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# ABSTRACT

## The Active Role of the Natural Rate of Unemployment during Cyclical Recoveries*

We propose that the natural rate of unemployment has an active role in the business cycle, in contrast to the prevailing view that the rate is essentially constant. We demonstrate that this tendency to treat the natural rate as near-constant would explain the surprisingly low slope of the Phillips curve. We show that the natural rate closely tracked the actual rate during the long recovery that began in 2009 and ended in 2020. We explain how the common finding of research in the Phillips-curve framework of low – often extremely low – response of inflation to unemployment could be the result of fairly close tracking of the natural rate and the actual rate in recoveries. Our interpretation of the data contrasts to that of most Phillips-curve studies, that conclude that inflation has little relation to unemployment. We suggest that the flat Phillips curve is an illusion caused by assuming that the natural rate of unemployment has little or no movement during recoveries.

**JEL Classification:** E32, J63, J64

**Keywords:** business cycle, recovery, unemployment, recession, monetary policy, natural rate of unemployment, inflation anchor, NAIRU

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# 1 Introduction

Following its introduction in Friedman (1968), the natural rate of unemployment became an essential element of thinking about the business cycle, about inflation, and about monetary policy. In the New Keynesian analysis that emerged from Friedman's insights, the natural rate is the unemployment rate that would hold if the economy were in equilibrium, free of transitory forces that raise inflation above its normal level or depress inflation below that level.

We investigate the relation between the actual rate of unemployment and the natural rate. We start by describing the relation between inflation and its two determinants in the Phillips curve. One is the *inflation anchor*, the equilibrium value described above. The other is *inflation pressure*, the gap between the actual unemployment rate and the natural unemployment rate. The natural rate of unemployment is used to construct the unemployment gap measure of inflation pressure.

We review existing evidence and provide new evidence that the natural rate is not a slow-moving measure reflecting demography, as often taken in the Phillips-curve literature. Rather, the natural rate glides down during cyclical recoveries similarly to actual unemployment. We conclude that the gap between the actual and the natural unemployment rates—inflation pressure—stays fairly close to zero during recoveries. Actual unemployment tracks natural unemployment.

Our first exercise involves the cyclical recovery beginning in 2009 and ending in early 2020. During that recovery, inflation stayed close to the Fed's target of two percent per year. We posit that the inflation anchor remained close to constant at two percent. Accordingly, the unemployment rate was close to the natural rate. Observed unemployment glided down from 10.0 percent at its maximum to 3.5 at its minimum. Our conclusion is that the natural rate glided down fairly closely to the same path.

Our second exercise extends the analysis to consider other cyclical recoveries, some with significant variations in actual inflation. Our additional evidence on the behavior of the natural rate of unemployment during recoveries builds on our finding that, historically, actual unemployment has moved in a systematic way (Hall and Kudlyak (2022a)), gliding smoothly downward at a low but steady proportional rate after an occasional sharp upward movement in an economic crisis. The fact that actual unemployment behaves the similarly in all recoveries makes it plausible that the natural rate behaves the same way.

We show that if the true natural rate is substantially positively correlated with the actual rate, a Phillips curves estimated with a constant in place of the true natural rate will inevitably appear to be close to flat. The same conclusion follows if the Phillips curve estimation uses a stochastic substitute for the natural rate that is uncorrelated with the actual rate.

We review existing time-varying measures of the natural rate and observe that many of these measures reflect long-run trend movement in actual unemployment rather than cyclical movements during recoveries. We also note that a more recent branch of research identifies a time-varying path of the natural rate as a component of a state-space model. This approach yields a path that is more volatile and correlated with the actual unemployment rate. Another approach that generates greater volatility in the natural rate is to use a general-equilibrium model to calculate the natural rate as the unemployment rate in a counterfactual solution of the model without sticky prices and wages.

With respect to the measurement of inflation pressure, we find that, during recoveries, unemployment by itself is not an indicator of inflation pressure. Rather, inflation pressure is the gap between the actual unemployment rate and the natural rate. This finding explains why recoveries can continue over extended periods, as long as a decade, when unemployment is gliding downward by many percentage points, while inflation is close to constant. Another way to express the same finding is that the labor market can gradually tighten in the sense of the Diamond-Mortensen-Pissarides (DMP) model's measure of labor-market tightness, the vacancy/unemployment ratio, while inflation remains at a constant low level.

In contrast to most research, we draw this conclusion not because the slope of the Phillips curve is close to zero, but because inflation pressure in recoveries is close to zero.

As we noted above, a large branch of Phillips-curve literature posits that the natural rate of unemployment is constant or weakly correlated with actual unemployment. Under that view, in recoveries with anchored inflation, inflation pressure must be highly variable, because unemployment falls so far during recoveries. In those times, the slope of the Phillips curve must be close to zero. Under our contrary view, inflation pressure in recoveries is low, while under the existing view, the inflation response is low. The differentiating factor between the two views is flexibility of prices. The literature finds quite sticky prices, whereas our view can be consistent with relatively flexible prices.

This paper is mainly about the labor market during cyclical recoveries. Most of the time, the market evolves smoothly during recoveries, which last a few years in some cases and more than a decade at the longest. Each recovery is ended by some important adverse aggregate development. During the ensuing recessions, unemployment rises rapidly. The paper has relatively little to say about recessions.

The paper is organized as follows:

Section 2 describes inflation's relation to the inflation anchor and inflation pressure through the Phillips curve.

Section 3 sets forth the definition of the natural rate of unemployment used in this paper, derived from Friedman's pioneering analysis. We note that the natural rate is close to the actual rate when inflation is close to its anchored value. A related empirical regularity is that the actual unemployment rate glides downward smoothly during recoveries until a crisis intervenes and unemployment jumps upward, or unemployment reaches the low value of around 3.5 percent. The behavior of unemployment during the recovery from 2009 to 2019 exemplifies this regularity.

Section 4 deals with the correlation of the actual and natural unemployment rates. Zero correlation is a common assumption. If correct, the estimated slope coefficient tends to have a low value. If mistaken, the estimated slope coefficient is likely to be severely biased toward zero. This section reviews existing time-varying measures of the natural rate and existing estimates of Phillips curve, focusing especially on more recent work.

Section 5 discusses the econometric identification of the slope parameter of the Phillips curve. Historically, most investigators identified that parameter by assuming that the natural rate of unemployment is constant over time or that it moves slowly on a path determined by changes in demography. Under that type of assumption, the Phillips curve is found to be quite flat, especially after 2000.

Section 6 discusses the potential effect of the pandemic on the Phillips curve. Inflation rose rapidly starting in the spring of 2021 to a peak of over 8 percent in the summer of 2022, then fell rapidly. Both of these movements were at rates far faster than almost any existing Phillips curve rate attributed to changes in inflation pressure. Within the basic logic of the New Keynesian model, however, a change in the economic environment of price-setting of the magnitude that occurred during the pandemic might play an important role in understanding the rapid changes in inflation.

# 2 The Inflation Process and the Phillips Curve

## 2.1 The sticky-price model of the inflation process

The New Keynesian inflation model views sellers as considering two forces in deciding how to set a price today that remains in force some time into the future. One is that inflation has a component that reflects the success of monetary policy in stabilizing inflation at a low and reasonably constant level. We call this component the inflation anchor. It is the result of an inference that a price-setter makes about how to set a price that will remain in effect for some time into the future. The basis for the inference includes information about the likely success of the central bank in stabilizing inflation in the near future, recent actual inflation, and the sources of that inflation. If monetary policy loses its grip and high inflation sets in, the anchor rises to reflect that development.

The second force operates at business-cycle frequencies. It captures the relation of inflation to economic activity. In the New Keynesian model, this force is measured by (the negative of) the gap between the actual rate of unemployment and the natural unemployment rate. This gap reflects inflation pressure. The inflation response is the increase in inflation that accompanies the upward pressure on inflation when some expansionary force raises or lowers the unemployment rate relative to the current natural rate. The level of inflation pressure is not observable directly because it depends on the natural rate, which is not observed.

The source of the positive relation between the unemployment gap and the rate of inflation in the basic New Keynesian model is the following: If unemployment is below the natural rate, it means that the price level is below its equilibrium value. The reason is that the sticky price level is below equilibrium and the consumers and firms, therefore, demand more than the equilibrium levels of output. Their demands are automatically satisfied by producers because, according to a basic Keynesian principle, demand determines output. Higher output implies lower unemployment. As time passes, the previously stuck prices become unstuck, prices free up, the price level rises from their low levels back toward equilibrium. Inflation continues higher than the anchored rate during this process.

## 2.2 Inflation and unemployment in the basic model

Figure 1 displays the basic model in a phase diagram. The unemployment rate $u$ is on the horizontal axis and the inflation rate $\pi$ is on the vertical axis. The natural unemployment rate, $u^*$, is marked on the $u$ axis and the anchored inflation rate, $\pi^*$, is marked on the $\pi$ axis. The natural rate and the anchored rate are two key parameters of the model.

A conspicuous dot with coordinates $[u^*, \pi^*]$ marks the resting point of the economy, with unchanging unemployment and inflation. A line rising from the high-unemployment, low-inflation region in the lower right describes the upward convergence toward the point of rest. A line declining from the low-unemployment, high inflation region describes convergence from that region. Both slopes are negative. Jointly, they trace out the Phillips curve of the model. Its slope is the inflation change divided by the unemployment change. That ratio, designated $\phi$, is the slope of the Phillips curve, the third key parameter of the model.

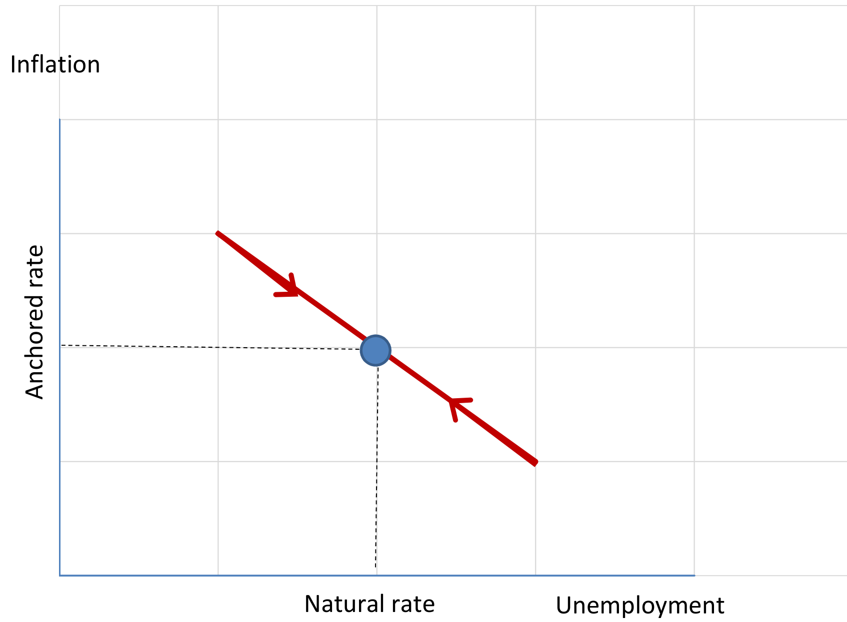


Figure 1: The New Keynesian Model in a Phase Diagram

### 2.3 The Phillips curve

The model sketched above defines the Phillips curve, an equation relating inflation to an inertial term and to a term involving economic activity. The Phillips curve is a key component of the New Keynesian class of macroeconomic models—see Woodford (2003) for a detailed exposition and Chapters 6 and 7 in Romer (2019) for a recent advanced textbook treatment of New Keynesian macroeconomics.

Following this logic, we consider a setup that embodies the properties described above:

$$\pi_t - \pi_t^* = -\phi \cdot (u_t - u_t^*). \tag{1}$$

For brevity, we omit random disturbances here. Note that we now consider the possibility that $\pi^*$ and $u^*$ change over time.

This equation is the Phillips curve. $\pi_t$ is the actual rate of inflation; $\pi_t^*$ is the inflation anchor; and $\phi$ is a non-negative coefficient governing the strength of the response of inflation to inflation pressure, measured as the gap between unemployment and the natural rate, $u_t - u_t^*$. If $\phi$ is small, prices are quite sticky and movements of $u_t - u_t^*$ are large and persistent. If $\phi$ is large, prices are flexible and $u_t - u_t^*$ returns quickly to its normal value of zero.

The Phillips curve has the property that $u_t = u_t^*$ when $\pi_t = \pi_t^*$, that is, when inflation is at its anchored level, unemployment is at its natural rate.

Inflation also fluctuates for reasons apart from unemployment, notably from fluctuations in the supply of energy and agricultural products, and, more recently, in the pandemic, products with supplies cut back by the shutdowns. These fluctuations could be included in an extended model. In most empirical Phillips curves, they enter as separate additive terms.

As in Figure 1, the Phillips curve is generally displayed as a graph with the unemployment gap, $u - u^*$, on the horizontal axis and the inflation gap, $-\phi \cdot (\pi - \pi^*)$ on the vertical axis. The slope of the Phillips curve is $\phi$ points of reduced inflation for each point of extra unemployment. We (and most other macroeconomists) talk about the slope of the curve being a positive number, $\phi$, but, to tell the truth, the slope is the negative number, $-\phi$.

Although it is conventional to display unemployment on the horizontal axis of a Phillips curve and to treat the unemployment gap informally as an exogenous determinant of inflation, the variables under discussion here are obviously jointly determined.

If $\phi$ is large, the Phillips curve is nearly vertical; even small values of the unemployment gap go with large effects on inflation. If $\phi$ is small, the Phillips curve is nearly flat.

## 2.4 Price flexibility

We can rewrite the Phillips curve in aggregate-supply form as

$$u_t = u_t^* - \frac{1}{\phi} \cdot (\pi_t - \pi_t^*). \tag{2}$$

The coefficient $\phi$, the downward slope of the Phillips curve, controls the influence of inflation on real activity, as measured by unemployment. Higher values of $\phi$ make the model more like the real-business-cycle model, where real activity is not influenced by monetary factors such as inflation. In one polar case, with full monetary neutrality, $\phi$ is large and unemployment tracks the natural rate. At the other end, $\phi$ is small, and the Phillips curve is nearly flat. Large movements of unemployment are paired with small movements in inflation.

## 2.5 Uncovering the natural rate of unemployment, the inflation anchor, and the slope of the Phillips curve, from the data

The model sketched out above involves some interesting challenges to the macroeconomist seeking to use it as the starting point for practical implementation. Only two of its variables are unambiguously observable–unemployment and inflation. The natural unemployment rate, the inflation anchor, and the slope of the Phillips curve, are theoretical constructs that have observable counterparts provided by modeling.

# 3 Defining and Measuring the Natural Rate of Unemployment

## 3.1 Our definition of the natural rate

The natural rate of unemployment, in our definition, is the unemployment rate at the point of rest in Figure 1. At this point, inflation equals the inflation anchor. The natural rate of unemployment also goes by the name *non-accelerating-inflation rate of unemployment* or NAIRU, which we take to be a synonym for the natural rate. The name captures a key property of that rate: Periods of stable inflation are times when the unemployment rate is at its natural level.

Our definition of the natural rate may differ from others based on a hypothesized absence of frictions in price and wage formation. Our definition recognizes that positive unemployment prevails when the economy is at rest, owing to normal turnover in the labor market. Defining the natural rate of unemployment is essential, given the myriad of definitions in the literature as summarized by Rogerson (1997).

## 3.2 Inferring the natural rate of unemployment during periods of anchored inflation

We can solve the Phillips curve for the natural rate of unemployment:

$$u_t^* = u_t + \frac{1}{\phi}(\pi_t - \pi_t^*). \tag{3}$$

Thus the natural rate is the actual unemployment rate with an adjustment for the effect on unemployment inferred from the inflation pressure, $\pi_t - \pi_t^*$.

This equation is only useful if there is information from some other source about the inflation anchor, $\pi_t^*$. Moreover, inflation pressure, $u_t - u_t^*$, is not observable directly. Had we known $\pi_t^*$ and $\phi$, we could have inferred the natural rate of unemployment $u_t^*$ from inflation and actual unemployment, from equation (3).

In this section, we start by noting a special case where outside information is arguably available that permits calculation of $u_t^*$ from equation (3).

For any admissible value of $\phi$, equation (3) shows that $u_t^* = u_t$ if actual inflation equals anchored inflation—the equation embodies our definition of the natural rate. The issue becomes, what configuration of theory and data would make a finding plausible that actual inflation was equal to anchored inflation at a particular time? Friedman himself associated the source of inertia in inflation with expectations of future inflation, and that theme has resonated in most discussions of inflation dynamics ever since. One determinant of expected inflation is the central bank's target rate of inflation. Another is the success of the bank in achieving its target. The place to look for actual inflation close to anchored is a moderately lengthy historical episode where inflation was close to constant at a low level consistent with the central bank's stated objective.

The recovery in the US economy starting in November 2009 and lasting until the pandemic terminated the recovery partway through March 2020, deserves consideration as an episode when inflation was close to the target rate of 2 percent, adopted formally by the Fed in 2012. The anchored rate coincided fairly closely with the target rate.

Under these two assumptions about the US labor market in 2009 to 2020, we construct Figure 2 to demonstrate the resulting inferences about the natural rate during that period. The figure plots the actual unemployment rate and indicates with red dots the months when inflation was close to the anchored rate of two percent. Specifically, it indicates the months when the year-over-year percentage change of the price of personal consumption expenditures (PCE) was between 1.5 and 2.5 percent. The figure shows that during the 2009-2020 period, there were numerous months when the natural rate closely tracked actual unemployment, according to the logic developed in this section.

Note that the figure applies only to the one recovery with exceptionally stable inflation. In the other recoveries, actual inflation was sufficiently variable that we cannot make a similar inference. And there is no case of a recession with constant inflation, so there is no direct information about the natural rate during recessions, using this approach. In Section 4.3.2, we discuss existing econometric approaches to solving this problem.

## 3.3 Inferring the path of the natural rate of unemployment from a model

General-equilibrium models provide another way to determine the path of the natural rate of unemployment. The idea is to construct a counterfactual solution to a version of the model that describes an economy satisfying a definition of the natural rate of unemployment. For example, the version could impose the condition that the actual rate of inflation equals the anchored rate and both equal the central bank's inflation target.

Galí, Smets and Wouters (2011) (GSW) is the leading example of this approach to extracting the natural rate of unemployment from a general-equilibrium model. In GSW's case, the model is a widely used New Keynesian model developed by Smets and Wouters. The model includes a sub-model that deals explicitly with unemployment along the lines of the DMP framework. The calculation runs as follows: "We [GSW] ...assess the role of wage rigidities as a factor underlying observed unemployment fluctuations by comparing the observed unemployment rate to its estimated natural counterpart, where the latter is

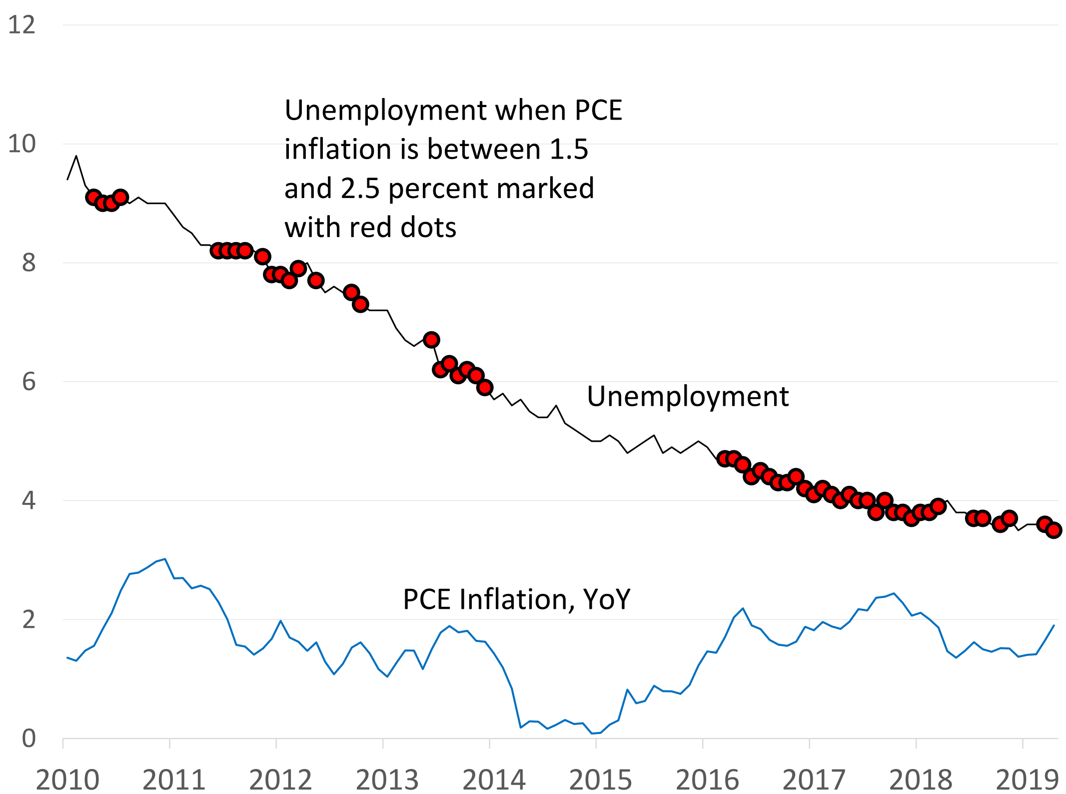


Figure 2: The Actual Unemployment Rate and the Estimated Natural Rate during the 2009-2019 Recovery

Note: Authors' calculations using monthly data from the CPS and the National Income and Product Accounts. October 2010—January 2020.

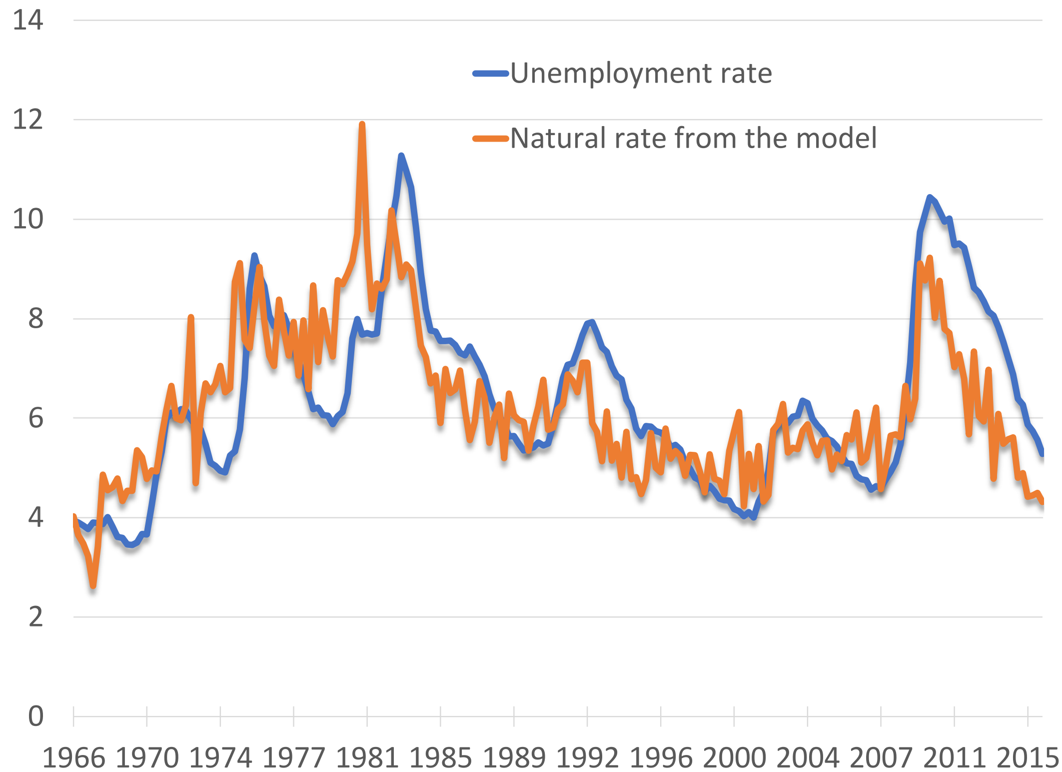


Figure 3: The Actual Unemployment Rate and the Natural Rate based on Galí et al. (2011)

Note: Data provided to the authors by Galí; quarterly, Q1 1966—Q3 2015.

defined as the unemployment rate that would be observed in the absence of nominal wage rigidities..." (pages 348-349). Their counterfactual is somewhat different from ours, but we believe that its results are indicative of one based on ours or other definitions of the natural rate.

Figure 3 plots the resulting calculated natural rate from 1966, first quarter, through 2015, third quarter.[1] The path of the natural rate captures the bulge of unemployment in the 1970s, and it also moves substantially in harmony with actual unemployment at other times, especially after the serious recessions starting in 1981 and 2007.

[1]This figure displays data supplied to us by Galí covering three more years than the published Figure 10.

### 3.4 Some evidence from the behavior of actual unemployment

Equation (2), replicated here,

$$u_t = u_t^* - \frac{1}{\phi} \cdot (\pi_t - \pi_t^*),$$

implies that systematic movements of actual unemployment resemble the corresponding movements of the natural rate. This holds unless the $\frac{1}{\phi} \cdot (\pi_t - \pi_t^*)$ component is large, which happens when inflation exhibits substantial deviations from the anchor, or when the coefficient $\phi$ is small. We proceed under the assumption that $\phi$ is not zero, that is, that inflation does respond to the unemployment gap—the Phillips curve is not completely flat.

We characterize the behavior of actual unemployment from Hall and Kudlyak (2022a), in which we undertake a close examination of the behavior of actual unemployment over the period from 1948 to 2019 using data from the Current Population Survey. We find that rather than vibrating around a fixed natural rate, the observed behavior of unemployment comprises (1) occasional sharp upward movements in times of economic crisis (recessions), and (2) an inexorable downward glide at a low but reliable proportional rate at all other times. The glide continues until unemployment reaches a low barrier of approximately 3.5 percent or until another economic crisis interrupts the glide.

Figure 4 shows our main evidence. It displays the log of the unemployment rate during the 10 completed recoveries since 1948, with the recession spells of sharply rising unemployment left blank. The key fact about recoveries is apparent in the figure: Unemployment declines smoothly but slowly throughout most recoveries most of the time. In the log plot, the recoveries appear as impressively close to straight lines. In our analysis, we confirm the linearity and estimate the slopes of these lines. We estimate that the annual reduction of log unemployment during recoveries over 1949-2019 was 0.10. The interpretation of this estimate is that during a recovery an expected annual reduction of unemployment is approximately 10 percent of the unemployment value at the beginning of the year.

The fact that actual unemployment behaves the same way in all recoveries makes it plausible that the natural rate behaves the same way.

A mechanism behind the steady but slow downward glide of actual unemployment during recoveries provides a clue why the natural rate cannot be too far from actual unemployment during that time. In Hall and Kudlyak (2022b), we show that despite high variation in monetary and fiscal policy, and in productivity and labor-force growth during the 70 years, there was little variation in the rate of decline of unemployment during recoveries. Why has the US economy recovered so consistently from every recession in the past 70 years? Our thesis is that the economy has a powerful tendency to self-recover from serious adverse shocks. A natural force causes job-seekers to match with available jobs and to lower unemployment.

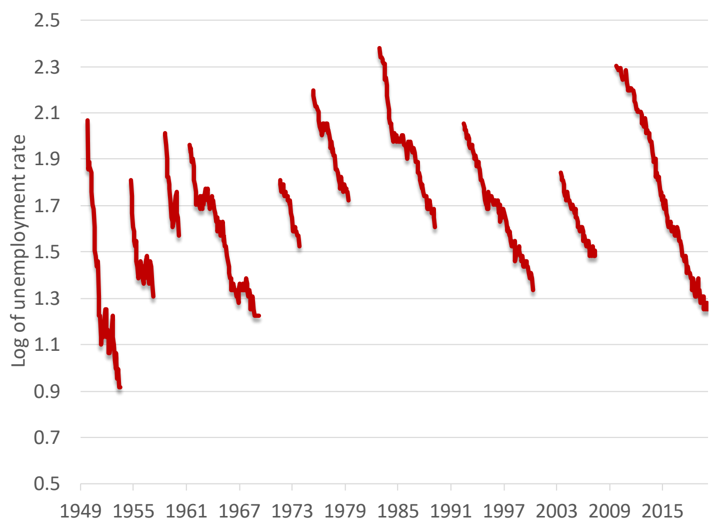


Figure 4: The Paths of Log-Unemployment During Recoveries

Source: Hall and Kudlyak (2022a), Figure 1. Monthy series, January 1949—February 2020.

The process is slow because a typical crisis breaks worker-firm employment relationships, and the process of creating new stable firm-worker relationships is time consuming. Workers who lost jobs often circle through short-term jobs, spells of unemployment, and spells out of the labor force, before finding stable employment (Hall and Kudlyak (2019)).

In Hall and Kudlyak (2022b), we suggest that high unemployment itself slows down the search and matching process through congestion. For example, in times of higher unemployment, employers incur higher costs to select suitable prospective workers from among the many applicants, thereby lowering job-finding for many other job-seekers in addition to those who lost jobs in the crisis. The bulge of unemployment created by a crisis at the beginning of a recovery creates a negative feedback to labor market tightness, endogenously slowing the recovery. Hall (2013) observes that an implication of the DMP model is that there is no fixed natural rate of unemployment which the actual unemployment rate revolves around; rather, the observed level of unemployment varies according to the driving forces.

To summarize, unemployment recovers steadily but slowly after each recession because of a self-recovery property of the labor market. The process is slow due to an impediment that arises from high unemployment itself. As an example, in the case of a recovery where the unemployment rate is 9 percent, following a serious crisis, no forces outside the labor market can lower unemployment to 5 percent next year. The natural rate must be close to 9 percent in this case.

# 4 The Natural Rate and the Flat Phillips Curve

Our discussion above makes it clear that determining the path of the natural unemployment is a challenge. But that path is an intrinsic element of the Phillips curve—every study of the Phillips curve rests on a specification of the natural rate. A simple specification, adopted by many authors, is to take the natural rate to be a constant, or, equivalently, to omit the natural rate altogether. Another specification takes a long-run trend in the actual unemployment rate as the natural rate of unemployment. Such time-varying measures do not capture the cyclical variation in the natural rate described in Sections 3.2-3.4. In this section, we discuss a potential bias toward understatement of the slope of the Phillips curve resulting from the absence of realistic variation over time in the natural rate of unemployment embodied in most specifications of the Phillips curve. In this section, we show that if the true natural rate of unemployment is highly correlated with the actual rate, Phillips curves estimated with constant natural rates or natural rates uncorrelated with actual rate will inevitably be close to flat.

## 4.1 Implications of mis-specifying the natural rate as uncorrelated with actual unemployment

We are studying the Phillips curve from Equation (1), replicated here

$$\pi_t - \pi_t^* = -\phi \cdot (u_t - u_t^*).$$

We presume that we have solved the problem of measuring $\pi_t^*$, and focus on the unobserved $u_t^*$.

We now demonstrate that taking the natural rate of unemployment as constant or near-constant has profound implications for estimation of the Phillips curve if the true model has a material positive correlation between the natural and actual rates.

Consider a Phillips-curve regression that does not include the natural unemployment rate:

$$\pi_t - \pi_t^* = -\widetilde{\phi} u_t. \tag{4}$$

The regression coefficient is

$$\widetilde{\phi} = \frac{-\mathrm{Cov}(\pi_t - \pi_t^*, u_t)}{\mathrm{V}(u_t)}. \tag{5}$$

Substituting the model for $\pi_t$ with the true time-varying natural rate of unemployment yields

$$\widetilde{\phi} = \frac{\mathrm{Cov}(u_t - u_t^*, u_t)}{\mathrm{V}(u_t)} \cdot \phi = \left(1 - \frac{\mathrm{Cov}(u_t^*, u_t)}{\mathrm{V}(u_t)}\right) \cdot \phi = (1 - R) \cdot \phi. \tag{6}$$

$R$ is the regression coefficient of $u_t^*$ on $u_t$. It is an index of the relevance of the natural rate. If $R = 0$, the natural rate is irrelevant and the regression coefficient $\widetilde{\phi}$ will be an unbiased estimate of the Phillips-curve slope, $\phi$. If $R = 1$, then $\widetilde{\phi} = 0$—the Phillips curve appears to be totally flat, even if the true slope is robustly positive. In that case, the natural rate is highly relevant.

Note that $R$ is not sensitive to the overall level of the natural rate, because the constant part of $R$ is absorbed by the constant that would normally be part of the functional form of the Phillips curve. $R$ is sensitive to the co-variation of the natural rate and the actual unemployment rate.

Our discussion of the omission of a time-varying natural rate from a regression for the slope of the Phillips curve, is an application of the standard analysis of the bias from an omitted right-hand variable.

The denominator in $R$, $\mathrm{V}(u_t)$, is observed directly and is robustly positive because unemployment rises briskly in recessions and falls reliably in recoveries. The big question is the covariance of the natural rate $u_t^*$ with actual unemployment. If the covariance is zero—possibly because $u_t^*$ is constant over time—there is no bias. If $u_t$ tracks the natural rate $u_t^*$ almost perfectly, $R$ will be almost 1, and the estimate of $\widetilde{\phi}$ will be essentially zero, even if the true value of $\phi$ is quite positive.

One particularly salient conclusion from this analysis is the following: If the true natural rate is highly correlated with the actual rate, Phillips curves estimated with constant or nearly constant natural rates of unemployment uncorrelated with the actual rate will inevitably be close to flat.

Our finding that, in the long expansion of 2009 through early 2020, actual unemployment tracked the natural rate closely, shows that the bias was almost certainly substantial during that period. $R$ for that period is very close to one.

We can use the results from Galí et al. (2011) to illustrate our analysis of the bias from failing to consider the movements of the natural rate. Suppose that we studied the Phillips curve in the GSW model by regressing an appropriate version of $\pi_t - \pi_t^*$ on $u_t$. The regression coefficient for $u_t^*$ on $u_t$ is $R = 0.60$ in the 1966-2015 sample, so the estimated slope of the Phillips curve is depressed to $1 - R = 0.40$ times its true value during that period.

The natural rate does not account for all of the movement of actual unemployment—inflation pressure accounts for some of the cyclical movements. The key conclusion is that the movements of the natural rate constitute a central factor in the economics of the Phillips curve and employment volatility. The natural rate and the actual rate of unemployment move together—in some episodes, notably in the recovery from the financial crisis, the regression coefficient is close to 1. On the average over a lengthy period, GSW's results set it to $R = 0.60$.

#### 4.1.1 Non-linear specifications

Nonlinear specifications for the Phillips curve can suffer from similar bias. The analysis of the bias follows standard results in econometrics.

### 4.2 Changes over time in the relevance of the natural rate of unemployment

Increases over time in the relevance of the natural rate might account for the apparent flattening of the Phillips curve. To check this possibility in the Galí et al. (2011) results, we replicated the estimates in Figure 3 in the form of coefficients from rolling regressions covering 20 forward quarters from the date shown on the horizontal axis. Figure 5 shows the movements of $R$ calculated in this way. In general, $R$, is cyclical but untrended over the long run. There is a sign of an increase in $R$ that would explain the flattening of the Phillips curve over the period from 1994 through 2010.

Another approach to the issue is based on del Negro, Lenza, Primiceri and Tambalotti (2020) (DLPT), a recent careful empirical study of the response of inflation and the unemployment rate to what they call an unemployment shock, in an 8-variable vector autoregression. The authors argue that the Phillips curve has flattened since 1990 and seek to explain the flattening. Lines 1, 2, and 3 of Table 1 display their results. Line 1 shows the response of unemployment to the shock for the first half of the sample through 1989, and for the second half, starting in 1990. The response is measured four quarters after the shock. Line 2 shows the responses of inflation, and line 3 shows ratio of the price response to the unemployment response. We interpret the ratio as the slope of the Phillips curve, in units of the percentage-point decline in inflation per percentage point increase in unemployment. According to these results, the estimated slope of the Phillips curve was 1.0 in the earlier period, and declined to 0.3 in the later period. These findings are in line with the substantial literature, which is surveyed extensively in the DLPT paper.

DLPT include CBO's time series for the natural rate of unemployment as one of its 8 basic indicators and as their measure of natural unemployment, but does not report the functions for the responses to the natural rate impulses. It does report the impulse response function from unemployment to the CBO series for the natural rate, which suggests almost no volatility.

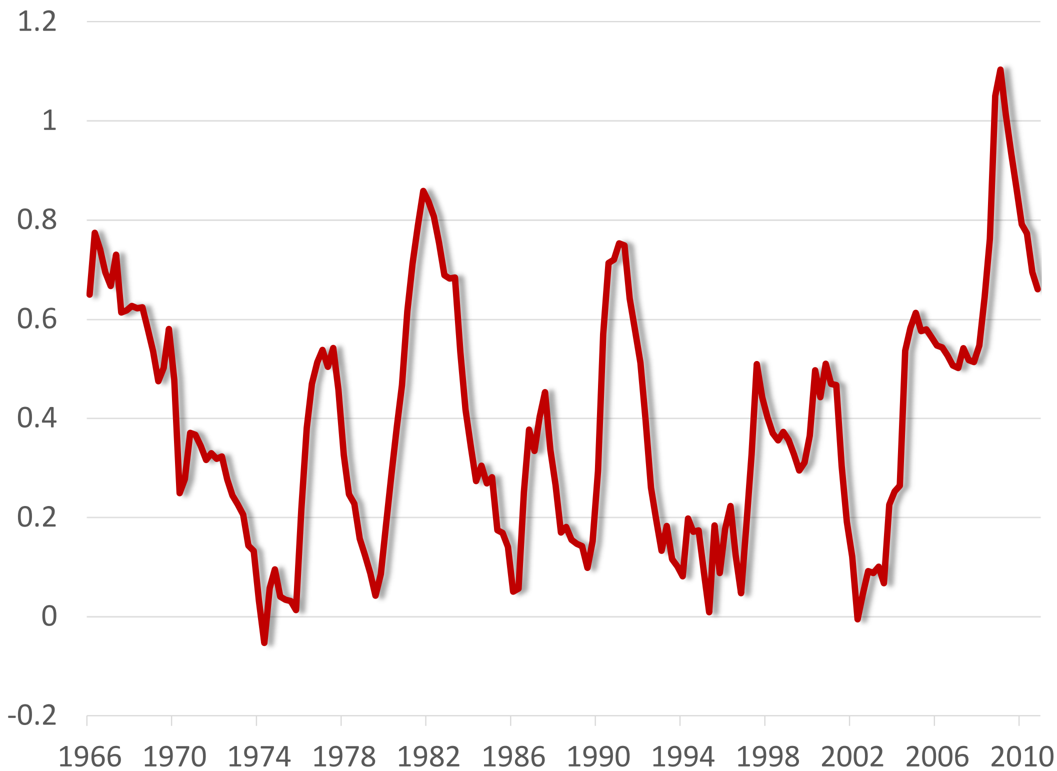


Figure 5: Estimated Values of $R$, the Relevance of the Natural Rate of Unemployment for Actual Unemployment, 20 Quarters Forward

Note: The figure shows the coefficients from rolling regressions covering 20 forward quarters from the date shown on the horizontal axis, Q1 1966—Q4 2010.

| *Line* | *Variable* | *Before 1990* | *After 1990* | *Source* |
|---|---|---|---|---|
| 1 | Unemployment (positive) | 0.3 | 0.3 | DLPT |
| 2 | Inflation (negative) | 0.3 | 0.1 | DLPT |
| 3 | Slope of Phillips curve regression | 1.0 | 0.3 | row 2/row 1 |
| 4 | Relevance of natural rate | 0.6 | 0.9 | GSW (see text) |
| 5 | True slope of Phillips curve | 2.5 | 2.5 | row 3/(1-row 4) |

Table 1: Calculations Relating to the Downward Bias in the Estimated Slope of the Phillips Curve, based on the estimates from del Negro et al. (2020) (DLPT) and Galí et al. (2011) (GSW)

We use our earlier derivations of the relevance of the natural rate based on leaving out the natural rate and the estimates of the natural rate series from GSW. Lines 4 and 5 of Table 1 provide information about the possibility that the econometric downward bias in the slope of the Phillips curve, caused by omitting data on the natural rate, accounts for the large decline in the slope of the estimated Phillips curve from the pre-1990 to post-1990 period. Line 4 reports the relevance statistic $R$, which we derived from GSW, as potentially applicable to the earlier period in DLPT's results. Line 5 shows the implied value of the slope of the Phillips curve adjusted for the given $R$. For the earlier period, the adjusted estimate, $\phi$, is $2.5 = 1.0/(1 - 0.6)$.

We then calculate what the value of $R$ should be for the later period so that the adjusted Phillips curve slopes are the same in both periods. With $R = 0.9$, the slope in the later period $\phi = 2.5 = 0.3/(1 - 0.9)$ equals the slope in the earlier period. While we do not know the value of $R$, the hypothesis that the relevance of the natural rate of unemployment for actual unemployment has increased in the later period has some support in Figure 5. It shows that during the long recovery from the 2007-09 recession, $R$, calculated from GSW data, reached its all-time maximum.

We propose that the US labor market experienced an increase in the relevance of the natural rate as measured by $R$, the regression coefficient for the natural rate on the actual rate of unemployment. This supports our hypothesis that the scenario in Table 1 accounts for at least some of the apparent decline in the slope of the Phillips curve after 1990.

## 4.3 Selected empirical studies

### 4.3.1 Estimates of the long-run path of the natural rate of unemployment

The notion that the natural unemployment rate changes slowly over time along with the composition of the labor force has been influential from the beginning. The Congressional Budget Office publishes a frequently updated estimate of the time path of the natural unemployment rate with demographic adjustment, and many Phillips-curve studies have adopted the CBO's path. That path rises gradually to a maximum in the 1970s and declines thereafter—see Figure 6.

In 2021, the CBO changed the name of the path, to the *noncyclical rate of unemployment*. The new name clarifies that the counterfactual underlying the calculation is the absence of cyclical movements, not the absence of all low and medium-frequency fluctuations. The clarification differentiates the CBO's estimate from those calculating the natural rate as the result of passing the actual rate through a band-pass filter, such as Hodrick-Prescott, and retaining only the low-frequency component. Other recent estimates of the long-run trend

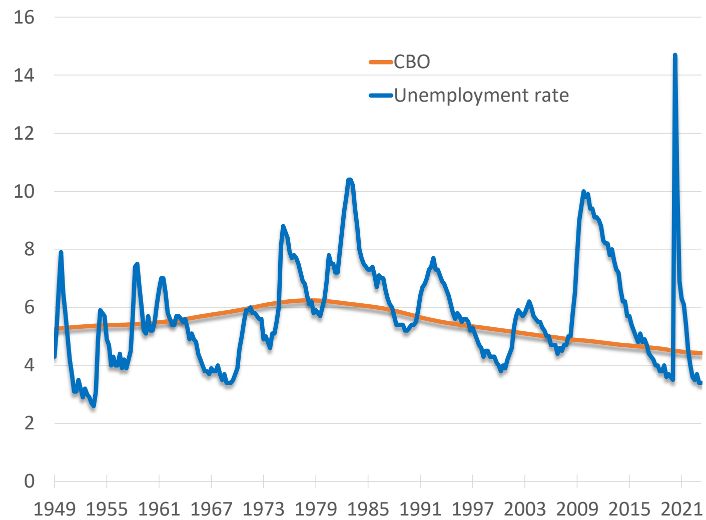


Figure 6: The Actual Unemployment Rate and the Noncyclical Rate of Unemployment from the Congressional Budget Office

in unemployment are Barnichon and Matthes (2017), Tasci (2018), Barnichon and Mesters (2018), and Hornstein and Kudlyak (2019).

### 4.3.2 Joint estimation of the natural rate and the Phillips curve

Gordon (1997) estimated a time-varying natural rate from a statistical model comprising an inflation equation with the inflation pressure and an equation for the natural rate, which follows a random walk. From 1955 through 1995, the estimated natural rate varied between 5.4 and 6.5 percent—see the series under the author's preferred smoothness parameter in his Figure 2. It declined by a percentage point between the mid-1980s and mid-1990s. See also Gordon (1998).

Bok, Crump, Nekarda and Petrosky-Nadeau (2023) (BCNP) estimate a model that combines a Phillips curve and the extraction of the natural rate of unemployment in a three-equation state-space model. The model draws upon earlier work by Laubach (2001); Crump, Eusepi, Giannoni and Sahin (2019); and Crump, Eusepi, Giannoni and Sahin (2022). In BCNP, the inflation pressure follows an AR(1) process and the natural rate follows a random walk.[2] Figure 2 of BCNP shows the unemployment rate and the estimated natural rate,

[2] Crump et al. (2019) take the natural rate of unemployment as the sum of a secular trend component and a cyclical component. Having estimated the trend, they estimate the cyclical component from a forward-looking Phillips curve model under the assumption of an AR(2) process for the unemployment gap.

labeled as the "preferred stable-price unemployment rate". Their estimated natural rate is substantially positively correlated with the actual rate as compared to the CBO's natural rate, so, in some years, the gap $u_t - u_t^*$ is small. However, the gap is large in the recovery following the recession of 2007 through 2009, a period where our Figure 2 shows that the gap appears to be quite small. Using BCNP's estimate of the natural rate, we find the regression coefficient for $u_t^*$ on $u_t$ to be $R = 0.47$ over 1985 through 2019. During the period from Q1 1985 to Q3 2015, for which we have both GSW's and BCNP's estimates of the natural rate, we find $R$ to be 0.46 and 0.45, respectively.

Such statistical models of the inflation pressure or the natural rate may overstate the pressure following recessions. As we suggest in Section 3.4, the natural rate may jump upward to come close to matching the high level of unemployment coming out of a crisis. Then, for a protracted period, the actual unemployment rate and the natural rate glide down together, implying that the pressure is small throughout the recovery. Accommodating such possibilities for the natural rate requires relaxing the assumption of an AR process with a constant-variance error for the inflation pressure.

#### 4.3.3 Policymakers' views

Policymakers have faced a great deal of uncertainty about the natural rate of unemployment. Orphanides (2002) and Orphanides and Williams (2013), discuss the magnitude of informational problems and disagreement over the natural rate of unemployment among policymakers in real time. Staiger, Stock and Watson (1997) find that it is measured quite imprecisely. A similar conclusion is reached in a comment by Davis (2019). Hetzel (2022), in a book on the history of the Federal Reserve system, expresses skepticism about the role of the Phillips curve as a structural model of the economy to guide the choice of monetary policy, evoking the Lucas critique (Lucas (1976)) and the lack of certainty about the natural rate of unemployment. Crump, Nekarda and Petrosky-Nadeau (2020) discuss the range of the natural unemployment rate benchmarks used by policymakers.

In a departure from the prevailing view among policymakers and other observers, that the natural rate moves slowly and does not track the cycle, Lacker (2012) took the opposite view: "There is a reference unemployment rate to which it's most appropriate to compare the current unemployment rate for the purposes of assessing current policy...The most common term for this reference rate is "the natural rate" of unemployment...There is a clear intuition for having the unemployment yardstick for monetary policy vary with economic conditions...Estimates of [the natural rate] invariably impose the assumption that it varies only slowly and does not respond to many transitory shocks...These are reasonable strategies for estimating the long-run unemployment rate, but by design, they will fail to capture important variations in the natural rate, especially variations over the business cycle."

### 4.3.4 Other aspects of the natural rate and the Phillips curve

As we noted earlier, a substantial literature starts from the assumption that the natural rate of unemployment is constant or only slow-moving, reflecting movements in a long-run trend in actual unemployment, and, therefore, uncorrelated or only weakly correlated with actual unemployment. Another branch of the literature notes that there have been several instances when large movements in the unemployment rate have coincided with small changes in the inflation rate. Using a smooth trend as the measure of the natural rate, studies typically find a flat Phillips curve, especially in recent decades. Some studies have invoked nonlinearities, whereby the slope differs when unemployment is high versus low, or time-variation of the Phillips curve across time-periods.

Hooper, Mishkin and Sufi (2020), review earlier research and contribute their own national and cross-state evidence on the flattening of the Phillips curve. They conclude that "national data going back to the 1950s and 60s yield strong evidence of negative slopes and significant nonlinearity in those slopes, with slopes much steeper in tight labor markets than in easy labor markets. The evidence of both slope and nonlinearity weakens dramatically based on macro data since the 1980s for the price Phillips curve." For the aggregate analysis, they use the CBO's measure of the national natural unemployment rate. For the state-level results, they use bins of actual state data because no estimates of the state-level natural rate were available at the time of their work.

Hazell, Herreno, Nakamura and Steinsson (2022) is a recent contribution that finds very flat state-level Phillips curves for non-tradeable goods and services at the state level, using state-level unemployment as the measure of the state-level gap and the CBO's natural-rate measure for the aggregate analysis.

Most recently, using the CBO's measure in the aggregate analysis and the unemployment rate rather than the unemployment gap in the MSA-level analysis (because "there are no estimates of the natural rate of unemployment ... for the city-level unemployment data"), Smith, Timmermann and Wright (2023) study time-variation in the slope of the Phillips curve and find two regime changes: prior to 1972, the estimated slope is 0.51 units of price decline per unit of unemployment increase; this slope steepens 1972-2001 to 0.87; after, 2001 break, the slope of the Phillips curve becomes essentially zero.

Leduc and Wilson (2017) measure the city-level natural rate of unemployment as a 10-year trailing average of the city's actual unemployment. They estimate the slope of the cross-city wage Phillips curve using seven-year rolling regressions and find a steady flattening of the Phillips curve slope starting with the 2001–2007 sample.

Dotsey, Fujita and Stark (2018), measuring the inflation pressure by the deviations of actual unemployment from its Hodrick–Prescott trend, conclude that "using the Phillips curve may add value to the monetary policy process during downturns...We find no evidence for relying on the Phillips curve during normal times, such as those currently facing the U.S. economy." (p.90) Ashley and Verbrugge (2023), using the long-run trend of unemployment from Tasci (2018) and from CBO as the measures of the natural rate, estimate what they call a "persistence-dependent" version of the Phillips curve that varies across three phases of the business cycle and find that in the recovery phase, inflation is unrelated to the unemployment gap. Using the CBO's measure, Doser, Nunes, Rao and Sheremirov (2023) estimate a piecewise-linear specification and document that the data favor a model with two regions, with the response of inflation to an increase in unemployment slower in the region where unemployment is already high. Barnes and Olivei (2003) estimate a piecewise linear specification of the Phillips curve. Using state fixed effects, Leduc, Marti and Wilson (2019) estimate nonlinear wage Phillips curve in the state-level unemployment data and find some evidence of steeper Phillips curve in hot labor markets. Other recent studies of the Phillips curve include Laubach (2001), Stella and Stock (2013), Coibion and Gorodnichenko (2015), Cecchetti, Feroli, Hooper, Kashyap and Schoenholtz (2017), and Jorgensen and Lansing (2019).

Stock and Watson (2010) present evidence consistent with our conclusion that, in recoveries, the natural rate follows the smoothly declining path of the actual rate. They show that inflation takes a step downward early in a recession, but then remains unrelated to unemployment changes as the business cycle progresses through recovery—see their Figure 2. Constant inflation with declining unemployment suggests that the natural rate of unemployment is declining in parallel with actual unemployment, according to equation (3).

# 5 Potential Ambiguity about the Slope of the Phillips Curve and the Magnitude of Inflation Pressure

## 5.1 Econometric identification of the natural rate and the slope parameter

The Phillips curve sets the excess of the rate of inflation over its anchor to the product of the slope parameter and the inflation pressure:

$$\pi_t - \pi_t^* = -\phi_t \cdot (u_t - u_t^*).$$

In this section, we explore the case that arises if $\pi_t - \pi_t^*$ and $u_t$ are observed with reasonable accuracy but $\phi_t$ and $u_t^*$ are unknown. This case interests investigators who are skeptical about the identification assumptions behind the natural rate, $u_t^*$. As our review of research on this topic shows, a common approach to identification posits a one-dimensional condition involving $u_t^*$ alone, often assuming that it is a constant or a function of slow-moving demographics, or, more recently, to posit a state-space model as in the BCNP's work and its predecessors.

Absent a convincing identifying condition, the available information identifies a range of paths of $\phi_t$ paired with the corresponding paths of $u_t^*$, that satisfy the Phillips curve, given the observed $\pi_t - \pi_t^*$ and $u_t$. Near one end, $\phi_t$ is close to zero and $\pi_t - \pi_t^*$ is correspondingly large. Near the other end, $\phi_t$ is large and the unemployment gap $u_t - u_t^*$ is small.

The range of opinions about the recovery from 2009 through early 2020 illustrates this point. One view, widely present in the literature, is that the Phillips-curve slope, $\phi_t$, was small and even vanishing during the recovery, while the inflation pressure, $u_t - u_t^*$ was large and negative. We call this the low-and-sticky view of the slope of the Philips curve.

The other view is that the slope, $\phi$, was material during the recovery, while the inflation pressure, $u_t - u_t^*$, was small and positive. We call this the flexible view of the Phillips curve. Section 3 of this paper makes the case for this interpretation.

Both views fit the specified data. Additional data helping to reveal $u_t^*$ or $\phi$ would be needed to determine which view is correct.

## 5.2 Further discussion of the flexible view

In the flexible view of the Phillips curve, low unemployment does not necessarily signal high inflation pressure. During recoveries, the pressure, $u_t - u_t^*$, is close to zero, based on the evidence that during recoveries natural rate of unemployment glides down together with actual unemployment and they are likely close to each other. Our finding means that the unemployment gap is close to zero and, therefore, inflation pressure is weak. We do not take a position on the gap during contractions when unemployment is rising rapidly.

Our finding is that it is hard or impossible to see the effects of the inflation pressure because, during recoveries, inflation pressure is essentially zero.

Under the flexible view, during recoveries, the pressure is zero. That is, during recoveries an economy resembles a real business cycles economy, with $\frac{1}{\phi_t}$ being relatively low and the Phillips curve being steep. The flexibility of prices is the key differentiating factor. Our view requires that prices are somewhat flexible, so the Phillips curve is reasonably steep, whereas the opposing low and sticky view posits stickier prices and a flatter Phillips curve.

The most radical potential conclusion about the relation between the observed rate of unemployment and the natural rate is that there is no difference—observed unemployment is at its natural level all the time. This conclusion would cut the heart out of the Phillips curve and the distinctive features of the New Keynesian model. It would deny unemployment any role as a measure of inflation pressure. That conclusion goes beyond the evidence, however. We have relatively few observations of stable inflation in times of rising or really high unemployment. We believe that a reasonable interpretation of the evidence is that, during long, slow, reliable recoveries with gradually declining unemployment, unemployment is close to its natural rate and is not a measure of inflation pressure. Under those conditions, there is no meaningful unemployment gap.

# 6 The Effect of the Pandemic on the Phillips Curve

During the long recovery from 2009 to 2019, inflation became powerfully anchored at just below 2 percent per year. In this stable environment, sellers adapted their price-setting procedures to stability. The Phillips curve became relatively flat because sellers tended to leave prices unchanged for extended periods—relatively few sellers responded to change each month.

The pandemic created a completely different environment for pricing decisions, with rapid new developments. A quick response to each new development was required. Policy responses to the pandemic included expansionary monetary and fiscal policy. Another important development was a reduction in output supply, due to idling of important sectors of the economy, notably hospitality. The turbulence that the pandemic brought to seller's economic situations made it necessary to make more frequent prices changes than in the tranquil pre-pandemic times.

In the Phillips curve framework, an increase in turbulence represents a regime change—the Phillips curve becomes more sensitive to changes in unemployment. That is, in tranquil times the Phillips curve is relatively flat—any shifts in demand show up mostly as quantity changes, while in turbulent times, the shifts in demand have large effects on inflation. One possible explanation behind the post-pandemic inflation is an increased volatility of the inflation anchor—$\pi_t^*$ in the Phillips curve. As discussed above, the inflation anchor also depends on the current speed of adjustment of prices —if information becomes more volatile, sellers will choose to change prices more frequently.

Hall (2023) suggests that the New Keynesian Phillips curve became steeper in the volatile conditions of the pandemic. He establishes that sectors with price stickiness are prone to rapid transitions from stickiness to flexibility, as sellers elect to reset their prices and abandon anchoring. He argues that the logic of the New Keynesian model of the Phillips curve links inflation to volatility, because a larger fraction of sellers are pushed out of their regions of inaction when volatility is elevated. See also Blanco, Boar, Jones and Midrigan (2023) and Cavallo, Lippi and Miyahara (2023), and work cited there.

With respect to the pandemic, this line of thought implies that the prices that we thought were sticky turned much more flexible. The pandemic economy is closer to a flexible-price economy, an economy with significantly less price inertia, compared to the sticky-price pre-pandemic economy.

As the environment changed toward lower inflation, because the constraints from the pandemic relaxed and the Fed adopted a contractionary policy to combat inflation, the steep slope of the Phillips curve became an advantage. Disinflation is occurring with a materially smaller bulge of unemployment than would occur under the pre-pandemic, more stickier-price economy.

Sargent (1982) studies the disinflations in four economies as they overcame high inflation rates and achieved price stability with little dislocation of economic activity. Their Phillips curves became vertical. The steeper is the Phillips curve, the less the cost of disinflation. In times of rapid change, especially those involving fiscal or monetary reforms, the real cost of disinflation can be smaller than in more tranquil times.

# 7 Concluding Remarks

For decades, Friedman's invention, the natural rate of unemployment, languished as an unimportant constant in the Phillips curve, or as a slow-moving trend. But puzzling evidence accumulated about the role of the resulting Phillips curve in the behavior of inflation and unemployment. In particular, the decade-long expansion starting in 2009 combined near-constant inflation with continuing declines in unemployment from 10 percent to 3.5 percent.

Phillips curves constructed with constant natural rates and constant slopes became untenable as this process unfolded. A few investigators reconsidered constancy of the natural rate in favor of a decline, but many concluded that it was the slope of the Phillips curve that had declined. Some estimates of the slope by the end of the recovery were close to zero.

Our investigation of the recovery starting in 2009 concludes in favor of a declining natural rate. The logical basis for this conclusion is that the anchored inflation rate must have converged to the Fed's target rate of two percent over such a long period of stable inflation so close to that target. A bedrock principle of the New Keynesian model is that in an economy with actual inflation equal to its anchor (sometimes called the expected rate), the observed unemployment rate must equal the natural rate.

Based on our earlier work on the behavior of unemployment in cyclical recoveries in the 10 recessions since 1948, we extend our conclusion, but with more wiggle room, to all of those recessions, because all of them share the pattern of the 2009-19 recovery. Unemployment starts at an elevated level and glides downward until interrupted by the next recession or unemployment hits bottom at around 3.5 percent.

We provide some surrounding analysis and evidence for our conclusion. The finding of declining estimated values of the slope of the Phillips curve could be an artifact of the omission of the natural rate from the regression or the use of a proxy for the natural rate that lacks the true correlation with the unemployment rate. Adjusting for this bias could disclose that the true slope of the Phillips curve remained the same or even increased as unemployment declined.

Another important conclusion is that the labor market may change dramatically upon changes in the macroeconomic environment. There may have been a substantial loosening so that prices rose instead of unemployment falling in 2021 and early 2022 followed by disinflation currently instead of rising unemployment. Prices may have had a bigger role in stabilizing output than is implied by sticky-price models.

# References


Ashley, Richard and Randal J. Verbrugge, “The Intermittent Phillips Curve: Finding a Stable (But Persistence-Dependent) Phillips Curve Model Specification,” 2023. Federal Reserve Bank of Cleveland Working Paper 19-09R2.

Barnes, Michelle L. and Giovanni P. Olivei, “Inside and Outside the Bounds: Threshold Estimates of the Phillips Curve,” *Federal Reserve Bank of Boston New England Economic Review*, 2003, pp. 3–18.

Barnichon, Regis and Christian Matthes, “The Natural Rate of Unemployment over the Past 100 Years,” *FRBSF Economic Letter*, 2017, *23.*

—— and Geert Mesters, “On the Demographic Adjustment of Unemployment,” *Review of Economics and Statistics*, 2018, *100*, 219–231.

Blanco, Andres, Corina Boar, Callum Jones, and Virgiliu Midrigan, “Nonlinear Inflation Dynamics in Menu Cost Economies,” Working Paper, NYU 2023.

Bok, Brandyn, Richard K. Crump, Christopher J. Nekarda, and Nicolas Petrosky-Nadeau, “Estimating Natural Rates of Unemployment: A Primer,” Working Paper 2023-25, Federal Reserve Bank of San Francisco 2023.

Cavallo, Alberto, Francesco Lippi, and Ken Miyahara, “Large Shocks Travel Fast,” Working Paper 31659, National Bureau of Economic Research 2023.

Cecchetti, Stephen, Michael Feroli, Peter Hooper, Anil Kashyap, and Kermit L. Schoenholtz, “Deflating Inflation Expectations: The Implications of Inflation’s Simple Dynamics,” Discussion Paper 11925, CEPR 2017.

Coibion, Olivier and Yuriy Gorodnichenko, “Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation,” *American Economic Journal: Macroeconomics*, 2015, *7* (1), 197–232.

Crump, Richard K., Christopher J. Nekarda, and Nicolas Petrosky-Nadeau, “Unemployment Rate Benchmarks,” Working Paper 2020-072, Finance and Economics Discussion Series, Federal Reserve Board 2020.

—— , Stefano Eusepi, Marc Giannoni, and Aysegul Sahin, “A Unified Approach to Measuring u*,” *Brookings Papers on Economic Activity*, 2019, pp. 143–214.

———, ———, ———, and ———, "The Unemployment-Inflation Trade-Off Revisited: The Phillips Curve in COVID Times," Working Paper 29785, National Bureau of Economic Research 2022.

Davis, Steven J., "Comment on "A Unified Approach to Measuring u* " by Richard K. Crump, Stefana Eusepi, Marc Giannoni and Aysegul Sahin," *Brookings Papers on Economic Activity*, 2019, *Spring*, 215–223.

del Negro, Marco, Michele Lenza, Giorgio Primiceri, and Andrea Tambalotti, "What's Up with the Phillips Curve?," *Brookings Papers on Economic Activity*, 2020, pp. 301–357.

Doser, Alexander, Ricardo Nunes, Nikhil Rao, and Viacheslav Sheremirov, "Inflation Expectations and Nonlinearities in the Phillips Curve," *Journal of Applied Econometrics*, 2023, *38*, 453–471.

Dotsey, Michael, Shigeru Fujita, and Tom Stark, "Do Phillips Curves Conditionally Help to Forecast Inflation?," *International Journal of Central Banking*, 2018, *14*, 43–92.

Friedman, Milton, "The Role of Monetary Policy. Presidential address delivered at the 80th Annual Meeting of the American Economic Association," *American Economic Review*, 1968, *58* (1), 1 – 17.

Galí, Jordi, Frank Smets, and Rafael Wouters, "Unemployment in an Estimated New Keynesian Model," *NBER Macroeconomics Annual*, 2011, *26*, 329–360.

Gordon, Robert J., "The Time-Varying NAIRU and Its Implications for Economic Policy," *Journal of Economic Perspectives*, 1997, *11* (1), 11–32.

———, "Foundations of the Goldilocks Economy: Supply Shocks and the Time-Varying NAIRU," *Brookings Papers on Economic Activity*, 1998, (2), 297–346.

Hall, Robert E., "The Routes Into and Out of the Zero Lower Bound," *Federal Reserve Bank of Kansas City Symposium (Jackson Hole)*, 2013.

———, "A Major Shock Makes Prices More Flexible and May Result in a Burst of Inflation or Deflation," Working Paper 31025, National Bureau of Economic Research 2023.

——— and Marianna Kudlyak, "Job-Finding and Job-Losing: A Comprehensive Model of Heterogeneous Individual Labor-Market Dynamics," Working Paper No. 25625, National Bureau of Economic Research 2019.

——— and ——— , "The Inexorable Recoveries of Unemployment," *Journal of Monetary Economics*, 2022, *131*, 15–25.

——— and ——— , "Why Has the US Economy Recovered So Consistently from Every Recession in the Past 70 Years?," *NBER Macro Annual*, 2022, *36.*

Hazell, Jonathon, Juan Herreno, Emi Nakamura, and Jon Steinsson, "The Slope of the Phillips Curve: Evidence from U.S. States," *Quarterly Journal of Economics*, 2022, *137* (3), 1299–1344.

Hetzel, Robert L., *The Federal Reserve System. A New History*, University of Chicago Press, 2022.

Hooper, Peter, Frederic S. Mishkin, and Amir Sufi, "Prospects for Inflation in a High Pressure Economy: Is the Phillips Curve Dead or Is It Just Hibernating?," *Research in Economics*, 2020, *74*, 26–62.

Hornstein, Andreas and Marianna Kudlyak, "Aggregate Labor Force Participation and Unemployment and Demographic Trends," Working Paper 2019-07, Federal Reserve Bank of San Francisco 2019.

Jorgensen, Peter Lihn and Kevin J. Lansing, "Anchored Inflation Expectations and the Flatter Phillips Curve," 2019. Federal Reserve Bank of San Francisco, Working Paper No. 2019-27.

Lacker, Jeffrey M., "Maximum Employment and Monetary Policy," Speech, The Money Marketeers of New York University, New York 2012.

Laubach, Thomas, "Measuring the NAIRU: Evidence from Seven Economies," *Review of Economics and Statistics*, 2001, *83* (2), 218–231.

Leduc, Sylvain and Daniel J. Wilson, "Has the Wage Phillips Curve Gone Dormant?," *Federal Reserve Bank of San Francisco Economic Letter*, 2017, (2017-30).

——— , Chitra Marti, and Daniel J. Wilson, "Does Ultra-Low Unemployment Spur Rapid Wage Growth?," *Federal Reserve Bank of San Francisco Economic Letter*, 2019, (2019-02).

Lucas, Robert E., "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference Series on Public Policy*, 1976, *1*, 19–46.

Orphanides, Athanasios, "Monetary Policy Rules Based on Real-Time Data," *American Economic Review*, 2002, *91* (4), 964–985.

—— and John C. Williams, "Monetary Policy Mistakes and the Evolution of Inflation Expectations," *in The Great Inflation: The Rebirth of Modern Central Banking, Bordo and Orphanides (Eds.)*, 2013.

Rogerson, Richard, "Theory Ahead of Language in the Economics of Unemployment," *Journal of Economic Perspectives*, 1997, *11* (1), 73–92.

Romer, David, *Advanced Macroeconomics*, McGraw-Hill Education; 5th edition, 2019.

Sargent, Thomas J., "The Ends of Four Big Inflations," *NBER Chapters, in: Inflation: Causes and Effects, National Bureau of Economic Research*, 1982, pp. 41–98.

Smith, Simon C., Allan Timmermann, and Jonathan H. Wright, "Breaks in the Phillips Curve: Evidence from Panel Data," Working Paper 2023-015, Federal Reserve Board, Washington, D.C., Finance and Economics Discussion Series 2023.

Staiger, Douglas O., James H. Stock, and Mark W. Watson, "How Precise Are Estimates of the Natural Rate of Unemployment?," *In Reducing Inflation: Motivation and Strategy, Christina D. Romer and David H. Romer, Editors*, 1997, pp. 195 – 246.

Stella, Andrea and James H. Stock, "A State-Dependent Model for Inflation Forecasting," 2013. Board of Governors of the Federal Reserve System, International Finance Discussion Papers No. 1062r.

Stock, James H. and Mark W. Watson, "Modeling Inflation after the Crisis," 2010. Prepared for the Federal Reserve Bank of Kansas City Symposium, "Macroeconomic Policy: Post-Crisis and Risks Ahead," Jackson Hole, Wyoming, August 26-28 2010.

Tasci, Murat, "The Ins and Outs of Unemployment in the Long Run: Unemployment Flows and the Natural Rate," Working Paper 12-24, Federal Reserve Bank of Cleveland 2018.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, NJ: Princeton Univ. Press, 2003.