-1.7)	1.0 (1.0)	0.0 (-1.5)
w_b (real)	-0.0 (0.7)	0.0 (-1.5)	-0.0 (0.8)	0.0 (-1.2)
real b in region a	-0.6	-2.5	-0.7	-2.3
real b in region b	0.9	-1.9	1.0	-1.6
Var. log income (real)	2.7 (1.5)	0.5 (1.2)	2.0 (1.3)	0.5 (0.8)
W_a^E	0.8	-1.8	1.0	-1.5
W_a^U	0.8	-1.8	1.0	-1.5
W_b^E	0.7	-1.5	0.8	-1.2
W_b^U	0.7	-1.5	0.8	-1.2

Table 8: Sensitivity analysis with $\sigma = 0.3$ and $\sigma = 0.7$ (percentage changes - for the unemployment rates variation in percentage points)