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DISCUSSION PAPER SERIES

IZA DP No. 16766

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ABSTRACT

Artificial Intelligence and the Discovery of New Ideas: Is an Economic Growth Explosion Imminent?

Theory predicts that global economic growth will stagnate and even come to an end due to slower and eventually negative growth in population. It has been claimed, however, that Artificial Intelligence (AI) may counter this and even cause an economic growth explosion. In this paper, we critically analyse this claim. We clarify how AI affects the ideas production function (IPF) and propose three models relating innovation, AI and population: AI as a *research-augmenting technology*; AI as *researcher scale enhancing technology*; and AI as a *facilitator of innovation*. We show, performing model simulations calibrated on USA data, that AI on its own may not be sufficient to accelerate the growth rate of ideas production indefinitely. Overall, our simulations suggests that an economic growth explosion would only be possible under very specific and perhaps unlikely combinations of parameter values. Hence we conclude that it is not imminent.

JEL Classification: O31, O33, O40, J11, J24

Keywords: automation, artificial intelligence, economic growth, innovation, ideas production function

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1 Introduction

How long will the global economy still be able to grow? Theory predicts that global economic growth will stagnate and even come to an end due to slower and eventually negative growth in population, the "empty planet" effect (Bricker and Ibbitson, 2020; Jones, 2022a,b). It has been claimed, however, that Artificial Intelligence (AI) may counter this and even cause *explosive economic growth*, that is, growth in Gross World Production (GWP) of 30% per annum or more (Erdil and Besiroglu, 2023; Davidson, 2021; Trammell and Korinek, 2020).

Even though, at present, the measured impact of AI on economic and aggregate productivity growth is modest (Parteka and Kordalska, 2023; Naudé, 2021), the expectation is that it is only a matter of time before AI has diffused wide enough and improved sufficiently to match or exceed the level of human intelligence (Mokyr, 2018; Brynjolfsson et al., 2017; Erdil and Besiroglu, 2023). At this time "rapid growth in information technology and artificial intelligence will cross some boundary, after which economic growth will rise rapidly as an ever-increasing pace of improvements cascade through the economy" (Nordhaus, 2021, p.299). The development of Large Language Models (LLMs) has fuelled these expectations (Naudé et al., 2024).

AI may accelerate economic growth by automating the production of goods and ideas. In this paper, our interest is in the latter. This is because, fundamentally, economic growth is driven by ideas, not objects (Romer, 2019). Moreover, from the point of view of explosive growth from AI, this seems the only possibility if one rules out the appearance of an Artificial General Intelligence (AGI), which still only exists in the realms of science fiction. Aghion et al. (2019), for instance, has shown how, with automation in the production of goods, even with "nearly complete automation" (p.274) Baumol's cost disease can lead to balanced growth with a constant capital share less than 100%. Gries and Naudé (2020) has shown that with binding demand constraints, for instance, due to high inequality resulting from AI automation in the production function for goods, economic growth would stagnate.

Therefore, whether or not explosive economic growth is imminent will hinge on how AI affects the *ideas production function* (IPF). As such, the contribution of this paper is twofold. First, we critically discuss the treatment of the IPF in the literature and highlight the centrality of population in economic growth. We show that AI is generally incorporated as a tool that enables higher efficiency in the production of knowledge but with no law of motion associated with it. Second, we address this lacuna and introduce three models that relate innovation, AI and population: AI as a *research-augmenting* technology, AI as *researcher scale enhancing* technology, and AI as a *facilitator of innovation* through a standing-on-shoulders effect.

We find that AI on its own may not be sufficient to accelerate the growth rate of ideas production indefinitely. Our simulations for the transitional dynamics when saddle-path stability occurs also show that the possibility of explosive growth due to AI may occur only in the very short-run in the case that AI idea-generation overshoots its long-run growth. Overall, our simulations indicate that an economic growth explosion would occur only under very particular and unlikely combinations of parameters, concluding that it is not imminent.

The paper is structured as follows. Section 2 discusses the relevant literature on the relationship between economic growth and population. Then, section 3 outlines and introduces the modelling of AI in innovation-driven growth. Sections 3.1, 3.2 and 3.3 contain the core contributions of the paper. First, we present a simple model of AI as a *researcher-augmenting* technology. This model can generate explosive growth or explosive de-growth only if the population increases or decreases respectively. This model illustrates the necessity to explicitly model AI innovations in the IPF to evaluate claims about its impact on economic growth.

In section 3.2, we describe a model of AI as a *researcher scale enhancing* technology. This shows that economic growth can explode, collapse, or be constant (negative or positive) in the long run, depending on how hard or easy new ideas are to find. We also find that AI on its own may not be sufficient to accelerate the growth rate of ideas production indefinitely.

In section 3.3, we model AI as a *facilitator* of the discovery of new innovations. We characterize a long-run equilibrium that can be semi-endogenous (with either fishing-out of ideas or positive spillovers existing) or fully endogenous (with or without scale effects). Section 4 concludes.

2 Related Literature

In this section, we discuss the literature on ideas as the ultimate determinant of economic growth; we outline the relationship between population growth and innovation on the one hand and economic growth on the other. From this, we show that declining population growth would lead to a decline in economic growth and that this decline could be faster if accompanied by a decline in the productivity of innovation - the rate at which new ideas are discovered. We provide an overview of empirical studies that have provided evidence that both population growth declines and research productivity declines are limiting economic growth in advanced countries. Then, we review the recent literature that has argued that the slowdown in economic growth may be reversed by AI progress, even to the extent of causing explosive growth.

2.1 The Ideas Production Function

The "secret sauce" of economic growth is *ideas* and not objects (Romer, 1993). As recognised by McCloskey (2017), "Our riches did not come from piling brick upon brick, or bachelor's degree on bachelor's degree, or bank balance on bank balance, but from piling idea on idea." An idea is "a design, a blueprint, or a set of instructions for starting with existing objects and transforming or using them in some way that generates either more output or more utility" (Jones, 2019, p.865). The power of ideas is based on their being non-rival in use, which means that they are infinitely usable, and their combinatorial nature[1] which implies that the world will never run out of ideas (Jones, 2019, 2022a,b; Koppl et al., 2019; Mokyr, 2007; Romer, 2019, 1990, 1993; Weitzman, 1998).

Where do "ideas" come from? An extensive literature dealing with so-called R&D models of endogenous growth, has attempted to explain this using the ideas production function (IPF) concept. The IPF specifies innovations, measured say by patents and trademarks, as outputs from several inputs (e.g., R&D activities and expenditures) into the process that generates new ideas (Ramani et al., 2008).

If we denote ideas by A, then the commonly used Cobb-Douglas specification of economic output is $Y_t = A_t {K_t}^{\alpha} {L_t}^{\beta}$, where GDP is denoted by Y_t, capital by K and labour by L and where where α and β are the shares of capital and labour in final output (GDP). Following Jones (2022b, p.127) a basic ideas production function (IPF) can be written as follows:

$$\Delta A_t = (sL_t)^{\lambda} A_t^{\phi} \tag{1}$$

where A_t is the stock of ideas (knowledge); ΔA_t is the growth in new ideas; L_t is the population and sL_t with $0 < s < 1$ the share of the population devoted to discover new ideas. Ideas are generated by people (L_t) who provide R&D work. The larger the population the more R&D work can be done. Moreover, in a larger population, there may be more "geniuses" born who discovers path-breaking new ideas.

Hence, according to Kuznets (1960, p.328) "The greatest factor in growth of output *per*

[1]Koppl et al. (2019, p.1) presents a combinatorial model of technological change which suggests that "the Industrial Revolution was a combinatorial explosion in an unchanging process of technological change that began with the origin of our species." With combinatorial growth, an industrial revolution is inevitable.

capita is, of course, the increasing stock of tested, useful knowledge. The producers of this stock are the scientists, inventors, engineers, managers, and explorers of various description [...] Population growth [...] would, therefore, produce an absolutely larger number of geniuses, talented men, and generally gifted contributors to new knowledge." And as more recently echoed by Jones (2022b, p.126) "new ideas are discovered through the hard work and serendipity of people. Just as more autoworkers will produce more cars, more researchers and innovators will produce more new ideas."

In (1), the parameter ϕ measures how much past ideas matter for new ideas. Formally, it reflects the extent and nature of intertemporal knowledge spillovers (Abdih and Joutz, 2005). If $\phi > 0$ then the spillovers are positive, meaning past knowledge or ideas increases the research productivity of future R&D workers. This has been called the "standing-on-shoulders" effect[2] (Abdih and Joutz (2005); Neves and Sequeira (2018)). With $\phi < 0$ the converse holds. Agrawal et al. (2023) presents a model wherein AI magnifies the standing-on-shoulders effect, acting as a "super" scientist, by predicting which innovations in a given pipeline of innovations are more likely to be successful.

If $\phi < 1$ ideas are becoming "fished-out" and "harder to find." In such a case, policies to increase the number of researchers would be ineffective to raise economic growth - it would be akin to having more fishermen to fish in a pond where there are no fish. While Romer (1990) assumed $\phi = 1$, which allowed economic growth with a constant number of researchers, Jones (1995) argued that this was inconsistent with the actual experience of the USA over the 20th century, which saw a rise in the number of people in R&D but with growth remaining fairly constant around 2%. He explained[3] this by arguing that $\phi < 1$.

The parameter λ measures the returns to R&D from the pool of researchers available. If

[2]For an interesting account of this effect, see Kragh (2024)'s discussion of how "Einstein built on the past to make his breakthroughs."

[3]As Trammell and Korinek (2020) remarks, growth models can be classified as endogenous when the assumption is that $\phi = 1$, and as semi-endogenous when it is assumed that $\phi < 1$.

$\lambda > 1$ researchers compliment each other (Trammell and Korinek, 2020). But if if $\lambda < 1$ is signifies diminishing returns, or what has been called the "stepping-on-toes" effect (Sequeira and Neves, 2020; Abdih and Joutz, 2005). It could for instance reflect duplication effects in R&D. With $\phi < 1$ the research productivity of research decline; hence increases in the number of researchers will not lead to a proportional increase in new ideas.

Note that if $\lambda = 1$ and $\phi = 0$ then new ideas grow proportional to the size of the population, L_t.

2.2 Population and Economic Growth

The gist of equation (1) is that a positive relation exists between the growth of new ideas and the growth rate of the population. ϕ is a measure of the productivity of innovation, in other words, the productivity of finding new ideas. The expectation is that $\phi > 0$, because unlike typically objects in the production function, ideas are non-rival in use. As such, (1) implies increasing returns to ideas: the more people, the more ideas there are, and the faster the stock of ideas will grow (Davidson, 2021). For Galor and Weil (2000, p.807) this means that "changes in the size of population can be taken as a direct measure of technological improvement."

If the total population remains constant, the economy will continue to grow at a constant rate because the stock of new ideas generated by that population grows at constant rate (Kremer, 1993; Jones, 2022a). However, if the population growth should decline, or even worse, turn negative, then economic growth will decline as well. This does not need to happen over the short-to-medium term. Jones (2022b) points out that over the past 75 years population growth explained only 0.3% of the average annual 2% growth that was achieved - the rest being due to improvements in education, more effective resource allocation and rising global research intensity.

These contingent factors however, are once-off improvements and cannot be sustained indefinitely. At some point as their contributions fades out, population growth becomes the binding constraint on growth. Jones (2022b, p.140) calculates that "once these sources have run their course, all that will remain is the 0.3 percentage point contribution from population growth. In other words, a natural implication of this framework is that long-run growth will be γn. With our estimate of $\gamma = 1/3$ and $n = 1\%$, the implication is that long-run growth in living standards will be 0.3% per year rather than 2% per year - an enormous slowdown!"

Jones (2022a) furthermore show that when population growth turns negative, that the stock of knowledge converges to some finite value, where " living standards stagnate for a population that vanishes" (Jones, 2022b, p.143). He calculates that with a 1% annual decline in population, that world GDP growth would drop to zero somewhere between 85 to 250 years (Jones, 2022a, p.9).

Thus, because economic growth depends on the population growth rate - and given the fertility rates are not infinite but limited, it implies that economic growth is "capped" by the population growth rate (Erdil and Besiroglu, 2023, p.23). Conversely, a larger population would, by providing more labour to do R&D, generate higher rates of long-run growth. Eden and Kuruc (2023, p.17) derives a growth model based on the above that illustrates that "a future that stabilizes at a larger global population will be richer per capita than a world that stabilizes at a smaller population."

But is a larger population necessarily associated with faster long-run economic growth?

According to Jones (1995) empirical evidence from the USA and other OECD countries over the 20th century was inconsistent with this expectation. While population, and the number of labourers devoted to R&D grew in absolute size over this period, there was no acceleration in economic growth. What explained this inconsistency is declines in research productivity over this period, which moreover, was partly caused by rising population. Hence, to understand the relationship between population and economic growth, and eventually grasp how AI may (or not) lead to explosive economic growth, is is necessary to understand the relationship between population and research productivity better. In the next sub-section we explain how the literature has developed a more nuanced approach to the IPF than the basic model in (2).

2.3 Research Productivity and Economic Growth

The IPF specifies not only a link between population and economic growth, but implies that research productivity and the availability of useful ideas affects economic growth, through respectively ϕ and λ.

Economic growth would slow down, at any given level of population, if the productivity of existing researchers would decline, i.e. if ϕ would become smaller. Empirical evidence in favour of declining research productivity (innovation effectiveness) is provided by Ang and Madsen (2015). Using an underlying Schumpeterian growth model, and drawing on Young (1998) who noted the negative relationship between R&D effectiveness and "product proliferation effects" as an economy increases in size, they (re) formulate the ideas production as follows (p.82):

$$\Delta A_t = s(\frac{X}{Q^B})^\lambda A^\phi \tag{2}$$

Where $0 < s \leq 1$; $\lambda \leq 1$.

In (2) X denotes research (R&D) effort, Q is population size, which is in this literature a proxy for product variety, which reflects innovation - more innovation means greater product variety (Madsen, 2008). λ is the duplication ("stepping-on-toes") parameter which $=0$ if innovations are duplications, and $= 1$ if not; ϕ is, as already explained, the returns to scale in the stock knowledge and B is a parameter of product proliferation. The ratio $\frac{X}{Q}$ reflects the research intensity going into the search for new ideas new (Madsen, 2008, p.3).

The essence of (2) is that is shows that with constant R&D effort and research productivity, a continued increase in population (as e.g. measured by product variety) would lead to a decline in the rate of new ideas generated. This is as Cowen (2010) explains, because all the "low-hanging fruit" of knowledge has been "eaten." This effect is known as "fishing-out" (Klüppel and Knott, 2023). Ang and Madsen (2015) took a logarithmic specification of (2) and estimate it, using a dataset covering 26 countries between 1870 and 2010. Their estimates for the crucial parameters of the ideas production function are $\phi = 0,98$ and $B = -0,013$. These estimates are consistent with Schumpeterian endogenous growth models: ϕ close to 1 suggests constant returns to scale to existing knowledge and $B < 0$ evidence that negative effects from population size are significant. Hence they conclude that "population-induced expansions in R&D were neutralized by a proportional increase in product variety" (Ang and Madsen, 2015, p.105).

Bloom et al. (2020, p.46) furthermore tested whether a "fishing-out" of ideas had taken place in the USA, estimating $\phi = -2.1$ which is empirical evidence for fishing out and for ideas getting harder to find. They concluded that this finding means that "*just to sustain constant growth in GDP per person, the U.S. must double the amount of research effort searching for new ideas every 18 years to offset the difficulty of finding new ideas.*"

More recently Klüppel and Knott (2023) argued that Bloom et al. (2020) did not directly test whether ideas have been fished out; they rectify this by constructing a direct measure of research productivity - which entail that they test whether in expression (2) above, $\phi \geq 1$, which would indicate positive intertemporal spillovers from ideas, meaning that "there is no fishing-out, and growth from R&D continues in perpetuity."

They estimate ϕ to be around 1.02 to 1.24, concluding from this that "*We found no evidence of fishing-out. Instead, we found increasing returns to the knowledge stock. This finding resonates with real-world evidence of several important, yet fairly recent, general-purpose innovations like the internet, GPS, and smartphones. We would not expect such important innovations to be so recent if ideas are being fished out*" (p.1).

However, they find (p.5) that $\lambda = -0.1$ which means a "stepping-on-toes effect" characterises the R&D process by researchers. They conclude that "while we fail to find evidence of fishing out, there is substantial evidence that aggregate research productivity has declined. What, other than fishing out, might explain that?"

They speculate that the decline in aggregate research productivity in the USA, which seems not to be because ideas are getting harder to find, are rather due to contingent factors, such as imbalances between research versus development, with more and more research being done and outsourced to universities and which results in "excess research;" and "deterioration" of R&D practices on the firm level.[4] The implication of their finding is that increases in the population is not yet associated with fishing out and hence with $\phi > 1$ further ideas production would increase economic growth. Moreover, if the pathologies in how R&D is organised can be improved, and the "stepping-on-toes" effect ameliorated, then the growth-enhancing effects of R&D on growth will be even more significant.

The upshot of the above is that the relationship between population and economic growth is not a simple positive linear relationship. A more nuanced relationship can be described as follows. As the population increases, so does knowledge and ideas. At first, this creates positive scale effects, given that innovation through R&D is subject to fixed costs, and therefore, the returns to R&D improves with a larger market. Over time, however, it becomes more difficult to create and use new valuable knowledge. It becomes more difficult to innovate,

[4] The suspicion that R&D practices are deteriorating, leading to reduced research productivity, is consistent with the findings of Arora et al. (2011) and Liu and Ma (2021, p.1), who found that "R&D misallocation accounts for about 0.68 percentage points of missing annual growth since the 2000s".

not only may ideas getting fished-out, but researchers may be stepping on each others toes metaphorically, and complexity brakes may slow down innovation (Naudé, 2022). In this latter regard Sequeira et al. (2018, p.103) put it as follows:

> "The difficulty of introducing new products and replacing old ones is proportional to the market size [...] the larger the market size, the larger the costs necessary to discover, develop and market the associated technology, e.g. costs pertaining to the construction of prototypes and samples, new assembly lines and training of workers, and generic coordination, organizational, marketing and transportation costs."

In sum, in this section so far we have shown that 1) population growth drives economic growth through innovation (new ideas) and the productivity with which a population of given size finds new ideas; 2) economic growth can stagnate if either population growth stagnate and/or the productivity of R&D declines; 3) population growth does not have simple, monotonic relationship with economic growth, after some level of population, product proliferation effects and "stepping-on-toes effects" will result in declining R&D productivity.

The question is, is the future of economic growth bleak, or can AI overturn (or perhaps reinforce) the importance of (1) and (2)?

2.4 How can AI help to Sustain and Even Accelerate Economic Growth?

In light of the somewhat bleak prognosis in the previous sub-sections, the question is: can AI lead to sustained or even *explosive* economic growth?

Several contributions have argued that it is possible, including Aghion et al. (2019), Davidson (2021), Erdil and Besiroglu (2023), Jones (2022b), and Trammell and Korinek (2020). According to Hanson (2024):

> "[The] innovation rate is limited by population, and due to falling fertility, population is going to peak soon, plausibly in about thirty years or so, and then decline [...] if we can make cheap-enough human-level AI before this fall gets too deep, the world economy can continue to grow exponentially, and thus innovate well, with machines replacing the disappearing humans."

Automation may already have helped to sustain economic growth in the face of population ageing, which had been widely expected to put a damper on economic growth. Acemoglu and Restrepo (2017) presents evidence that since the 1990s there has not been a negative relationship between population ageing and economic growth in advanced economies. They ascribe this to the fact that the extent of automation has been faster in countries with older populations.

Jones (2022b) explains how AI may make up for the decline in the generation of new ideas as a result of population growth declines and sustain or even accelerate growth. This could be through 1) the automation of the discovery of new ideas, and 2) through an AGI automating all human labour in production - making labour accumulable (which is highly speculative, as an AGI is still confined to science fiction, and the fears of AI doomsters).

2.4.1 Automating the Discovery of New Ideas

According to Jones (2022b) AI can automate[5] the discovery of new ideas. He gives examples of technologies that have in the past helped expand new ideas: "Microscopes, computers,

[5]Automation refers to "the process of introducing technologies to automatically execute a task previously performed by a human or impossible to perform by a human" (van der Zande et al., 2020, p.33). Gries and Naudé (2022) provides an economic model of AI as a complementing factor in the production functions for goods and services.

DNA sequencing machines, and the Internet itself are examples of automation that enhance the production of ideas" (Jones, 2022b, p.146). Indeed, the ideas expanding potential of AI has been hyped by attention-grabbing headlines, such as that "*A Google AI has discovered 2.2m materials unknown to science*".[6] Another example is in (bio) chemistry, where DeepMind's AlphaFold AI system has been used to predict the 3D structure of proteins (Jumper et al., 2021). It has been called "the most important achievement in AI - ever" (Tunyasuvunakool et al., 2021; McMahon, 2022). In energy, AI models have begun to "identify potential molecules and materials for flow batteries, organic light-emitting diodes, organic photovoltaic cells and carbon dioxide conversion catalysts" (De Luna et al., 2017, p.24). And in astrophysics and cosmology, an AI system has used the information from a single galaxy to infer the structure of the universe (Villaescusa-Navarro et al., 2022). Nordhaus (2021, p.305) refers to this channel of AI-driven economic growth as the "superintelligent technology mechanism." For a broad review of the use of artificial intelligence in scientific discovery, see Wang et al. (2023).

While these examples are impressive and suggests that AI holds great promise in invention, it has been limited so far in that fundamental contributions to theory by an AI has remained elusive. As He and Burtsev (2024, p.241) point out, if one applies the Birch test ("a mathematical analogue of the Turing Test"), "no AI has yet made such a discovery." Thus, at least with AI capabilities at the time of writing, the greatest potential in innovation may lie in AI augmenting human researchers and inventors.

AI automation and augmentation of the innovation process is indeed widely seen as being a potentially valuable tool to help firms innovate. According to Haefner et al. (2021, p.2) "supporting the innovation process with AI could generate real value for firms by reducing both the riskiness and the costliness of innovation processes." They make the case that AI can augment human decision-making in innovation in four areas, namely "(1) developing

[6]See The Economist, 29 November 2023

ideas by overcoming information processing constraints; (2) generating ideas by overcoming information processing constraints; (3) developing ideas by overcoming local search routines; and (4) generating ideas by overcoming local search routines" (Haefner et al., 2021, p.3).

Antons and Breidbach (2018) and Verganti et al. (2020) discusses how AI and big data can help firms perform more effective service innovation and design innovation. According to Verganti et al. (2020, p.212) "AI enables an organization to overcome many past limitations of human-intensive design processes, by improving the scalability of the process, broadening its scope across traditional boundaries, and enhancing its ability to learn and adapt on the fly." Bilgram and Laarmann (2023) use a Large Language Model (ChatGPT) to illustrate how AI can assist firms in the early stages of innovation, in exploration, ideation and digital prototyping. They conclude that "generative AI may be a game changer in early prototyping as the delegation of tasks to an artificial agent can result in faster iterations and reduced costs" (Bilgram and Laarmann, 2023, p.24).

Bouschery et al. (2023) ran an experiment wherein they compare the performance of teams in using brainstorming to generate new ideas, finding that hybrid groups consisting of humans augmented by generative AI (GPT-3), "outperformed nominal and interactive groups in terms of both the total number of ideas generated and the number of creative ideas" (p. 4). Generally, generative AI has the capacity to expand the problem and solution spaces where researchers operate (Bouschery et al., 2023; Piller et al., 2023) which means that LLMs have the ability to improve the amount of research questions that are posed regarding a specific subject and also provide additional plausible answers that were not initially contemplated by a research team that aims at optimizing a particular product feature.

To see the potential effect of the automation of the discovery of new ideas on economic growth, consider the ideas production function in 1 slightly modified to write the "standing on shoulders" parameter ϕ as $\phi = 1 - (\beta - \alpha)$, where β is a measure of the extent to which research productivity is declining (ideas getting harder to find) and α the extent of automation, which refers to the share of capital in (1) as automation is the "development and adoption of new technologies that enable capital to be substituted for labor" (Acemoglu and Restrepo, 2019, p.3):

$$\Delta A_t = (sL_t)^{\lambda} A_t^{1-(\beta-\alpha)} \tag{3}$$

Now, with a population growth rate denoted by n (as $L_t = L_0 e^{nt}$), the growth rate in new ideas production (g_A) is simply (Jones, 2022b, p.146):

$$g_A = \frac{n}{\beta - \alpha} \tag{4}$$

Expression 4 implies that as α increase, for any given population growth rate, the growth rate in ideas (and hence economic growth) will increase. Explosive growth is possible, as when α approaches β expression 4 will approach a mathematical singularity. Jones (2022b) points out that if $\beta = 0,66$ then "only" 66% of research tasks need to be automated for growth to explode. The automation of research tasks by AI counters the declining productivity (β) of research because "compared to humans, AI systems can act faster, absorb more knowledge, and communicate at a far higher bandwidth" (Bengio et al., 2023, p.2).

Aghion et al. (2019) proposes a model with AI affecting the ideas production function (IPF). They use a CES-IPF following the task-approach, by splitting the ideas discovery process into various tasks (X_{it}), of which some may be automated by AI (note that they omit λ, the potential "stepping-on-toes effect"):

$$\Delta A_t = A_t^{\phi}\left(\int_0^{\rho} X_{it}\right)^{\frac{1}{\rho}} \tag{5}$$

With $\rho < 0$. They show that, assuming a CES production function, that if $\phi < 1$ then automation leaves "the long-run growth rate of the economy unchanged" and if $\phi \geq 1$ automation will increase the rate of long-run growth (Aghion et al., 2019, p.251).

Aghion et al. (2019, p.255) discusses two types of growth explosions due to automation : a "Type I" growth explosion, which is when economic growth rates explode (i.e. growth exceeds 30%) but remains finite; and "Type II" growth explosion, which is when the growth rates becomes infinite. These results are subject to the implicit assumption that there is no stepping on toes effect.

2.4.2 Artificial General Intelligence and Accumulable Labor

In addition to the above where AI automate the discovery of new ideas, Jones (2022b) also considers the possibility of a future Artificial General Intelligence (AGI) as a technology that may sustain and even accelerate economic growth. In such a case, because it is an AGI, it can substitute (automate) for all human labour in the production function, leaving only capital. This channel of AI-driven growth is broadly consistent the "capital deepening mechanism" discussed by Nordhaus (2021). In this case the production function would boil down to $Y_t = A_t K_t$ with

$$\Delta A_t = (K_t) A_t^{\phi} \tag{6}$$

Which implies explosive growth as long as A_t is unlimited, which may well be the case given that new ideas consists of combinations of existing ideas. This explosive growth is Type II explosive growth, as it goes to infinity in finite time (Aghion et al., 2019).

What makes economy growth explode in the case of an AGI is that AGI makes labour, which is currently non-accumulable, accumulable - like capital (Aghion et al., 2019). As Erdil and Besiroglu (2023, p.17) put it "Labor is unique in that it's an input that's both a key driver of economic production and growth and cannot be increased by reinvestment of economic output the same way physical, compute, energy etc. production can be. In other words, labor is non-accumulable, while other factors of production that are of comparable importance to labor are accumulable. If we also assume that the cost of producing or maintaining this stock of accumulable labor inputs is not prohibitively expensive, almost all conventional growth models will predict explosive growth in this situation." This is not much different as looking as human capital, an accumulable factor, as input in the IPF. In this case Boikos et al. (2023) showed that an accumulable factor in the IPF could sustain positive economic growth (and possibly innovation also), also in a declining population situation.

At the core of the expectation that AGI will unleash a flood of growth-enhancing new ideas is the belief that AGI represents not just a tool for making existing business models more efficient and competitive, but an innovation in the method of innovation (IMI). It is seen as a General Purpose Technology (GPT) that will alter the "playbook" of innovation (Cockburn et al., 2019). Perhaps "the first ultraintelligent machine is the last invention that man need ever make, provided that the machine is docile enough to tell us how to keep it under control" (Good, 1965, p.33).

It need to be stressed that the possibility of explosive economic growth through AI that turns labour accumulable, can only be entertained under the assumption of an AGI, and not under the rather "narrow" AI that currently exist. Thus, it belongs to the realm of science fiction. Opinions and beliefs as to the eventual likelihood and time of arrival of an AGI widely differs. For instance, Jones (2022b, p.147) considers the likelihood of an AGI to be "quite far-fetched," and Nordhaus (2021, p.300) conclude that a growth "Singularity" from an AGI is "not near." Others seems more optimistic, such as Erdil and Besiroglu (2023, p.18) who believes that "AGI deployment this century seems plausible based on estimates of how much resources would be needed for the creation of an AGI system and how much we can expect effective investment into AI to get scaled up by the end of this century."

2.4.3 Limitations and Constraints on AI in the Ideas Production Function

The aforegoing paragraphs discussed various ways in which AI could help to improve the efficiency and effectiveness of innovation, and hence raise the outputs from the ideas production function. It has however, also to be recognised that AI may not only have positive impacts, but possible could also burden or limit the discovery of new ideas. AI may accentuate a "stepping-on-toes" in research. In this subsection we mention several mechanisms, without attempting to be exhaustive.

One mechanism is through what Ribeiro et al. (2023) terms increasing "mundane knowledge work" which are "practices of checking, sharing and standardising data; and preparing, repairing and supervising laboratory robots." By raising the mundane knowledge work in R&D, they found from a study of bioscience researchers that the introduction of AI takes up more of their time and do not simplify their R&D work processes.

Another way in which AI may limit the production of new ideas is through the winner-take all effects it creates, and the inequalities that it may generate, as a result of the network effects and demand economies of scale characterising the technology. In this regard Torre et al. (2020, p.123) finds that the diffusion and implementation of AI on a firm level for innovation "varies greatly across organizations" and that this generates a "performance gap between AI pioneers that appear to be pulling further away from organizations that are still lagging behind." And van Roy et al. (2020, p.33) concludes from studying a global set of 155,000 AI patents that there is a "very high concentration of AI intellectual property rights to a few countries and organizations at a time when the technology is still in its emerging phase." Igna and Venturini (2023) also finds from analysing data on 20,192 firms that applied for 219,835 patents in digital technologies, that firms who are leading in innovating in AI tend to be those who have prior experience in ICT and that there seem to be strong learning effects, which would suggest winner-take-all effects and best-vs-the rest dynamics in the use of cutting edge AI.

Yet a further way in which AI may limit the production of new ideas is through resulting in unreliable, wrong and unverifiable research outputs (Krämer, 2023). Ball (2023) discusses in this regard the danger that naive use of AI in research is threatening a reproducible crisis, and refers to the problem of data leakage, where data used to train the AI is also present in data used to test the AI, as being very frequent.

Finally, AI may produce new ideas that are dangerous, poses informational hazards, and that may pose an existential risk (for example in facilitating the spread of the formulas for making deadly new pathogens). Jones (2023) explores the risk-return trade-offs between an AI that may promise high growth (e.g. 10%) but poses an existential risk (e.g. of 2%). He concludes that although the form of the utility function and assumed rates of risk aversion and the value of a life affect the results, it can be shown that "AI is too risky to make even 10% growth worthwhile" and that "even singularities that deliver infinite consumption immediately are not as valuable as one might have thought" in the presence of existential risk.

3 Modelling AI in Innovation-Driven Growth

In the IPF literature reviewed in the previous sections, AI is generally incorporated as a tool that enables higher efficiency in the production of knowledge but with no *law of motion* associated with it. In this section we contribute and explore how AI can augment the IPF and raise R&D productivity by addressing this lacuna. We propose and discuss three alternative models for the IPF incorporating AI in the research process as well as its implications in terms of steady-state growth rates of technology and transitional dynamics. With these models in mind, we are able to understand, for example, which configuration allows AI to generate steady and positive knowledge creation, or how reasonable is to expect explosive economic growth from AI.

3.1 AI as researcher scale enhancing technology

Let us consider the traditional *law of motion* for new ideas à la Jones (1995):

$$\frac{\dot{A}_L}{A_L} = R^{\lambda} A_L^{-\alpha} \tag{7}$$

Lambda (λ) regulates the returns to scale of researchers in the production of general knowledge (A_L). Since this parameter is strictly positive, we could have either decreasing ($\lambda < 1$), constant ($\lambda = 1$), or increasing ($\lambda > 1$) returns to scale to researchers. The parameter α regulates how hard it is to find new ideas given the stock of knowledge that exists in the economy.

AI may enhance the returns to scale of researchers by allowing them to do some sort of parallel simulation of ideas, and increase the rate at which "dead ends" are excluded from the pipeline. This is broadly similar to the prediction function which Agrawal et al. (2023) models for AI in innovation. If we assume that AI are algorithms that are applied to a particular database to be functional, we can establish a relationship between λ and the order of magnitude of AI output:

$$\lambda = \psi \ln(A_{IT}) \tag{8}$$

Where ψ governs the strength of AI algorithms in facilitating the researchers' productivity in finding new ideas. Hence (7) can be reformulated as:

$$\frac{\dot{A}_L}{A_L} = e^{\lambda \ln R} A_L^{-\alpha} \tag{9}$$

and consequently,

$$\frac{\dot{A}_L}{A_L} = e^{\psi \ln(A_{AI}) \ln R} A_L^{-\alpha} \tag{10}$$

Defining $\frac{\dot{A}_L}{A_L} = g_{AL}$:

$$g_{AL} = e^{\psi \ln(A_{AI}) \ln R} A_(L)^{-\alpha} \tag{11}$$

The rate of knowledge creation over time becomes clear by taking logarithms

$$\ln g_{A_L} = \psi \ln(A_{A_{IT}}) \ln R - \alpha \ln A_{A_L} \tag{12}$$

and its derivative with respect to time:

$$\frac{\dot{g}_{A_L}}{g_{A_L}} = \psi[\ln(A_{_{IT}})\frac{\dot{R}}{R} + \ln R\dot{g}_{A_{IT}}] - \alpha g_{A_L} \tag{13}$$

If all the parameters in (13) are positive, then with positive population growth the growth in the knowledge stock approaches infinity in the limit. Keeping in mind that the terms $\ln(A_{A_{IT}})$ and $\ln R$ are also growing, and assuming a normal exponential behavior for each of those processes, results in:

$$\frac{\dot{g}_{A_L}}{g_{A_L}} = \psi[(A_{IT})_0 + g_{A_{IT}}t]g_r + [R_0 + g_r t]g_{A_{IT}} - \alpha g_{A_L} \tag{14}$$

On the other hand, if we assume a steady decline in population, where $g_r < 0$, and a positive and constant growth rate for $A_I T$, we obtain the complete opposite result, where the growth rate of innovation is collapsing - towards a negative infinity in the limit.

In the intermediate case ($g_r = 0$), in which population is static, we can determine some steady and finite value for g_{A_L} as a function of $g_{A_{IT}}$.

In the case where we have a static population, the expression becomes:

$$\frac{\dot{g}_{A_L}}{g_{A_L}} = \psi R_0 g_{A_L} - \alpha g_{A_{IT}} \tag{15}$$

In this case the model has a steady-state solution given by :

$$g_{A_L} = \frac{\alpha g_{A_{IT}}}{\psi R_0} \tag{16}$$

This expression indicates that a steady state for knowledge creation is possible whenever there is a static population and the growth rate of traditional knowledge is a specific multiple of the exogenous growth rate of AI knowledge production.

In sum the model presented in this sub-section, in which productivity of researchers are affected by AI by the elimination of "dead ends" in the innovation process, shows that AI can at most determine how far the economy is from a growth explosion or stagnation. These outcomes are only dependent on the growth rate of population (tied to the growth rate of researchers g_r) and a steady state is only possible when population is static and the growth rate of traditional knowledge comes out to be a multiple of the exogenous growth rate of AI knowledge. In that case, for each level of the growth rate of AI knowledge, we have a steady-state growth rate for traditional knowledge.

By implying that growth outcomes hinges on the dynamics of AI knowledge, these results suggest the the need to model AI innovations explicitly in the IPF.

3.2 AI as a Research-Augmenting Technology

We now introduce the possibility of having a system of IPFs in which one IPF governs the evolution of AI (A_{IT}) and the other governs traditional knowledge creation which typically augments labour in the production of goods and services (A_L).

The assumption is that both types of ideas are essential for economic growth. The laws of motion describing how ideas are created in each of the IPFs (or research sectors) can be stated as:

$$\frac{\dot{A}_L}{A_L} = (A_{IT} R_1)^{\lambda} A_L^{-\beta} \tag{17}$$

$$\frac{\dot{A}_{IT}}{A_{IT}} = (A_L R_2)^{\gamma} A_{IT}^{-\theta} \tag{18}$$

In equations (17) and (18), the parameters λ and γ are related to the returns to scale of researchers in the production ideas and capture "stepping-on-toes" effects. At the same time, the parameters β and θ are related to whether ideas are getting easier or harder to find, with higher values related to more difficulty in finding new ideas, and R_1 and R_2 reflecting the number of researchers in each industry.

Taking the logarithm of expressions (17) and (18):

$$\ln\left(\frac{\dot{A}_L}{A_L}\right) = \lambda(\ln A_{IT} + \ln R_1) - \beta \ln A_L \tag{19}$$

$$\ln\left(\frac{\dot{A}_{IT}}{A_{IT}}\right) = \gamma(\ln A_L + \ln R_2) - \theta \ln A_{IT} \tag{20}$$

And differentiating them with respect to time, we have:

$$\frac{\dot{g}_{A_L}}{g_{A_L}} = \lambda\left(g_{A_{IT}} + \frac{\dot{R}_1}{R_1}\right) - \beta g_{A_L} \tag{21}$$

$$\frac{\dot{g}_{A_{IT}}}{g_{A_{IT}}} = -\theta g_{A_{IT}} + \gamma \left(g_{A_L} + \frac{\dot{R}_2}{R_2} \right) \tag{22}$$

Now, with the system expressed in terms of the growth rate of ideas, we can solve the model for the equilibrium flow of ideas produced in each industry. We do that by setting the former expressions to zero and generating the associated null-spaces (isoclines):

$$g_{A_L} = \frac{\lambda}{\beta} \left(g_{A_{IT}} + \frac{\dot{R}_1}{R_1} \right) \tag{23}$$

$$g_{A_{IT}} = \frac{\gamma}{\theta} \left(g_{A_L} + \frac{\dot{R}_2}{R_2} \right) \tag{24}$$

The isoclines for both $g_{A_{IT}}$ and g_{A_L} form a linear system of equations. If they are linearly independent, we are able to find only one equilibrium vector, the stead-state of the system:

$$g^*_{A_L} = \left(1 - \frac{\lambda}{\beta}\frac{\gamma}{\theta} \right)^{-1} \frac{\lambda}{\beta} \left[\frac{\gamma}{\theta} \left(\frac{\dot{R}_2}{R_2} \right) + \left(\frac{\dot{R}_1}{R_1} \right) \right] \tag{25}$$

$$g^*_{A_{IT}} = \frac{\gamma}{\theta} \left[\frac{\lambda}{\beta}\frac{\gamma}{\theta} \left(1 - \frac{\lambda}{\beta}\frac{\gamma}{\theta} \right)^{-1} + 1 \right] \left(\frac{\dot{R}_2}{R_2} \right) + \frac{\gamma}{\theta}\frac{\lambda}{\beta} \left(1 - \frac{\lambda}{\beta}\frac{\gamma}{\theta} \right)^{-1} \left(\frac{\dot{R}_1}{R_1} \right) \tag{26}$$

Now that we have an equilibrium solution, we can analyze the dynamics of the system. We do that by analyzing the associated homogeneous system of differential equations:

$$\begin{bmatrix} \frac{\dot{g}_{A_L}}{g_{A_L}} \\ \frac{\dot{g}_{A_{IT}}}{g_{A_{IT}}} \end{bmatrix} = \begin{bmatrix} -\beta g^*_{A_L} & \lambda g^*_{A_{IT}} \\ \gamma g^*_{A_L} & -\theta g^*_{A_{IT}} \end{bmatrix} \begin{bmatrix} \ln g_{A_L} - \ln g^*_{A} \\ \ln g_{A_{IT}} - \ln g^*_{A_{IT}} \end{bmatrix}$$

To understand the dynamics of this system, we first calculate the structure of the characteristic equation that follows from the Jacobian matrix of the log-linearized system:

$$\left| \begin{bmatrix} -\beta g^*_{A_L} - \lambda^e & \lambda g^*_{A_{IT}} \\ \gamma g^*_{A_L} & -\theta g^*_{A_{IT}} - \lambda^e \end{bmatrix} \right| = 0$$

$$\lambda^e = \frac{-(\beta g^*_{A_L} + \theta g^*_{A_{IT}}) \pm \sqrt{(\beta g^*_{A_L} + \theta g^*_{A_{IT}})^2 - 4(\beta\theta g^*_{A_L} g^*_{A_{IT}} - \gamma\lambda g^*_{A_L} g^*_{A_{IT}})}}{2}$$

The system converges to a steady-state independently from initial conditions (a *sink*) whenever these the following conditions hold simultaneously:

1. $(\beta g^*_{A_L} + \theta g^*_{A_{IT}})^2 - 4(\beta\theta g^*_{A_L} g^*_{A_{IT}} - \gamma\lambda g^*_{A_L} g^*_{A_{IT}}) > 0$

2. $(\beta g^*_{A_L} + \theta g^*_{A_{IT}}) > \sqrt{(\beta g^*_{A_L} + \theta g^*_{A_{IT}})^2 - 4(\beta\theta g^*_{A_L} g^*_{A_{IT}} - \gamma\lambda g^*_{A_L} g^*_{A_{IT}})}$

These two conditions guarantee that the system has two negative eigenvalues.

The system converges to a steady-state only under the condition that it starts on a stable arm (saddle-path stability) whenever the following conditions hold:

1. $(\beta g^*_{A_L} + \theta g^*_{A_{IT}})^2 - 4(\beta\theta g^*_{A_L} g^*_{A_{IT}} - \gamma\lambda g^*_{A_L} g^*_{A_{IT}}) > 0$

2. $(\beta g^*_{A_L} + \theta g^*_{A_{IT}}) < \sqrt{(\beta g^*_{A_L} + \theta g^*_{A_{IT}})^2 - 4(\beta\theta g^*_{A_L} g^*_{A_{IT}} - \gamma\lambda g^*_{A_L} g^*_{A_{IT}})}$

These two conditions make sure that we have one positive and one negative eigenvalue. For this reason, the system only converges to the steady-state equilibrium if the initial condition, i.e. a pair of growth rates for knowledge, is an element of the eigenspace related to the negative eigenvalue. Whenever the initial condition is out of the eigenspace, i.e. outside the stable arm, the system diverges as the positive eigenvalue dominates when time goes to infinity.

In all other cases, we have a *source*, which means that the system always diverges independently from where it starts in the phase space. Another test that can be more readily conducted to determine whether the system is saddle-path stable or not is to analyze the sign of the determinant. If the determinant of the Jacobian matrix that characterizes the system is positive, the system could be a sink or a source; if the determinant is negative, it is always saddle-path stable.

In our context, this condition can be stated mathematically in the following way:

1. *Sink or source*

$$\frac{\beta}{\lambda} > \frac{\gamma}{\theta} \implies \beta\theta > \lambda\gamma \tag{27}$$

If this is the case, the characteristic equation has to be solved in order to determine the sign of the eigenvalues. But, if this condition is met, the system will be a sink whenever both $\beta < 0$ and $\theta < 0$.

2. *Saddle-path stability*

$$\frac{\beta}{\lambda} < \frac{\gamma}{\theta} \implies \beta\theta < \lambda\gamma \tag{28}$$

Under these conditions we can now simulate the transitional dynamics and steady-state equilibrium values for the growth rate of ideas production in each ideas creating sector for what we consider interesting scenarios. In sub-section 3.2.1 below we calibrate based on USA data.

3.2.1 Calibration and quantitative results

The model developed so far is semi-endogenous in nature and, as we can see from equations (25) and (26), the growth rates of knowledge in both sectors hinge on the growth rates of researchers, which in the long-run has to converge to the population growth rate.

Scenario 1. We simulate an economy where traditional ideas are harder to find than new algorithms (parameters β and θ), taking the parameters values from Bloom et al. (2020) for the US economy. For traditional ideas, we use the estimates for the economy as a whole and for new algorithms the estimations regarding the semiconductor industry. The returns to scale of researchers are assumed to be equal across knowledge producing industries and below one and the population growth is positive (in line with the current trend).

Given the values:

$$\beta = 3.1, \quad \theta = 0.4, \quad \lambda = 0.8, \quad \gamma = 0.8, \quad \frac{\dot{R}_1}{R_1} = 0.01, \quad \frac{\dot{R}_2}{R_2} = 0.01$$

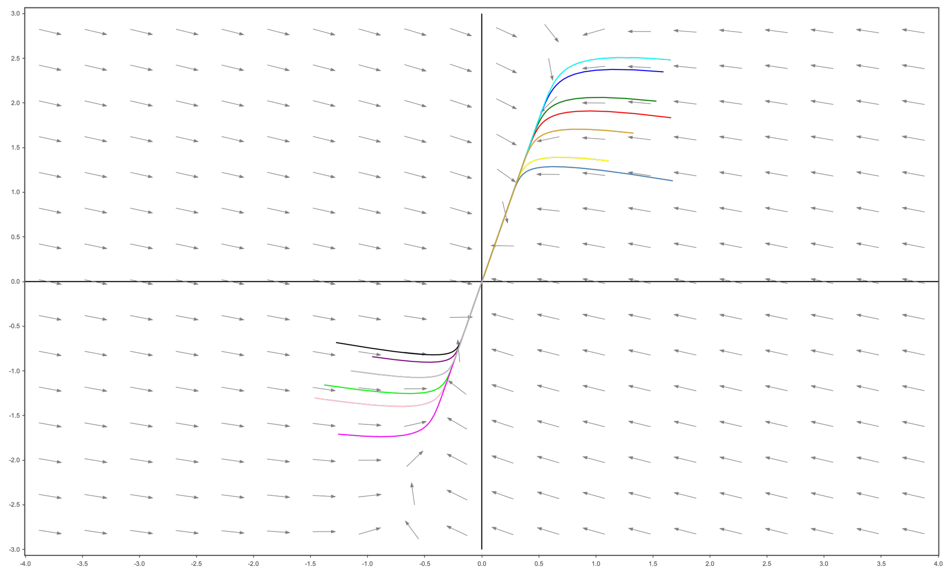
Checking the condition for the determinant of the Jacobian matrix:

$$\beta\theta > \lambda\gamma \quad \Rightarrow \quad 3.1 \times 0.4 > 0.8 \times 0.8$$

It is possible to observe from the vector field that this parameterization generates a *sink*, which means that no matter where we start in the phase space, the system always converges to the steady-state equilibrium. In this case, the rate of knowledge creation in the traditional sector is 1.6% p.a. and the rate of growth of new algorithms is 5.2% p.a.

In this scenario, we can see that the knowledge equilibrium growth rate in the AI industry is higher than in the traditional sector. This is due to the fact that the stock of knowledge does not impose as much of a drag in the creation of new ideas in the AI sector as it does in

Figure 1: Scenario 1



Source: Authors' compilation

the traditional sector.

Scenario 2. In this second scenario, we simulate an economy where traditional ideas are still harder to find than new algorithms, the returns to scale of researchers in both knowledge creating sectors are below one but population growth is negative:

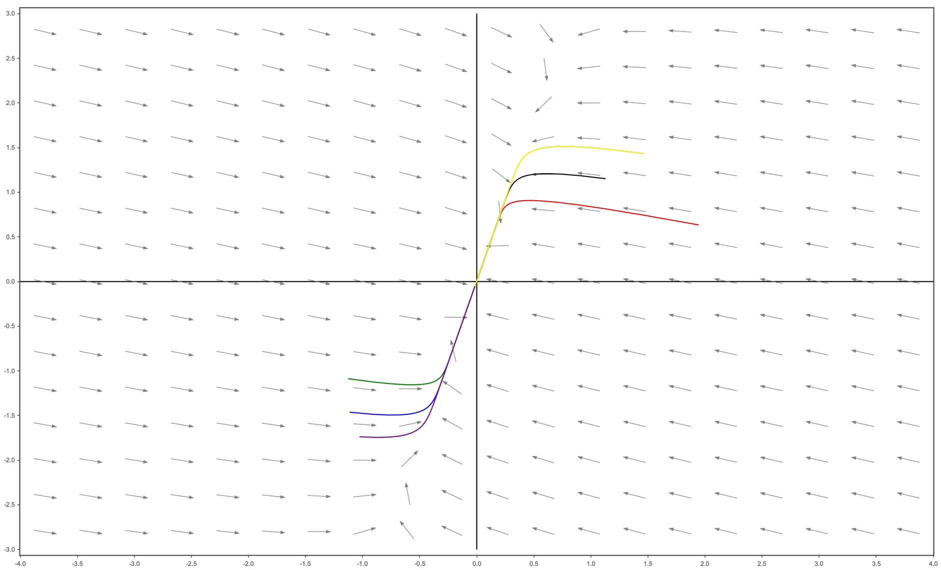
Given the values:

$$\beta = 3.1, \quad \theta = 0.4, \quad \lambda = 0.8, \quad \gamma = 0.8, \quad \frac{\dot{R}_1}{R_1} = -0.01, \quad \frac{\dot{R}_2}{R_2} = -0.01$$

In this case, the vector field is exactly the same as before but the steady-state equilibrium for the rate of knowledge creation in the traditional sector is -1.6% p.a. and the rate of growth of new algorithms is -5.2% p.a., i.e. exactly the opposite of scenario 1.

From the illustrative results above we can conclude that population dynamics, even in the presence of AI as a research-augmenting technology, continues to play a central role in determining the sign and intensity of the growth rate of knowledge creation.

Figure 2: Scenario 2



Source: Authors' compilation

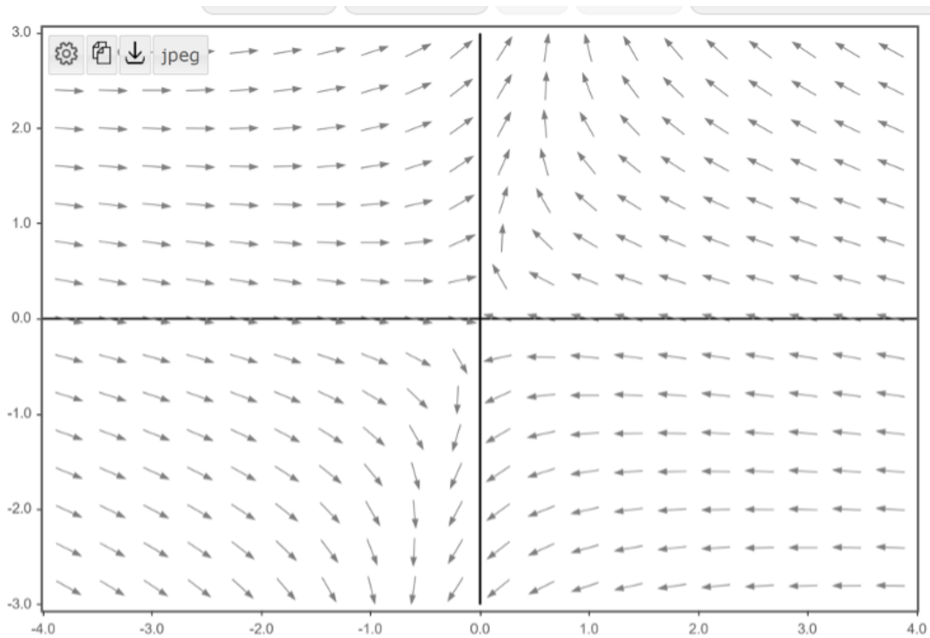
Scenario 3. In third scenario, we simulate an economy where traditional ideas are harder to find and algorithm production benefits from the state of total algorithm knowledge, the returns to scale of researchers in both knowledge creating sectors are below one but population growth is negative:

Given the values:

$$\beta = 3.1, \quad \theta = -0.4, \quad \lambda = 0.8, \quad \gamma = 0.8, \quad \frac{\dot{R}_1}{R_1} = -0.01, \quad \frac{\dot{R}_2}{R_2} = -0.01$$

In this case, what the simulation and the analysis of the Jacobian matrix show is that we have saddle-path stability with one negative and one positive eigenvalue.The convergence of the system towards a steady-state value relies on it starting at the eigenspace associated with the negative eigenvalue, i.e. a very particular situation that would only happen by chance in the absence of a control variable. If the initial conditions of the system are out of this stable-arm, then as time goes by, the system diverges from the long-run equilibrium, meaning an explosive behavior even in the presence of negative population growth.

Figure 3: Saddle-path stability



Source: Authors' compilation

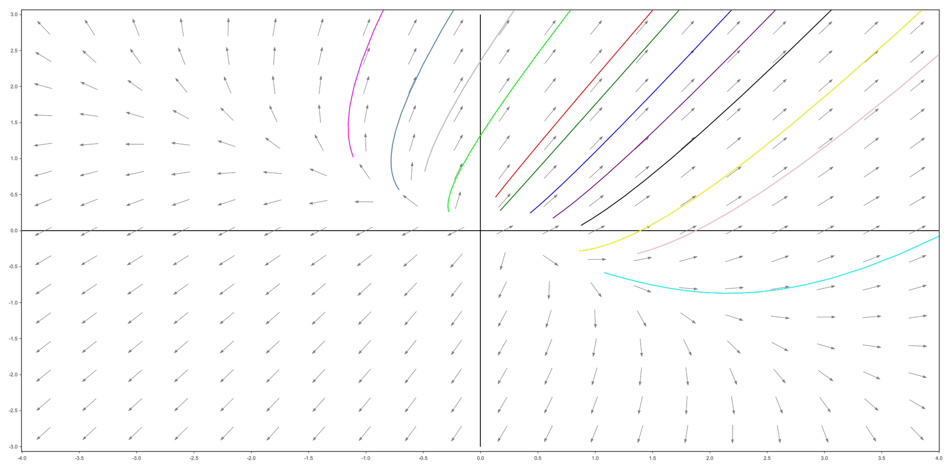
Scenario 4. In this fourth scenario, we simulate an economy where both traditional ideas and algorithm production benefit from the stock of knowledge, i.e. there are positive spillover effects, so the parameters β and θ are negative. We start with the case where the returns to scale of researchers in both knowledge creating sectors are still below one and population growth is positive:

Given the values:

$$\beta = -1, \quad \theta = -2, \quad \lambda = 0.8, \quad \gamma = 0.8, \quad \frac{\dot{R}_1}{R_1} = 0.01, \quad \frac{\dot{R}_2}{R_2} = 0.01$$

The fact that the spillover effect measured by the pair (-β,-θ) is now positive creates an explosive dynamics for the growth rate of ideas creation in both sectors. The steady-state is the same as in Scenario 1, but the because the system is now a source, it never converges to equilibrium.

Figure 4: Scenario 4



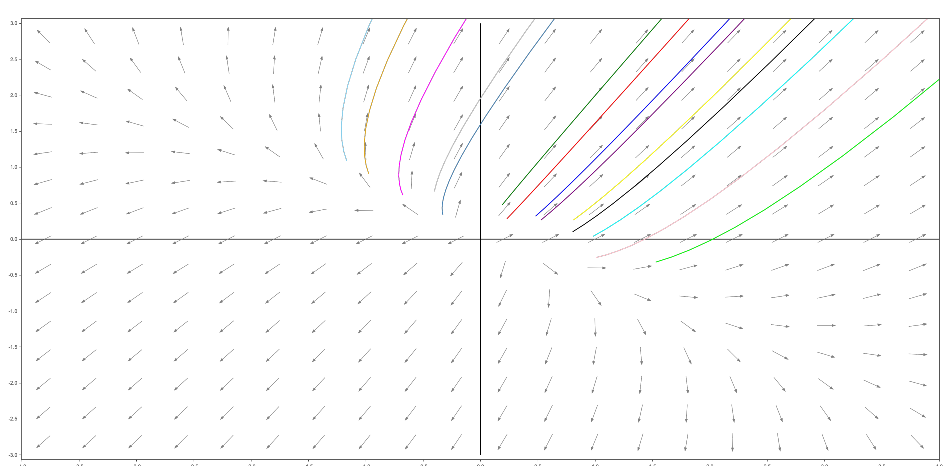
Source: Authors' compilation

Scenario 5. In this fifth scenario, we simulate an economy where both traditional ideas and algorithm production benefit from the stock of knowledge, but now we assume a negative population growth rate

Given the values:

$$\beta = -1, \quad \theta = -2, \quad \lambda = 0.8, \quad \gamma = 0.8, \quad \frac{\dot{R}_1}{R_1} = -0.01, \quad \frac{\dot{R}_2}{R_2} = -0.01$$

Figure 5: Scenario 4



Source: Authors' compilation

Even though the growth rate of population is negative, the explosive dynamics is the same as in Scenario 4.

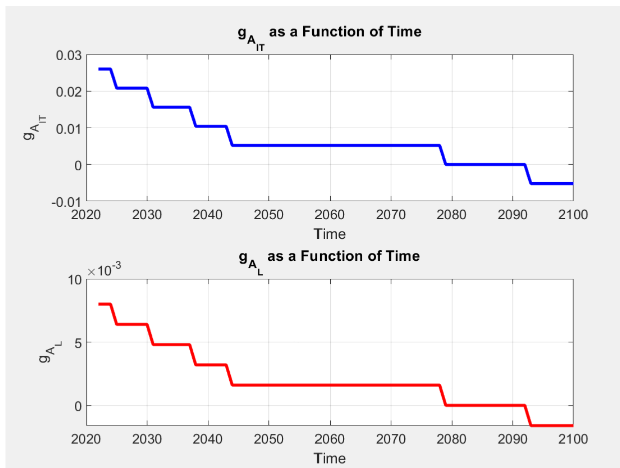
The results obtained thus far shows that, for reasonable values for the parameters, population dynamics is a fundamental element that determine the sign and partially the intensity of the steady-state value for the growth rates of ideas production. In fact, the parameters β and θ, which are related to the "standing-on-shoulders effect" are key to determine the type of dynamics the system will display.

Given that population dynamics is a central determinant of whether aggregate knowledge increases or decreases whenever the model converges to a steady-state over time, it is important to highlight how demography may develop in a context of automation. Almeida and Sequeira (2023) show that the optimal fertility rate can either increase or decrease depending on how much the relative price of labor intensive versus capital intensive tasks increase following an automation shock. If the increase in relative prices is substantial, a version of Baumol's cost disease argument, the households are discouraged to have additional children which may lead to a declining population in the future. Costanzo (2022) develops a model that links automation and fertility and concludes that further automation postpones decisions of having children.

If we join the prospects of fertility dynamics (i.e. population growth) in a context where ideas are getting harder to find, the result is a steady-state equilibrium where the growth rate of ideas is negative. The only parameters that, in the presence of negative population growth, could avert a decline in the production of ideas are the elasticities of knowledge production with respect to the stock of ideas. We have seen from the model, that both parameters β and θ govern the dynamics of the system, meaning that AI would only counterbalance a population decrease by making ideas easier to find, which is the case of scenarios 3 and 4, where at least one of the elasticities of knowledge production with respect to the stock of ideas is positive.

Next we use the estimates provided by Bloom et al. (2020) to calibrate for the elasticity of knowledge production with respect to the stock of ideas and the projection for population growth rates made by the U.S. Census until the end of this century. We use the steady-state equations 25 and 26 to calculate the equilibrium growth rates of both traditional knowledge and AI algorithms.

Figure 6: Long-run knowledge growth rates for the U.S. economy in the steady-state

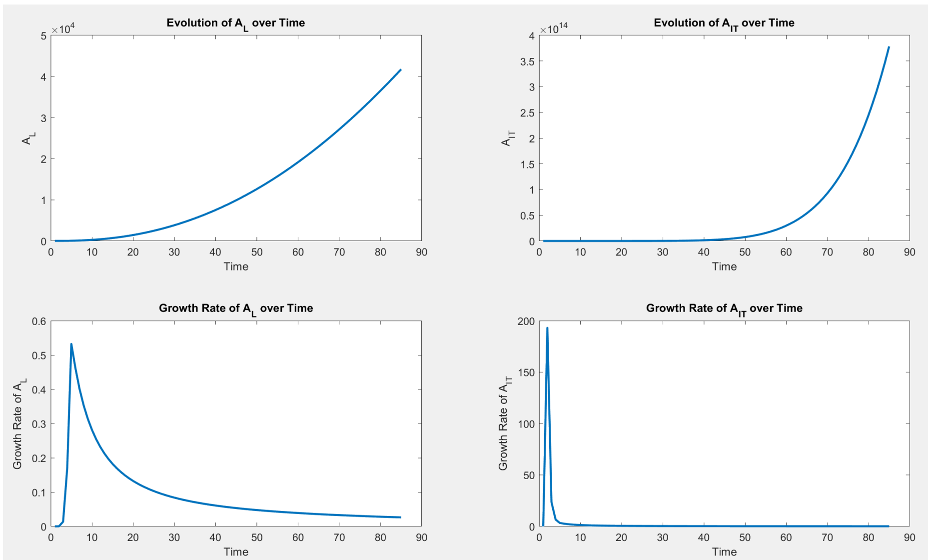


Source: Authors' compilation. *Parameters*: β=3.1; θ=0.4;λ=γ=0.8.

The growth trajectories of AI and traditional knowledge in Figure 6 show progressive deceleration as the projected population growth rate for the U.S. declines from 0.5% to -1.0% over the course of the century. From the point of view of AI knowledge, the growth rate would be expected to decline from close to 3.0% p.a. to -0.5%, while in the case of traditional knowledge, the growth rate would be expected to fall from 0.8% to something close to -0.1% over the course of the same period.

The estimations above (Figure 6) use the long-run equilibrium conditions of the model to provide values for the growth rates of knowledge in each period. But being those long-run estimates, we are not taking into consideration the transition path to those equilibrium values. Below, using the same values for the parameters, we calculate the growth rates of knowledge creation using the transition equations (17) and (18) instead of the long-run equilibrium conditions. We use as an initial condition for the ratio of the stock of knowledge in the traditional sector to the AI sector of 32.1/0.81, which values are from Igna and Venturini (2023).

Figure 7: Transition knowledge growth rates for the U.S. economy



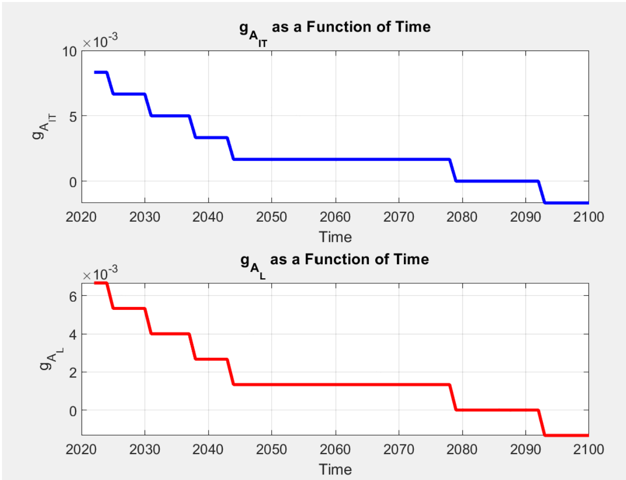
Source: Authors' compilation. *Parameters*: β=3.1; θ=0.4;λ=γ=0.8.

What we observe is that, after a period of rapid expansion from their respective initial condition (much stronger for AI when compared to traditional technology sector), both knowledge growth rates decelerate. By the end of the period, knowledge growth rate for the traditional sector is expected to reach 2.6% and for the AI sector 9.0%.

We also conduct these simulations using a different set of parameters to describe the "stepping-on-toes" and "standing-on-shoulders" effects from Neves and Sequeira (2018) and Igna and Venturini (2023) instead of those in Bloom et al. (2020). Again we use equations (25) and (26) to calculate the steady-state growth rates of knowledge production in each sector as the population growth rates estimates from the U.S. Census changes.

With the new set of parameters, we observe that the general pattern is maintained for the long-run equilibrium, with knowledge growth rates decreasing in both sectors as population growth declines. In the AI sector, growth rates fall from 0.8% in the beginning of the time horizon to -0.2%, while in the traditional industry, knowledge growth rate decreases from

Figure 8: Long-run knowledge growth rates for the U.S. economy in the steady-state

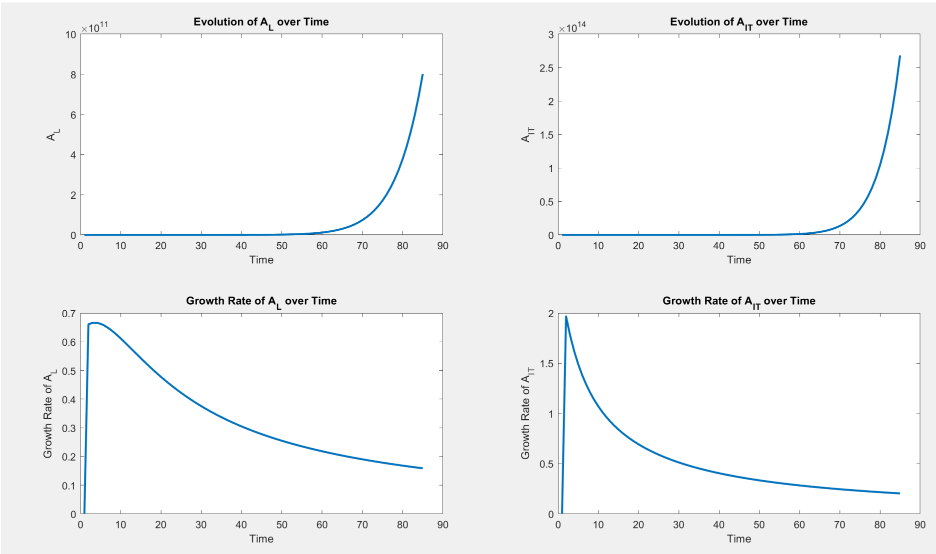


Source: Authors' compilation. *Parameters*: β=0.2; θ=0.14;λ=γ=0.1.

0.7% to -0.1% at the end of the century.

As before, we can use the transition equations (17) and (18) with the new set of parameters to calculate the growth rates of knowledge for each sector while outside the steady-state. Figure 9 shows the time-path for the two states, A_{IT} and traditional knowledge A_L, as well as growth rates.

Figure 9: Transition knowledge growth rates for the U.S. economy



Source: Authors' compilation. *Parameters*: β=0.2; θ=0.14;=γ=0.1.

The upshot from the above simulations with different parameters and dynamic equations is that *AI alone may not be sufficient to accelerate the growth rate of ideas production indefinitely.* In all reasonable variations of the model, we observe that the growth rate of ideas follows closely the population dynamics in the long-run, even though there is an overshoot in the short-run. Both Figures 7 and 9 depicts explosive growth in AI technology, which then rapidly declines. When there are fishing-out effects in the traditional ideas production function and positive spillovers in the AI ideas production function (as in Figure 7) the overshooting is substantial, but of short duration - around 10 years. When there are positive spillovers in both ideas production functions (as in Figure 9) the overshooting is smaller, but of longer duration. The simulations indicate that the standing-on-the-shoulders effect in the traditional IPF has a crucial role in the transitional dynamics.

3.3 AI used to improve the use of previous knowledge

In this sub-section we propose an ideas production function in which general technologies benefit more from past research, the higher the relative artificial intelligence stock available. This relies on the idea that AI technologies help to combine, summarize and organize previous existing knowledge that can be used and combined to generate new ideas. For instance Agrawal et al. (2018) and Agrawal et al. (2023) argues that AI improves the possibilities of combinations of previous technologies that enhance the productivity of innovations. They argue that AI dramatically improves prediction accuracy about which combinations have the highest potential.

We adapt their idea and propose the following IPF:

$$\frac{\dot{A}_L}{A_L} = \delta(R_1)^{\lambda} A_L^{ln(c+\frac{A_{IT}}{A_L})-1} \tag{29}$$

$$\frac{\dot{A}_{IT}}{A_{IT}} = \omega(R_2)^{\gamma} A_{IT}^{-\theta} \quad (30)$$

where δ and ω are productivity parameters and variables. Note that the exponent $ln(c + \frac{A_{IT}}{A_L}) - 1$ in equation (29) means that the standing-on-shoulders effect depends positively on the ratio between the AI stock and the general technology stock. Intuitively this means that AI facilitates the use of previous technology knowledge in finding new ideas of the general type. Theoretically the implication is that as AI technologies accumulate we will encounter first the fishing-out of ideas; pass through a semi-endogenous stage characterised by positive spillovers; and eventually end up in a fully endogenous setting in the general technology IPF with possible scale effects.[7]

It cannot be concluded *a priori* which regime will dominate in the steady-state as this depend on the particular value of the $(\frac{A_{IT}}{A_L})$ and of the arbitrary constant c. However, as in the steady-state, both growth rates should equalize, the stationary variable is the above mentioned ratio $(\frac{A_{IT}}{A_L})$.

In what follows we will characterize the steady-state and its features. As we did previously we will also perform simulations using USA data.

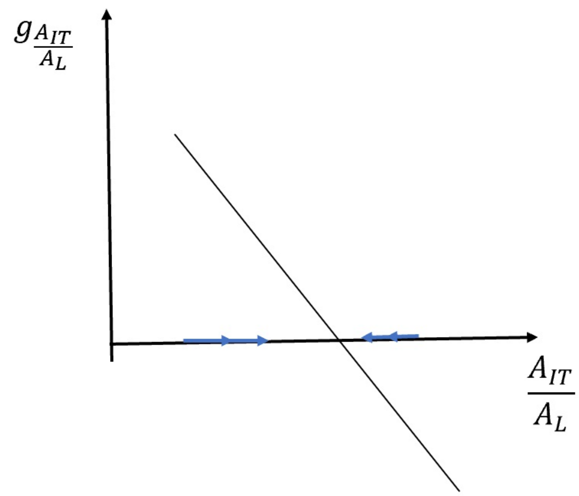
In order to draw the phase diagram we define $F((\frac{A_L}{A_{IT}}), A_{IT}, A_L)$ as (29)-(30). Without loss of generality we set $c = 0$ and take the derivative $F(.)$ in order to $u = \frac{A_{IT}}{A_L})$ using the chain rule. This can be shown to result in:

$$\partial F/\partial(\frac{A_{IT}}{A_L})) = \partial F/\partial u.(1/A_L) + \partial F/\partial u.(-A_{IT}/A_L^2)^2 \quad (31)$$

[7] While the first two models are semi-endogenous in its nature (à lá Jones (1995)), this one can present constant or increasing returns to scale in researchers or population (à lá Romer (1990)) in the particular situation in which $ln(c + \frac{A_{IT}}{A_L}) >= 1$. The partial or total elimination of scale effects can be introduced in the model using product proliferation (see a discussion e.g. in Sequeira et al. (2018)). The possibility that AI facilitates more the vertical innovation or horizontal innovations – even considering some possibility of harmful innovations (see e.g. Jones (2023)) is left for future research.

Since the two stocks of technologies must be strictly positive the sign of $(\partial F/\partial(\frac{A_{IT}}{A_L}))) = (\partial F/\partial u)$ is < 0 implying that the isocline is negative, as depicted in Figure (10). In this case

Figure 10: Phase Diagram



Source: Authors' compilation

there will *not* be explosive growth due to AI. In the case of a steady state with increasing returns (Romer-type) and scale effects, population growth do cause explosive growth, but this is not driven by AI.

The steady-state expression of the ratio of technologies, in the case the model end in a semi-endogenous steady state, is as follows:

$$g_A = \frac{\lambda n}{1 - (A_{IT_0} - A_{L_0})}, \tag{32}$$

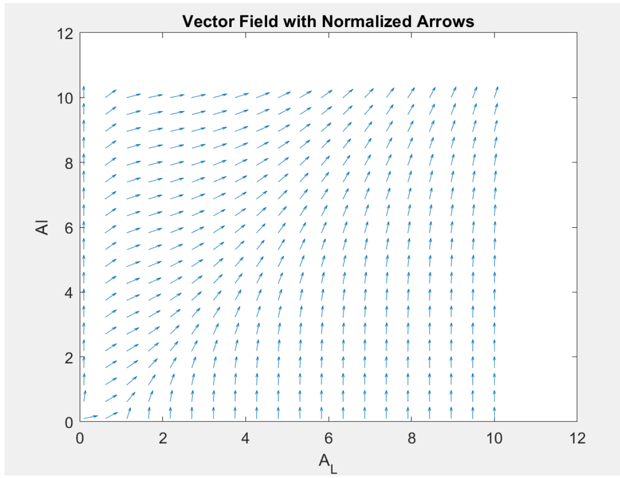
which is straightforward to see that is strictly positive since $A_{IT_0} < A_{L_0}$.

3.3.1 Calibration and quantitative results

In order to understand how the system behaves over time and compare this to the results obtained in sub-section 3.2 above, we assume the following values for the parameters, obtained from Igna and Venturini (2023) and Sequeira and Neves (2020): λ=γ=0.1; θ=0.14.

For simplicity δ and ω=1 and n=0.01.

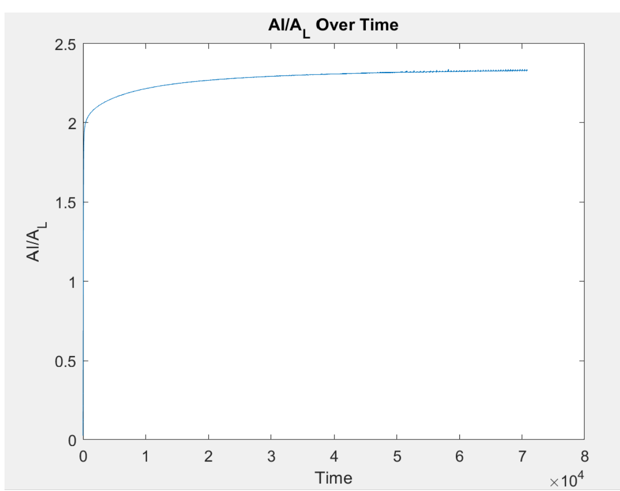
Figure 11: Vector Field in the AI-A$_L$*space*



Source: Authors' compilation *Parameters*: θ=0.14;λ=γ=0.1.

The system produces the above vector field in the A_L/A_{IT} space, which shows that starting from any point in the state space, the stock of knowledge in the traditional and AI sectors grow unevenly until they converge to one the eigenspaces and continues growing at the same rate. Along that explosive eigenspace, the behaviour of the ratio A_L/A_{IT} is stable and we have both A_L and A_{IT} (AI) growing at a constant and equal rate in equilibrium (see Figure 12), as we also may expect from Figure 10.

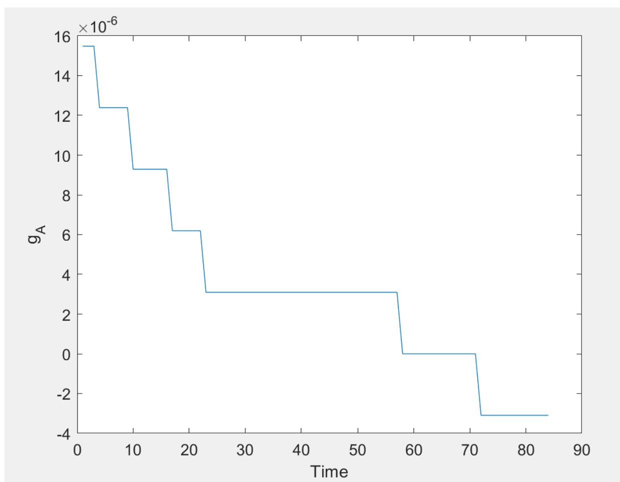
Figure 12: Dynamic behavior of the ratio AI/A$_L$



Source: Authors' compilation. *Parameters*: θ=0.14;λ=γ=0.1.

If we, as we did in sub-section 3.2, apply the ratio A_L/A_{IT}=32.1/0.81 obtained from Igna and Venturini (2023), we are able to generate steady-state values for the growth rate knowledge in the traditional sector of the economy with equation 32. Because in a steady-state equilibrium, the ratio A_L/A_{IT} is set to be constant, we can take the growth rate of ideas production in the traditional IPF as being equal to the growth rate of ideas in the AI IPF. We perform this simulation incorporating the estimates of U.S. Census for population growth throughout this century (see Figure 13).

Figure 13: Long-run knowledge growth rates for the U.S. economy in the steady-state



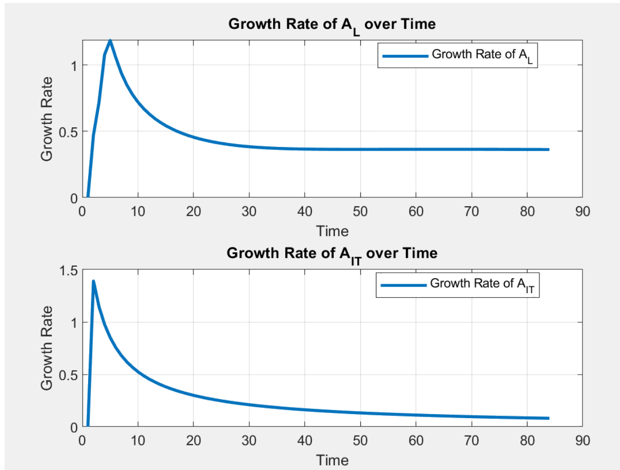
Source: Authors' compilation. *Parameters*: θ=0.14; λ=γ=0.1.

According to Figure 13 we the unique steady-state new ideas growth rate starts close to 15% and decrease to a negative growth rate of nearly -3%. These changes are all due to the projected evolution of population, as equation (32) anticipates.

Using now the system of differential equations (29) and (30), instead of the steady-state condition (32), we can calculate the growth rates of ideas production in both IPFs as population changes according to the U.S. Census. The results are shown in Figure 14.

This transitional dynamics behaviour of both technologies also shows the overshooting behaviour which is now more similar between the two types of IPFs. It implies that the possibility of explosive growth due to AI may occur only in the very short-run in the case

Figure 14: Transition knowledge growth rates for the U.S. economy



Source: Authors' compilation. *Parameters*: θ=0.14;λ=γ=0.1.

that AI idea-generation overshoots its long-run growth.

4 Concluding Remarks

Theory predicts that global economic growth will stagnate and even come to an end due to slower and eventually negative growth in population. It has been claimed, however, that AI may counter this and even cause an economic growth explosion. In this paper, we critically analyse this claim.

We did so by focusing on how AI affects the ideas production function (IPF). First, we critically discussed the literature on economic growth, population dynamics, and AI. In this literature, the "secret sauce" of economic growth is ideas, and hence the ideas production function (IPF) is accorded central importance in economic growth theory. Standard IPFs specify the production of new ideas as a function of population growth, and parameters measuring the efficiency of the R&D process for discovering new ideas. This includes the extent of intertemporal spillovers (standing-on-shoulders) and congestion (standing-on-toes) effects in performing and organizing R&D.

From an overview of the literature, we concluded that 1) population growth drives economic growth through innovation (new ideas) and the productivity with which a population of given size finds new ideas; 2) economic growth can stagnate if either population growth stagnate and/or the productivity of R&D declines; 3) population growth does not have simple, monotonic relationship with economic growth, after some level of population, product proliferation effects and "stepping-on-toes effects" will result in declining R&D productivity.

To reverse this, and even generate explosive economic growth, it has been claimed that AI can make up for the decline in human population by automating tasks in the IPF and improving the extent of intertemporal spillovers in the search for new ideas. We reviewed the literature exploring how AI can be included in the IPF, as well as the empirical literature on the actual magnitudes of the parameters in the IPF that reflects the extent of intertemporal spillovers and standing-on-toes effects. Several ways in which AI can complement human researchers in the discovery of new ideas were discussed, and it was pointed out that AI may not only have positive impacts on new idea discovery, but possible could also burden or limit the discovery of new ideas. AI may actually worsen any potential "stepping-on-toes" effects in research.

We moreover showed that AI is generally incorporated as a tool that enables higher efficiency in the production of knowledge but with no *law of motion* associated with it.

Hence in the remained of the paper we contributed to the literature by exploring how AI can augment the IPF and raise R&D productivity by addressing this lacuna. We proposed and discussed three alternative models for the IPF incorporating AI in the research process as well as its implications in terms of steady-state growth rates of technology and transitional dynamics. With these models in mind, we are able to understand, for example, which configuration allows AI to generate steady and positive knowledge creation, or how reasonable is to expect explosive economic growth from AI.

The three models of the IPF we proposed were calibrated by data from the USA and used to generate several simulations. The three models described AI as (1) a *research-augmenting* technology; (2) a *researcher scale enhancing* technology; and (3) as a *facilitator of innovation.*

Our simulations results showed that AI on its own may not be sufficient to accelerate the growth rate of ideas production indefinitely and that an economic growth explosion would only be possible under very specific and perhaps unlikely combinations of parameter values. These modelling choices are not the only possible and several combinations between AI and other technologies more traditional may be tested in the future. Our simulations for the transitional dynamics when saddle-path stability occurs also show that the possibility of explosive growth due to AI may occur only in the very short-run in the case that AI idea-generation overshoots its long-run growth.

Based on these IPFs and the simulation results conduction using them, we can conclude that an AI-induced growth explosion is not imminent.

Finally, our paper opens prospects for future research. First, the possibility that AI can enter in different ways in quality improvement innovations than in product proliferation can lead to new results concerning the possibility that AI contributes to the renewal of the so-called scale effect or to dilute it. The consideration of some possible harmful new products can also be studied in a setting that considers simultaneously vertical and horizontal innovations. Second, adding the demand side of the economy (households) and more traditional accumulable factors, such as physical capital, may enrich the analysis and give more precise quantitative outcomes in a full growth model. Third, to better quantify the possible effects of AI via the IPF, and to select the best model that describe the innovation process in the AI age, empirical estimations of the AI-enriched IPFs we proposed here would be useful.

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