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ABSTRACT

A Minimum Wage May Increase Exports and Firm Size Even with a Competitive Labor Market

This paper explores how a minimum wage affects a firm's behavior with a competitive labor market and an uncertain export cost. The model provides several novel insights which are consistent with recent empirical evidence. Thus, a minimum wage increases an exporter's foreign-market size and may cause a non-exporter to start exporting. The foreign-market size may increase so much that, although the home-market size decreases, the overall firm size increases.

JEL Classification: J30

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1 Introduction

According to standard economic theory, the introduction of a binding minimum wage with a competitive labor market will increase a firm's production cost and thereby cause the firm to shrink. Moreover, a firm engaged in international trade will scale back its exports and may stop exporting altogether. A common feature of most models in the minimum wage literature is that they abstract from uncertainty in the firm's product market.[1] In contrast, uncertainty plays an important role in many international-trade models.[2] However, while incorporating the influence of uncertainty on firms' export behavior, the international-trade literature is largely oblivious to the impact of a minimum wage.

Since neither strand of theoretical literature addresses the interaction between the minimum wage and uncertainty in detail, we are not aware of any analysis of the joint impact of the minimum wage and uncertainty on the firm's sales at home and abroad or on the firm's overall size. We are also not aware of empirical investigations that include both the minimum wage and uncertainty as explanatory variables.

The purpose of the present paper is to start filling this gap by exploring how a minimum wage affects firm behavior with a competitive labor market and an uncertain export cost.[3] We will show that the presence of uncertainty leads to several novel insights which are consistent with recent empirical evidence. In particular, while the minimum wage decreases a firm's incentive to sell at home, surprisingly it also increases the firm's incentive to sell abroad. Indeed, the empirical investigation in Hau, Huang and Wang (2020, pp. 2666-7) documents that for Chinese manufacturing firms, a positive minimum wage shock increases export quantities (but not export prices as would occur with a pass-through mechanism).[4]

[1] See Basu, Chau and Kanbur, 2010; Lee and Saez, 2012; Cahuc and Laroque, 2014; and Gerritsen and Jacobs, 2020. Uncertainty is considered in Danziger, 2009, and Bennett and Chioveanu, 2017.

[2] This includes Eaton, Kortum and Kramarz, 2011; Vannoorenberghe, 2012; and Vannoorenberghe, Wang and Yu, 2016.

[3] The uncertain export cost may stem from, among other things, foreign policy shocks (Handley and Limão, 2017, and Crowley, Exton and Han, 2020); volatile exchange rates (Das, Roberts and Tybout, 2007); and fluctuating transportation costs (Brancaccio, Kalouptsidi and Papageorgiou, 2020).

[4] Hau, Huang and Wang (2020) explain the increase in exports as being due to productivity catching

Relatedly, the direction of the effect of the minimum wage on a firm's overall size depends on the firm's productivity. Specifically, the minimum wage reduces the size of a low-productivity firm and, if the minimum wage is not too large, increases the size of a high-productivity firm. Since firm size is positively related to firm productivity, the minimum wage therefore makes a small firm smaller and a large firm larger.

In our model, the firm invests in the size of its home and foreign markets that it wishes to have access to, sets its wage (which is either competitively determined or equal to the minimum wage) and hires its desired number of workers before the realization of the uncertain export cost. Depending on the realization of the export cost, the firm may end up employing only part of its hired workers and laying off the rest. Specifically, if the firm has invested in the foreign market and the realized export cost is low, then the firm will employ all its hired workers, but if the realized export cost is high, then the firm will lay off some of them. Thus, the vagaries of the uncertain export cost create income uncertainty for workers. They will therefore agree to be hired only if the firm offers them a wage that, after taking into account their probability of being laid off, provides them with an expected income at least equal to what they can obtain elsewhere.[5]

We begin by considering the baseline case with a competitive labor market in which the firm offers the lowest wage that will attract workers. The workers' expected income will therefore equal their reservation wage. Thus, if the firm were to increase its export share and thereby its workers' layoff probability without increasing their wage, the workers' expected income would fall below their reservation wage. As a result, when choosing its home- and foreign-market sizes, the firm takes into account that a higher export share must

up of low-productivity firms. Ni and Kurita (2020) show that in Indonesia the probability of exporting increases with the minimum wage, and Nguyen (2021) that in Vietnam the export sales increase with the minimum wage. In contrast, however, Gan, Hernandez and Ma (2016) estimate that in China a minimum wage increase is associated with a decrease in the probability of exporting and in export sales, conditional on exporting. None of the empirical papers include export market uncertainty, which is an essential ingredient of our model, as an explanatory variable.

[5]The assumption that the hiring of workers takes place before the resolution of uncertainty and that this uncertainty is costly to the firm receive empirical support from the finding that firms may choose to reduce uncertainty by paying extra to ship goods faster (Evans and Harrigan, 2005) or to locate closer to the point of sales (Hummels and Schaur, 2010).

be associated with offering a higher wage.

Introducing a binding minimum wage changes this calculation for the firm. We show that being paid the minimum wage if employed may more than compensate workers for their layoff risk. As a result, even though for a given wage an increase in the firm's export share reduces workers' expected income due to the increased layoff risk, if paid the minimum wage when employed the workers' expected income may exceed their reservation wage. Therefore, with a minimum wage, unlike in the baseline case, the firm may be able to increase its export share without simultaneously having to increase workers' wages.[6]

The firm's ability to increase the export share without increasing wages above the minimum wage is the mechanism by which the minimum wage, despite *raising* the total cost of production, *lowers* the cost of production for export relative to the baseline case for any given level of production for the home market. Therefore, while, as would be expected, the minimum wage makes it *costlier to produce for the certain home market*, the minimum wage counterintuitively also makes it *cheaper to produce for the uncertain foreign market.*

That the minimum wage affects the cost of production for the home and foreign market in opposite directions constitutes the basis for our findings in this paper. This includes the abovementioned results that the minimum wage incentivizes an exporter to enlarge its foreign market and may incentivize a non-exporter to start exporting, thereby causing a small (low-productivity) firm to shrink and a large (high-productivity) firm to expand.

2 The Baseline

Consider a firm that produces a unique good which it sells domestically and may also export. The firm's productivity is φ so that one worker can produce φ units of output. The firm can choose its home- and foreign-market sizes, that is, the number of consumers

[6] Our model is particularly relevant for China where the labor market is competitive, layoffs are common, and the minimum wage is of local nature so that even the firm's nearby competitors are not exposed to a similar increase in labor cost (Hau, Huang and Wang, 2020).

it has access to in each market, by paying an investment cost that is convex in the market size chosen by the firm.[7] Specifically, to acquire a home market of size H, the firm must invest $\frac{1}{2}H^2$ units of the numeraire good, and to acquire a foreign market of size F, the firm must invest $\frac{1}{2}F^2$ units of the numeraire good.

In each market, consumers' reservation price is q units of the numeraire good. As a result, if consumers in a particular market can obtain the firm's product for a price that does not exceed q, demand for the firm's good equals that market's size (i.e., H for the home market and F for the foreign market), while if the price exceeds q, the demand is zero in that market.

In addition to the investment cost of acquiring the foreign market, exporting involves an uncertain export cost that is low with probability θ and high with probability $1-\theta$, where $\theta \in (0,1)$. In particular, the reservation price less the export cost is α when the realized export cost is low and β when it is high, where $0 < \beta < \alpha < q$. Thus, the firm will set the price at home to be q and the FOB price for goods sold to the foreign market to be either α or β depending on the realized export cost.[8]

The firm makes its decisions in two stages. The first stage takes place before the realization of the uncertain export cost and the second stage after its realization. At the first stage, the firm chooses its home- and foreign-market sizes; sets the wage, w, denominated in units of the numeraire good; and decides how many workers to hire, N. At the second stage, the firm takes the H, F, w, and N chosen at the first stage as given and, having learned its realized export cost, chooses how much to sell at home, H_x, and abroad, F_x, for $x \in \{\alpha, \beta\}$. The corresponding employment of workers will be $L_x = (H_x + F_x)/\varphi$,

[7] Arkolakis (2010) provides microfoundations for the increasing marginal investment cost building on the evidence of how the marketing cost of reaching a given number of consumers depends on the population size of the market and on the cost of reaching more consumers for a given population size.

[8] The assumption of reservation prices is consistent with the finding in Hau, Huang and Wang (2020) that there is no pass-through of the minimum wage to export prices. It simplifies the analysis but is not essential to many of our results. What matters is that if the realized FOB price is high, then the demand abroad is so low that the firm will lay off some of its workers. This would also be the case with, e.g., CES demands. Thus, in Appendix B we extend the model to CES demands and among other things show that, similar to the finding in the model with reservation prices, with CES demands a minimum wage can increase a firm's foreign-market size and cause a non-exporter to start exporting.

where the L_x employed workers are randomly chosen from the N workers hired at the first stage. These L_x workers get paid w at the second stage, while the remaining $N - L_x$ hired workers will be laid off and get paid nothing. Formally, the firm's second-stage choice of how much to be sold at home and abroad must satisfy the capacity constraints

$$H_x \leq H; \; F_x \leq F; \text{ and } L_x \leq N \text{ for } x \in \{\alpha, \beta\}. \tag{1}$$

Workers know the firm's productivity and can observe its choice of market sizes, wage, and number of hirees. They are risk neutral and laid-off workers' alternative income is zero since they cannot immediately get another job.[9] The hired workers' expected income is therefore $w\underset{x}{\mathrm{E}}L_x$. The labor market is competitive and workers' reservation wage, i.e., the expected income they can obtain elsewhere in the economy, is r units of the numeraire good. Thus, the firm can hire as many workers at it wishes so long as it satisfies the workers' participation constraint (WPC)

$$w\underset{x}{\mathrm{E}}L_x \geq rN. \tag{2}$$

The firm's revenue per unit of output sold at home is q, and its net revenue per unit of output sold abroad is the FOB price, i.e., either α or β. So, the firm's expected profit is

$$q\underset{x}{\mathrm{E}}H_x + \underset{x}{\mathrm{E}}\left(xF_x\right) - w\underset{x}{\mathrm{E}}L_x - \tfrac{1}{2}H^2 - \tfrac{1}{2}F^2. \tag{3}$$

The firm's objective in the two-stage optimization problem is to maximize its expected profit (3) subject to the capacity constraints (1) and the WPC (2). At the first stage the firm chooses H, F, w, and N, while at the second stage it chooses H_x and F_x for $x \in \{\alpha, \beta\}$.

Since we wish to study the effects of employment volatility induced by exporting, we

[9] Our main results would not change if workers were risk averse or if laid-off workers' alternative income were positive. They would also not change if there were a fixed employment cost due to, e.g., a time-consuming investment in firm-specific human capital.

focus on a firm that, if it invests in the foreign market, will lay off workers when the realized export cost is high. To this end, we limit our analysis to a firm with productivity less than r/β, which assures that $F_\beta = 0$.[10] We further assume that $\theta\alpha > \beta$, which assures that exporting is sufficiently attractive that there are productivities below r/β for which the firm will export when the realized export cost is low.

3 Firm Optimization

At the first stage, the firm sets the lowest possible wage that allows it to hire its desired number of workers. The WPC therefore binds so that the expected income of each hired worker is r. As shown in Appendix A, the firm's optimization problem can then be simplified to at the first stage choosing (H, F) to maximize the expected profit

$$\left(q - \frac{r}{\varphi}\right) H + \theta\left(\alpha - \frac{r}{\theta\varphi}\right) F - \tfrac{1}{2}H^2 - \tfrac{1}{2}F^2 \tag{4}$$

and hiring exactly the number of workers it would need to employ in order to satisfy all demand in its home and foreign markets, $N = (H + F)/\varphi$. At the second stage the firm always satisfies demand in the home market, $H_\alpha = H_\beta = H$, and satisfies demand in the foreign market if the realized export cost is low but not if it is high, $F_\alpha = F$ and $F_\beta = 0$. The expected number of employed workers is then $(H + \theta F)/\varphi$, and at the first stage the firm therefore sets the wage to satisfy the binding WPC

$$w(H + \theta F) = r(H + F). \tag{5}$$

Since the expected profit (4) is additively separable in H and F, the firm's choice of

[10] If $\varphi < r/\beta$, the unit cost of production, w/φ, will exceed the FOB price of a unit sold abroad when the realized export cost is high. The firm would therefore rather lay off workers than produce for export. To avoid cutting off high firm productivities, the model could be extended by assuming that β is either probabilistic or decreases with the foreign-market size. It would also be avoided if the model were extended to CES demands (see footnote 8).

home- and foreign-market sizes are independent of each other. We will write the chosen market sizes as functions of φ to emphasize that they depend on the firm's productivity. Proposition 1 characterizes $H(\varphi)$ and $F(\varphi)$.[11]

Proposition 1

The home- and foreign-market sizes are

$$H(\varphi) = \begin{cases} 0 & \text{if} \quad \varphi \leq \varphi_a; \\ q - \dfrac{r}{\varphi} & \text{if} \quad \varphi > \varphi_a; \end{cases} \qquad \text{and} \quad F(\varphi) = \begin{cases} 0 & \text{if} \quad \varphi \leq \varphi_x; \\ \theta\left(\alpha - \dfrac{r}{\theta\varphi}\right) & \text{if} \quad \varphi > \varphi_x; \end{cases}$$

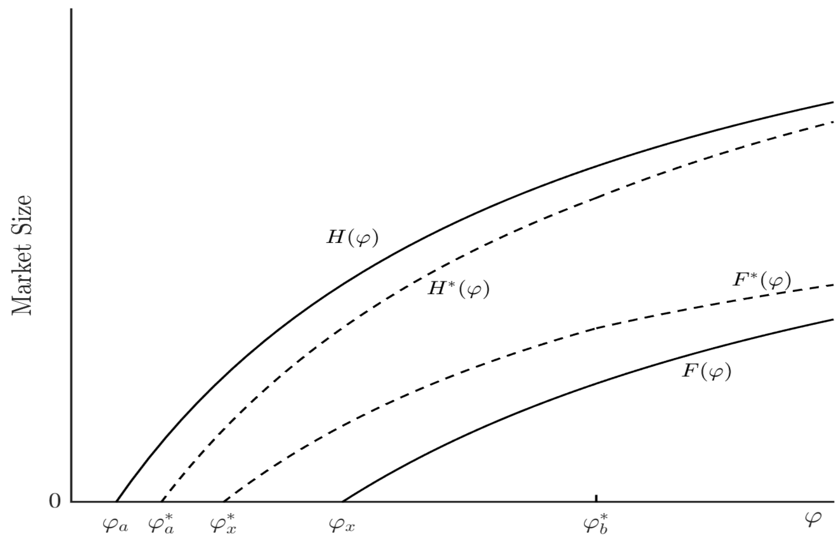
where $\varphi_a = r/q$ is the activity cutoff and $\varphi_x = r/(\theta\alpha)$ is the export cutoff.

The expression for the firm's expected profit in (4) reveals that the unit variable profit at home, $q - r/\varphi$, exceeds the unit variable profit abroad, $\alpha - r/(\theta\varphi)$. This reflects both that the price at home, q, exceeds the FOB price abroad, α, and that the cost of producing a unit for the certain home market, r/φ, is less than the cost of producing a unit for the uncertain foreign market, $r/(\theta\varphi)$. The higher cost of producing for the foreign market stems from the need for the firm to compensate its hired workers for the layoff risk that exposure to the foreign market brings with it. The upshot is that selling in the home market is more attractive to the firm than selling in the foreign market.

The market sizes equalize the constant marginal (expected) variable profit from having the option to sell an additional unit in a given market with the increasing marginal investment cost of expanding that market. That is, $q - r/\varphi = d\left(\frac{1}{2}H^2\right)/dH = H$ at home and $\theta\left[\alpha - r/(\theta\varphi)\right] = d\left(\frac{1}{2}F^2\right)/dF = F$ abroad. Since the marginal variable profit from access to a larger home market exceeds the expected marginal variable profit from access to a larger foreign market, the firm invests in a larger home market than foreign market. In Figure 1 the home- and foreign-market sizes as functions of the firm's productivity are shown by the solid curves labeled $H(\varphi)$ and $F(\varphi)$.

[11] The proofs of the propositions are in Appendix A.

Figure 1: Home- and Foreign-Market Sizes



Notes: The solid (dashed) curves show the home- and foreign-market sizes in the baseline case (with the minimum wage) as functions of firm productivity. The curves in this and subsequent figures are computed using the following parameter values: $q = 9/4$, $\theta = 3/4$, $\alpha = 2$, $\beta = 0.2$, $r = 1$ and $m = 1.1$. Hence, $\varphi_b^* = 11/9$ and $r/\beta = 5$, which implies that $\varphi_b^* < r/\beta$.

As we have just seen, the marginal investment cost of expanding the home (foreign) market is equal to H (F); so the marginal cost of beginning to invest in a market is zero. As a result, at the first stage, the firm will invest in a market if the (expected) variable profit from being able to sell a unit in that market (which is constant for all units in the market) is positive. This yields $\varphi_a = r/q$ as the activity cutoff beyond which the firm invests in the home market and the higher $\varphi_x = r/(\theta\alpha)$ as the export cutoff beyond which the firm invests in the foreign market as well.[12]

If the firm exports, the difference between the home- and foreign-market sizes is $q - \theta\alpha$, which is independent of the firm's productivity. This, together with the fact that the home-market size is larger than the foreign-market size, implies that the foreign-market size increases *relative* to the home-market size with firm productivity. Hence, if the firm exports, then the export share, defined as the firm's expected foreign sales relative to its expected

[12]That $\theta\alpha > \beta$ implies $\varphi_a < r/\beta$.

total sales, i.e., $S(\varphi) = \theta F(\varphi) / [H(\varphi) + \theta F(\varphi)]$, increases with firm productivity.

To determine the baseline wage, $w(\varphi)$, we substitute the market sizes into the binding WPC (5) to obtain that[13]

$$w(\varphi) = \begin{cases} r & \text{if} \quad \varphi_a < \varphi \leq \varphi_x; \\ r[1 - S(\varphi)] + \dfrac{r}{\theta} S(\varphi) & \text{if} \quad \varphi > \varphi_x. \end{cases} \tag{6}$$

If the firm sells only at home it pays its workers the reservation wage r since it does not subject them to any uncertainty, while if, hypothetically, the firm would sell only abroad it would need to pay a higher wage equal to r/θ to compensate the workers for accepting a layoff probability of $1 - \theta$. In reality, however, if the firm sells abroad, then it also sells at home and pays a wage equal to the weighted average of r and r/θ, where the weight of r is one minus the export share and the weight of r/θ is the export share. As a result, the firm's wage increases with the firm's export share which in turns increases with the firm's productivity. The consequent positive relationship between the wage and firm productivity represents a wage differential that compensates workers for their additional risk of being laid off, and is not due to rent accrual by workers whose expected income is independent of the firm's productivity, export status and size.[14]

4 Minimum Wage

Suppose that the firm is required to pay a minimum wage that equals m units of the numeraire good. We assume that $w(\varphi) < m < r/\theta$. That is, the minimum wage exceeds the firm's baseline wage and is thus binding, but less than the wage it would have to pay if

[13] If $\varphi \leq \varphi_a$, then the firm does not hire any workers since $H(\varphi) = F(\varphi) = 0$ as per Proposition 1.

[14] The layoff probability is $1 - r/w(\varphi)$. Consistent with the empirical evidence, a low-productivity firm pays a low wage and does not export; a high-productivity firm pays a higher wage and exports; and the higher the exporter's productivity, the higher is the wage it pays and the larger is its export market and export share. The variability of the firm's employment and production, revenue, and profit also increases with its productivity.

it, hypothetically, produced only for the foreign market. The firm's two-stage optimization problem remains the same as in the baseline case except for the additional constraint that the firm must pay (at least) the minimum wage to the workers it ends up employing.[15] Substituting m for w in (2) and (3), the WPC is

$$m\underset{x}{\mathrm{E}}L_x \geq rN \tag{7}$$

and the firm's expected profit is

$$q\underset{x}{\mathrm{E}}H_x + \underset{x}{\mathrm{E}}\left(xF_x\right) - m\underset{x}{\mathrm{E}}L_x - \tfrac{1}{2}H^2 - \tfrac{1}{2}F^2. \tag{8}$$

As in the baseline case, at the first stage the firm will always hire exactly the number of workers needed were it to satisfy all demand in its chosen home and foreign markets, i.e., $N = (H + F)/\varphi$. Moreover, just as in the baseline case, at the second stage the firm always satisfies the demand in the home market, $H_\alpha = H_\beta = H$, and satisfies the demand in the foreign market if the realized export cost is low and does not export if it is high, $F_\alpha = F$ and $F_\beta = 0$. Therefore, (8) can be written as

$$\left(q - \frac{m}{\varphi}\right) H + \theta \left(\alpha - \frac{m}{\varphi}\right) F - \tfrac{1}{2}H^2 - \tfrac{1}{2}F^2. \tag{9}$$

Furthermore, since expected employment is $\left(H + \theta F\right)/\varphi$, the WPC (7) can be written as

$$vH \geq F, \tag{10}$$

where $v \equiv (m - r)/(r - \theta m)$.

Unlike in the baseline case, however, the WPC need not bind. Indeed, if in the baseline case the WPC were slack, the firm could increase its expected profit by offering workers a

[15] The firm will not offer a wage higher than the minimum wage (see the proof of Proposition 2).

lower wage. With a minimum wage this is no longer true because the firm cannot tighten a slack WPC by lowering the wage it offers. Therefore, we must consider both cases where the WPC is slack and cases where it binds.

If the WPC (10) is slack, then the firm can determine the home- and foreign-market sizes independently of each other, just as in the baseline case, since (9) is additively separable in H and F. However, if instead the WPC binds, then the home- and foreign-market sizes must satisfy $vH = F$ and cannot be determined independently of each other.

In what follows, an asterisk denotes a variable that depends on the firm's choices with a minimum wage and, as such, depends on m. In addition to characterizing the market sizes $H^*(\varphi)$ and $F^*(\varphi)$, Proposition 2 shows that the WPC is slack if $\varphi \leq \varphi_b^*$ and binding if $\varphi > \varphi_b^*$, where

$$\varphi_b^* \equiv \begin{cases} \dfrac{(\theta - v)m}{\theta\alpha - vq} & \text{if} \quad m < \bar{m}; \\ \infty & \text{if} \quad m \geq \bar{m}; \end{cases} \tag{11}$$

and $\bar{m} \equiv (q + \theta\alpha)\, r / (q + \theta^2\alpha)$.[16]

Proposition 2

(a) *If $\varphi \leq \varphi_b^*$, then the WPC is slack, and the home- and foreign-market sizes are*

$$H^*(\varphi) = \begin{cases} 0 & \textit{if} \quad \varphi \leq \varphi_a^*; \\ q - \dfrac{m}{\varphi} & \textit{if} \quad \varphi > \varphi_a^*; \end{cases} \qquad \textit{and } F^*(\varphi) = \begin{cases} 0 & \textit{if} \quad \varphi \leq \varphi_x^*; \\ \theta\left(\alpha - \dfrac{m}{\varphi}\right) & \textit{if} \quad \varphi > \varphi_x^*; \end{cases}$$

where $\varphi_a^* = m/q$ is the activity cutoff and $\varphi_x^* = m/\alpha$ is the export cutoff.

(b) *If $\varphi > \varphi_b^*$, then the WPC binds, and the home- and foreign-market sizes are*

$$H^*(\varphi) = \frac{q - m/\varphi + v\theta\,(\alpha - m/\varphi)}{1 + v^2} \quad \textit{and} \quad F^*(\varphi) = vH^*(\varphi)\,.$$

[16] It follows from (6) that the assumption that the minimum wage exceeds the baseline wage is equivalent to an assumption that the firm's productivity is less than φ_m, where $\varphi_m \equiv (1 - v)r/(\theta\alpha - vq)$ if $m < \bar{m}$ and $\varphi_m = \infty$ if $m \geq \bar{m}$. Note that $m < \bar{m}$ implies that $\theta\alpha - vq > 0$ and $1 - v > 0$, and hence that $\varphi_m > 0$.
.

In Figure 1 the dashed curves labeled $H^*(\varphi)$ and $F^*(\varphi)$ depict the home- and foreign-market sizes with a minimum wage as functions of the firm's productivity, both when the WPC is slack and when it binds. We now explain each part of Proposition 2 in turn.

Part (a) The WPC is slack If the firm's productivity is at most φ_b^*, then the WPC is slack. The expression for the firm's expected profit in (9) shows that with the minimum wage the unit cost of producing for either market is *the same*, m/φ, even though production for the foreign market happens only with probability θ. Therefore, compared to the baseline case, the minimum wage *increases* the unit cost of producing for home (from r/φ to m/φ), but *decreases* the unit cost of producing for export (from $r/(\theta\varphi)$ to m/φ).

The reason that the cost of producing for home increases is that the firm is forced to pay the higher minimum wage even though selling at home does not entail a layoff risk. The reason that the cost of producing for export decreases is that, as long as the WPC is slack, the firm can increase its foreign-market size without having to compensate its workers for the increased layoff risk by offering a higher wage. In other words, since the workers' expected income with the minimum wage exceeds their reservation wage, the additional layoff risk stemming from an increase in the firm's foreign-market size will not cause a violation of the WPC. In contrast, in the baseline case where the WPC binds and the workers' expected income equals their reservation wage, the firm cannot increase the layoff risk stemming from exporting without also offering a higher wage. It is this ability of the firm paying the minimum wage to increase its workers' layoff risk without having to increase their wage that makes production for export cheaper with a minimum wage than without.

By increasing the cost of producing for home and decreasing the cost of producing for abroad, the minimum wage reduces the variable profit from selling a unit at home and increases the expected variable profit from being able to sell a unit abroad. As in the baseline case, the (expected) marginal variable profit from increased access to the home (foreign) market equals H (F). As a result, the activity cutoff increases (from $\varphi_a = r/q$ to $\varphi_a^* = m/q$), and, for productivities above this cutoff, the size of the home market shrinks to $q - m/\varphi$. Concurrently, the export cutoff decreases (from $\varphi_x = r/(\theta\alpha)$ to $\varphi_x^* = m/\alpha$), and, for productivities above this cutoff, the size of the export market increases to $\theta(\alpha - m/\varphi)$. Thus, similar to what a standard minimum wage model would predict, the minimum wage causes the firm to produce less for the home market and to stop producing for the home market altogether if the firm's productivity is low. However, in contrast to what a standard minimum wage model would predict, the minimum wage causes an exporter to export more and may even cause a non-exporter to start exporting.

Even though the minimum wage reduces the profitability of selling at home while raising the expected profitability of exporting, the export cost continues to make exporting less attractive than selling at home. Consequently, it is still the case that the activity cutoff is less than the export cutoff and that a firm's home market is bigger than its foreign market. Likewise, it is still the case that the more productive is an exporting firm, the larger is its foreign market relative to its home market. The export share, $S^*(\varphi) = \theta F^*(\varphi) / [H^*(\varphi) + \theta F^*(\varphi)]$, therefore increases with the firm's productivity.

Since the layoff probability increases with the firm's export share which itself increases with the firm's productivity, it follows that workers' expected income decreases with the firm's productivity if the WPC is slack. Whether or not the decrease in expected income will eventually cause the WPC to bind depends on the size of the minimum wage. If the minimum wage is high, then the WPC will not bind for any export share chosen by the firm and hence not for any productivity level. However, for lower minimum wages, the WPC will bind for a sufficiently high export share and hence productivity level. We now turn to those cases.

Part (b) The WPC binds If the firm's productivity exceeds φ_b^*, then the WPC binds. That is, the choice of home- and foreign-market sizes provides a layoff probability that ensures that with the minimum wage workers' expected income equals their reservation wage. Specifically, this layoff probability is achieved when the firm chooses the foreign-market size to equal v times the home-market size. As a result, the two market sizes equalize the constant expected variable profit from selling an additional unit at home plus an additional vth of a unit abroad if the export cost is low, i.e., $q - m/\varphi + v\theta(\alpha - m/\varphi)$, with the increasing marginal investment cost of expanding the two markets subject to $F = vH$, i.e., $d\left(\frac{1}{2}H^2 + \frac{1}{2}F^2\right)/dH = H + vF$. That is,

$$H^*(\varphi) + vF^*(\varphi) = q - \frac{m}{\varphi} + v\theta\left(\alpha - \frac{m}{\varphi}\right), \tag{12}$$

which, since $F^*(\varphi) = vH^*(\varphi)$, yields the firm's choice of home- and foreign-market sizes.

The proposition shows that (12) holds independently of whether the WPC is slack or binds, that is, the sum $H^*(\varphi) + vF^*(\varphi)$ is unaffected by the binding WPC. Nevertheless, the firm's choice of its individual market sizes, $H^*(\varphi)$ and $F^*(\varphi)$, is affected by the binding WPC as it forces the firm to choose a larger home-market size and a smaller foreign-market size *relative* to what it would have chosen absent the WPC.[17] As a result, the export share, which equals $S^*(\varphi_b^*) = v\theta/(1 + v\theta)$ and is independent of the firm's productivity when the WPC binds, is less than what the export share would have been absent the WPC.

5 Market Sizes

The above analysis shows that the minimum wage decreases the attractiveness of the home market but increases the attractiveness of the foreign market. As a result, the change in the home-market size caused by the minimum wage, $\Delta H(\varphi) \equiv H^*(\varphi) - H(\varphi)$, is negative (if $\varphi > \varphi_a$ so that the firm is active in the baseline case), while the change in the foreign-market size, $\Delta F(\varphi) \equiv F^*(\varphi) - F(\varphi)$, is positive (if $\varphi > \varphi_x^*$ so that the firm exports with the minimum wage). This is stated in Proposition 3 which further characterizes how the

[17] That is, if $\varphi > \varphi_b^*$, then $H^*(\varphi) > q - m/\varphi$ and $F^*(\varphi) < \theta(\alpha - m/\varphi)$.

effect of the minimum wage on the market sizes depends on the firm's productivity.

Proposition 3

(a) *If* $\varphi > \varphi_a$, *then* $\Delta H(\varphi)$ *is negative, decreases until reaching its minimum at* $\varphi = \varphi_a^*$, *and increases thereafter.*

(b) *If* $\varphi > \varphi_x^*$, *then* $\Delta F(\varphi)$ *is positive, increases until reaching its maximum at* $\varphi = \varphi_x$, *and decreases thereafter.*

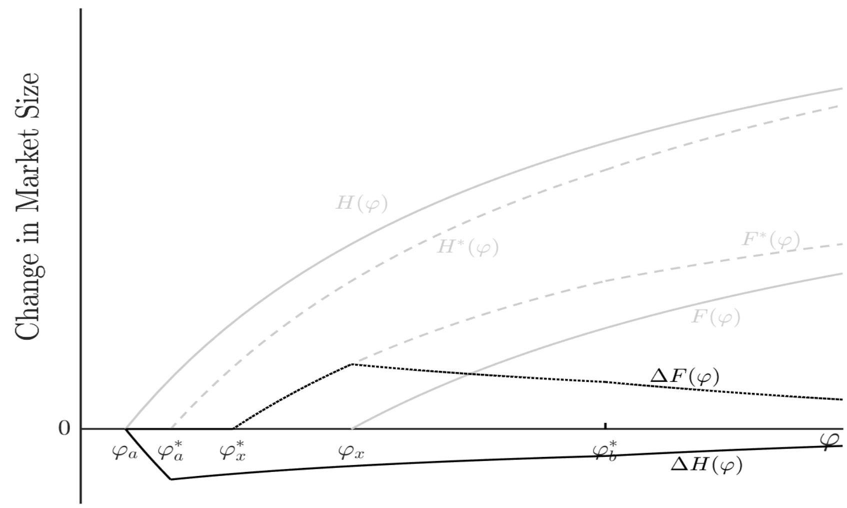
In Figure 2, we illustrate $\Delta H(\varphi)$ by the solid black curve and $\Delta F(\varphi)$ by the dotted black curve. If the firm is productive enough to be active in the baseline case but not so productive that it is active with the minimum wage (i.e., $\varphi_a < \varphi \leq \varphi_a^*$), then the whole effect of the minimum wage on $\Delta H(\varphi)$ stems from the increase in $H(\varphi)$, which explains why $\Delta H(\varphi)$ decreases in this range. If the firm is so productive that it is also active with the minimum wage (i.e., $\varphi > \varphi_a^*$), then $H^*(\varphi) < H(\varphi)$ together with the convex investment cost makes it cheaper to increase $H^*(\varphi)$ than to increase $H(\varphi)$, which explains why $\Delta H(\varphi)$ increases in this range.

The explanation for the shape of $\Delta F(\varphi)$ is analogous to the explanation for the shape of $\Delta H(\varphi)$. If the firm is productive enough to export with the minimum wage but not so productive that it exports with the minimum wage (i.e., $\varphi_x^* < \varphi \leq \varphi_x$), then the whole effect on $\Delta F(\varphi)$ stems from the increase in $F^*(\varphi)$, which explains why $\Delta F(\varphi)$ increases in this range. If the firm is so productive that it exports in the baseline case as well (i.e., $\varphi > \varphi_x$), then $F^*(\varphi) > F(\varphi)$ together with the convex investment cost makes it cheaper to increase $F(\varphi)$ than to increase $F^*(\varphi)$, which explains why $\Delta F(\varphi)$ decreases in this range.

6 Firm Size

We define the firm's size as the sum of its chosen home- and foreign-market sizes, i.e., $Z(\varphi) \equiv H(\varphi) + F(\varphi)$ in the baseline case and $Z^*(\varphi) \equiv H^*(\varphi) + F^*(\varphi)$ with the minimum

Figure 2: Effect of a Minimum Wage on Home- and Foreign-Market Sizes



Notes: The solid (dotted) black curve shows the change in the home- (foreign-) market size caused by the minimum wage as a function of firm productivity. The gray curves, included for scale, show the home- and foreign-market sizes both with and without the minimum wage, as in Figure 1.

wage. The effect of the minimum wage on the firm's size is $\Delta Z(\varphi) \equiv Z^*(\varphi) - Z(\varphi)$ and equals $\Delta H(\varphi) + \Delta F(\varphi)$, that is, the sum of the changes in the market sizes due to the minimum wage.

Proposition 4 establishes how the opposing effects of the minimum wage on the home- and foreign-market sizes are combined into the effect on firm size. It shows that the direction of the effect on firm size depends on how large the minimum wage is as well as on the firm's productivity. In particular, a minimum wage that is not too high increases the size of a high-productivity firm.[18] Defining $\tilde{m} \equiv 2r/(1+\theta)$, we obtain

Proposition 4

Suppose that $\varphi > \varphi_a$.

(a) *If $m < \tilde{m}$, then $\Delta Z(\varphi) \lesseqgtr 0$ as $\varphi \lesseqgtr \varphi_0$, where $\varphi_0 \in (\varphi_x^*, \varphi_x)$.*

[18] The proposition will remain unchanged if, instead of defining the firm's size as the sum of its home- and foreign-market sizes, we define the firm's size by either its number of hirees, expected output or expected employment.

(b) *If* $m = \tilde{m}$*, then* $\Delta Z(\varphi) < 0$ *if* $\varphi < \varphi_x$ *and* $\Delta Z(\varphi) = 0$ *if* $\varphi \geq \varphi_x$*.*

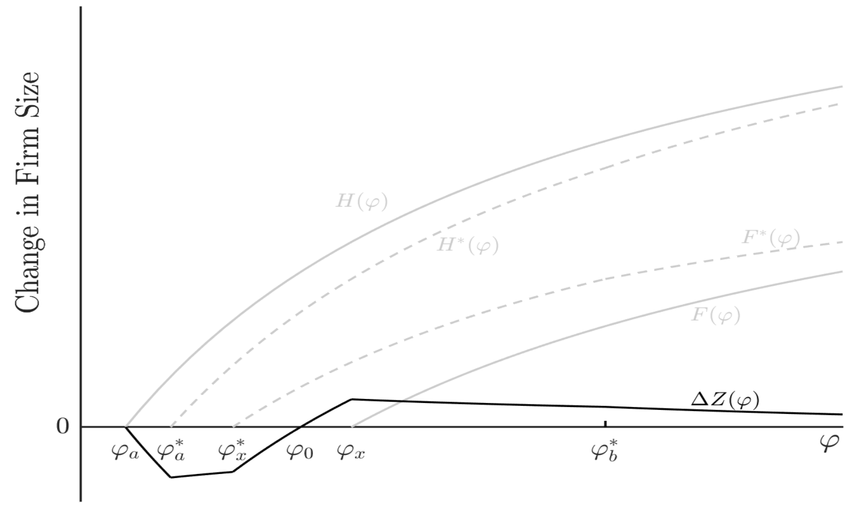
(c) *If* $m > \tilde{m}$*, then* $\Delta Z(\varphi) < 0$ *for all* φ*.*

To understand the proposition, note that if the firm's productivity is such that it does not export in either the baseline case or with the minimum wage (i.e., $\varphi_a < \varphi \leq \varphi_x^*$), then $\Delta Z(\varphi) = \Delta H(\varphi)$. Since $\Delta H(\varphi) < 0$ (see Proposition 3), it follows that also $\Delta Z(\varphi) < 0$ in this range. If the firm's productivity is high enough to export with the minimum wage but not high enough to export in the baseline case (i.e., $\varphi_x^* < \varphi \leq \varphi_x$), then $\Delta Z(\varphi) = \Delta H(\varphi) + F^*(\varphi)$. Since both $\Delta H(\varphi)$ and $F^*(\varphi)$ increase with the firm's productivity in this range (see Propositions 2 and 3), it follows that also $\Delta Z(\varphi)$ increases with firm productivity.

The proposition shows that whether or not $\Delta Z(\varphi)$ becomes positive, and hence whether or not the minimum wage can increase firm size, depends on the size of the minimum wage. In particular, case (a) shows that if the minimum wage is small, i.e. $m < \tilde{m}$, then $\Delta Z(\varphi)$ increases so much with firm productivity in the range $\varphi_x^* < \varphi \leq \varphi_x$ that it becomes positive. Thus, as illustrated in Figure 3, if the firm's productivity exceeds φ_0, then the minimum wage increases the firm's size. A major finding is therefore that a small minimum wage increases the size of a high-productivity firm. However, if the minimum wage is not small, i.e., $m \geq \tilde{m}$, then the firm's size does not increase whatever the firm's productivity. Indeed, if $m = \tilde{m}$, then $\Delta Z(\varphi)$ increases until reaching zero at φ_x and remains zero thereafter, and if $m > \tilde{m}$, then $\Delta Z(\varphi)$ remains negative for all productivities.

One way to understand the impact of the magnitude of the minimum wage on the change in firm size is by considering the change in the activity cutoff (from $\varphi_a = r/q$ to $\varphi_a^* = m/q$) and in the export cutoff (from $\varphi_x = r/(\theta\alpha)$ to $\varphi_x^* = m/\alpha$). As m approaches r/θ, the activity (export) cutoff with the minimum wage diverges from (converges to) the activity (export) cutoff in the baseline case. This is a reflection of the fact that if the minimum wage is large, the increase in the cost of producing for home is large but the reduction in the cost of producing for export is small. The implication is that with a large minimum wage, the reduction in the home-market size dominates the increase in the foreign-market

Figure 3: Effect of a Minimum Wage on Firm Size



Notes: The black curve shows the change in firm size caused by the minimum wage as a function of firm productivity. The gray curves, included for scale, show the home- and foreign-market sizes both with and without the minimum wage, as in Figure 1.

size, leading to a smaller overall firm size.

However, the effect of a minimum wage on firm size is reversed for small minimum wages if the firm's productivity is sufficiently high. Indeed, the smaller the minimum wage, the smaller is the increase in the activity cutoff and the larger is the decrease in the export cutoff. Thus, a smaller minimum wage manifests itself as a smaller reduction in the home-market size and a larger increase in the foreign-market size. In fact, if the minimum wage is small enough and the firm sufficiently productive, then the gain in foreign-market size outweighs the loss in home-market size, leading to an increase in overall firm size.

7 Conclusion

In this paper, we study the impact of a minimum wage on a firm that chooses its domestic and foreign-market sizes as well as the number of workers to hire before the realization of an uncertain export cost. We show that the introduction of a minimum wage raises the cost of producing for home but *lowers* the cost of producing for abroad. The firm therefore reduces the size of its home market. An exporting firm also increases the size of its foreign market and a non-exporter may be prompted to start to export. In fact, the minimum wage may increase the firm's foreign-market size so much that the firm's size increases. Therefore, the minimum wage may generate an increase in both exports and firm size.

A key insight in this paper is that a minimum wage increases the relative attractiveness of risky ventures for the firm and is relevant also beyond our focus on how much a firm exports. For instance, just as the minimum wage increases the attractiveness of the risky foreign market relative to the safe home market, a multiproduct firm may find that the minimum wage increases the attractiveness of investing in riskier innovative products relative to investing in its safer core products. Similarly, the minimum wage may make investment in newer but riskier technologies more attractive.

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Appendix A

Firm Optimization in the Baseline Case

Substituting w from the WPC (2), which is binding, together with $F_\beta = 0$ into (3), the firm's expected profit is

$$q\underset{x}{\mathrm{E}}H_x + \theta\alpha F_\alpha - rN - \tfrac{1}{2}H^2 - \tfrac{1}{2}F^2. \tag{13}$$

At the second stage where H, F, and N are given, the firm earns more by selling a unit of output at home than abroad. Therefore, $H_a = H_\beta$. Further, at the first stage the firm will not hire fewer workers than necessary to satisfy demand in its markets, since if the firm at the second stage cannot satisfy demand if the realized export cost is low, it might as well have saved part of the investment costs by choosing smaller market sizes. Hence, $N \geq (H + F)/\varphi$. At the first stage the firm will also not hire so many workers that some of them will be laid off at the second stage if the realized export cost is low. Hence, $N \leq (H + F)/\varphi$. As a result, the firm will hire exactly the number of workers needed to satisfy all demand in its home and foreign markets, i.e., $N = (H + F)/\varphi$. It follows that $H_a = H_\beta = \underset{x}{\mathrm{E}}H_x = H$ and $F_\alpha = F$. Substituting into (13), the firm's expected profit is

$$\left(q - \frac{r}{\varphi}\right)H + \theta\left(\alpha - \frac{r}{\theta\varphi}\right)F - \tfrac{1}{2}H^2 - \tfrac{1}{2}F^2. \tag{14}$$

The firm's optimization problem is therefore simplified to at the first stage choosing (H, F) to maximize (14) and hiring $N = (H + F)/\varphi$ workers. At the second stage the firm chooses $H_\alpha = H_\beta = H$, $F_\alpha = F$, and $F_\beta = 0$. Since the export cost is low with probability θ, the expected employment is $(H + \theta F)/\varphi$. The binding WPC is therefore $w(H + \theta F) = r(H + F)$, and at the first stage the firm sets the wage to satisfy the WPC.

Proof of Proposition 1

The derivative of the firm's expected profit (4) with respect to H is $q - r/\varphi - H$, which is negative for $H > 0$ if $\varphi \leq \varphi_a$ and is zero for $H = q - r/\varphi$ if $\varphi > \varphi_a$. Hence, $H(\varphi) = 0$ if $\varphi \leq \varphi_a$ and $H(\varphi) = q - r/\varphi$ if $\varphi > \varphi_a$. The derivative of (4) with respect to F is $\theta[\alpha - r/(\theta\varphi)] - F$, which is negative for $F > 0$ if $\varphi \leq \varphi_x$ and is zero for $F = \theta[\alpha - r/(\theta\varphi)]$ if $\varphi > \varphi_x$. Therefore, $F(\varphi) = 0$ if $\varphi \leq \varphi_x$ and $F(\varphi) = \theta[\alpha - r/(\theta\varphi)]$ if $\varphi > \varphi_x$. $\square$

Proof of Proposition 2

Let $\mathcal{H}(\varphi)$ and $\mathcal{F}(\varphi)$ denote the home- and foreign-market sizes absent the WPC. The derivative of the firm's expected profit (9) with respect to H is $q - m/\varphi - H$. Accordingly, $\mathcal{H}(\varphi) = 0$ if $\varphi \leq \varphi_a^*$ and $\mathcal{H}(\varphi) = q - m/\varphi$ if $\varphi > \varphi_a^*$. The derivative of (9) with respect to F is $\theta(\alpha - m/\varphi) - F$. Accordingly, $\mathcal{F}(\varphi) = 0$ if $\varphi \leq \varphi_x^*$ and $\mathcal{F}(\varphi) = \theta(\alpha - m/\varphi)$ if $\varphi > \varphi_a^*$.

(i) If $\varphi \leq \varphi_b^*$, then either $m < \bar{m}$ and hence

$$\begin{aligned} \varphi &\leq \frac{(\theta - v)\,m}{\theta\alpha - vq} \\ \Leftrightarrow \quad v\left(q - \frac{m}{\varphi}\right) &\geq \theta\left(\alpha - \frac{m}{\varphi}\right) \\ \Leftrightarrow \quad v\mathcal{H}(\varphi) &\geq \mathcal{F}(\varphi); \end{aligned}$$

or $m \geq \bar{m}$ and hence

$$\begin{aligned} m &\geq \frac{(q + \theta\alpha)r}{q + \theta^2\alpha} \\ \Leftrightarrow \quad v\left(q - \frac{m}{\varphi}\right) &> \theta\left(\alpha - \frac{m}{\varphi}\right) \\ \Leftrightarrow \quad v\mathcal{H}(\varphi) &> \mathcal{F}(\varphi). \end{aligned}$$

It follows that in both cases the WPC is slack. Accordingly, $H^*(\varphi) = \mathcal{H}(\varphi)$ and $F^*(\varphi) = \mathcal{F}(\varphi)$.

(ii) If $\varphi > \varphi_b^*$, then $m < \bar{m}$ and hence

$$\begin{aligned} \varphi &> \frac{(\theta - v)\, m}{\theta\alpha - vq} \\ \Leftrightarrow \quad v\left(q - \frac{m}{\varphi}\right) &< \theta\left(\alpha - \frac{m}{\varphi}\right) \\ \Leftrightarrow \quad v\mathcal{H}(\varphi) &< \mathcal{F}(\varphi). \end{aligned}$$

It follows that the WPC binds. By substituting for $F = vH$ into (9), the firm's expected profit can be written to depend on only H,

$$\left[q - \frac{m}{\varphi} + v\theta\left(\alpha - \frac{m}{\varphi}\right)\right] H - \tfrac{1}{2}\left(1 + v^2\right) H^2. \tag{15}$$

The derivative with respect to H is $q - m/\varphi + v\theta\,(\alpha - m/\varphi) - (1 + v^2)\, H$. Accordingly, $H^*(\varphi) = [q - m/\varphi + v\theta\,(\alpha - m/\varphi)]\,/\,(1 + v^2)$ and $F^*\,(\varphi) = vH^*\,(\varphi)$.

Together, (i)-(ii) prove Proposition 2. □

Proof of Proposition 3

The effect on $\Delta H\,(\varphi)$

(i) If $\varphi_a < \varphi \le \varphi_a^*$, then $H\,(\varphi) = q - r/\varphi$ and $H^*\,(\varphi) = 0$. Hence, $\Delta H\,(\varphi) = -H\,(\varphi) < 0$ and $d\Delta H\,(\varphi)\,/d\varphi = -r/\varphi^2 < 0$.

(ii) If $\varphi_a^* < \varphi \le \varphi_b^*$, then $H\,(\varphi) = q - r/\varphi$ and $H^*\,(\varphi) = q - m/\varphi$. Hence, $\Delta H\,(\varphi) = -(m - r)/\varphi < 0$ and $d\Delta H\,(\varphi)\,/d\varphi = (m - r)/\varphi^2 > 0$.

(iii) If $\varphi > \varphi_b^*$, then $H\,(\varphi) = q - r/\varphi$ and $H^*\,(\varphi) = [q - m/\varphi + v\theta\,(\alpha - m/\varphi)]\,/\,(1 + v^2)$. Accordingly,

$$\begin{aligned} \Delta H\,(\varphi) &= \frac{q - m/\varphi + v\theta\,(\alpha - m/\varphi)}{1 + v^2} - q + \frac{r}{\varphi} \\ &= -\frac{v\,(\theta\alpha - vq)\,(\varphi_m/\varphi - 1)}{1 + v^2}, \end{aligned}$$

where $\varphi_m = r(1-v)/(\theta\alpha - vq) > \varphi$. Hence, $\Delta H(\varphi) < 0$ and $d\Delta H(\varphi)/d\varphi > 0$.

It follows from (i)-(iii) that if $\varphi > \varphi_a$, then $\Delta H(\varphi)$ is negative, decreases in φ until reaching its minimum at $\varphi = \varphi_a^*$, and increases in φ thereafter.

The effect on $\Delta F(\varphi)$

First, assume that $\varphi_b^* > \varphi_x$

(iv) If $\varphi_x^* < \varphi \leq \varphi_x$, then $F(\varphi) = 0$ and $F^*(\varphi) = \theta(\alpha - m/\varphi)$. Hence, $\Delta F(\varphi) = F^*(\varphi) > 0$ and $d\Delta F(\varphi) = \theta m/\varphi^2 > 0$.

(v) If $\varphi_x < \varphi \leq \varphi_b^*$, then $F(\varphi) = \theta[\alpha - r/(\theta\varphi)]$ and $F^*(\varphi) = \theta(\alpha - m/\varphi)$. Hence, $\Delta F(\varphi) = (r - \theta m)/\varphi > 0$ and $d\Delta F(\varphi)/d\varphi = -(r-\theta m)/\varphi^2 < 0$.

(vi) If $\varphi > \varphi_b^*$, then $F(\varphi) = \theta[\alpha - r/(\theta\varphi)]$ and $F^*(\varphi) = v[q - m/\varphi + v\theta(\alpha - m/\varphi)] / (1+v^2)$. Hence,

$$\begin{aligned}
\Delta F(\varphi) &= \frac{v[q - m/\varphi + v\theta(\alpha - m/\varphi)]}{1+v^2} - \theta\alpha + \frac{r}{\varphi} \\
&= \frac{(\theta\alpha - vq)(\varphi_m/\varphi - 1)}{1+v^2} \\
&> 0
\end{aligned}$$

and $d\Delta F(\varphi)/d\varphi < 0$.

Next, assume that $\varphi_b^* \leq \varphi_x$

(vii) If $\varphi_x^* < \varphi \leq \varphi_b^*$, then $F(\varphi) = 0$ and $F^*(\varphi) = \theta(\alpha - m/\varphi)$. Hence, $\Delta F(\varphi) = F^*(\varphi) > 0$ and $d\Delta F(\varphi)/d\varphi = \theta m/\varphi^2 > 0$.

$(viii)$ If $\varphi_b^* < \varphi \leq \varphi_x$ (which is empty if $\varphi_b^* = \varphi_x$), then $F(\varphi) = 0$ and $F^*(\varphi) = v[q - m/\varphi + v\theta(\alpha - m/\varphi)]/(1+v^2)$. Hence, $\Delta F(\varphi) = F^*(\varphi) > 0$ and $d\Delta F(\varphi)/d\varphi = vm(1+v\theta)/[\varphi^2(1+v^2)] > 0$.

(ix) If $\varphi > \varphi_x$, then $F(\varphi) = \theta[\alpha - r/(\theta\varphi)]$ and $F^*(\varphi) = v[q - m/\varphi + v\theta(\alpha - m/\varphi)] / (1+v^2)$. As in (vi), we have that $\Delta F(\varphi) > 0$ and $d\Delta F(\varphi)/d\varphi < 0$.

It follows from (iv)-(ix) that if $\varphi > \varphi_x^*$, then $\Delta F(\varphi)$ is positive, increases in φ until reaching its maximum at $\varphi = \varphi_x$, and decreases in φ thereafter. $\square$

Proof of Proposition 4

(i) If $\varphi_a < \varphi \leq \varphi_x^*$, then $H(\varphi) > 0$, $H^*(\varphi) \geq 0$, and $F(\varphi) = F^*(\varphi) = 0$. So, $\Delta Z(\varphi) = \Delta H(\varphi) < 0$.

(ii) If $\varphi_x^* < \varphi \leq \varphi_x$, then $H(\varphi) > 0$, $H^*(\varphi) > 0$, $F(\varphi) = 0$, and $F^*(\varphi) > 0$. So, $\Delta Z(\varphi) = \Delta H + F^*(\varphi)$ which increases with firm productivity (since both $\Delta H(\varphi)$ and $F^*(\varphi)$ increase with firm productivity).

(iii) If $\varphi > \varphi_x$, then $H(\varphi) > 0$, $H^*(\varphi) > 0$, $F(\varphi) > 0$, and $F^*(\varphi) > 0$. Now,

$$H(\varphi) + vF(\varphi) = q - \frac{r}{\varphi} + v\theta\left(\alpha - \frac{r}{\theta\varphi}\right), \tag{16}$$

while $H^*(\varphi) + vF^*(\varphi)$ is given by (12). By subtracting (16) from (12), we obtain $\Delta H(\varphi) + v\Delta F(\varphi) = 0$ and hence $\Delta Z(\varphi) = \Delta H(\varphi) + \Delta F(\varphi) = (1-v)\Delta F(\varphi)$. Since $\Delta F(\varphi) > 0$, it follows that $\Delta Z(\varphi) \gtreqless 0$ as $v \lesseqgtr 1$ or, equivalently, that $\Delta Z(\varphi) \gtreqless 0$ as $m \lesseqgtr \tilde{m}$. Continuity of $\Delta Z(\varphi)$ implies that $\Delta Z(\varphi) \gtreqless 0$ as $m \lesseqgtr \tilde{m}$ also at $\varphi = \varphi_x$.

(iv) If $\varphi_x^* < \varphi < \varphi_x$, then (ii) and (iii) imply that if $m < \tilde{m}$, then there exists a firm productivity, $\varphi_0(m) \in (\varphi_x^*, \varphi_x)$, such that $\Delta Z(\varphi) \lesseqgtr 0$ as $\varphi \lesseqgtr \varphi_0(m)$, and that if $m \geq \tilde{m}$, then $\Delta Z(\varphi) < 0$.

Together, (i), (iii) and (iv) prove Proposition 4. $\square$

Appendix B

Extension to CES Demands

The Baseline

We here assume that consumers' demand, rather than being determined by a reservation price, is given by a CES function. Thus, if p_h and p_f denote the prices in terms of the numeraire good in the home and foreign markets, respectively, then $Hp_h^{-\sigma}$ and $Fp_f^{-\sigma}$, $\sigma > 1$, are the demands in the two markets. Since the CES demand is positive for arbitrarily high prices, if the investments costs of acquiring home and foreign markets of sizes H and F are given by $\frac{1}{2}H^2$ and $\frac{1}{2}F^2$ of the numeraire good, respectively, any firm no matter how low its productivity will make positive investments in both markets (as $\lim_{H\to 0}\left[d(\frac{1}{2}H^2)/dH\right] = \lim_{F\to 0}\left[d(\frac{1}{2}F^2)/dF\right] = 0$). Hence, in order to have positive activity and export cutoffs, we let the investment cost of acquiring the home and foreign markets be $H + \frac{1}{2}H^2$ and $F + \frac{1}{2}F^2$ of the numeraire good, respectively (as $\lim_{H\to 0}\left[d(H + \frac{1}{2}H^2)/dH\right] = \lim_{F\to 0}\left[d(F + \frac{1}{2}F^2)/dF\right] = 1 > 0$).

The export cost constitutes a fraction of the price charged in the foreign market. If the realized export cost is low (high) the FOB price for the firm's export is $\alpha p_{f\alpha}$ ($\beta p_{f\beta}$), where $0 < \beta < \alpha < 1$ and $p_{f\alpha}$ ($p_{f\beta}$) is the price charged in the foreign market.[19] The corresponding price in the home market is $p_{h\alpha}$ ($p_{h\beta}$). It follows that for a given $x \in \{\alpha, \beta\}$, the firm's revenue from sales at home and abroad minus its cost of production is $H\left[p_{hx}^{1-\sigma} - (w/\phi)p_{hx}^{-\sigma}\right] + F\left[xp_{fx}^{1-\sigma} - (w/\phi)p_{fx}^{-\sigma}\right]$.

The firm's expected profit taking the investment costs for the home and foreign markets

[19] For convenience, α and β are defined differently from in the model with reservation prices.

into account is

$$
\begin{aligned}
&\theta\left[H\left(p_{h\alpha}^{1-\sigma}-\frac{w}{\phi}p_{h\alpha}^{-\sigma}\right)+F\left(\alpha p_{f\alpha}^{1-\sigma}-\frac{w}{\phi}p_{f\alpha}^{-\sigma}\right)\right]\\
&+(1-\theta)\left[H\left(p_{h\beta}^{1-\sigma}-\frac{w}{\phi}p_{h\beta}^{-\sigma}\right)+F\left(\beta p_{f\beta}^{1-\sigma}-\frac{w}{\phi}p_{f\beta}^{-\sigma}\right)\right]\\
&-H-\tfrac{1}{2}H^2-F-\tfrac{1}{2}F^2.
\end{aligned}
\tag{17}
$$

The firm's capacity constraint is

$$
\frac{1}{\phi}\left(Hp_{hx}^{-\sigma}+Fp_{fx}^{-\sigma}\right)\leq N \text{ for } x\in\{\alpha,\beta\}, \tag{18}
$$

and the WPC is

$$
\frac{w}{\phi}\left[\theta\left(Hp_{h\alpha}^{-\sigma}+Fp_{f\alpha}^{-\sigma}\right)+(1-\theta)\left(Hp_{h\beta}^{-\sigma}+Fp_{f\beta}^{-\sigma}\right)\right]\geq rN. \tag{19}
$$

In order to maximize its expected profit (17) subject to the capacity constraint (18) and the WPC (19), the firm at the first stage (i.e., before the realization of the export cost) chooses H, F, w, and N, and at the second stage (i.e., after the realization of the export cost) chooses p_{hx} and p_{fx} for $x \in \{\alpha, \beta\}$ and employes the workers necessary to satisfy the demand.

At the second stage, due to the demand at home and the unit cost of production being independent of the export cost, the firm's price at home is independent of the export cost, i.e., $p_h = p_{h\alpha} = p_{h\beta}$. Hence, the firm's income from output sold at home less the cost of producing this output is $H\left[p_h^{1-\sigma} - (w/\phi)p_h^{-\sigma}\right]$. Further, if the firm exports, due to the elasticity of demand and the unit cost of production being identical for the two markets, the firm's prices satisfy $p_{fx} = p_h/x$ for $x \in \{\alpha, \beta\}$. Hence, the firm's income from output sold abroad less the cost of producing this output is $F\left[xp_{fx}^{1-\sigma} - (w/\phi)p_{fx}^{-\sigma}\right] =$

$x^\sigma F\left[p_h^{1-\sigma}-(w/\phi)p_h^{-\sigma}\right]$ for $x\in\{\alpha,\beta\}$. The firm's expected profit (17) is

$$(H+\ell F)\left(p_h^{1-\sigma}-\frac{w}{\phi}p_h^{-\sigma}\right)-H-\tfrac{1}{2}H^2-F-\tfrac{1}{2}F^2, \tag{20}$$

where $\ell\equiv\theta\alpha^\sigma+(1-\theta)\beta^\sigma$. Since $\alpha>\beta$, the capacity constraint (18) is

$$\frac{1}{\phi}(H+\alpha^\sigma F)p_h^{-\sigma}\leq N, \tag{21}$$

and the WPC (19) is

$$\frac{w}{\phi}\left(H+\ell F\right)p_h^{-\sigma}\geq rN. \tag{22}$$

At the second stage the firm will want to set $p_h=w/\left(\phi\rho\right)$ and employ the corresponding $(H+x^\sigma F)\phi^{\sigma-1}(\rho/w)^\sigma$ workers for $x\in\{\alpha,\beta\}$. The expected profit will be

$$(H+\ell F)\left(\frac{\phi\rho}{w}\right)^{\sigma-1}\frac{1}{\sigma}-H-\tfrac{1}{2}H^2-F-\tfrac{1}{2}F^2. \tag{23}$$

In order to be able to employ $(H+x^\sigma F)\phi^{\sigma-1}(\rho/w)^\sigma$ workers at the second stage, the firm must have hired that many workers at the first stage. Hence, the firm's choice of H, F, w, and N at the first stage must satisfy the capacity constraint (21) and the WPC (22) for $p=w/(\phi\rho)$. As both constraints will be binding for the firm to maximize its expected profit, the capacity constraint together with the WPC show that the baseline wage and the number of hired workers satisfy

$$w=\frac{r(H+\alpha^\sigma F)}{H+\ell F}, \tag{24}$$

$$N=\frac{(H+\ell F)^\sigma\varphi^{\sigma-1}\rho^\sigma}{(H+\alpha^\sigma F)^{\sigma-1}r^\sigma}. \tag{25}$$

If the realized export cost is low the firm employs more workers than if the realized export cost is high. The consequence is that some workers are laid off if the realized export cost is high.

Substituting w from (24) into (23) yields that as a function of H and F the expected profit is

$$(H+\ell F)^{\sigma}\left[\frac{\phi\rho}{r\,(H+\alpha^{\sigma}F)}\right]^{\sigma-1}\frac{1}{\sigma}-H-\tfrac{1}{2}H^{2}-F-\tfrac{1}{2}F^{2}. \tag{26}$$

At the first stage the firm therefore chooses H and F to maximize (26), and w and N to satisfy (24) and (25).

Differentiating (26) with respect to H and F shows that in the baseline case a positive H satisfies

$$\left(\frac{H+\ell F}{H+\alpha^{\sigma}F}\right)^{\sigma-1}\left[1-\rho\left(\frac{H+\ell F}{H+\alpha^{\sigma}F}\right)\right]\left(\frac{\phi\rho}{r}\right)^{\sigma-1}-1-H=0, \tag{27}$$

and a positive F satisfies

$$\left(\frac{H+\ell F}{H+\alpha^{\sigma}F}\right)^{\sigma-1}\left[\ell-\rho\alpha^{\sigma}\left(\frac{H+\ell F}{H+\alpha^{\sigma}F}\right)\right]\left(\frac{\phi\rho}{r}\right)^{\sigma-1}-1-F=0. \tag{28}$$

Since $\ell<\alpha^{\sigma}<1$, for given H and F the bracketed term in (27) exceeds the bracketed term in (28), which implies that the baseline activity cutoff, φ_a, is less than the baseline export cutoff, φ_x, and also that $H>F$ for $\varphi>\varphi_a$. Hence, if $\varphi\in[\varphi_a,\varphi_x]$, then (27) shows that

$$H=\frac{1}{\sigma}\left(\frac{\phi\rho}{r}\right)^{\sigma-1}-1. \tag{29}$$

The baseline activity and export cutoffs are[20]

$$\varphi_a=\frac{r\sigma^{1/(\sigma-1)}}{\rho};$$
$$\varphi_x=\frac{r}{\rho\,(\ell-\rho\alpha^{\sigma})^{1/(\sigma-1)}}.$$

[20] The baseline activity cutoff is obtained from (29) by setting $H=0$, and the baseline export cutoff from (28) by setting $F=0$.

Minimum Wage

Suppose that the firm can only hire workers by paying the minimum wage, m, which exceeds the baseline wage. The firm then maximizes its expected profit, which is (17) with m substituted for w, subject to the capacity constraint (18) and the WPC, which is (19) with m substituted for w. At the first stage the firm chooses H, F, and N, and at the second stage p_{hx} and p_{fx} for $x \in \{\alpha, \beta\}$ and produces to satisfy the demand.

Analogous to the baseline case, at the second stage the firm wants to set $p_{h\alpha} = p_{h\beta} = m/(\phi\rho)$; if the firm exports, to set $p_{fx} = m/(\phi\rho x)$ for $x \in \{\alpha, \beta\}$; and to employ the corresponding $(H + x^{\sigma}F)\phi^{\sigma-1}(\rho/m)^{\sigma}$ workers for $x \in \{\alpha, \beta\}$. By (23) its expected profit will be

$$(H + \ell F)\left(\frac{\phi\rho}{m}\right)^{\sigma-1}\frac{1}{\sigma} - H - \tfrac{1}{2}H^2 - F - \tfrac{1}{2}F^2. \tag{30}$$

At the first stage the firm hires the number of workers needed to produce the output demanded if the realized export cost is low, i.e., the binding capacity constraint is

$$N = \frac{(H + \alpha^{\sigma}F)\varphi^{\sigma-1}\rho^{\sigma}}{m^{\sigma}}. \tag{31}$$

Using (22) with the price at home being $m/(\phi\rho)$ and (31), in terms of H and F the WPC is

$$m \geq \frac{r(H + \alpha^{\sigma}F)}{H + \ell F}. \tag{32}$$

Therefore, at the first stage the firm chooses H and F to maximize (30) while satisfying (32), and N to satisfy (31).

If the WPC is slack, differentiating (30) with respect to H and F shows that with the minimum wage H and F, if positive, are given by

$$H^* = \frac{1}{\sigma}\left(\frac{\phi\rho}{m}\right)^{\sigma-1} - 1; \tag{33}$$

$$F^* = \frac{\ell}{\sigma}\left(\frac{\phi\rho}{m}\right)^{\sigma-1} - 1. \tag{34}$$

The activity and export cutoffs with the minimum wage are therefore

$$\varphi_a^* = \frac{m\sigma^{1/(\sigma-1)}}{\rho};$$

$$\varphi_x^* = \frac{m\sigma^{1/(\sigma-1)}}{\rho\ell^{1/(\sigma-1)}}.$$

Both increase with m and $\varphi_a^* < \varphi_x^*$.

Effects of the Minimum Wage

Since $\varphi_a^* - \varphi_a = (m - r)\,\sigma^{1/(\sigma-1)}/\rho > 0$, the activity cutoff with a minimum wage exceeds the activity cutoff in the baseline case. As $F^* = 0$ at φ_x^*, there exists a $\varphi_b^* > \varphi_x^*$ such that the WPC is slack for $\varphi \leq \varphi_b^*$. To show that the export cutoff with a minimum wage may be less than the export cutoff in the baseline case, consider the special case where $\sigma = 2$ and $\theta = \frac{1}{2}$. Then

$$\varphi_x - \varphi_x^* = \frac{4\left[r(1 + \alpha^2/\beta^2) - 2m\right]}{\alpha^2 + \beta^2}.$$

It follows that if $m < \frac{1}{2}r(1+\alpha^2/\beta^2)$, then $\varphi_x^* < \varphi_x$.[21] That is, the export cutoff with a minimum wage is less than the export cutoff in the baseline case. Consequently, the introduction of a minimum wage increases an exporter's foreign-market size for some productivities above φ_x and causes a non-exporter to start exporting for productivities $\varphi \in (\varphi_x^*, \varphi_x]$.

□

[21] The minimum wage must exceed the baseline wage. This is possible since (24) together with the fact that $H > F$ for $\varphi > \varphi_a$ imply that the baseline wage is less than $2r\alpha^2/\left(\alpha^2 + \beta^2\right)$ and hence less than $\frac{1}{2}r\left(1 + \alpha^2/\beta^2\right)$.