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ABSTRACT

Competitive Search with Private Information: Can Price Signal Quality?*

This paper considers competitive search equilibrium in a market for a good whose quality differs across sellers. Each seller knows the quality of the good that he or she is offering for sale, but buyers cannot observe quality directly. We thus have a "market for lemons" with competitive search frictions. In contrast to Akerlof (1970), we prove the existence of a unique equilibrium, which is separating. Higher-quality sellers post higher prices, so price signals quality. The arrival rate of buyers is lower in submarkets with higher prices, but this is less costly for higher-quality sellers given their higher continuation values. For some parameter values, higher-quality sellers post the full-information price; for other values these sellers have to post a higher price to keep lower-quality sellers from mimicking them. In an extension, we show that if sellers compete with auctions, the reserve price can also act as a signal.

JEL Classification: C78, D82, D83
Keywords: competitive search, signaling

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1 Introduction

This paper considers competitive search equilibrium in a market for a good whose quality differs across sellers. Each seller knows the quality of the good that he or she is offering for sale, but buyers cannot observe quality directly. We thus have a "market for lemons" (Akerlof, 1970) with competitive search frictions, and the question we address is whether the price a seller posts can signal quality in this environment.

Assuming that the value of retaining a high-quality good is greater than that of retaining a low-quality good,[1] we prove the existence of a unique equilibrium. This unique equilibrium is separating. Higher-quality sellers post higher prices, so price signals quality. The arrival rate of buyers is lower in submarkets with higher prices, but this is less costly for higher-quality sellers given their higher continuation values. That is, sellers can use posted prices as a signaling device.

The existence of (competitive) search frictions is important for our result because a seller who posts a higher price *endogenously* has a smaller trading probability, which is the cost of signaling. In the absence of search frictions, there is no reason why firms with higher prices meet fewer buyers.

In our benchmark model, we assume that seller types are discrete, and meetings are bilateral (each agent can meet at most one counterparty). We show that when the difference between the values of high-quality goods and low-quality goods is large, high-quality sellers post the full-information price. When the difference is small, sellers of high-quality goods post a higher price to keep sellers of low-quality goods from mimicking them. There is an interesting technical challenge in proving the existence of the unique separating equilibrium, namely, the isoprofit curves in price - (perceived) quality space are non-monotonic. Nonetheless, we prove single-crossing.

We then extend our model to the case of a continuum of seller types and show that the equilibrium can be characterized by a differential equation. In this case, sellers never post the full-information price. Finally, we relax the assumption that

[1] Why would the value of retaining the good be higher for high-quality goods? First, in many markets, the seller continues to consume the good if it is not sold. Second, even if a seller does not consume the good, a characteristic that is hidden today can become public knowledge in the future and this makes the continuation value of carrying an unsold high-quality good larger than that of carrying an unsold low-quality good.

sellers can meet at most one buyer and allow sellers to meet multiple buyers and compete with auctions. The separating equilibrium arises again: sellers of higher-quality goods post higher reserve prices and accept a lower trading probability.

There are several other papers that combine search and information frictions, but in most of these papers, the terms of trade are posted by agents on the uninformed side of the market. For example, Guerrieri et al. (2010) show that with bilateral meetings, when the uninformed party screens the other side by posting contracts, the unique equilibrium is separating.[2] Auster and Gottardi (2019) show that with many-on-one (urn-ball, specifically) meetings, agents on the uninformed side all post the same contract, and a pooling equilibrium arises.

The nature of equilibrium when the terms of trade are posted on the informed side of the market is less well understood. For example, Guerrieri et al. (2010) conjecture in their conclusion that when the informed party can signal their type that, as in competitive markets, multiple equilibria exist. Their conjecture is in line with the literature on adverse selection without search frictions. Wilson (1980) and Hellwig (1987) show that the predictions of a model with asymmetric information and no search frictions depend on details such as the order of decisions and who posts prices.[3] In contrast, in our competitive search setting, we get a unique separating equilibrium when the informed party sets the terms of trade.

There are also a few papers with search frictions and asymmetric information in which the informed side posts the terms of trade, but these are based on very different models than ours. Introducing search frictions, but using a random search model, Barsanetti and Camargo (2022) show that the outcome is similar to that of a frictionless model and that it matters whether the informed (signaling) or uninformed party (screening) sets the terms of trade. Delacroix and Shi (2013) analyze a model with competitive search in which all sellers choose between two qualities and have the same continuation value regardless of the quality of the good they offer. They show that there exists pooling equilibria, some in which all firms produce the low quality and some in which all produce the high quality. Menzio (2007), Kim (2012), and Kim and Kircher (2015) assume that the informed side

[2] Similarly, Inderst and Muller (2002) assume that there exist a large number of market makers who compete in creating submarkets as in Moen (1997), which, they argue, is equivalent to the uninformed side posting the prices. Chang (2018) considers a screening model in which the informed side has two-dimensional private information.

[3] Absent search frictions, equilibrium need not exist (Rothschild and Stiglitz, 1976).

announces cheap talk messages about quality. Depending on the details of the setup, cheap-talk messages can be fully or partially informative.

In finance, there is a substantial literature starting with Leland and Pyle (1977) that shows that retention, i.e., the fraction of an asset that the seller wants to retain, can act as a signal for the quality of an asset. In Guerrieri and Shimer (2014), high-quality assets trade at higher prices but with a lower price-dividend ratio and a lower probability of trade. However, in these models there are no search frictions.[4]

Finally, there are papers that combine signaling and competitive search in which the uncertainty is not about the quality of the good, e.g., Albrecht et al. (2016) (uncertainty is about how eager the seller is to sell) and Moon (2023) (uncertainty is about seller advertising intensity and the corresponding queue length). There are also competitive models, e.g., Milgrom and Roberts (1986) in which introductory prices and advertising intensity can, in the presence of repeat sales, signal quality.

The paper is organized as follows. First, in Section 2 we present the model, define equilibrium and derive the (constrained) efficient Planner's solution for the full-information case. Then, in Section 3, we prove that a unique separating equilibrium exists and derive conditions under which separation of high-quality sellers is efficient. We do this both for a finite number of seller types and for a continuum of types. In section 3.4, we show that our results also hold for many-on-one meetings in which sellers compete with auctions. In section 4, we discuss why a unique separating equilibrium exists in our model with search frictions, while absent search frictions, a separating equilibrium only exists in a very special knife-edge case, and then only among a continuum of other equilibria. Finally, Section 5 concludes.

2 The Model

Agents. We consider a static economy with a continuum of risk-neutral homogeneous buyers and heterogeneous sellers. Each seller owns a single unit of an indivisible good, for which each buyer has unit demand. The good's quality can differ, and a fraction z_i of sellers are of type i, where type refers to the quality of the good they offer. A good with quality i has a value c_i to the seller and a value

[4] Williams (2021) presents a screening model of retention and liquidity (the probability of selling a security) in which there are search frictions.

$v_i = \nu(c_i)$ to the buyers, where $c_i \in [\underline{c}, \overline{c}]$ and $c_1 < \cdots < c_I$. We assume $\nu'(c) > 0$ for all c.[5] That is, both sellers and buyers prefer the high-quality goods over the low-quality goods. The quality of the good is a seller's private information. The measure of sellers is 1, and the measure of buyers is determined by free entry in the sense that there is a large measure of buyers each of whom can participate in the market if and only if they pay a cost $k > 0$. We denote the gains from trade by $s_i = v_i - c_i$ and assume that $s_i > k$ for each i, so there are always gains from trade.

Search. In the first stage, each seller posts and commits to a price. After observing all posted prices, each buyer chooses one seller to visit. We refer to all buyers and sellers who choose a particular price as a *submarket*. Within a submarket, meetings are bilateral; that is, a seller meets at most one buyer and a buyer meets at most one seller. The total number of meetings are given by a constant-returns-to-scale matching function $M(N_s, N_b)$ where N_s and N_b are respectively the measures of sellers and buyers within the submarket. Define $\lambda = N_b/N_s$ as the *queue length* in the submarket. Then, $m(\lambda) \equiv M(1, \lambda)$ is the probability that a seller meets a buyer, and similarly, $q(\lambda) \equiv m(\lambda)/\lambda$ is the probability that a buyer meets a seller. We assume that $m(\lambda)$ is strictly increasing and strictly concave, which implies that $q(\lambda)$ is strictly decreasing.

Payoffs. Let $\Lambda(p)$ be the queue length and $\Gamma(p) = (\gamma_1(p), \ldots, \gamma_I(p))$ be the expected seller composition in submarket p, where $\gamma_i(p)$ is the expected fraction of type i sellers in the submarket.[6] Buyers' beliefs are then formally represented by $\Gamma(p)$. Note that we require $\Lambda(p)$ and $\Gamma(p)$ to also be defined for prices that are not posted in equilibrium.

The problem of a seller with a good of quality i is given by

$$\pi_i^* = \max_{p > c_i} m\left(\Lambda(p)\right)(p - c_i), \tag{1}$$

where π_i^* is the equilibrium payoff of type i sellers.

[5] We assume that $\nu(c)$ is strictly increasing to simplify the exposition. Our model also allows for constant $\nu(c)$. In that case, sellers' private information is irrelevant for buyers' decisions, and the equilibrium is the same as the one with complete information (see Example 1 after Proposition 1). Note also that in our benchmark model of discrete seller types, the values of $\nu(c)$ for $c \notin \{c_1, \ldots, c_I\}$ are irrelevant. However, later we consider the case of a continuum of seller types and therefore we introduce the function $\nu(c)$ from the start.

[6] Note that $\Gamma(p)$ is a probability simplex. That is, $\gamma_i(p) \geq 0$ and $\sum_1^I \gamma_i(p) = 1$.

Given beliefs $\Gamma(p)$, for each price p (both on and off the equilibrium path), the queue length $\Lambda(p)$ is determined by the buyers' free-entry condition.

$$k = q\left(\Lambda(p)\right)\left(\sum_{i=1}^{I} \gamma_i(p) v_i - p\right), \tag{2}$$

where $\hat{v} = \sum_{i=1}^{I} \gamma_i(p) v_i$ is the expected value of the good associated with price p. If $k \geq q(0)(\sum_1^I \gamma_i(p) v_i - p)$, then $\Lambda(p) = 0$. Note that, for a given price, the queue length is longer if buyers believe that it is more likely that the seller offers a high-quality good. Similarly, if we fix the buyers' belief $\hat{v}$ about the quality of the good, the queue length is shorter if a seller posts a higher price.

Equilibrium Definition. Let $F_i(p)$ be the distribution of prices posted by sellers of type i. Since the highest possible price is v_I and the lowest possible price is c_i, we have $F_i(c_i) = 0$ and $F_i(v_I) = 1$ for each i.[7]

We can now formally define a competitive search equilibrium as follows.

Definition 1. *A competitive search equilibrium is a tuple* $(\Lambda(p), \Gamma(p), F_i(p), \pi_i^*)$ *with the following properties:*

1. *Optimal Search: Given beliefs* $\Gamma(p)$*, for each price* p *(both on and off the equilibrium path), the queue length* $\Lambda(p)$ *is determined by equation* (2).

2. *Profit Maximization: Each* p *in the support of* $F_i(p)$ *solves the maximization problem given in equation* (1).

3. *Beliefs: For prices that are posted in equilibrium,* $\Gamma(p)$ *is given by Bayes' rule:*

$$\gamma_i(p) = \frac{z_i dF_i(p)}{\sum_{i=1}^{I} z_i dF_i(p)},$$

where the numerator is the measure of type i *sellers who post a price equal to* p*, and the denominator is the corresponding measure for all sellers.*[8]

For prices that are not posted in equilibrium, $\Gamma(p)$ *is such that for each* i*,* $\pi_i^* \geq m\left(\Lambda(p)\right)(p - c_i)$*, with equality if* $\Lambda(p) > 0$ *and* $\gamma_i(p) > 0$.

[7] We assume that sellers always choose to be active by posting some price, which is without loss of generality since posting a price is costless.

[8] Formally, $\gamma_i(p)$ is a Radon–Nikodym derivative.

Our definition of competitive search equilibrium is standard, except that we need to impose an additional assumption on how buyers' beliefs are formed off the equilibrium path. Note that, given c_i, the expected profit of a type i seller who posts a deviant price p' depends only on the queue length $\Lambda(p')$. Hence, for each type of seller there exists a threshold queue length above which posting p' will yield a profit strictly higher than their equilibrium payoff.[9] Then $\Gamma(p')$ puts all weight on the seller type who has the lowest threshold. That is, $\gamma_\iota(p) = 1$ if ι minimizes $\pi_i^*/(p' - c_i)$ for $i \in \{1, \dots, I\}$ with $p' > c_i$.

Following Guerrieri et al. (2010), we can alternatively think of the following hypothetical adjustment process, which pins down buyers' belief $\Gamma(p')$ for some deviant price p'. Suppose that a measure zero of sellers accidentally post p' and a small, positive measure of buyers mechanically join the submarket. Then, we start with $\Lambda(p') = \infty$. If no seller wants to post a price p' even with $\Lambda(p') = \infty$ (the most favorable situation for sellers), then $\Gamma(p')$ is irrelevant. Otherwise, some additional sellers are willing to join this submarket, which then decreases the buyer-seller ratio $\Lambda(p')$. As sellers join and $\Lambda(p')$ decreases, some seller types will leave. We continue this process until only one type of seller is left in this submarket and is willing to offer p'. Hence, this adjustment process ends with a degenerate $\Gamma(p')$. Loosely speaking, $\Gamma(p')$ puts all the weight on the seller type that can tolerate the lowest possible queue length at price p'. As noted by Guerrieri et al. (2010), this is equivalent to the D1 refinement which is commonly used in signaling models (see Banks and Sobel (1987)).

The Social Planner's Problem. Before analyzing the decentralized market equilibrium, we now briefly write down and solve the social planner's problem. The planner faces the same coordination frictions (that are standard in competitive search models) as the market participants but faces no information constraints about quality. The planner's problem reduces to choosing an optimal queue length λ_i^p for each seller type i taking into account that the buyer participation cost is k.

[9] Note that if $p' < c_i + \pi_i^*$ for some i, then we can set this threshold to infinity for sellers of type i.

That is, λ_i^p, $i = 1, \ldots, I$, solves the following maximization problem:

$$\max_{\lambda_1,\ldots,\lambda_I} \quad \sum_{i=1}^{I} z_i s_i m(\lambda_i) - k \sum_{i=1}^{I} z_i \lambda_i.$$

Note that the measure of buyers in submarket i is equal to the buyer-seller ratio times the measure of sellers, $z_i\lambda_i$, for each i. Since the above problem is strictly concave, the following first-order condition is both necessary and sufficient, and admits a unique solution: for all i,

$$s_i m'(\lambda_i^p) = k.$$

3 Market Equilibrium

3.1 Competitive Search Equilibrium

We start with a bird's-eye view of the main force that drives equilibrium. Without loss of generality, we assume that $\Lambda(p)$ is strictly decreasing.[10] Furthermore, given that buyers can only observe prices, it seems natural to require buyers' beliefs to be such that $\Lambda(p)$ is strictly decreasing, which is also implied by the D1 refinement on beliefs. The sellers' problem in equation (1) is supermodular in (p, c), which implies that sellers of a higher type always choose (weakly) higher prices. Below we show that sellers of different types will not choose the same price in equilibrium and therefore the equilibrium is always separating.

Consider a type i seller who posts a price p. If this seller's good is believed to have value $\hat{v}$, then his or her expected profit is

$$\pi(c_i, p, \hat{v}) = m\left(q^{-1}\left(\frac{k}{\hat{v} - p}\right)\right)(p - c_i) \tag{3}$$

where we used equation (2) to solve for the queue length associated with $(p, \hat{v})$: $\lambda = q^{-1}(\frac{k}{\hat{v}-p})$. Note that here the queue length λ is associated with a hypothetical situation in which the posted price is p and the perceived value of the good is $\hat{v}$, and is not related to the equilibrium object $\Lambda(p)$. If the seller's iso-profit curves satisfy the single-crossing condition in the p-$\hat{v}$ plane, and if sellers of a higher type

[10] Suppose $p_a > p_b$ and $\Lambda(p_a) \geq \Lambda(p_b)$, then no sellers would choose price p_b.

have flatter iso-profit curves, then the least-cost separating equilibriun is the unique equilibrium and sellers of a higher type post a strictly higher price. [11] The seller's profit function $\pi(c_i, p, \hat{v})$ in equation (3) seems complicated. We now simplify the seller problem by treating queue length as the choice variable instead of price. For this purpose, we rewrite $\pi(c_i, p, \hat{v})$ as a function of c_i, λ, and $\hat{v}$,

$$\widetilde{\pi}(c_i, \lambda, \hat{v}) = m(\lambda)(\hat{v} - c_i) - \lambda k, \tag{4}$$

where we used $q(\lambda)(\hat{v}-p) = k$ and $m(\lambda) = \lambda q(\lambda)$ to substitute out p in equation (2) . Intuitively, a higher posted price (both on and off the equilibrium path) corresponds to a shorter queue length. Hence, we can think of sellers choosing queue length as a signaling device instead of the price. Below, we formally show that both approaches are equivalent.

To better understand equation (4), it is helpful to contrast it with the case in which a seller's type is public information. In that case, equation (4) becomes $m(\lambda)(v_i - c_i) - \lambda k$, where the first term is the expected surplus, the second term is the part of surplus attributed to the buyers (the cost of the queue), and the seller's profit is then the difference between the two terms.

When seller type is private information, the seller's profit in equation (4) has a similar interpretation. The total gain from trade is $\hat{v} - c_i$ instead of $v_i - c_i$ since when the transaction price is p, the seller's gain is $p - c_i$ and the buyer's perceived gain is $\hat{v} - p$. Here the queue only depends on the buyer's perceived payoff, not the actual payoff.

The seller's profit $\widetilde{\pi}(c_i, \lambda, \hat{v})$ defined by equation (4) is strictly increasing in $\hat{v}$, and is first increasing and then decreasing in λ. The slope of $\widetilde{\pi}(c_i, \lambda, \hat{v})$ in the λ-$\widehat{v}$ plane is given by

$$-\frac{\widetilde{\pi}_\lambda(c_i, \lambda, \hat{v})}{\widetilde{\pi}_{\hat{v}}(c_i, \lambda, \hat{v})} = -\frac{m'(\lambda)(\hat{v} - c_i) - k}{m(\lambda)}, \tag{5}$$

which is strictly *increasing* in c_i. Hence, $\widetilde{\pi}(c_i, \lambda, \hat{v})$ satisfies the single-crossing con-

[11] That is, the equilibrium in which there is no distortion at the bottom and type i sellers choose a price to maximize their expected profit under the constraints that i) they are perceived to be type i sellers, and ii) type $i-1$ sellers find it not profitable to mimic type i sellers. In this case, asymmetric information imposes minimal cost on type i sellers' expected profit assuming that the equilibrium is separating.

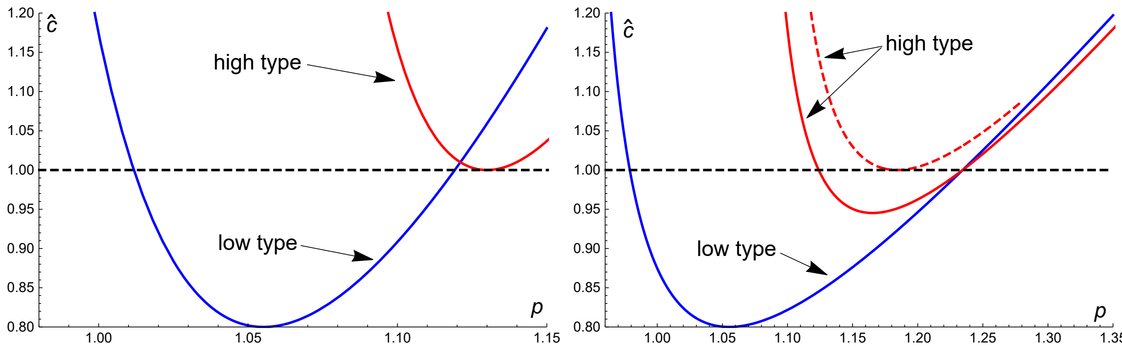
dition in the λ-$\widehat{v}$ plane: If $\widetilde{\pi}(c_i, \lambda_a, \hat{v}_a) = \widetilde{\pi}(c_i, \lambda_b, \hat{v}_b)$ for two points $(\lambda_a, \hat{v}_a)$ and $(\lambda_b, \hat{v}_b)$ with $\lambda_a < \lambda_b$, then $\widetilde{\pi}(c_j, \lambda_a, \hat{v}_a) > \widetilde{\pi}(c_j, \lambda_b, \hat{v}_b)$ for $c_j > c_i$.

Since sellers choose p as the signal instead of λ, we still need to show that $\pi(c_i, p, \hat{v})$ also satisfies the single-crossing condition (in the p-$\hat{v}$ plane).[12] We first show that the iso-profit curves of a seller are well-defined functions in the p-$\hat{v}$ plane. Holding p fixed, $\pi(c_i, p, \hat{v})$ is strictly increasing in $\hat{v}$, since a higher $\hat{v}$ increases λ in equation (3). That is, for a fixed price, a seller's expected profit is higher, the greater the likelihood that buyers attach to the seller offering a high-quality good. Holding $\hat{v}$ fixed, the price p affects sellers' expected profit solely through the queue length λ (see equation (4)). Since λ is strictly decreasing in p, $\pi(c_i, p, \hat{v})$ is first increasing and then decreasing in p. Therefore, the iso-profit curves are first decreasing and then increasing in p.

Next, we show that $\pi(c_i, p, \hat{v})$ also satisfies the single-crossing condition in the p-$\hat{v}$ plane, which follows simply from the correspondence $\pi(c, p, \hat{v}) = \widetilde{\pi}(c, q^{-1}(\frac{k}{\hat{v}-p}), \hat{v})$. To see this formally, suppose that we have $\pi(c_i, p_a, \hat{v}_a) = \pi(c_i, p_b, \hat{v}_b)$ for some seller type i and two points $(p_a, \hat{v}_a)$ and $(p_b, \hat{v}_b)$ with $p_a < p_b$. The corresponding queue lengths are $\lambda_j = q^{-1}(\frac{k}{\hat{v}_j - p_j})$ for $j = a, b$. Since $p_a < p_b$, by equation (3) we must have $\lambda_a > \lambda_b$ for sellers of type c_i to have the same profit between the two points. Since $\widetilde{\pi}(c_i, \lambda, \hat{v})$ satisfies the single-crossing condition and $\lambda_a > \lambda_b$, $\widetilde{\pi}(c_i, \lambda_a, \hat{v}_a) = \widetilde{\pi}(c_i, \lambda_b, \hat{v}_b)$ implies that $\widetilde{\pi}(c_j, \lambda_a, \hat{v}_a) < \widetilde{\pi}(c_j, \lambda_b, \hat{v}_b)$ for $c_j > c_i$. Hence $\pi(c_j, p_a, \hat{v}_a) < \pi(c_j, p_b, \hat{v}_b)$. So if seller c_i is indifferent between $\hat{v}_a$ and $\hat{v}_b$ with $p_a < p_b$, then seller $c_j > c_i$ prefers $\hat{v}_b$ with a corresponding higher p_b and a shorter queue. Figure 1 illustrates the above results by showing the sellers' iso-profit curves in the $(p, \hat{v})$ plane, when there are just two types of goods: low and high quality. Note that in the p-$\hat{v}$ plane, the iso-profit curves of sellers with a higher-quality product are flatter than the iso-profit curves of sellers with a lower-quality product. The opposite is true in the λ-$\hat{v}$ plane, since a higher p corresponds to a lower λ.

By the standard logic of signaling models, single crossing of sellers' iso-profit curves and the D1 criterion jointly imply that the equilibrium must be least-cost separating. Furthermore, the sellers of the lowest type c_1 choose an optimal price as

[12] In Appendix A.6. we give a direct, general proof that $\pi(c_i, p, \hat{v})$ satisfies the single-crossing condition in the p-$\hat{v}$ plane. The proof shows that our main result also applies to the case in which workers are risk-averse. We analyze our model in the $\lambda - \hat{v}$ plane since the characterization of the seller problem is easier in this case. See equations (8) and (9) below.



(a) Efficient equilibrium: $\beta = 0.1$ (b) Inefficient equilibrium: $\beta = 0.5$

Figure 1: Illustration of the competitive search equilibrium with two types of sellers: $m(\lambda) = 1 - e^{-\lambda}$, $k = 0.3$, $c_1 = 0.8, c_2 = 1, v_1 = 1.5$ and $\nu(c) = v_1 + \beta(c - c_1)$. To help compare figures (a) and (b), we set $\hat{c}$ instead of $\hat{v}$ as the y-axis where $\hat{v} = \nu(\hat{c})$ so that the positions of the black dashed lines in the two figures are the same.

if their type were publicly observable (no distortion at the bottom), and the sellers of type c_i choose an optimal price subject to the constraint that the sellers of type c_{i-1} do not want to mimic them. Since it does not matter whether we assume that sellers choose p or choose λ as the signal, we can adopt the simpler case which is the latter.

Denote the sellers' optimal queue length by λ_i^* and their equilibrium payoff by π_i^* for $i \geq 1$. Since there are always no distortions at the bottom, the problem of the sellers with the lowest-quality product is

$$\pi_1^* \equiv \max_{\lambda_1} s_1 m(\lambda_1) - \lambda_1 k, \tag{6}$$

where $s_1 = v_1 - c_1$. The optimal solution λ_1 is characterized by the FOC: $s_1 m'(\lambda_1) = k$, which implies that

$$\lambda_1^* = \Lambda_o(\frac{k}{s_1}), \tag{7}$$

where $\Lambda_o(\cdot)$ is the inverse function of $m'(\cdot)$ which represents the socially optimal λ.

The problem of sellers of type c_i with $i \geq 2$ is to choose a queue length that maximizes their profit subject to the constraint that it is not in the interest of sellers of type c_{i-1} to mimic them. That is,

$$\pi_i^* \equiv \max_{\lambda_i} s_i m(\lambda_i) - \lambda_i k, \tag{8}$$

subject to the incentive compatibility constraint (ICC) that λ_i must be such that

$$\pi_{i-1}^* = m(\lambda_{i-1}^*)(v_{i-1} - c_{i-1}) - \lambda_{i-1}^* k \geq m(\lambda_i)(v_i - c_{i-1}) - \lambda_i k, \tag{9}$$

where the right-hand side of (9) is the payoff from mimicking for a seller of type $i-1$, which follows from equation (4).

We now analyze the solution to the sellers' problem above. In typical signaling models, the iso-profit curves of sellers are monotonic and the ICC is binding. However, in our model the sellers' iso-profit curves are first decreasing and then increasing (both in the $(p, \hat{v})$ plane and the $(\lambda, \hat{v})$ plane), which implies that there are two solutions for λ_i such that the ICC constraint is binding, i.e., such that equation (9) holds with equality. To see this, note that the payoff from mimicking by sellers of type $i-1$, the right-hand side of equation (9), is strictly concave in λ_i. When $\lambda_i = \lambda_{i-1}^*$, the ICC is violated since $v_i > v_{i-1}$, and when $\lambda_i = 0$ or greater than $(v_i - c_{i-1})/k$, the ICC is satisfied trivially. Hence, the smaller solution is *strictly* between 0 and λ_{i-1}^* and the larger solution is *strictly* between λ_{i-1}^* and $(v_i - c_{i-1})/k$. The smaller solution of λ_i (that corresponds to a higher p_i) yields a higher profit for sellers of type i, since the slopes of their iso-profit curves (in the $\lambda - \hat{v}$ plane) are steeper. From now on, we denote by Λ_{icc}^i the smaller solution such that the ICC constraint (9) is binding. Then,

$$m(\lambda_{i-1}^*)(v_{i-1} - c_{i-1}) - \lambda_{i-1}^* k = m(\Lambda_{icc}^i)(v_i - c_{i-1}) - \Lambda_{icc}^i k, \text{ and } \Lambda_{icc}^i < \lambda_{i-1}^*. \tag{10}$$

Another consequence of sellers' iso-profit curves being non-monotonic is that the ICC constraint may not be violated at $\Lambda_o(\frac{k}{s_i})$, the optimal choice of λ_i under complete information. That is, $\Lambda_o(\frac{k}{s_i}) \leq \Lambda_{icc}^i$ (see the proof of Proposition 1 for more details). In this case, λ_i^* is simply $\Lambda_o(\frac{k}{s_i})$.

To sum up, the solution to the problem of sellers of type i is given by

$$\lambda_i^* = \min\{\Lambda_o(\frac{k}{s_i}), \Lambda_{icc}^i\}. \tag{11}$$

Figure 1 illustrates the above issues in the p-$\hat{v}$ plane (instead of in the λ-$\hat{v}$ plane) with two types of sellers ($I = 2$). The two intersections between the blue line and the black dashed line are the two prices where the ICC constraint is binding. In Figure 1a, the ICC constraint is not violated even when high-type sellers choose their unconstrained optimal value ($\lambda_2^* = \Lambda_o(\frac{k}{s_2})$). In Figure 1b, the opposite is true, and $\lambda_2^* = \Lambda_{icc}^2$, which corresponds to a price equal to 1.23 (the larger intersection point). Note also that the larger solution of λ_2 for which the ICC constraint is binding corresponds to a price equal to 0.98 (the smaller intersection point), which will yield a negative profit for the high-type sellers since $c_2 = 1$.

Thus, the sellers' problem can be solved recursively, starting from the problem of c_1 sellers where $\lambda_1^* = \Lambda_o(k/s_1)$. This can be summarized by the following proposition.

Proposition 1. *There exists a unique equilibrium, which is separating. In equilibrium, all sellers of type i post the same price p_i^* and attract queue length λ_i^* with $\lambda_1^* > \cdots > \lambda_I^* > 0$ and $p_1^* < \cdots < p_I^*$, where λ_1^* is given by equation (7), λ_i^* is recursively determined by equation (11) for $2 \leq i \leq I$, and $p_i^* = v_i - k/q(\lambda_i^*)$.*

Proof. See Appendix A.1. □

Signaling versus screening. Instead of having the informed side post prices to signal their quality, Guerrieri et al. (2010) analyze the case in which the uninformed side post contracts to screen different types of sellers. In the classic asymmetric information literature without search frictions, equilibrium outcomes can depend on which side posts trading arrangements. Notably, there is no separating equilibrium in the Akerlof (1970) lemons model. Thus, it is surprising that prices and allocations in equilibrium are the same in our signaling model and the screening model of Guerrieri et al. (2010).[13] Hence, in competitive search models, it does not matter whether buyers, sellers, or some third party (market makers) post the prices; the equilibrium outcomes are the same.

[13] Guerrieri et al. (2010) noted in their conclusion that "It may also be interesting to study the case opposite to the one analyzed here, where the informed instead of the uninformed parties post contracts." One reason that the equivalence between the two models is not trivial a priori may be a technical one. To establish separating equilibrium in a large class of models, Guerrieri et al. (2010) adopted a sorting condition, which is different from the standard single-crossing condition used in our paper. One consequence of this difference is that their characterizations are less sharp: the constraint faced by type i sellers in their model is that all lower types (from type 1 to $i-1$) will not mimic; in our model we only need to check that type $i-1$ sellers will not mimic.

When is the ICC binding? We present two special cases to illustrate the issues at stake. In the first example, the ICC constraint is never binding while in the second example, it is always binding. In both examples, we fully characterize the equilibrium.

Example 1: Constant v_i. Things are particularly simple in the extreme case where v_i is constant.[14] Then, buyers' values are the same across all sellers regardless of their c_i, making buyers' beliefs about sellers' types irrelevant. Sellers who post the same price will have the same expected queue length, regardless of their types. The model is equivalent to the standard competitive search equilibrium without private information, where sellers can buy queues in a competitive market at a price equal to k. This implies that sellers of type $i-1$ will not mimic sellers of type i when the latter choose their unconstrained optimal price. To see this formally, consider the ICC constraint (9). Assume that $\lambda_{i-1}^* = \Lambda_o(k/s_{i-1})$, so sellers of type $i-1$ reach their unconstrained maximum profit. Then, the left-hand side of (9) is always greater than the right-hand side since $v_i = v_{i-1}$, i.e., the ICC constraint is always satisfied. By continuity, the same conclusion holds when v_i's are sufficiently close. Figure 1a illustrates this scenario for the case of two seller types. $\square$

Example 2: Weakly increasing s_i. Suppose that the surplus from trade is weakly increasing in the type of sellers: $s_1 \leq \cdots \leq s_I$. Consider the ICC constraint (9) and assume that $\lambda_i^* = \Lambda_o(k/s_i)$. The right-hand side of (9) can be rewritten as

$$(m(\lambda_i^*)s_i - \lambda_i^* k) + m(\lambda_i^*)(c_i - c_{i-1}) > \max_{\lambda} m(\lambda)s_{i-1} - \lambda k$$

where the strict inequality follows from i) the first term on the left-hand side corresponds to $\max_\lambda m(\lambda)s_i - \lambda k$ and $s_i \geq s_{i-1}$, and ii) $c_i > c_{i-1}$. Therefore, the ICC constraint (9) is violated at $\lambda_i = \Lambda_o(k/s_i)$, which implies that the ICC constraint is always binding and $\lambda_i^* = \Lambda_{icc}^i$ in equation (11). $\square$

[14] This example is similar to the model of Albrecht et al. (2016) in which there are two types of sellers; low c (or high s) sellers, who are labeled motivated, and high c (or low s) sellers, and the value of the good to the buyers is independent of seller type.

3.2 The Case of Two Seller Types

We now characterize in detail the equilibrium for the case of two seller types ($I = 2$). A byproduct of this analysis is that the ICC constraint is binding in equilibrium when c_2 is sufficiently close to c_1, which greatly simplifies the analysis for the case of a continuum of seller types in the next subsection.

Below we solve the model by comparing $\Lambda_o(k/s_2)$ and Λ^2_{icc}. Assume $s_1 > s_2$, since for the other case $s_1 \leq s_2$, the ICC constraint is always binding (see Example 2 in the previous subsection). The proposition below confirms the insights from the two special examples above. For low-type sellers, the gain from mimicking high-type sellers is $v_2 - v_1$, while the cost is a lower matching probability. Hence, if $v_2 - v_1$ is small, the ICC constraint is not binding. Sellers' private information then does not matter, and the equilibrium is the same as the case without private information and is socially efficient. On the other hand, if $v_2 - v_1$ is large, the ICC constraint is binding. In this case, high-quality sellers must charge a higher price than in the full-information case to distinguish themselves from low-quality sellers, resulting in a shorter queue length, i.e., $\lambda_2^* < \Lambda_o(k/s_2)$. The following proposition presents the formal statement of the above results.

Proposition 2. *Assume $I = 2$ and $s_1 > s_2$, where $s_i = v_i - c_i$ for $i = 1, 2$. Define*

$$\Psi(k, s_1, s_2) \equiv \int_{s_2}^{s_1} \left(\frac{m(\Lambda_o(k/s))}{m(\Lambda_o(k/s_2))} - 1 \right) ds. \tag{12}$$

If $v_2 - v_1 \leq \Psi(k, s_1, s_2)$, then in equilibrium $\lambda_2^* = \Lambda_o(k/s_2)$; otherwise, $\lambda_2^* < \Lambda_o(k/s_2)$.

Proof. See Appendix A.2. □

The definition of $\Psi(k, s_1, s_2)$ seems complicated. However, it reduces to simple expressions for common meeting technologies. Normalize $s_1 = 1$. When $m(\lambda) = 1 - e^{-\lambda}$, $\Psi(k, 1, s_2) = \frac{k}{s_2 - k}(1 - s_2 + s_2 \log s_2)$. Similarly, when $m(\lambda) = \lambda/(1+\lambda)$, $\Psi(k, 1, s_2) = (1 - \sqrt{s_2})^2/(\sqrt{s_2/k} - 1)$. Note that $\Psi(k, s_1, s_2)$ is homogeneous of degree 1, since doubling both the cost k and the match values s_1 and s_2 naturally double Ψ, the threshold of the value difference (see Appendix A.2 for a detailed argument).

Next, we analyze what happens when seller types become arbitrarily close. Let $c_2 = c_1 + \Delta c$ and $s_2 = s_1 - \Delta s$. Since $s_1 > s_2$ and $v_2 - v_1 = \Delta c - \Delta s > 0$, we have $0 < \Delta s < \Delta c$. As $\Delta s \to 0$ (because $\Delta c \to 0$), the integrand on the right-hand side of (12) approaches zero, which implies that

$$\lim_{\Delta s \to 0} \frac{\Psi(k, s_1, s_2)}{\Delta s} = \lim_{\Delta s \to 0} \frac{m(\Lambda_o(k/\widetilde{s}))}{m(\Lambda_o(k/s_2))} - 1 = 0$$

where the first equality follows from the mean value theorem with some $\widetilde{s} \in [s_2, s_1]$, and the second from continuity. As an example, let $m(\lambda) = 1 - e^{-\lambda}$. Then $\Psi(k, 1, 1 - \Delta s) \approx \frac{k}{2(1-k)} \Delta s^2$ (second-order approximation), where we normalize $s_1 = 1$.

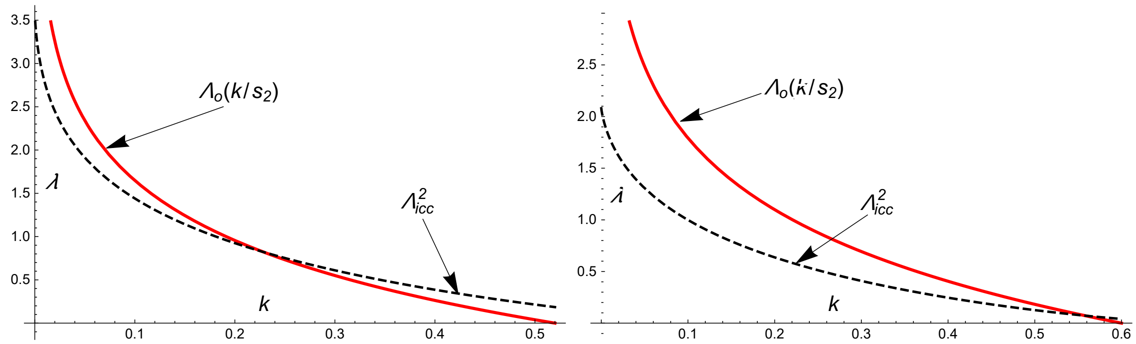
The above result has a striking consequence. For the equilibrium to be efficient (non-binding ICC), we need $\Delta v \leq \Psi(k, s_1, s_1 - \Delta s)$, which implies that $\Delta v / \Delta c \leq \Psi(k, s_1, s_1 - \Delta s)/\Delta c \leq \Psi(k, s_1, s_1 - \Delta s)/\Delta s \to 0$. Hence, for the ICC constraint to be non-binding when Δc is sufficiently small, Δv must be an order smaller than Δc, which then contradicts $\nu'(c_1) > 0$. The following proposition summarizes this result.

Proposition 3. *Assume $I = 2$ and let $c_2 = c_1 + \Delta c$. When Δc is sufficiently small, then the ICC constraint is binding for the high-type sellers, and hence the equilibrium is not socially efficient.*

Proof. See the above discussion. □

Next, we analyze how the equilibrium varies with buyers' entry cost k. Figure 2 shows $\Lambda_o(k/s_2)$ (red solid line) and Λ^2_{icc} (black dashed line) for the parameter values of Figure 1, except now we let k vary (x-axis). Note that $\Lambda_o(k/s)$ crosses Λ^2_{icc} exactly once and from above. That is, when k is small, $\Lambda_o(k/s_2) > \Lambda^2_{icc}$ (the ICC constraint is binding) and the opposite holds when k is large. Note that when $k = 0.3$ (the value in Figure 1), $\Lambda_o(k/s_2) < \Lambda^2_{icc}$ in Figure 2a and the opposite holds in Figure 2b. Thus, the equilibrium corresponds to the full-information equilibrium in Figure 1a and is inefficient in Figure 1b.

Below we show that the above results are true in general: The ICC constraint is binding if and only if k is small (so there are many buyers). For this we need to impose an assumption on the meeting technology, which is equivalent to assuming that $\Psi(k, s_1, s_2)$ is strictly increasing in k.



(a) $\beta = 0.1$ (b) $\beta = 0.5$

Figure 2: Illustration of $\Lambda_o(k/s_2)$ and Λ^2_{icc} as functions of k; note $\nu(c) = v_1 + \beta(c - c_1)$

Assumption 1. *The ratio* $\varepsilon_m(\lambda)/\varepsilon_2(\lambda)$ *is strictly decreasing in* λ*, where* $\varepsilon_m(\lambda) = \lambda m'(\lambda)/m(\lambda)$ *and* $\varepsilon_2(\lambda) = -\lambda m''(\lambda)/m'(\lambda)$*, which is the absolute value of the elasticity of* $m'(\lambda)$*.*

For common meeting technologies, $\varepsilon_2(\lambda)$ is strictly increasing. For example, when $m(\lambda) = 1 - e^{-\lambda}$ (urn-ball), $\varepsilon_2(\lambda) = \lambda$; when $m(\lambda) = \lambda/(1 + \lambda)$ (geometric), $\varepsilon_2(\lambda) = 2\lambda/(1 + \lambda)$. Since $\varepsilon_m(\lambda)$ is strictly decreasing for these common meeting technologies, the above assumption holds trivially.

Proposition 4. *Under Assumption 1,* $\Psi(k, s_1, s_2)$*, which is defined by equation* (12)*, is strictly increasing in* k *with* $\lim_{k\to 0} \Psi(k, s_1, s_2) = 0$ *and* $\lim_{k\to s_2} \Psi(k, s_1, s_2) = \infty$*.*

Proof. See Appendix A.3. □

Together with Proposition 2, the above result implies that the equilibrium is efficient if and only if k is above a certain threshold. Intuitively, when k is large, there are relatively few buyers. The queue length in the high-quality submarket is already small under full information, so low-quality sellers will choose not to join.

3.3 Continuum of Seller Types.

We now consider the case of a continuum of seller types and show that the equilibrium can be characterized as a differential equation. Define $s(c) = \nu(c) - c$ (surplus) with $c \in [\underline{c}, \overline{c}]$, $\Delta v_i = v_i - v_{i-1}$, and $\Delta\lambda^*_i = \lambda^*_i - \lambda^*_{i-1}$. Finally, set $c_i - c_{i-1} = \Delta c$ (equal distance).

The ICC constraint (9) is satisfied if and only if

$$m(\lambda^*_{i-1})\Delta v_i \le -\Delta\lambda^*_i \left(m'(\lambda^*_{i-1})s_{i-1} - k\right),$$

where we applied a first-order approximation to the right-hand side of the ICC constraint (9). Dividing both sides above by Δc and letting $\Delta c \to 0$ yields

$$m(\lambda(c))\nu'(c) \le -\lambda'(c)\left(m'(\lambda(c))s(c) - k\right), \tag{13}$$

where $\lambda(c)$ is the equilibrium queue length of type c sellers and $\Delta\lambda^*_i/\Delta c \to \lambda'(c)$ as $\Delta c \to 0$. If the ICC constraint is not binding for sellers of type c, then $m'(\lambda(c))s(c) = k$, and the right-hand side above is zero. Therefore, if $\nu'(c) > 0$, then the ICC constraint is always binding, which we already saw for the case of two seller types. Since the ICC is always binding, (13) holds with equality, which yields equation (14) below, a differential equation for $\lambda(c)$. We thus have the following proposition.

Proposition 5. *Consider the case of a continuum of seller types. Define $s(c) = \nu(c) - c$ (trade surplus). The equilibrium queue length $\lambda(c)$ is uniquely determined by the following differential equation*

$$\lambda'(c) = -\frac{m(\lambda(c))\nu'(c)}{m'(\lambda(c))s(c) - k}, \tag{14}$$

with $m'(\lambda(c))s(c) < k$ and the initial condition $\lambda(\underline{c}) = \Lambda_o(k/s(\underline{c}))$.

Proof. See Appendix A.4. □

In the special case where $s(c) = s_0$ (a constant) or equivalently $\nu(c) = c + s_0$ and the meeting rate is Poisson, $m(\lambda) = 1 - e^{-\lambda}$, we can solve the above differential equation analytically. In this case, the right-hand side of equation (14) depends only on $\lambda(c)$. We then use the inverse function of $\lambda(c)$: $c(\lambda)$ (note $c'(\lambda) = 1/\lambda'(c)$). Solving the above differential equation yields

$$c(\lambda) = \underline{c} + k\lambda - (s_0 - k)\log(1 - e^{-\lambda}) - s_0 \log\frac{s_0}{s_0 - k} + k\log\frac{k}{s_0 - k}. \tag{15}$$

The corresponding equilibrium price is then $p(c) = \nu(c) - k/q(\lambda(c))$. Figure 3 below compares the equilibrium prices (blue sold line) to the optimal prices, i.e., the prices that generate the socially optimal queue lengths (red dashed line). We see that there is no distortion at the bottom and that the distance between the market price that needs to satisfy the ICC and the socially optimal price under full information is increasing in seller type c.

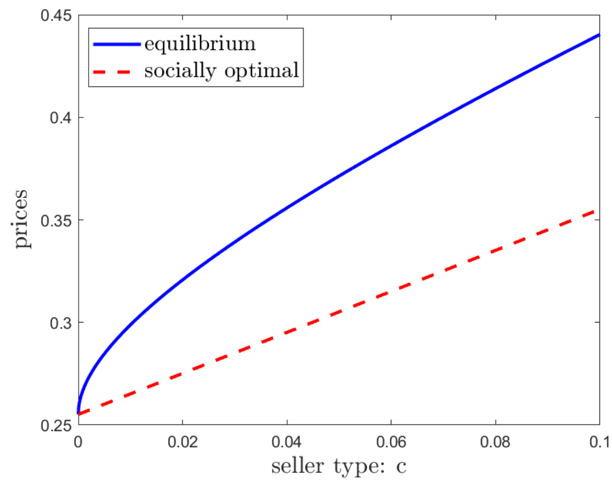


Figure 3: Comparison between equilibrium and socially optimal prices: $s(c) = 0.7$ for $c \in [0, 1]$, $k = 0.3$, and $m(\lambda) = 1 - e^{-\lambda}$

3.4 Competing Auctions

In some markets, like the housing market, it is more reasonable to allow sellers to use auctions rather than posted prices, since sellers can gather multiple buyers. We now show that our insights and analysis continue to hold in this environment.

Assume that the probability that a seller meets n buyers is $P_n(\lambda)$ for $n \geq 0$. Again let $m(\lambda) = 1 - P_0(\lambda)$, the probability that a seller meets at least one buyer. We assume that each seller posts a second-price auction with a reserve price r (since revenue equivalence holds, the exact format of the auction is irrelevant).

First, consider the counterpart of equation (3). The payoff of a seller who posts a reserve price r and is perceived to have value $\hat{v}$ is then

$$\pi(c_i, r, \hat{v}) = P_1(\lambda)(r - c) + (m(\lambda) - P_1(\lambda))(\hat{v} - c). \tag{16}$$

With probability $P_1(\lambda)$, the seller meets exactly one buyer, and the product is sold at the reserve price r; with probability $m(\lambda) - P_1(\lambda)$, the seller meets two or more buyers and the product is sold at price $\hat{v}$. Note that since in this section all buyers are homogeneous, only $m(\lambda) - P_1(\lambda)$ matters; the individual values of $P_n(\lambda)$ for $n \geq 2$ are irrelevant here.

The (expected) queue length λ faced by the above seller is determined by the buyers' indifference condition

$$k = \frac{P_1(\lambda)}{\lambda}(\hat{v} - r). \tag{17}$$

A buyer obtains a positive payoff if and only if this buyer is the only bidder in the auction, which happens with probability $P_1(\lambda)/\lambda$.

Combining the above two equations yields again equation (4). Hence the previous analysis of sellers' choice in the λ-$\hat{v}$ plane is unchanged. Our previous analysis shows that we only need to show that the sellers' iso-profit curves are well-defined in the r-$\hat{v}$ plane. For this we need the following assumption.

Assumption 2. *$P_1(\lambda)/\lambda$ is strictly decreasing, and $P_1(\lambda)/m(\lambda)$ is weakly decreasing.*

The above assumption states that when λ is higher, the probability that a buyer is the only bidder becomes smaller, and a seller is more likely to meet multiple buyers conditional on meeting at least one buyer.

By equation (4), $\pi(c, r, \hat{v})$ is first increasing and then decreasing in r. However, the effect of $\hat{v}$ on $\pi(c, r, \hat{v})$ is not clear immediately, since equation (16) is more complicated than equation (3). The following lemma shows that $\pi(c, r, \hat{v})$ is strictly increasing in $\hat{v}$ so that the sellers' iso-profit curves are well-defined in the $r-\hat{v}$ plane.

Lemma 1. *Under Assumption 2, $\pi(c, r, \hat{v})$ is strictly increasing in $\hat{v}$.*

Proof. See Appendix A.5. □

Therefore, $\pi(c, r, \hat{v})$ satisfies the single-crossing condition. The equilibrium features least-cost separation, and the allocation of buyers and sellers is exactly the same as before. The equilibrium queue lengths are determined recursively by equations (7) and (11).

When the meeting technology is urn-ball where $P_n(\lambda) = e^{-\lambda}\lambda^n/n!$, as is well-known in the literature, without private information sellers always post a reserve price equal to their reservation value c_i.[15] With private information, when the ICC constraint is not binding, the equilibrium has $r_i^* = c_i$, otherwise $r_i^* > c_i$.

Instead of allowing the informed side (sellers) to post, Auster and Gottardi (2019) assumes that the uninformed side (buyers) can meet multiple counterparties and post general trading mechanisms. They find that all buyers post the same mechanism, resulting in a pooling equilibrium. When adverse selection is severe (the ICC is binding), the equilibrium mechanism is inefficient, meaning that trade is not realized even when a buyer is in contact with one or more sellers. In contrast, this type of inefficiency never arises in our model since sellers post auctions, which are always efficient ex post. However, the inefficiency in our model is ex ante: the queue lengths attracted by high-type sellers are inefficiently low.

4 Understanding the role of search frictions

In order to understand the role of search frictions, in this section, we compare our results with the case in which search frictions are absent. Suppose that sellers can post prices and that the measure of buyers is greater than that of sellers so that a buyer's payoff is zero in equilibrium. Following Akerlof (1970), in the absence of search frictions, the most natural equilibrium is the single-price equilibrium in which supply equals demand.

However, there also exists a continuum of other equilibria in which sellers of type c post a price $p = p(c)$ and a seller who posts price p is accepted with probability $\alpha(p)$. Since a buyer's payoff is always zero, buyers are indifferent across all $\alpha(p)$ between zero and one. In particular, we can construct an $\alpha(p)$ such that sellers of type c find it optimal to post $p = \nu(c) - k$.[16]

We consider this to be an uninteresting equilibrium because there is no reason for buyers to choose $\alpha(p) \in (0, 1)$ when their payoff is zero anyway, let alone the $\alpha(p)$ that generates a separating equilibrium. Furthermore, it is unclear how to set up the (Walrasian) market so that sellers who post price p trade with probability $\alpha(p)$. The least unreasonable case is when there is only one seller and one buyer,

[15] See, for example, McAfee (1993), Albrecht et al. (2014), and Cai et al. (2023).

[16] The seller's problem is $\max_p (p-c)\alpha(p)$, and the corresponding FOC is $(p-c)\alpha'(p)+\alpha(p) = 0$. Since we require $p = \nu(c) - k$ in equilibrium, the FOC can be rewritten as $(p - \nu^{-1}(p+k))\alpha'(p) + \alpha(p) = 0$, from which we can solve $\alpha(p)$. Note that $\alpha(p)$ is not uniquely determined; we can multiply it by a scalar smaller than 1, and the same result holds.

where the buyer accepts the good with probability $\alpha(p)$ even though this is still arbitrary. This case is analyzed by Kreps (2023, pp. 115–117), who constructs an example with a continuum of seller types in which trade surplus $\nu(c) - c$ is constant in c. Kreps (2023) also argues that there is no good reason why in this case the separating equilibrium would be selected out of the continuum of equilibria. In contrast, in the competitive search equilibrium of the previous section, it follows from buyers' optimal search that a higher price corresponds to a shorter buyer queue so that a unique $\alpha(p)$ arises endogenously.

Similar to Kreps (2023), Cai et al. (2007) studies a stand-alone auction with one seller and n buyers in which the seller posts a reserve price. However, they assume that each buyer has some idiosyncratic preference for the product, i.e., the value of a product with type i for buyer j is $v_i + \varepsilon_j$ where $i = 1, \ldots, I$ (as in our model) and $j = 1, \ldots, n$. The idiosyncratic part ε_j is iid across buyers. If the dispersion of the idiosyncratic part is large enough, then the sellers' optimal reserve price under complete information induces a trading probability strictly smaller than 1. Under asymmetric information, high-type sellers increase their reserve price even further so that low-type sellers will not mimic. Hence, the tradeoff is again that a higher reserve price leads to a lower trading probability (as in our model). However, their result depends crucially on the dispersion of the idiosyncratic part. If it is small, then their model is equivalent to the one in Kreps (2023), and there does not exist a separating equilibrium. With search frictions, there always is a probability that sellers do not sell and a separating equilibrium always exists even when buyers have no idiosyncratic preferences for the good.

5 Conclusion

In this paper, we consider a market for a good whose quality differs across sellers and in which there is competitive search. There is asymmetric information in the sense that each seller knows the quality of the good that he or she is offering for sale, but buyers cannot observe quality directly. In other words, we have a "market for lemons" with competitive search frictions.

We prove the existence of a unique equilibrium, which is separating unlike the case in (Akerlof, 1970). Price serves as a signal of quality in that higher-quality sellers post higher prices. This is shown in a market with a discrete number of sellers and also for one with a continuum of seller types. Finally, we also show that our model has a separating equilibrium in a market in which the sellers can post auctions.

Our results are in stark contrast with those of frictionless asymmetric information models. In those models, an equilibrum sometimes does not exist, or multiple equilibria can exist with properties that depend on the details of the model such as the order of decisions.

Appendix A Proofs

A.1 Proof of Proposition 1

Denote by $\overline{\Lambda}^i_{icc}$ the larger solution of λ_i for which the ICC constraint (9) is binding. Then we have $\pi^*_{i-1} = \widetilde{\pi}(c_{i-1}, \Lambda^i_{icc}, v_i) = \widetilde{\pi}(c_{i-1}, \lambda^*_{i-1}, v_{i-1}) = \widetilde{\pi}(c_{i-1}, \overline{\Lambda}^i_{icc}, v_i)$ with $\Lambda^i_{icc} < \lambda^*_{i-1} < \overline{\Lambda}^i_{icc}$ (see the discussions before Proposition 1). Because the slope of the iso-profit curve of sellers with a higher type is steeper, we have $\widetilde{\pi}(c_i, \Lambda^i_{icc}, v_i) > \widetilde{\pi}(c_i, \lambda^*_{i-1}, v_{i-1}) > \widetilde{\pi}(c_i, \overline{\Lambda}^i_{icc}, v_i)$.

If the maximum of $\widetilde{\pi}(c_i, \lambda^i, v_i)$ were reached on the right-hand side of $\overline{\Lambda}^i_{icc}$, i.e., $\Lambda_o(\frac{k}{s_i}) \geq \overline{\Lambda}^i_{icc}$, then we would have $\widetilde{\pi}(c_i, \Lambda^i_{icc}, v_i) < \widetilde{\pi}(c_i, \overline{\Lambda}^i_{icc}, v_i)$, since $\widetilde{\pi}(c_i, \lambda_i, v_i)$ is strictly concave in λ_i. This is a contradiction so we have $\Lambda_o(\frac{k}{s_i}) < \overline{\Lambda}^i_{icc}$. The ICC (9) implies that the sellers of type i need to choose $\lambda_i \leq \Lambda^i_{icc}$ or $\lambda_i \geq \overline{\Lambda}^i_{icc}$. If $\Lambda_o(\frac{k}{s_i}) \leq \Lambda^i_{icc}$, then $\lambda^*_i = \Lambda_o(\frac{k}{s_i})$; while if $\Lambda_o(\frac{k}{s_i}) \in (\Lambda^i_{icc}, \overline{\Lambda}^i_{icc})$, then $\lambda^*_i = \Lambda^i_{icc}$. Thus, λ^*_i is given by equation (11), which implies $\lambda^*_i > 0$. Moreover, since $\Lambda^i_{icc} < \lambda^*_{i-1}$, we have $\lambda^*_i < \lambda^*_{i-1}$. Finally, if $p^*_i \leq p^*_{i-1}$, then sellers of type i would strictly prefer posting p^*_{i-1} since $\lambda^*_i < \lambda^*_{i-1}$, which contradicts the fact that the equilibrium is separating. Hence we have $p^*_i > p^*_{i-1}$. □

A.2 Proof of Proposition 2

Since the decentralized equilibrium is unique, we can first assume that the equilibrium is efficient and then check for the configurations in which the low-type sellers do not have an incentive to mimic the high-type sellers.

The ICC constraint (9) is satisfied if and only if

$$v_2 - v_1 \leq \frac{\pi^*_1 - \pi^*_2}{m(\lambda^*_2)} - (s_1 - s_2). \quad (18)$$

Define $\widetilde{\pi}(s) = \max_\lambda(sm(\lambda) - \lambda k)$. Since the equilibrium is assumed to be efficient, $\pi_i^* = \widetilde{\pi}(s_i)$ and $\lambda_i^* = \Lambda_o(k/s_i)$ for $i = 1, 2$. By the envelope theorem, $\widetilde{\pi}'(s) = m(\Lambda_o(k/s))$, which implies that $\pi_1^* - \pi_2^*$ can be rewritten in the integral form: $\int_{s_2}^{s_1} m(\Lambda_o(k/s))ds$. Substituting this for $\pi_1^* - \pi_2^*$ in the above inequality yields the expression for Ψ in equation (12). Finally, note that doubling k, s_1, and s_2 will double π_1^* and π_2^* without affecting λ_2^* on the right-hand side of (18), which implies that Ψ is also doubled. Hence, $\Psi(k, s_1, s_2)$ is homogeneous of degree 1. □

A.3 Proof of Proposition 4

Given the integral form of equation (12), we just need to show that $m(\Lambda_o(k/s))/m(\Lambda_o(k/s_2))$, or equivalently $\log m(\Lambda_o(k/s)) - \log m(\Lambda_o(k/s_2))$, is strictly increasing in k. Note that

$$\frac{\partial}{\partial k}\left[\log m(\Lambda_o(k/s)) - \log m(\Lambda_o(k/s_2))\right] = \int_{s_2}^{s} \frac{\partial^2 \log m(\Lambda_o(k/\tilde{s}))}{\partial k \partial \tilde{s}} d\tilde{s}$$

Hence, $m(\Lambda_o(k/s))/m(\Lambda_o(k/s_2))$ is strictly increasing in k if and only if $\log m(\Lambda_o(k/\tilde{s}))$ is strictly supermodular. Next, we have

$$\frac{\partial \log m(\Lambda_o(k/\tilde{s}))}{\partial k} = \frac{m'(\Lambda_o(k/\tilde{s}))}{m(\Lambda_o(k/\tilde{s}))}\Lambda_o'(k/\tilde{s})\frac{1}{\tilde{s}} = \frac{m'(\Lambda_o(k/\tilde{s}))}{m(\Lambda_o(k/\tilde{s}))}\frac{m'(\Lambda_o(k/\tilde{s}))}{m''(\Lambda_o(k/\tilde{s}))}\frac{1}{k}$$

where for the last equality, we used $\tilde{s}m'(\Lambda_o(k/\tilde{s})) = k$, which defines $\Lambda_o(k/\tilde{s})$. Hence, $m(\Lambda_o(k/s))/m(\Lambda_o(k/s_2))$ is strictly increasing in k if and only if the last term on the right-hand side is strictly decreasing in s, i.e., Assumption 1 holds.

Recall that on the right-hand side of equation (18), $\pi_i^* = \max_\lambda(s_i m(\lambda) - \lambda k)$ and λ_i^* is the optimal solution for this maximization problem. When $k \to 0$, λ_1^* and λ_2^* both converge to ∞, and π_1^* and π_2^* converge to s_1 and s_2, respectively. Thus, $\Psi(k, s_1, s_2)$, the right-hand side of equation (18) converges to zero. When $k \to s_2$, we have $\lambda_2^* \to 0$ and λ_1^* converges to some finite limit since $s_1 > s_2$, which implies that the right-hand side of equation (18), $\Psi(k, s_1, s_2)$, converges to ∞. □

A.4 Proof of Proposition 5

The derivation of equation (11) follows from the discussion before Proposition 5. We still must show that a unique solution $\lambda(c)$ to equation (11) exists. Since the right-hand side of equation (11) is continuous, existence and uniqueness follow from the Picard-Lindelöf theorem (see Theorem 1.3.1 of Coddington and Levinson (1955)). There is, however, one issue in applying the theorem, At the initial point $\lambda'(\underline{c}) = -\infty$. To address this, we apply the Picard-Lindelöf theorem to its inverse function $c(\lambda)$, which is well-defined since $\lambda(c)$ is strictly decreasing. At the initial point, $c'(\lambda(\underline{c})) = 0$ (thus bounded), so a unique function $c(\lambda)$ exists around the initial point $\lambda(\underline{c})$, i.e., defined in $[\lambda(\underline{c}), \lambda(\underline{c}) - \varepsilon]$ with $\underline{c} + \varepsilon' = c(\lambda(\underline{c}) - \varepsilon)$. From $\underline{c}+\varepsilon'$ onwards, we can apply the Picard-Lindelöf theorem to the differential equation of $\lambda(c)$. $\square$

A.5 Proof of Lemma 1

First, differentiating equation (17) with respect to $\hat{v}$ yields,

$$\lambda_{\hat{v}} = \frac{\lambda P_1(\lambda)}{(\hat{v} - r)\left(P_1(\lambda) - \lambda P_1'(\lambda)\right)}$$

Note that the denominator is strictly positive since $P_1(\lambda)/\lambda$ is strictly decreasing.

Differentiating equation (4) with respect to $\hat{v}$ and substituting the above equation into the resulting equation yields

$$\pi_{\hat{v}}(c, r, \hat{v}) = m(\lambda) + \frac{P_1(\lambda)\left((\hat{v} - c)\lambda m'(\lambda) + (r - \hat{v})P_1(\lambda)\right)}{(\hat{v} - r)\left(P_1(\lambda) - \lambda P_1'(\lambda)\right)}$$

where we used equation (17) to substitute out k. Note that the above equation is strictly increasing in r (keeping $\hat{v}$ and λ fixed). Since the smallest possible r is $\underline{r} = c - \frac{m(\lambda) - P_1(\lambda)}{P_1(\lambda)}(\hat{v} - c)$ at which $\pi(c, r, \hat{v}) = 0$, $\pi_{\hat{v}}(c, r, \hat{v}) > 0$ if $\pi_{\hat{v}}(c, \underline{r}, \hat{v}) \geq 0$, which is given by

$$\pi_{\hat{v}}(c, \underline{r}, \hat{v}) = m(\lambda)\left(1 - \frac{P_1(\lambda)^2\left(m(\lambda) - \lambda m'(\lambda)\right)}{m(\lambda)^2\left(P_1(\lambda) - \lambda P_1'(\lambda)\right)}\right) \geq 0$$

where the inequality follows from i) $m(\lambda) \geq P_1(\lambda)$, and ii) $1 - \lambda P_1'(\lambda)/P_1(\lambda) \geq 1 - \lambda m'(\lambda)/m(\lambda)$, which is equivalent to $P_1(\lambda)/m(\lambda)$ being weakly decreasing.$\square$

A.6 Direct Proof of Single-Crossing in the $(p, \hat{v})$ Plane

Suppose that the queue length associated with $(p, \hat{v})$ is denoted by the function $\widetilde{\lambda}(p, \hat{v})$. Note that in our model, $\widetilde{\lambda}(p, \hat{v}) = q^{-1}(\frac{k}{\hat{v} - p})$. Since $\pi(c, p, \hat{v}) =$

$m(\widetilde{\lambda}(p,\hat{v}))(p-c)$,

$$\frac{\partial}{\partial c}\left(-\frac{\pi_p(c,p,\hat{v})}{\pi_{\hat{v}}(c,p,\hat{v})}\right) = -\frac{m(\widetilde{\lambda}(p,\hat{v}))}{(p-c)^2 m'(\widetilde{\lambda}(p,\hat{v}))\widetilde{\lambda}_{\hat{v}}(p,\hat{v})},$$

which implies that the single-crossing condition holds whenever $\widetilde{\lambda}(p,\hat{v})$ is strictly increasing in $\hat{v}$, i.e., $\widetilde{\lambda}_{\hat{v}}(p,\hat{v}) > 0$.

Risk-averse Buyers. Suppose that buyers are risk averse and all have utility function $u(\cdot)$ with initial wealth W_0. Then $\widetilde{\lambda}(p,\hat{v})$ is given by the following equation

$$q(\lambda)\left(u(\hat{v}-p+W_0-k) - u(W_0-k)\right) = u(W_0) - u(W_0-k)$$

where $u(W_0-k)$ is the payoff from entering the market and failing to trade, and $u(\hat{v}-p+W_0-k)$ is the utility from entering the market and purchasing the good with quality $\hat{v}$ and price p. Clearly, $\widetilde{\lambda}(p,\hat{v})$ is again strictly increasing in $\hat{v}$. Hence, the equlibrium is again separating with risk-averse buyers.

References

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84:488–500.

Albrecht, J. W., Gautier, P. A., and Vroman, S. B. (2014). Efficient entry with competing auctions. *American Economic Review*, 104(10):3288–3296.

Albrecht, J. W., Gautier, P. A., and Vroman, S. B. (2016). Directed search in the housing market. *Review of Economic Dynamics*, 19:218–231.

Auster, S. and Gottardi, P. (2019). Competing mechanisms in markets for lemons. *Theoretical Economics*, 14(3):927–970.

Banks, J. S. and Sobel, J. (1987). Equilibrium selection in signaling games. *Econometrica*, pages 647–661.

Barsanetti, B. and Camargo, B. (2022). Signaling in dynamic markets with adverse selection. *Journal of Economic Theory*, 206:105558.

Cai, H., Riley, J., and Ye, L. (2007). Reserve price signaling. *Journal of Economic Theory*, 135(1):253–268.

Cai, X., Gautier, P., and Wolthoff, R. (2023). Meetings and mechanisms. *International Economic Review*, 64(1):155–185.

Chang, B. (2018). Adverse Selection and Liquidity Distortion. *The Review of Economic Studies*, 85(1):275–306.

Coddington, A. and Levinson, N. (1955). *Theory of Ordinary Differential Equations*. International series in pure and applied mathematics. McGraw-Hill.

Delacroix, A. and Shi, S. (2013). Pricing and signaling with frictions. *Journal of Economic Theory*, 148:1301–1332.

Guerrieri, V. and Shimer, R. (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review*, 104(7):1875–1908.

Guerrieri, V., Shimer, R., and Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862.

Hellwig, M. (1987). Some recent developments in the theory of competition in markets with adverse selection. *European Economic Review*, 31(1):319–325.

Inderst, R. and Muller, H. M. (2002). Competitive search markets for durable goods. *Economic Theory*, 19:599–622.

Kim, K. (2012). Endogenous market segmentation for lemons. *The RAND Journal of Economics*, 43(3):562–576.

Kim, K. and Kircher, P. (2015). Efficient competition through cheap talk: The case of competing auctions. *Econometrica*, 83(5):1849–1875.

Kreps, D. M. (2023). *Microeconomic foundations II: Imperfect competition, information, and strategic interaction*. Princeton University Press.

Leland, H. E. and Pyle, D. H. (1977). Informational asymmetries, financial structure, and financial intermediation. *The Journal of Finance*, 32(2):371–387.

McAfee, R. P. (1993). Mechanism design by competing sellers. *Econometrica*, 61(6):pp. 1281–1312.

Menzio, G. (2007). A theory of partially directed search. *Journal of Political Economy*, 115:748–769.

Milgrom, P. and Roberts, J. (1986). Price and advertising signals of product quality. *Journal of Political economy*, 94(4):796–821.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of Political Economy*, 105:385–411.

Moon, J.-W. (2023). Hidden advertisement, signaling, and directed search. *Economic Theory*.

Rothschild, M. and Stiglitz, J. (1976). Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics*, 90(4):629–649.

Williams, B. (2021). Search, Liquidity, and Retention: Screening Multidimensional Private Information. *Journal of Political Economy*, 129(5):1487–1507.

Wilson, C. (1980). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *Bell Journal of Economics*, 11(1):108–130.