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ABSTRACT

Families' Career Investments and Firms' Promotion Decisions*

This paper studies how family and firm investments interact to explain gender gaps in career achievement. Using Danish administrative data, we first document novel evidence of this interaction through a "spousal effect" on firm-side career investments. This effect is accounted for by family labor supply choices that shape worker characteristics, which then influence firms' training and promotion decisions. Our main theoretical contribution is to develop a quantitative life cycle model that captures these family-firm interactions through household formation, families' joint career and fertility choices, and firms' managerial training and promotion decisions. We then use the estimated model to show that the interaction between families and firms in the joint equilibrium of labor and marriage markets is important when evaluating firm-side and family-side policy interventions. We find that gender-equal parental leave and a managerial quota can both improve gender equality, but leave implies costly skill depreciation, whereas the quota raises aggregate welfare, in part through adjustments in marital sorting towards families that invest in women.

Keywords: gender inequality, career investments, firm training, management promotions, marriage market matching

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1 Introduction

Gender gaps in career achievement are the result of decisions made by both families and firms. Families decide on spousal labor supply, and women typically shoulder more household responsibilities than their husbands. Therefore, women may accumulate labor market skills at a slower pace. Firms, in turn, select workers for career development opportunities based on their potential. If firms perceive women as less committed to their jobs, women may experience fewer opportunities to climb the career ladder than their male colleagues. Importantly, firms' differential perception may result from families investing more in men, which may, in turn, be influenced by lower anticipated returns to women's careers. This *interaction* between family and firm decisions could be essential for understanding persistent gender inequality, but has not been studied before.

In this paper, we investigate how career investments within two uncoordinated groups—families and firms—interact in shaping gender gaps in career achievement. We focus on managerial positions, which exhibit a big and persistence gender gap (Bertrand and Hallock, 2001; Gayle, Golan, and Miller, 2012; Bronson and Thoursie, 2021; Hampole, Truffa, and Wong, 2024; Haegele, 2024; Frederiksen, Kato, and Smith, Forthcoming) and are of key importance to both firms and workers. On the one hand, promoting suitable candidates into management is crucial for firm success because managers make strategic decisions that drive firm productivity (Bertrand and Schoar, 2003; Lazear, Shaw, and Stanton, 2015). On the other hand, promotion into management is also a key career step for employees associated with a substantial increase in responsibilities and compensation (Gibbons and Waldman, 1999b). If this interplay between families and firms is mediated by information frictions that prevent firms from identifying promising women and, in turn, signal to families that women's likelihood of success is lower, there might be room for policy interventions that induce a more efficient allocation of talent. Hence, understanding how families and firms jointly shape persistent gender gaps is crucial to designing welfare-improving policies that reduce the gap in access to these leadership opportunities.

Our paper makes three main contributions that map to the core parts of the analysis. First, we show novel evidence that firm-side and family-side investments in workers' human capital indeed interact in shaping gender gaps in career achievement (Section 2). To do this, we leverage Danish administrative data that allows us to link workers to their families and firms. Second, we specify (Section 3) and estimate (Section 4) a quantitative life cycle model that captures these family-firm interactions in a joint equilibrium of the marriage market and the labor market. Our key theoretical contribution is to model the interplay between families and firms through household formation, families' joint career and fertility decisions, and an active role of the firm in making managerial training and promotion decisions. Third, we use the estimated model to show the role of equilibrium adjustments by families and firms in response to commonly discussed policy interventions, such as parental leave programs and managerial quotas, aimed at reducing gender gaps (Section 5). Our analysis shows the importance of considering the joint reactions of both families and firms in policy design.

Using rich administrative data from Denmark, we first show novel evidence that firm-side and family-side investments in workers' human capital interact. We follow the cohort who completed their education between 1991 and 1995 from the moment of marriage and labor market entry throughout their life cycle. Crucially, we link workers to their spouses and employers and observe their labor market and family trajectories. We categorize workers into four education groups that capture how *ambitious* their program of education is based on the average starting wages and wage growth of program graduates, and we consider marital sorting in terms of the educational ambition type (Almar, Friedrich, Reynoso, Schulz, and Vejlin, 2024). This allows us to distinguish couples in which both spouses have similar career ambition from couples in which one spouse has higher ambition than the other. We also observe firms' investment in workers' human capital through our novel measure of *on-the-job managerial training*, which considers employees' formal managerial training and the probability that their occupational path leads to a managerial promotion.

We report sizable gender gaps in both managerial training and promotion, which, interestingly, vary with the ambition type of workers' spouses—a characteristic that firms do not perfectly observe. As expected, we find that highly ambitious workers have substantially higher chances of receiving managerial training and promotions, but women experience a smaller advantage. Moreover, among individuals of a given ambition type, those who are married to a highly ambitious spouse receive more managerial training and promotions than those who are married to less ambitious spouses. Again, this advantage is smaller for women. This "spousal effect" is drastically reduced when we account for workers' labor supply histories, suggesting that joint family choices translate into observable characteristics that firms act on. In addition, we document a close link between household types, fertility, and career investment choices. More ambitious women delay fertility, and their households display a lower and less persistent child penalty in labor supply. In contrast, this penalty is particularly pronounced for couples in which the husband is of the highest ambition type and the wife of lower ambition. This can help explain that firm investment is higher for women of higher ambition type. Taken together, this evidence suggests that the spousal type affects workers' human capital through labor market and fertility choices, which in turn affect workers' attractiveness for managerial training and promotions within the firm.

To understand the underlying economic mechanisms leading to these gender gaps in career achievement and inform the design of effective policies, we develop a life cycle model of marriage, fertility, individuals' investments in their careers, and firms' investments in training and promoting workers.

In the model, men and women meet in a competitive marriage market and select whether to marry and, if so, the ambition type of their partner. After forming their households, single and married individuals enter the labor market in which they choose between two alternative career ladders that differ in career opportunities and flexibility. Over their life cycle, individuals consume private goods and choose how to divide their time between producing a household public consumption good (for example, investing in children's education) and improving their labor market skills (for example, working full time in their careers). In addition, couples decide whether to have children early or later in their careers or to remain childless.

On the labor market side, a representative firm employs workers in production or managerial jobs on each of the two career ladders. To prepare workers for managerial jobs, the firm selects the most promising workers to participate in a firm-specific managerial training program. The firm makes this investment in part based on workers' observed accumulated labor market skills. In the final career stage, the firm offers a manager promotion to some of its trained workers, a position that requires a strong time commitment and offers high earnings.

An important assumption in our baseline model is that the firm does not observe family characteristics of workers. This is important because to make training and promotion decisions, the firm must form beliefs about workers' future labor supply, which workers decide jointly with their spouse. The firm forms these beliefs about optimal household choices by inferring spousal characteristics from the observed history of career investments of a worker and knowledge of the distribution of household types, which follows from the marriage market equilibrium. Through this inference, the model generates a link between spousal characteristics and firm investments conditional on a worker's own type to match the observed patterns in the data. Firms can internalize how the different roles of men and women in the household may influence future labor supply, which implies the possibility of gender-based statistical discrimination. In additional analysis, we explore the alternative case of strictly history-based firm policies that cannot differentiate by gender, and the case of full information about each workers' family.

We estimate our model by disciplining its parameters with key identifying moments that we construct from the Danish register data and that have a model counterpart. We match well the targeted labor supply patterns, earnings dynamics by ambition types and experience, marital sorting by ambition, fertility patterns, and firm training and promotions by gender and ladder.

The estimated model captures the reinforcing relationship between families and firms in shaping gender gaps in career achievement. We closely replicate the small initial gaps in participation between men and women of the same ambition type. These initial differences amplify over the life cycle through the interaction of subsequent household choices and firm investment choices. Indeed, firms who evaluate workers for training are more likely to support men—who are observed to have accumulated higher skills—than women to advance in their careers. At the same time, because women receive less on-the-job training, families find it optimal for the wife to allocate more time to home production than the husband, even later in the life cycle when the return on home production decreases.

Through these mechanisms, our model accurately captures a set of gender gaps in career development and achievement which, remarkably, we do not target in estimation. In particular, our estimated model matches gender gaps in promotion, training, labor supply, and lifetime income. Focusing on differences in gender gaps across household types, the model captures well the reinforcing nature of family- and firm-side investments. Specifically, our estimated model delivers the untargeted feature that families in which women marry more ambitious men show the highest gender gaps not only in lifetime labor supply but also in firm training and manager promotion.

We use our model to evaluate the equilibrium effects of widely discussed policy interventions. We study which policies incentivize investments in women's human capital within their families and their firms, and increase welfare for households. We find that a policy that imposes quotas for female representation in managerial positions achieves both goals. First, the main effect of this policy is that it incentivizes firms to train more women, whereas adjustments in household labor supply are smaller. Second, this higher firm investment raises welfare for families with ambitious women and the frequency of these couples increases in the new marriage market equilibrium. In contrast, families with the most ambitious men who marry less ambitious women lose welfare because fewer men are promoted. But these households now form less frequently in the presence of a quota. Together, these adjustments in marital sorting contribute to the overall aggregate welfare gain of this policy. Family-side policies show more mixed results. We find that mandating paid parental leave amplify gender differences if they are targeted only to mothers, or reduce them if they are targeted to both mothers and fathers. However, both types of parental leave policies expose individuals to the costs of skill depreciation, and therefore reduce welfare for most households. When we do not allow individuals to change their partner choice in response to leave policies, we find higher gender gaps, meaning that sorting in the marriage market facilitates gender convergence.

In developing our novel equilibrium framework, we combine several strands of literature.

First, we build on the literature on the consequences of workers' choices for career outcomes and the implications for the persistence of gender gaps in labor market performance. One fast growing strand quantifies the career impacts of children for women, and shows that gender gaps persist even after childcare responsibilities decrease (Adda, Dustmann, and Stevens, 2017; Angelov, Johansson, and Lindahl, 2016; Kleven, Landais, and Søgaard, 2019). Another influential literature shows the role of occupational choices. Importantly, there are significant returns to working long hours in certain occupations and home production responsibilities restrict women's access to these occupations (Goldin, 2014; Cortés and Pan, 2019; Erosa, Fuster, Kambourov, and Rogerson, 2022). This is consistent with women's higher willingness to pay for work flexibility (Mas and Pallais, 2017; Sorkin, 2017; Wiswall and Zafar, 2017). Firms play no active role in these frameworks, and neither does the marriage market.[1] Hence, we contribute to this literature by adding two empirically relevant actors: the family and the firm in joint equilibrium.

When incorporating the family in our dynamic equilibrium framework, we build on the established fact that workers' investments in their careers depend on who they marry. Therefore, the equilibrium configuration of couples formed in the marriage market interacts with workers' labor supply and human capital investments (Chiappori, Costa Dias, and Meghir, 2018; Gayle and Shephard, 2019; Lafortune and Low, 2023; Calvo, 2023; Reynoso, 2024). As in this literature, our paper studies marital sorting and joint household career investments over the life cycle, but we add the role of the firm in shaping human capital. Specifically, we model the mechanisms by which sorting in the marriage market influences how firms decide which workers to invest in.

When incorporating the role of the firm in our equilibrium framework, we build on the labor literature on firms' investments in their workforce. This literature has long emphasized that firms face

[1] Angelov et al. (2016) and Erosa et al. (2022) consider within-household decisions but not household formation.

uncertainty about workers and use observable characteristics to screen potential candidates (Spence, 1973; Stiglitz, 1975). As a consequence, firms may engage in statistical discrimination (Phelps, 1972; Arrow, 1973), building expectations about workers' potential at the hiring stage (Coate and Loury, 1993), and gradually learning, i.e., updating these beliefs over time, for wage and promotion decisions (Farber and Gibbons, 1996; Gibbons and Katz, 1991; Gibbons and Waldman, 1999a; Altonji and Pierret, 2001). More recent work has incorporated statistical discrimination and employer learning into quantitative models of worker careers (Gayle and Golan, 2012; Kahn and Lange, 2014; Pastorino, 2024; Xiao, 2024). In addition, the literature has also focused on the role of on-the-job training (Becker, 1962; Altonji and Spletzer, 1991; Demougin and Siow, 1994; Blundell, Costa Dias, Goll, and Meghir, 2021), but lack of data on firm training has limited empirical research (Black, Skipper, and Smith, 2023). Focusing on the market for managers, Friedrich (2023) models the strategic decisions of heterogeneous firms related to hiring, training and promotion of managers in an equilibrium framework with information frictions. Our model adds the fact that family formation and joint family choices interact with firm choices, in particular for training and promotion decisions that depend on workers' skill level and their commitment to the firm. Empirically, we show the link between families' and firms' investment choices using a novel measure of firm training.

In modeling the dynamic interactions between families' and firms' career investments in workers, we extend the literature on the interplay between marriage and labor markets. This literature captures this interaction in various ways. One strand considers the response of marital sorting and household labor market choices to exogenous changes in the wage process, job displacement, home production technology, and family attitudes (Fernández, Guner, and Knowles, 2005; Greenwood, Guner, Kocharkov, and Santos, 2016; Goussé, Jacquemet, and Robin, 2017; Chiappori, Salanié, and Weiss, 2017; Foerster, Obermeier, and Schulz, 2024). Another strand studies joint spousal job search behavior (Flabbi and Mabli, 2018; Flabbi, Flinn, and Salazar-Saenz, 2020; Pilossoph and Wee, 2021) and how it relates to household formation and dynamics (Pilossoph and Wee, 2023; Holzner and Schulz, 2023). These papers keep the labor market in partial equilibrium. Recently, Calvo, Lindenlaub, and Reynoso (2024) developed a static framework that combines both markets in equilibrium to study how gender and household inequality arises from the link between family labor supply and sorting in the labor and the marriage markets. While we only allow for labor market sorting on two career ladders, we extend this literature to a dynamic equilibrium framework of the two markets in which, importantly, not only families make human capital investments but also firms play an active role in training workers and offering managerial jobs. As said before, a novel feature of our model is that these firm-side decisions depend on sorting in the marriage market through the firm's beliefs about their workers' future labor investments—which are made within the household.

Finally, our unified equilibrium framework offers a fresh approach to the evaluation of policies aimed at reducing gender inequality in career outcomes. We focus on parental leave policies that have become a prominent feature of labor markets in developed countries (Olivetti and Petrongolo, 2017) and on affirmative action policies, such as board or managerial quotas, that have recently been in the public debate (McKinsey & Company, 2024). Yet, the empirical evidence suggests small (and sometimes negative) effects of these policies on women's careers.[2] Our framework allows us to explain these small—and potentially unintended—policy effects by accounting for joint *equilibrium* responses by families and firms. For example, policies that incentivize families to extend women's time on parental leave may also cause firms to invest less in all women if they expect women to work fewer hours. Thomas (2016), Tô (2018) and Xiao (2024) similarly emphasize firm beliefs in the context of parental leave, but do not incorporate the role of the family or marriage market sorting. Our analysis considers feedback effects between family and firm adjustments in joint equilibrium and we can quantify the role of the marriage market in the overall effects on gender gaps in career achievement. Moreover, our analysis is able to uncover interesting heterogeneity in policy effects depending on the couple type that may help explain the observed blanket effects.

To the best of our knowledge, our paper provides the first *equilibrium dynamic* framework of the interplay between firms and families in shaping gender inequality in career outcomes. By combining marriage market matching, labor supply, and fertility—on the family side—with on-the-job training and managerial promotions—on the firm side—we offer a novel approach to understanding the persistence of gender gaps and to study interesting policy implications.

2 Empirical Evidence

Even though previously undocumented, the interaction between families' and firms' investments in workers' human capital is salient in the data. We show this using Danish register data, in which we observe the education, family history, and career path at the individual level for the full population of residents.

2.1 Data and sample

Our main data source is administrative register data provided by Statistics Denmark. The registers contain information on the universe of residents at an annual frequency. We complement the registers with the Danish Labor Force Survey (LFS), which provides detailed information about hours of work and allows us to identify labor supply *in excess of full-time work*, which could be important for managerial career paths. We describe our data sources in detail in Online Appendix OA.1.

Our sample consists of the cohort of residents who graduated with their highest educational degree between 1991 and 1995, which we can follow over their careers. For each resident, we observe their history of employers, occupations, and labor supply, and their family history. Crucial for our project, we link each individual to their *spouse*. We aim to focus on the *decisive domestic partner* (married or

[2] For parental leave, recent studies find a range from small positive to negative labor market effects for mothers (Baker and Milligan, 2008; Schönberg and Ludsteck, 2014; Das and Polachek, 2015; Dahl, Løken, Mogstad, and Salvanes, 2016; Patnaik, 2019; Lassen, 2023; Bailey, Byker, Patel, and Ramnath, Forthcoming; Corekcioglu, Francesconi, and Kunze, 2024), with increasing gender gaps especially for highly skilled women (Olivetti and Petrongolo, 2017). Bertrand, Black, Jensen, and Lleras-Muney (2018) show positive effects of a board quota policy on the appointed women but no effects on female employees at large.

cohabiting) who participates in *joint* career and family decisions. To this end, we identify relationships lasting at least five years and starting around the time of labor market entry.[3]

In sum, our sample consists of 152,390 individuals who are linked to their spouses (if any) and all of their employers, and observed from the moment of household formation and labor market entry for about 25 years. In Appendix OA.2, we provide more details about the sample selection process.

2.2 Key variables

The empirical evidence shown in this section and the model estimation in Section 4 require the construction of a number of key variables. In Appendix OA.3, we provide all details on the measurement of these key variables. Here, we highlight the most essential information for interpreting our results.

Ambition types. Regarding our analysis of the marriage market, we focus on marital sorting on (exogenous) education. Hence, we build on the literature that improves the definition of *education-based types* and highlights the importance of *educational programs* for marital sorting (Almar et al., 2024; Wiswall and Zafar, 2021) and labor market performance (Altonji, Kahn, and Speer, 2014, 2016; Kirkeboen, Leuven, and Mogstad, 2016). Thus, we capture education-based types based on the labor market prospects of an individual's highest completed program of education, following the methodology developed in Almar et al. (2024) to construct *ambition types*. Intuitively, the expected labor market outcomes associated with the educational program that an individual graduates from signal *career ambition* and, accordingly, limited time commitments to the family. Thus, career ambition matters in the marriage market.

Specifically, we compute average starting wages (denoted by w_0) and ten-year-wage growth (denoted by g) across all graduates for each educational program and use k-means clustering (Steinley, 2006) to group these programs into 4 ambition types based on whether starting wages and wage growth are high or low. We formally denote the ambition type of individual i by θ_i and the set of four ambition types by Θ, defined as:

$$\Theta = \{\theta_1 = (\textit{low } w_0, \textit{low } g), \theta_2 = (\textit{high } w_0, \textit{low } g), \theta_3 = (\textit{low } w_0, \textit{high } g), \theta_4 = (\textit{high } w_0, \textit{high } g)\}$$

Our measure of education refines the standard definition based on educational *levels*. We observe 1,108 educational programs across all educational levels: primary, secondary, bachelor, and master & PhD. For example, Law and Architecture are two master's degree programs with high starting wages but very different wage growth, so they are categorized into different ambition types (θ_4 and θ_2, respectively).

Career ladders. We aim to capture the fact that individuals who graduate from similar programs (and thus have the same ambition type) sort into more or less labor-demanding career paths, which we refer to as *ladders*. To this end, we distinguish between *steep* (demanding) and *flat* (flexible) ladders

[3] In Online Appendix OA.3.2, we describe how we identify this *decisive domestic partner* and deal with cases of multiple partnerships.

in the data. Specifically, we split *occupation-by-firm* cells into two groups based on the average wage growth of all labor market entrants who start in each cell. We define as steep career ladders the occupation-firm cells in which the workers' hourly wage growth over the first 10 years of their career is above the 80th percentile of the across-cell wage-growth distribution. For example, a law graduate can be on a steep career ladder in a private law firm or a flat ladder in the public sector.[4]

Labor supply. Our measure of family investments in the careers of workers is the choice of how many hours to work in the labor market. We distinguish between four states of labor supply: non-participation, part-time work, full-time work, and *super-full-time work*.[5] The super-full-time category is constructed from the Danish LFS as working more than the full-time typical hours of 37 hours per week or irregular hours, such as weekend or nighttime work (excluding shift workers).

Managers and promotions. Our registers identify occupational codes for "Management" such as *top management*, *management within administration*, *management within production*, and *management within services*. We define a transition between occupational codes as a *promotion* into management if two conditions are met: 1) it is the worker's first transition into one of these managerial occupation codes; and 2) the worker keeps the occupational code for at least two consecutive years. For the typical worker, a promotion comes with a significant wage increase. In our data, managers earn on average 40% more than non-managers conditional on training, which we define next.

On-the-job managerial training. Finally, we measure firm's investments into their employees through proxies for managerial training facilitated by the firm. We aim to capture the type of training that workers need to qualify for a management position by combining two observable characteristics. First, we observe workers' participation in specialized tertiary management training programs offered by universities (e.g., MBA degrees). We interpret this as a measure of on-the-job training because, in the Danish context, enrollment into these programs requires that the student is formally employed, and tuition—which is relatively high—is typically paid by the employer. Second, we observe workers' (nonmanagerial) occupations, and identify which ones are most predictive of a later promotion into management using a logistic regression detailed in Appendix OA.3.8. Our interpretation is that firms will employ promising candidates in various roles and business functions to let them learn about the company and prepare them for management. Examples of occupations that are predictive of a promotion into management are occupations that require advanced knowledge in business (e.g., in accounting, finance, consultancy) and natural science, engineering, and information technology (statistical methods, software development, data management). Taken together, we assign a worker as a trainee if they complete a managerial training program or if they ever held an occupation that is a statistically significant predictor of being promoted to manager.

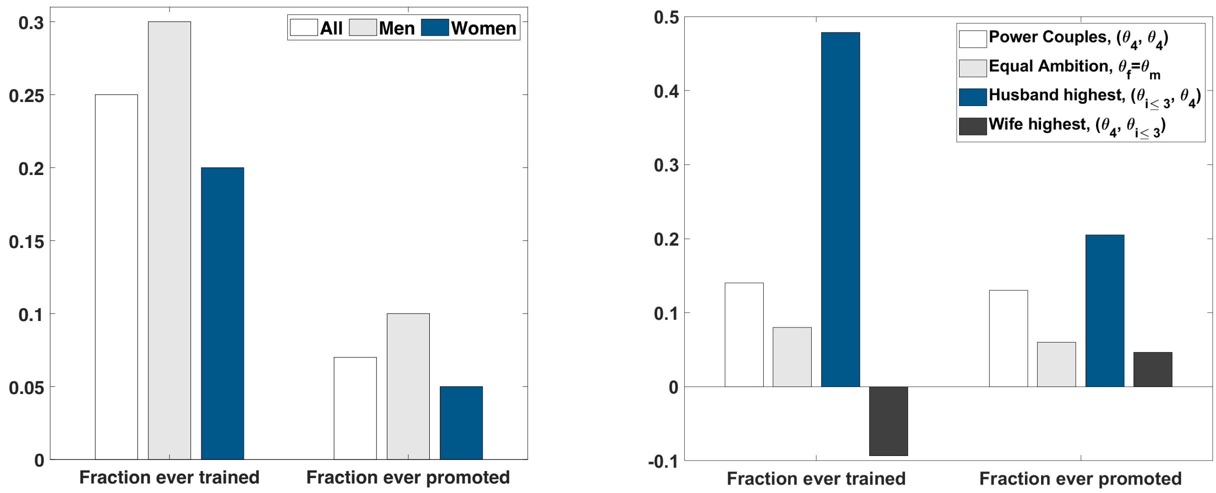
[4]Figure OA.1 in Appendix OA.3.10 shows that there is variability in sorting into ladders by ambition type.
[5]We thank Alessandra Voena for suggesting this partition and terminology.

2.3 Evidence on the interaction between families' and firms' career investments

Armed with our panel of families, firms, and the key variables constructed as described above, we document for the first time that career investments made within families and firms—two uncoordinated groups—interact in shaping gender gaps in career achievement.

Gender gaps in training and promotion. There are large gender gaps in training and promotion rates, which we interpret as firm-side investments into a worker's career. Although gender gaps in holding managerial positions have been extensively discussed in industry reports (McKinsey & Company, 2024), academic research has been held back by difficulties in measuring job titles and promotions. Notable exceptions include Bronson and Thoursie (2021) and Haegele (2024) who both emphasize the "broken rung" argument: women face particular difficulties in moving into their first managerial position. We contribute to this literature by presenting new evidence on how gender gaps in training and promotion rates are shaped by career ambition and the marriage market, i.e., the composition of households. Our findings corroborate the "broken rung" argument and shed light on the underlying mechanisms.

Figure 1: Gender gaps in training & promotion (left) are large and heterogeneous across couples (right)



Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

The left panel of Figure 1 plots the probability of having received training and of ever being promoted to management for all workers in our sample (white bars), men (gray bars) and women (blue bars). First, we observe large gender gaps in training and promotion: men are 30% more likely than women to receive on-the-job management training and 50% more likely to become managers. Second, not all trainees become managers, which we incorporate in our model in the form of a capacity constraint faced by firms when filling managerial positions. We complement these figures with a regression analysis (in Appendix Table A.1), which shows that these gender gaps persist when controlling for firm-by-ladder fixed effects. That is, our results are not driven by selection into different types of firms and career paths.

Firm-side investments vary with worker's family type. Gender gaps in firm-side investments (training and promotion) exhibit heterogeneity depending on the type of couple workers belong to, a worker characteristic that is either unobserved or presumably irrelevant for the firm. The right panel of Figure 1 plots the gaps in probabilities (husband minus wife) of training and promotion for workers in different couple types: (i) both spouses are of the highest ambition type, i.e., *power couples* (white bars); both spouses have equal ambition (gray); the husband is of the highest ambition type and the wife of a different type (blue); the wife is of the highest ambition type and the husband of a different type (black).[6] Gender gaps are the largest in couples with high-type husbands and wives of a lower type. They are smallest in couples with high-type wives and husbands of a lower type. For the latter group, gender gaps in training are indeed negative as one might expect, but the absolute value of the gap is much lower than in couples with high-type husbands and wives of a lower type. Interestingly, gender gaps in promotion rates favor the husband even in couples in which the wife has higher career ambition. But even for equal-type couples, and power couples in particular, we observe positive gender gaps.

We further explore the role of the spousal ambition type in explaining firm investment using regression analysis by estimating the following model:

$$\begin{aligned}\mathcal{O}_{ilt} =& \beta_0 + \beta_1 \cdot female_i + \beta_2 \cdot \textit{high-ambition}_i + \beta_3 \cdot \textit{high-ambition}_i \cdot female_i \\ &+ \beta_4 \cdot \textit{high-ambition spouse}_i + \beta_5 \cdot \textit{high-ambition spouse}_i \cdot female_i + X'_{ilt}\gamma + \epsilon_{ilt}\end{aligned}$$

Specifically, for worker i, in firm-ladder l, and period t, we regress indicators for having received training or manager promotion, $\mathcal{O}_{ilt}$, on the spousal ambition type, controlling for other characteristics. For parsimony, we define an indicator *high-ambition* if an individual's ambition type is $\theta = \{\theta_3, \theta_4\}$. This summarizes the high-wage-growth ambition types, so the low-wage-growth ambition types θ_1 and θ_2 form the baseline. We also explore the differential role of the spouse by gender by including full interactions of an indicator for *female* with spousal and own ambition type. The coefficient β_5 describes the differential role of the spousal ambition type for female workers.

Columns (1) and (3) of Table 1 report results for training and managerial employment without further controls X. The coefficient on female confirms the baseline gender gaps in training and managerial employment even for low-ambition workers, which are amplified among highly ambitious workers. Conditional on the worker's own type, receiving training and working as a manager is more common for workers with highly ambitious spouses. This relationship is significantly smaller for women, see the estimated β_5, indicating that gender gaps widen for women with ambitious husbands compared to men with ambitious wives.

Further, we explore the mechanisms through which the type of the spouse has an influence on firm investment. To this end, we add controls for worker characteristics that may vary with the

[6] While we can interpret type θ_1 as being "lower" than type θ_4 because the former comprises education programs with lower starting wage and lower wage growth, we remain agnostic of the order between mixed types θ_2 and θ_3. For interpretation in the remainder of the paper, we often focus on a single dimension of ambition, e.g., wage growth, and compare high-wage-growth types θ_3 and θ_4 to the low-wage-growth types θ_1 and θ_2.

Table 1: Training and promotions vary with worker's family type

	(1)	(2)	(3)	(4)
	Training		Manager	
female	-0.0287***	-0.0150***	-0.0090***	-0.0043***
	(0.002)	(0.003)	(0.001)	(0.001)
high-ambition	0.4302***	0.2992***	0.0475***	0.0401***
	(0.005)	(0.005)	(0.002)	(0.002)
high-ambition * female	-0.0738***	-0.0628***	-0.0150***	-0.0140***
	(0.007)	(0.007)	(0.002)	(0.002)
high-ambition spouse	0.1318***	0.0824***	0.0386***	0.0322***
	(0.007)	(0.007)	(0.003)	(0.003)
high-ambition spouse * female	-0.0652***	-0.0372***	-0.0326***	-0.0256***
	(0.008)	(0.008)	(0.003)	(0.003)
FE for Firm-Ladder, Age, LS History	No	Yes	No	Yes
Observations	1,860,063	1,827,942	1,860,063	1,827,942
R-squared	0.199	0.428	0.020	0.245
Total Effect, high-ambition spouse for female	0.0666	0.0453	0.00603	0.00660
P-Value	<0.0001	<0.0001	<0.0001	<0.0001

Notes: Standard errors clustered at the worker level in parentheses. *** Significant at the 1% level.

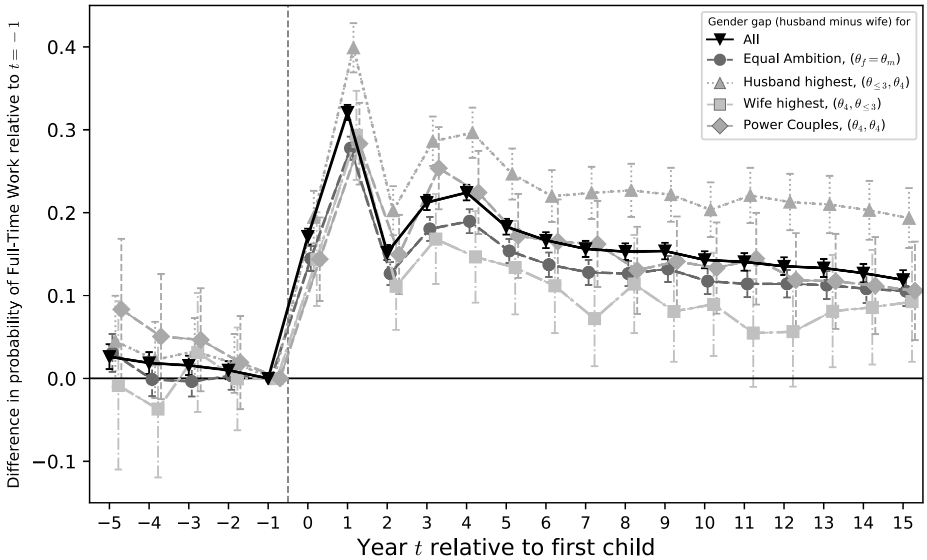
spousal type and that are observed by the firm. Specifically, columns (2) and (4) of Table 1 add flexible controls for each worker's labor supply choices, measured by the numbers of years working full-time, part-time, or in non-participation. In addition, we include controls for worker age, years since labor market entry, as well as fixed effects for firm-by-career-ladder. Conditional on these rich controls, the role of the spouse remains statistically significant but declines substantially. This suggests that households' joint time allocation and career investment choices influence firms' decisions through observable worker characteristics. This finding holds separately for ambitious (spousal) types θ_3 and θ_4, and when excluding singles from the sample (see Tables A.2 and A.3).[7]

Fertility and spousal time allocation within the household Finally, we show that family-side investments (labor supply) change differentially across couple types after child birth—an important observation that may again link back to firm-side policies and differential labor market returns across couples. To this end, we rely on an event study analysis around the birth of the first child. Figure 2 shows the gender gap in the probability of full-time work (men minus women). We report estimates of event-time dummies relative to the year before the birth of the first child and find a large and persistent "child penalty" (Angelov et al., 2016; Kleven et al., 2019) in the probability of working full-time across all couple types (dark down-pointing triangle plot).[8] That is, mothers' probability of working full time decreases relative to the same probability for fathers. Women married to a highly-ambitious husband exhibit the largest and most persistent penalty (up-pointing triangle plot). In contrast, we measure the smallest—but still sizable—penalty for highly-ambitious women who are married to husbands of a lower type (square plot). For power couples (diamond plot), the gap is not statistically different from the average gap in the population. Our results show that families in which the mother is of the

[7] We also test for the role of correlated career paths of spouses and find that a spouse in the same field of study is associated with a slightly higher chance of training and promotion, in particular for men, see Table A.4.

[8] All event studies include fixed effects for calendar year, age of the mother, and age difference between the spouses. The sample includes observations of couples where both spouses have completed their formal education. We show the same analysis for earnings as the outcome in Appendix Figure A.1.

Figure 2: Heterogeneous child penalty in labor supply by couple type



Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

highest ambition type, θ_4, experience a significantly lower child penalty relative to households in which the husband is of the highest ambition type and the wife of lower. The fact that highly ambitious women adjust their labor supply the least upon the arrival of their first child may, in part, reflect a response to their better chances of being trained and promoted to manager. Indeed, Figure A.2 in the appendix shows that women of the high-wage-growth ambition types delay fertility until after the ages at which employees typically receive training and promotions to manager, suggesting that firm policies feed back to fertility decisions and to how the family adjusts after having children.

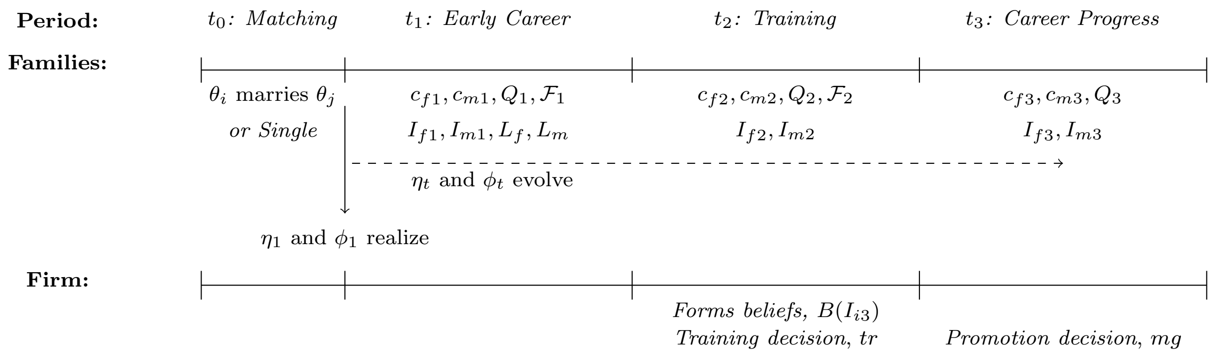
3 A Model of Families' Investments and Internal Labor Markets

To understand the mechanisms through which firms' decisions about on-the-job managerial training and promotions to management vary with workers' family composition, we develop a model in which family-side and firm-side investments interact. We first introduce the general setup in Section 3.1 and we formally present the details of families' collective problem in Section 3.2, the firm's optimization problem in Section 3.3, and partner choice in Section 3.4. In Section 3.5, we define the equilibrium of the model. Finally, we discuss the empirical implementation in Section 3.6.

3.1 Setup

We study an economy populated by an infinitely lived representative firm and overlapping cohorts of individuals. Each cohort has an equal mass of women and men and lasts for four periods denoted by t. In the following, we focus on the life cycle of one cohort of individuals. We denote the set of women by $\mathcal{X}$ and the set of men by $\mathcal{Y}$, and we index individuals in set $\mathcal{X}$ with f and in set $\mathcal{Y}$ with m. We denote the gender of individual $i \in \{f, m\}$ by $\mathcal{G}_i \in \{\mathcal{X}, \mathcal{Y}\}$. Individuals are characterized by their exogenous ambition type, $\theta_i \in \Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, and participate in two markets, the marriage and

Figure 3: The life cycle of individual θ_i and the representative firm



the labor market.

In the marriage market, individuals choose whether to marry and, if so, the ambition type of their partner. We only consider opposite-sex couples. In the labor market, workers develop their careers on one of the two career *ladders*. We think of ladders as proxies for occupations (as in the empirical section) and we allow them to differ in returns to skill and human capital accumulation as we detail below. We denote a worker's ladder choice by $L_i \in \{L_1, L_2\}$. To focus on the role that different types of households—in terms of marital status and marital sorting on ambition—play in explaining gender differences in on-the-job managerial training and promotions, we consider a competitive labor market with a representative firm. The simplifying assumption of a representative firm allows us to abstract from worker-firm sorting and from interactions between training and poaching decisions across heterogeneous firms.[9] Over their life cycle, individuals accumulate two types of human capital: market human capital, η_{it}—which is valuable in the labor market—and family human capital, ϕ_{it}—which is valuable to their households.

Figure 3 depicts the life cycle of individual i of ambition type θ_i along with the choices of the representative firm over the four periods. The notable features of each period are the following. In the *Matching* period, households form and, afterwards, the stochastic initial values of market and family human capital are realized. Thus, these realizations are not known prior to marriage. In the *Early Career* period, individuals sort into the firm's ladders and initiate their labor supply. Thereafter, market and family human capital evolve depending on the labor supply choices that we specify below. In the *Training* period, firms select the most promising workers to participate in the managerial training program. Finally, in the *Career Progress* period, firms select who to promote to manager among their trained workers, and workers develop their careers further. We describe these model features in more detail in the following paragraphs.

The marriage market. In the *Matching* period, individuals meet in a static marriage market and decide whether to marry a partner of ambition type $\theta_j \in \Theta$ or to remain single. In the latter case, we denote $\theta_j = \emptyset$. Formally, the marriage choice set is

[9] Friedrich (2023) develops a model in which heterogeneous firms face a trade-off between training internal candidates for managerial positions (similar to our setup) and hiring external managers.

$$\Theta_0 = \emptyset \cup \Theta = \{\emptyset, \theta_1, \theta_2, \theta_3, \theta_4\}.$$

There are 24 *types* of households that could form: 16 types of couples, which are pairs of ambition types $(\theta_f, \theta_m) \in \{\Theta_0 \times \Theta_0 \setminus \{(\emptyset, \emptyset)\}\}$, four types of single households with a woman of ambition type $\theta_f \in \Theta$, and four types of single households with a man of ambition type $\theta_m \in \Theta$. We adopt the convention that the first argument always refers to the female and the second, to the male. We denote the set of households by

$$\mathcal{H}_0 = \mathcal{X} \cup \emptyset \times \mathcal{Y} \cup \emptyset \setminus \{(\emptyset, \emptyset)\},$$

with element $h = (f, m)$ for couples and $h = (f, \emptyset)$ and $h = (\emptyset, m)$ for single women and men, respectively.

We build on frictionless marriage market matching models (Choo and Siow, 2006; Chiappori et al., 2017, 2018) and assume that the total value of joining household type $(\theta_f, \theta_m) \in \Theta_0^2 \setminus \{(\emptyset, \emptyset)\}$ for individual i is the sum of two components. First, an *endogenous* economic value common to all individuals of the same gender in the same type of household. Second, an idiosyncratic taste shock for a particular marital alternative that is assumed to follow an extreme value distribution of type I (Gumbel) with scale parameter σ^β. Formally, the values of choosing the marital alternatives $\theta_j \in \Theta_0$ for woman f or man m are, respectively,

$$\overline{U}_{\mathcal{X}}^{\theta_f \theta_j} + \beta_f^{\theta_f \theta_j} \text{ and } \overline{U}_{\mathcal{Y}}^{\theta_j \theta_m} + \beta_m^{\theta_j \theta_m}.$$

The endogenous economic components for each type of household, $\overline{U}_{\mathcal{X}}^{\theta_f \theta_j}$ and $\overline{U}_{\mathcal{Y}}^{\theta_j \theta_m}$, are determined by the choices of families and firms in the life cycle periods t_1 to t_3, which we describe in the next subsections 3.2 and 3.3, while $\beta_f^{\theta_f \theta_j}$ and $\beta_m^{\theta_j \theta_m}$ are the taste shocks.

The family. Households that form in the marriage market enter the 3-period life cycle. Married individuals jointly make consumption, fertility, and career investment choices. In our sample, divorce occurs most frequently later in the life cycle, after these career decisions have already been made. Thus, we abstract from divorce.[10] Spouses jointly choose which career ladders to join at the beginning of the period t_1, whether to remain childless or have the first child in periods t_1 or t_2—$(\mathcal{F}_1, \mathcal{F}_2) \in \{0, 1\}^2 \setminus \{(1, 1)\}$, their private and public consumption in each period—$\{(c_{ft}, c_{mt}, Q_t) \in R^3\}_{t=1}^3$, and their labor supply in each period—$\{(I_{ft}, I_{mt}) \in I^2\}_{t=1}^3$. We consider four labor supply choices: nonparticipation (NP), part-time (PT), full-time (FT), and super-full-time (SFT), which are defined by set $I = \{NP, PT, FT, SFT\} = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$. Households do not make a fertility choice in period t_3, so we define whether there are children in the household in period t by $\overline{\mathcal{F}}_t \in \{0, 1\}$:

$$\overline{\mathcal{F}}_t = \sum_{\tilde{t}=1}^{\min\{t,2\}} \mathcal{F}_{\tilde{t}}.$$

In each period, married individuals value their consumption according to the flow utility function

[10] 91.5% of divorces happen in the training and career progress periods. Supported by this observation, we assume a static one-shot marriage market with full commitment in our model.

$$u_{it} = \begin{cases} c_{it} Q_t & \text{if } \overline{\mathcal{F}}_t = 0 \\ c_{it} Q_t \chi^u & \text{if } \overline{\mathcal{F}}_t = 1, \end{cases}$$

where factor $\chi^u > 1$ denotes a utility boost if children are present in the household.

The household public good, Q_t, in married households is produced in every period with market goods c_{Qt} and spouses' time inputs according to the production function

$$Q_t = c_{Qt} + \phi_{ft}(1 - I_{ft}) + \phi_{mt}(1 - I_{mt}) - \chi^Q \cdot \overline{\mathcal{F}}_t.$$

We normalize a unit of time to be equal to 1, so our parametrization of labor supply set I implies that individuals working SFT have no time left for home production. Goods and time are perfect substitutes in producing the public good and the *family* human capital of each spouse, ϕ_{it}, captures the productivity of spousal time relative to market goods. Moreover, we assume that a minimum level of public good production is required when there are children in the household, which we model with the term $\chi^Q > 0$.[11]

While perfect substitution between market goods and time at home yields a force towards specialization of each spouse in either market or home production, our assumption about the role of public goods for individual utility provides a counteracting mechanism. The public good can be produced using market goods (for example, ready-to-eat meals or formal childcare), and public and private consumption are complementary in individuals' utility. This structure generates incentives towards equal incomes of spouses to maximize total household utility. This, in turn, is a force toward positive sorting based on ambition, i.e. earnings potential, in the marriage market.

Singles face all the choices that married individuals face over the life cycle except the fertility choice. The flow utility and public good production technology for singles are, respectively,

$$u_{it} = c_{it} Q_t \text{ and } Q_t = c_{Qt} + \phi_{it}(1 - I_{it}).$$

The representative firm. Within each ladder, the firm employs individuals in two types of jobs, producer or manager, denoted by $J_i = \{p, mg\}$. In each period, each worker i produces output *per unit of time* in job J_i and ladder L_i according to production technology

$$y_{L_i,J_i}(\eta_{it}) = a_{L_i,J_i} + b_{L_i,J_i} \eta_{it},$$

which depends on the worker's *market* human capital, η_{it}.

All workers start their careers as producers. In period t_2, the firm selects a subset of their producers on each ladder whom they offer firm-specific managerial training, tr. We denote the size of the on-the-job training program in ladder L by N^L_{tr}. In period t_3, the firm promotes a subset of their trainees to be managers, mg, whose position requires super-full-time work.[12]

[11] This is inspired by Goussé et al. (2017) who assume that home production is determined by a Stone–Geary function and estimate the minimal inputs. We tie these minimal inputs to the presence of children, $\chi^Q = 0$ for childless couples.

[12] This assumption reflects our empirical evidence on high and irregular working hours of managers: 87.5% of managers observed in the labor force survey (LFS) work super-full-time, compared to only 41% among never-managers.

Training and promotions are selective, which excludes the possibility that the firm screens workers by offering training to all. First, the number of management promotions must match the fixed capacity for managerial slots on each ladder, N^L_{mg}. Second, firms face a convex cost of training on each ladder, denoted by C^L_{OJT} and parameterized by factor ζ

$$C^L_{OJT} = \zeta \left(N^L_{tr}\right)^2.$$

Intuitively, training requires direct one-to-one interactions between each trainee and current managers (mentors). Training is specific to the firm and represents, for example, specific knowledge about the firm's clients and communication structure that a candidate acquires on the job by accompanying the current manager to important internal and external meetings. Hence, the convex cost function reflects the managerial time constraint and increasing opportunity costs.[13]

Market human capital. Market human capital evolves over the worker's life cycle and depends on both firm-side and family-side investments. First, after family formation and at the beginning of period t_1, individuals draw an initial level of market human capital from a distribution F^η. The mean of this distribution depends on the individual's ambition type:

$$\eta_{1i}(\theta_i) \overset{\text{iid}}{\sim} F^\eta(\mu^\eta_{\theta_i}, \sigma^\eta) \quad \forall \theta_i \in \Theta.$$

In subsequent periods, the *beginning-of-period-t* market human capital, η_{it}, depends on the human capital level and labor supply in the previous period (by ambition type and career ladder) and training:

$$\eta_{it} = [\eta_{it-1} + \alpha_{L_i,\theta_i} + \delta^S_{L_i} \mathbb{1}_{\{I_{it-1}=SFT\}} - \delta^P_{L_i} \mathbb{1}_{\{I_{it-1}=PT\}} - \delta^N_{L_i} \mathbb{1}_{\{I_{it-1}=NT\}}] \cdot \tau.$$

When worker i of type θ_i on ladder L_i worked full-time in the previous period $t-1$, their human capital increases by α_{L_i,θ_i}. Having worked super-full-time further boosts human capital by δ^S_L. In contrast, a period of part-time work or non-participation reduces the accumulation of market human capital according to δ^P_L or δ^N_L, respectively. On top of this, if the firm trains the worker in period t_2, the market human capital is multiplied by τ in period t_3. Formally:

$$\tau \begin{cases} = 1 & \text{if } t = \{t_0, t_1, t_2\} \\ > 1 & \text{if } t = t_3 \ \ \& \ \ tr = 1 \end{cases}.$$

Thus, market human capital evolves due to investments by both families (labor supply, ladder) and firms (training).

Family human capital. The relative productivity of time in home production also evolves over time. After households form, individuals draw their initial level of family human capital from distribution F^ϕ. For couples, the initial value is common to both spouses. This shock rationalizes the

[13] This intuition is reflected in the cost function by rewriting it as the sum of training costs per manager, $C^L_{OJT} = N^L_{mg} \cdot c^L_{OJT} = N^L_{mg} \cdot \hat{\zeta}^L \left(\frac{N^L_{tr}}{N^L_{mg}}\right)^2$, where $\hat{\zeta}^L = \zeta \cdot N^L_{mg}$ is the training cost parameter at the level of individual manager teams, see Appendix B.

observed heterogeneity in time use across otherwise similar families. Additionally, we allow for the possibility that married women have a productivity premium in home production, $\kappa \geq 1$. This is in line with recent findings that married women tend to spend more time at home than men after controlling for observable characteristics (Hancock, Lafortune, and Low, 2024). Formally, we define the initial family human capital of individual i in household $h = (f, m) \in \mathcal{H}_0$ as

$$\phi_{i1} = \begin{cases} \bar{\phi}_h \kappa & \text{if } i = f \;\;\&\;\; m \neq \emptyset \\ \bar{\phi}_h & \text{if } i = m \vee (i = f \;\;\&\;\; m = \emptyset) \end{cases},$$

with $\bar{\phi}_h \overset{\text{iid}}{\sim} F^{\phi}(\mu^{\phi}, \sigma^{\phi})$. Over time, family skills depreciate (relative to private goods) at rate γ,

$$\phi_{it} = \phi_{it-1}\gamma,$$

where $\gamma > 0$. Note that despite this depreciation in individual family skills, the relative gap in spousal skills persists over time.

Family Type vs. Worker Type at a given time t. A crucial and novel feature of our model is that individuals belong to two types at any period.

The individual i's *worker type* at time t, which we denote by ω_{it}, is a collection of all worker characteristics that their employer observes. These include exogenous traits (gender, ambition type, and initial human capital), which we denote by $\omega_i = (\mathcal{G}_i, \theta_i, \eta_{1i})$, and the history of endogenous choices (ladder, labor supply history, and training), which we denote by $\tilde{\omega}_{it} = (\{L_i\}_{\forall t \geq 1}, \{I_{ir}\}_{r=1}^{t}, \{tr_i\}_{t=t_3})$:

$$\omega_{it} = (\mathcal{G}_i, \theta_i, \eta_{1i}, \{L_i\}_{\forall t \geq 1}, \{I_{ir}\}_{r=1}^{t}, \{tr_i\}_{t=t_3}) = (\omega_i, \tilde{\omega}_{it}), \quad \omega_{it} \in \Omega_t$$

In addition, worker i of gender $\mathcal{G}_i$ in household $h \in \mathcal{H}_0$ has a *family type*, which we denote by φ_{ht}. The family type is a collection of the worker's and the spouse's (if present) worker types and the family human capital. For example, the family type of married woman f in couple $h = (f, m)$ at time t consists of her *own* worker type, her *spouse's* worker type, and their common family human capital:

$$\varphi_{ht} = (\underbrace{\omega_f, \tilde{\omega}_{ft}}_{\omega_{ft}}, \underbrace{\omega_m, \tilde{\omega}_{mt}}_{\omega_{mt}}, \bar{\phi}_h), \quad \textit{for } h = (f, m).$$

It is again convenient to distinguish between exogenous objects and choices in the definition of the family type. We therefore use the notation $\varphi_h = (\omega_f, \omega_m, \bar{\phi}_h)$ for exogenous family traits and $\tilde{\varphi}_{ht} = (\tilde{\omega}_{ft}, \tilde{\omega}_{mt})$ for endogenous family choices, and express:

$$\varphi_{ht} = (\varphi_h, \tilde{\varphi}_{ht}).$$

The family type of singles is denoted by $\varphi_{h=(f,\emptyset)t} = (\omega_{ft}, \emptyset, \bar{\phi}_{(f,\emptyset)})$ for women and $\varphi_{h=(\emptyset,m)t} = (\emptyset, \omega_{mt}, \bar{\phi}_{(\emptyset,m)})$ for men.

We emphasize here a crucial feature of our model: while workers make choices based on their *family* type, the firm in our baseline model only knows with certainty their workers' *worker* type, and needs to form beliefs about the family type of their workers (φ_{ht}).[14] In other words, the firm faces uncertainty about workers' future labor supply decisions, which also depend on their marital status and their spouse's characteristics unknown to the firm.

3.2 The family problem

Couples that formed in the marriage market choose a contingent—on family type—contract of ladders, fertility, labor supply, and consumption for each spouse. Adopting the notation that in period t_0—before families make any choices—the endogenous part of the family type is $\tilde{\varphi}_{h0} = \emptyset$, we define a contingent contract for household $h = (f, m)$ of worker i as:

$$x(\varphi) = \left\{x_t(\varphi_h, \tilde{\varphi}_{ht-1})\right\}_{t=1}^{3} =$$
$$\left\{L_f(\varphi_{ht-1}), L_m(\varphi_{ht-1}), \mathcal{F}_t(\varphi_{ht-1}), I_{ft}(\varphi_{ht-1}), I_{mt}(\varphi_{ht-1}), c_{ft}(\varphi_{ht-1}), c_{mt}(\varphi_{ht-1}), c_{Qt}(\varphi_{ht-1})\right\}_{t=1}^{3}.$$

Couple h chooses the contract that solves the following collective life cycle problem (HP_h)

$$\begin{aligned} \overline{U}_{\mathcal{Y}}^{\theta_f\theta_m} = \max_{x(\varphi)} \quad & E_{\varphi_h}\sum_{t=1}^{T=3}\rho^{t-1}\Big\{u_m(x_t(\varphi_{ht-1}))\Big\} \\ s.t. \quad & E_{\varphi_h}\sum_{t=1}^{T=3}\rho^{t-1}\Big\{u_f(x_t(\varphi_{ht-1}))\Big\} \geq \overline{U}_{\mathcal{X}}^{\theta_f\theta_m} \\ \forall t>0, \forall \varphi_{ht}: \quad & c_{ft}+c_{mt}+c_{Qt} = w_{L_f,J_{ft}}(\omega_{ft-1})I_{ft} + w_{L_m,J_{mt}}(\omega_{mt-1})I_{mt} \end{aligned} \tag{HP_h}$$

where E_{φ_h} denotes the expectation from the perspective of the matching stage over the distribution of household h's exogenous family type φ_h, and ρ is the discount factor.[15] That is, the household chooses the contract $x(\varphi)$ that maximizes the expected lifetime utility of the husband subject to the wife's attaining a lifetime expected utility of at least value $\overline{U}_{\mathcal{X}}^{\theta_f\theta_m}$ (the first constraint in the problem).[16] This value is a utility price common to all women of ambition type θ_f married to men of ambition θ_m, endogenously determined in the marriage market equilibrium (as we formalize in Section 3.4) but taken as given when households make their choices.

Furthermore, for every period and level of family human capital, the contingent contract must satisfy the period-state budget constraint (the second constraint in the problem). As it is standard, the budget constraint restricts the contingent plan to allocations such that total expenditures on market goods (used for private and public consumption) equal total household resources. In our model, resources are endogenously determined by household choices and, importantly, depend in part on firm policies. Specifically, note that at any period t, the optimal family contract determines the endogenous component of the worker types for all household members: $\tilde{\omega}_{it} \subset x_t(\varphi_{ht-1}), \forall i \in \{f, m\}$. When families make their choices over their members' worker types, they take as given firm's decisions that affect total family resources. In particular, families take as given the firm's policies of what *worker*

[14] We also consider an alternative information structure where firms observe the family type in Section 4.4.
[15] Because our periods are of roughly equal length we assume the same discount factor between each period.
[16] Without loss of generality, we present the couple's problem from the perspective of the husband.

types to train and promote to manager, as well as the wage rate function that specifies the pay that each worker type receives in their ladder and job assignment, $w_{L,J}(\omega)$. Even though taken as given by households, wages and training and promotion policies are endogenously determined in the equilibrium in the labor market (formalized in Section 3.3).

Individuals who remain single solve a similar problem but without the first constraint of the contract being incentive-compatible for the spouse.

3.3 Firm's problem

Wages. Firms compete for workers in a competitive labor market. They offer wages based on differences in observed worker type ω but cannot write long-term contracts that condition on future choices. This implies that workers are paid a wage rate, w, equal to their productivity per unit of time, which is determined by the level of human capital at the beginning of each period. That is, the wage function is given by

$$w_{L,J}(\omega_{it-1}) = y_{L,J}(\eta_{it}).$$

Training and promotion policies. Firms can offer firm-specific training to their workers to boost their match-specific productivity and prepare them for managerial positions. Only workers with completed training can be promoted to managerial positions and trainees generate higher production output in period 3 even if they remain in a producer job. We denote the skill level after successful training by η^{τ}. Firms pay for the training and share the rents from the additional future production output with workers according to rent-sharing parameter λ, as detailed below. This rent sharing will help capture the pay premium for managers. It incentivizes households' participation in firm-specific training and ensures employee retention after completing training.

The firm maximizes profits by choosing the optimal training and promotion policies for each ladder. Specifically, the firm chooses the share of trainees for each worker type in period t_2, $\{tr(\omega_2) \in [0,1]\}\ \forall\ \omega_2 \in \Omega_2$ and their promotion offer policy, $\{mg(\omega_2) \in [0,1]\}\ \forall\ \omega_2 \in \Omega_2$ to maximize total profits from training on each ladder, subject to the capacity constraints,

$$\max_{\{tr(\omega),mg(\omega)\}_{\forall \omega(L)\in\Omega_2}} \quad \Pi^L_{tr} = \sum_{\omega(L)\in\Omega_2} \Big(tr(\omega) N_\omega \big[mg(\omega) E[\pi_{mg}(\omega)] + (1-mg(\omega)) E[\pi_p(\omega)] \big] \Big) - C^L_{OJT}(N^L_{tr}) \tag{FP}$$

$$s.t. \qquad N^L_{tr} = \sum_{\omega(L)\in\Omega_2} tr(\omega) N_\omega$$

$$N^L_{mg} \geq \sum_{\omega(L)\in\Omega_2} tr(\omega) N_\omega \cdot mg(\omega) \cdot B_3(SFT \mid tr,\ \omega).$$

Expected profits from training and promotion depend on several components. First, the firm determines the number of trainees of each worker type ω based on its training policy $tr(\omega)$ and the total number of individuals of this type, denoted by N_ω. The firm takes this level N_ω as given, but it is determined in equilibrium from aggregating over the optimal behavior of all households (Section 3.2).

Second, among the trained workers, the firm *offers* manager promotions at rate $mg(\omega)$. Manager promotion requires completed training and working super-full-time in period t_3. Reflecting potential uncertainty about workers' future labor supply, the firm considers the expected revenue gain from offering the promotion, denoted by $E[\pi_{mg}(\omega)]$, or not, which implies remaining a producer, $E[\pi_p(\omega)]$. In case of a promotion offer, these expected gains are given by

$$E[\pi_{mg}(\omega)] = (1-\lambda)\left(\Big(y_{L,mg}(\eta^\tau)-y_{L,p}(\eta)\Big)SFT\cdot B_3(SFT\mid\omega)+\sum_{I\in\{PT,FT\}}\Big(y_{L,p}(\eta^\tau)-y_{L,p}(\eta)\Big)I\cdot B_3(I\mid\omega)\right).$$

Intuitively, total expected revenue gains are equal to a weighted sum of gains for each potential future labor supply choice of the worker by their household, including the possibility of producer assignment if future labor supply is too low. The size of the revenue increase in each job J is determined by the additional output per unit of time at trained skill level η^τ compared to untrained skill level η, multiplied by respective labor supply in period t_3. These gains are shared with the worker according to rent-sharing parameter λ, and the firm captures share $1-\lambda$.[17] The weights are given by the firm's beliefs about period t_3 labor supply conditional on observable worker type ω, $B_3(I|\omega)$. Specifically, the firm forms these beliefs by anticipating optimal future choices of different family types that are consistent with an individual's observed worker type ω_{i2},

$$B_3(\tilde{I}\mid\omega_{i2})\equiv B\left(I_{i3}(\omega_{i2})=\tilde{I}\mid tr_i,\ \omega_{i2}\right)=\sum_{h\text{ s.t. }\omega_{i2}\in argmax(HP_h)}\frac{\Gamma(\theta_f,\theta_m)}{\Gamma_{\omega_{i2}}}E_{\varphi_h}\left[\tilde{I}\in argmax(HP_h)\middle|tr_i\right],$$

where $\Gamma_{\omega_{i2}}=\sum_h\Gamma(\theta_f,\theta_m)\mathbb{1}\{\omega_{i2}\in argmax(HP_h)\}$ is the total share of households who optimally make choices that produce the worker's observed type ω_{i2}. To form this belief, the firm takes as given the frequency of (θ_f,θ_m) households, denoted by $\Gamma(\theta_f,\theta_m)$, which will be determined in equilibrium from the marriage market (as formalized in Section 3.4). The expectation E_{φ_h} is taken over the set of possible family types of the worker, which integrates over family human capital, as well as career investments of worker i's spouse including the spouse's training chances that may impact future labor supply of worker i.[18]

We note that the firm beliefs and policies are specific to each worker type, which includes the individual's gender. This suggests the potential for statistical discrimination, as firms may take into account how differing household roles between men and women could impact future labor supply. In section 4.4, we further explore the role of the firm's information and gender asymmetries. Specifically, we consider the alternative case of strictly history-based firm policies that cannot differentiate by

[17] Specifically, period-t_3-hourly wages for individuals who were trained in period t_2 and are employed in job J in period t_3 are given by the baseline productivity at skill level η without training plus the shared rent from the additional output in assignment J at trained skill level η^τ,

$$w_{L,J}(\omega_{i2}|tr_i=1)=y_{L,P}(\eta_{i3})+\lambda\cdot\big(y_{L,J}(\eta^\tau_{i3})-y_{L,P}(\eta_{i3})\big).$$

[18] Similarly, gains in case of subsequent producer assignment are determined by

$$E[\pi_p(\omega)]=(1-\lambda)\sum_{I\in\{SFT,PT,FT\}}\Big(y_{L,p}(\eta^\tau)-y_{L,p}(\eta)\Big)\cdot I\cdot B\left(I\mid\omega\right),$$

gender, and the case in which firms have full information about each workers' family.

The firm weighs the revenue gains from training against the total cost of the training program, which depends on the total size of the training program. This size is determined by the first constraint in the firm problem (FP) which sums the number of trainees across all worker types. The second constraint describes the capacity constraint for managers on ladder L. We treat the total number of manager slots on ladder L, N^{L}_{mg}, as exogenously given and require that the number of expected manager promotions based on training, promotion offers, and firm beliefs about period-t_3 labor supply does not exceed this capacity.

In sum, optimal behavior of the firm pins down policies $tr(\omega)$ and $mg(\omega)$ that households take as given when deciding on their optimal career investment choices.

3.4 The partner choice problem

Potential partners participate in a competitive static marriage market. Individuals in the marriage market take as given (i) the realizations of their idiosyncratic taste shocks for all marital alternatives, $\{\beta_i^{\theta_f\theta_m}\}_{(\theta_f,\theta_m)\in\{\Theta_0^2\setminus(\emptyset,\emptyset)\}}$; (ii) the wage function, $w_{L,J}(\omega_{it-1}), \forall t$; (iii) the firm's training and promotion policies, $tr(\omega_2)\in(0,1)$ and $mg(\omega_2)\in(0,1)$, $\forall\omega_2\in\Omega_2$; and (iv) the set of all utility prices that women of ambition type θ_f and men of ambition type θ_m receive in a marriage to any ambition type, $\{\overline{U}_{\mathcal{X}}^{\theta_f\theta_m}, \overline{U}_{\mathcal{Y}}^{\theta_f\theta_m}\}_{(\theta_f,\theta_m)\in\Theta^2}$.

These elements allow any individual of ambition type θ_i to anticipate the value of any potential type of household they may form. For example, a man of type θ_m expects—from the solution of problem (HP_h)—to gain $\overline{U}_{\mathcal{Y}}^{\theta_f\theta_m}(\overline{U}_{\mathcal{X}}^{\theta_f\theta_m})$ from marrying a woman of type θ_f when he commits to allocate a lifetime utility of $\overline{U}_{\mathcal{X}}^{\theta_f\theta_m}$ to her.

The man θ_m's partner-choice problem is to choose the marital alternative Θ_0 that maximizes his lifetime utility:

$$\max\Big\{\overline{U}_{\mathcal{Y}}^{\emptyset\theta_m} + \beta_m^{\emptyset\theta_m}, \Big\{\overline{U}_{\mathcal{Y}}^{\theta_f\theta_m} + \beta_m^{\theta_f\theta_m}\Big\}_{\theta_f\in\Theta}\Big\}$$

All types of women solve an analogous partner-choice problem. A competitive equilibrium in the marriage market obtains at the matrix of utility prices such that the market clears in the sense that all types of couples are chosen by the same amount of men and women, and the sum of married and single individuals equals the mass of women and men in the market. We denote the (equilibrium) distribution of couple types by matching function $\Gamma(\theta_f,\theta_m):\Theta^2\rightarrow(0,1)$.

3.5 Equilibrium

We solve for a competitive equilibrium, which is defined by marriage market matching frequencies and utility prices, labor market wage rates, households' ladder, fertility, labor supply, and consumption choices, as well as the firm's training and promotion policies, such that firms maximize profits, households maximize lifetime utility, and marriage markets by ambition type and labor markets by skill level clear. Formally:

Definition 1 *A competitive equilibrium is a tuple of (i) an assignment of women's types to men's types, $\Gamma(\theta_f, \theta_m)$; (ii) a matrix of marriage market prices, $\{\overline{U}_{\mathcal{X}}^{\theta_f\theta_m}, \overline{U}_{\mathcal{Y}}^{\theta_f\theta_m}\}_{(\theta_f,\theta_m)\in\Theta^2}$, (iii) a contingent-contracts-function $x(\varphi)$ prescribing ladders, fertility, labor supply, and consumption decisions for each family type φ; (iv) a distribution of worker types, N_ω $\forall t, \forall\omega \in \Omega_t$; (v) wage functions by job and ladder $w_{L,J}(\omega_{it-1})$ $\forall t, \forall\omega \in \Omega_t$; (vi) the firm's beliefs $B_3(I \mid \omega_2)$; (vii) a training policy $tr(\omega_2) \in (0,1)$; (viii) a promotion policy $mg(\omega_2) \in (0,1)$;*[19] *such that:*
The marriage market clears, married and single households solve problem (HP_h), the firm maximizes profits (FP), and the firm's beliefs are consistent with household behavior.

In this equilibrium, families' decisions and the firm's beliefs interact to endogenously generate gender gaps in human capital accumulation and in the probability to get a manager promotion. For example, when families consider states of the world in which women are trained and promoted less than men, families optimally decide to invest less in the human capital of wives. This reinforces the firm's beliefs that it is less profitable to train women, so the firm optimally invests less in its female employees, which again feeds back into families' decisions. This feedback loop determines the value of different types of households for individuals in the marriage market. In this example, it might incentivize the formation of asymmetric couples who specialize in the husband's career. This marital sorting may further exacerbate gender gaps in firm investments since the firm forms beliefs on workers' profitability based on marriage frequencies and optimal household decisions.[20]

3.6 Empirical implementation

The estimation of our model with three periods in the labor market requires an aggregation of our annual data into corresponding time periods. To this end, we first group years in the data into three periods that resemble career periods in the model. In Appendix D.1, we describe in detail how we use cutoffs of individuals' age to split observed workers' life cycles into the model's three career periods. Figure A.3 illustrates that our processing of the data leads to periods that reflect very well the salient characteristics of each period in the model regarding, e.g., the timing of marriage, fertility, training and promotions. In the data, workers' characteristics and choices may vary over time, also within period. We assign them to each model stage as follows.

First, workers' marital status, decisive domestic partner (if married), ambition type, year of first childbirth, receiving on-the-job training, and holding a managerial occupation do not change over time

[19] We guarantee these open intervals for the training and promotion policies by applying a small, nonzero trembling error rate ϵ to cases where the firm optimally chooses training or promotion policies at the boundaries of 0 or 1. This ensures that households will specify a full set of contingent choices on the equilibrium path. We thank Hector Chade for this suggestion.

[20] Two aspects of our model reduce the possibility of multiple equilibria. First, men and women are sufficiently different because of the technological advantage of women in home production, captured by κ, and this tends to prevent alternative equilibria where gender gaps favoring women arise (Coate and Loury, 1993). Second, we use an equilibrium refinement with trembling such that all possible histories receive (at least small) positive probability of training and promotion.

by construction (as explained in Section 2.2). In case the of fertility, we assign the first childbirth to the model stage in which we observe the woman having her first child.

Second, labor supply, earnings, and career ladder choices may change for individuals over the years within each model period (Figure A.3). For these variables, we aggregate their values within period. The labor supply status in period t is defined as the *mode* of (potentially different) labor supply statuses across years. Similarly, we measure earnings in each period as the *mean* of deflated earnings across years within the period. Finally, in cases where an individual has both flat and steep ladder positions within the same period, we use the most frequent ladder to characterize the entire period.

In addition, we specify the distributions of initial family and market human capital as normal distributions and discretize their support. Specifically, we assume that $F^{\phi} \sim N(\mu_{\phi}, \sigma_{\phi}^2)$ and $F^{\eta} \sim N(\mu_{\eta}, \sigma_{\eta}^2)$ and we use three and five equally spaced grid points, respectively.[21]

4 Estimation and Identification

To estimate our model, we first set the distribution of ambition types by gender to those directly observed in the data. Table A.5 shows that women are more represented in programs of the lowest ambition type, while men are overrepresented in programs of the highest ambition type. We also take the observed number of managers on each ladder to define the firm's managerial capacity constraints,[22] and set the discount factor $\rho = 1$, and the rent-sharing parameter for firm-specific training $\lambda = 0.5$.

We estimate the remaining 38 structural parameters within the model using the simulated method of moments. Our choice of targeted moments is based on a heuristic proof of how each structural parameter is pinned down by moments we observe in our data and that are also produced from the model.[23] Appendix Table A.6 presents the 56 targeted empirical moments. The first column shows a label for each moment which indicates the group they belong to; the second column provides their definition; and the third and fourth columns, their data and model values, respectively.

Based on the identification argument, we target 28 moments of the earnings process including mean initial earnings and earnings growth by ladder and ambition type (presented in Panel A of Table A.6 and labeled as group EP) ; 8 Marriage Market frequencies (Panel B, group MM); 4 moments of the fertility process (Panel C, group FP); 10 moments of labor supply (Panel D, group LS); and 6 moments pertaining firm-side investments (Panel E, group FI).

[21] We chose to be parsimonious for tractability because a higher number of possible human capital values would greatly expand the state space. In particular, firms' policies condition on each value of η and families' choices condition on the values of ϕ in our baseline model.

[22] The share of the population who are managers on ladder L_1 is 3.8% and that share on ladder L_2 is 2.8%, which together yields the total fraction promoted, see Figure 1.

[23] At any structural parameter vector, we simulate our model to produce the vector of 56 moments, mom_{sim}, that have a data counterpart, mom_{data}, and search for the 38-dimensional parameter vector, Π, that solves

$$\min_{\Pi} \quad [mom_{sim}(\Pi) - mom_{data}]'\mathcal{V}[mom_{sim}(\Pi) - mom_{data}], \tag{1}$$

where $\mathcal{V}$ is a positive semi definite weighting matrix. Because of the concerns raised by Altonji and Segal (1996), we normalize all of our moments to be between 0 and 1 and weigh them equally in the estimation. That is, $\mathcal{V}$ is an identity matrix. Computationally, we first use a genetic algorithm to search globally and then run a local search around the global minimum to improve precision.

While we present our identification proof in Appendix C, we give a brief summary here. First, the 8 parameters of the firm's production function—namely, intercepts and slopes of producers and managers by ladder ($\{(a_{L,J}, b_{L,J})\}_{L\in\{L_1,L_2\}\wedge J\in\{p,m\}}$) and the cost of training ζ—are related to earnings growth within ladder and job and to the fraction of trained workers. Second, we use differences across and within ambition types in starting wages to identify the four initial levels of market human capital by ambition type ($\{\mu^{\eta}_{\theta}\}_{\theta\in\Theta}$) and the common variance of initial human capital (σ^{η}). Moreover, the 8 parameters that capture the accumulation of human capital for full-time work ($\{\alpha_{L,\theta}\}_{L\in\{L_1,L_2\}\ \&\ \theta\in\Theta}$), the 2 super-full-time work premia by ladder ($\{\delta^{S}_{L}\}_{L\in\{L_1,L_2\}}$), and the 4 depreciation rates by ladder due to part-time work ($\{\delta^{P}_{L}\}_{L\in\{L_1,L_2\}}$) and non-participation ($\{\delta^{N}_{L}\}_{L\in\{L_1,L_2\}}$) govern the different labor supply choices and the implied earnings differences. Similarly, growth in earnings after training identifies the training skill boost (τ). We also estimate four parameters of the family human capital process: the initial level of family human capital (μ^{ϕ}) and its dispersion (σ^{ϕ}) to match initial participation rates and the variance of labor supply; the advantage of married women's time in home production (κ), which affects gender gaps in participation, training, and promotion; and the depreciation rate of family human capital (γ), which is related to the probability of re-entering the labor force after a period of non-participation. In addition, the utility boost from having children (χ^{u}) and the household's home production floor when children are present (χ^{Q}) govern the rate and timing of fertility. Finally, singlehood rates and marriage patterns discipline the dispersion of marriage market shocks (σ^{β}) and three additional parameters needed to match marital choices: two non-economic values from singlehood for low-wage-growth and high-wage-growth ambition types ($\chi^{\emptyset}_{1,2}$ and $\chi^{\emptyset}_{3,4}$), and a household penalty if both spouses work SFT in $t1$ or t_2 (χ^{S}).

4.1 Estimates and their implication for model fit

We present estimation results in Table 2, which groups estimates by their main role in the model: firm's production function (Panel A), market human capital (Panel B), marriage and fertility processes (Panel C), and family human capital (Panel D). The first and second columns show the symbol by which we denote the parameter in our model and its description, respectively; the column labeled *Par.* shows the estimated value of the corresponding parameter; the column labeled *s.e.* displays the standard errors; and the last three columns show the three most sensitive moments in estimation and their group—whose definitions and names correspond to those in Table A.6 and which we compute following Andrews, Gentzkow, and Shapiro (2017).[24]

In general, the parameters are estimated with high precision and are disciplined by their identifying moments. With the exception of two parameters, standard errors represent less than half the value

[24] We calculate standard errors as the square root of the variance estimator $\left[D_m'\mathcal{V}D_m\right]^{-1}D_m'\mathcal{V}C\mathcal{V}'D_m\left[D_m'\mathcal{V}D_m\right]^{-1}$, where D_m is the 56×38 matrix of the simple average of the backward and the forward numerical derivatives of moment conditions with respect to each parameter at the estimate—and C is the covariance matrix of the data moments, which we compute by bootstrapping. For the bootstrap, we use 1,000 repetitions and resample at the household level (both couples and singles are households). If we sample a couple then both spouses are resampled. We define the LFS as an independent stratum to keep the same share of surveyed individuals across repetitions. We obtain the three moments with the highest value of $|Sensitivity| = |-\left[D_m'\mathcal{V}D_m\right]^{-1}D_m'\mathcal{V}|$ and report the groups to which they belong.

Table 2: Estimates of parameters, standard errors, and top-3 sensitivity moments

Symbol	Description	Par.	s.e.	Sensitivity Moments		
Panel A. Estimates of firm's production function						
$b_{L_1,p}$	**Slope (producers) in L_1**	0.020	0.000	EP1	MM4	MM3
$a_{L_1,mg}$	**Intercept (managers) in L_1**	0.110	0.019	EP11	EP24	EP19
$b_{L_1,mg}$	**Slope (managers) in L_1**	0.022	0.000	EP4	EP19	EP17
$a_{L_2,p}$	**Intercept (producers) in L_2**	-0.070	0.001	EP13	MM4	EP11
$b_{L_2,p}$	**Slope (producers) in L_2**	0.025	0.000	MM4	EP1	MM3
$a_{L_2,mg}$	**Intercept (managers) in L_2**	0.000	0.051	EP26	EP22	EP19
$b_{L_2,mg}$	**Slope (managers) in L_2**	0.025	0.000	EP3	LS9	EP22
ζ	**Cost of training**	5.0018e-07	0.000	MM4	LS9	EP17
Panel B. Estimates of market human capital						
μ_1^η	**Mean of initial hk draw of θ_1**	9.909	0.041	LS1	EP1	LS3
μ_2^η	**Mean of initial hk draw of θ_2**	10.504	0.046	EP11	EP6	LS9
μ_3^η	**Mean of initial hk draw of θ_3**	9.360	0.190	EP4	EP7	EP17
μ_4^η	**Mean of initial hk draw of θ_4**	11.200	0.088	MM4	MM1	EP1
σ^η	**Variance of initial hk draw**	4.145	0.203	EP1	EP17	MM4
$\alpha_{L_1,1}$	**Accumulation rate in L_1 for θ_1**	1.199	0.071	EP11	EP1	EP21
$\alpha_{L_1,2}$	**Accumulation rate in L_1 for θ_2**	2.271	0.163	EP6	EP16	EP24
$\alpha_{L_1,3}$	**Accumulation rate in L_1 for θ_3**	4.709	0.216	EP17	EP13	EP25
$\alpha_{L_1,4}$	**Accumulation rate in L_1 for θ_4**	5.446	0.119	EP19	EP18	MM1
$\alpha_{L_2,1}$	**Accumulation rate in L_2 for θ_1**	1.511	0.166	EP15	EP21	EP10
$\alpha_{L_2,2}$	**Accumulation rate in L_2 for θ_2**	2.383	0.035	LS10	MM1	EP3
$\alpha_{L_2,3}$	**Accumulation rate in L_2 for θ_3**	4.714	0.189	EP17	EP13	MM3
$\alpha_{L_2,4}$	**Accumulation rate in L_2 for θ_4**	5.215	0.088	EP19	EP18	MM1
$\delta_{L_1}^P$	**PT Depreciation rate in L_1**	0.200	0.070	LS9	EP28	FI2
$\delta_{L_2}^P$	**PT Depreciation rate in L_2**	0.290	0.034	EP11	EP1	EP21
$\delta_{L_1}^N$	**NT Depreciation rate in L_1**	1.588	0.026	MM4	MM3	EP4
$\delta_{L_2}^N$	**NT Depreciation rate in L_2**	1.402	0.279	EP11	EP1	EP24
$\delta_{L_1}^S$	**Skill boost from working SFT in L_1**	0.002	0.074	FI1	MM4	LS9
$\delta_{L_2}^S$	**Skill boost from working SFT in L_2**	0.002	0.001	FI6	EP24	EP19
τ	**Skill boost from training**	1.235	0.021	MM4	EP5	EP22
Panel C. Estimates of marriage and fertility processes						
σ^β	**MM preference shock (scale)**	0.003	0.000	EP6	EP27	EP18
$\chi_{1,2}^\emptyset$	**Value of singlehood for θ_1 and θ_2**	1.351	0.002	EP11	EP27	EP18
$\chi_{3,4}^\emptyset$	**Value of singlehood for θ_3 and θ_4**	1.441	0.017	MM4	EP4	MM3
χ^u	**Utility boost with children**	1.169	0.003	FP1	MM3	FP4
χ^Q	**Min HP with children**	0.037	0.000	FP1	FP3	EP6
χ^S	**Penalty both SFT**	0.087	0.034	LS9	FI5	MM1
Panel D. Estimates of family human capital						
μ^ϕ	**Mean of initial family shock**	0.162	0.001	EP7	EP17	EP1
σ^ϕ	**Variance of initial family shock**	0.030	0.002	EP11	EP1	EP21
κ	**Biological advantage of women**	1.161	0.026	EP11	EP20	EP6
γ	**Depreciation rate**	0.681	0.025	EP1	EP28	EP11

Notes: The definitions of the moments listed under *Sensitivity Moments* are in Appendix Table A.6.

of the estimate and for 29 of our estimates their errors represent less than 10%.[25] Moreover, our sensitivity analysis is very consistent with our identification argument. For example, firm's technology parameters are mostly disciplined by initial earnings and earnings growth and in the case of managers' productivity, by the earnings boost after being promoted or trained. Similarly, the mean draws of initial human capital are mostly sensitive to initial mean earnings, and the accumulation rates are mostly driven by earnings growth (for full-time premia) and by firm-side investments (for super-full-

[25]The parameters with high standard errors are the intercept of managers' production in the steep ladder ($a_{L_2,mg}$ in Panel A) and the super-full-time premium in the flat ladder ($\delta_{L_1}^S$ in Panel B), both of which are estimated close to zero.

time premia). On the family side, earnings processes and marriage patterns discipline parameters of the value of households and fertility rates inform the value and constraints from having children. Finally, the parameters of family human capital are mostly related to differences in earnings between different labor supply choices.

In addition, our estimates imply a really good model fit to the 56 targeted moments, as shown in the last two columns of Table A.6. We expand on this next by highlighting how our estimates give rise to some important model features that reflect key facts in our data.

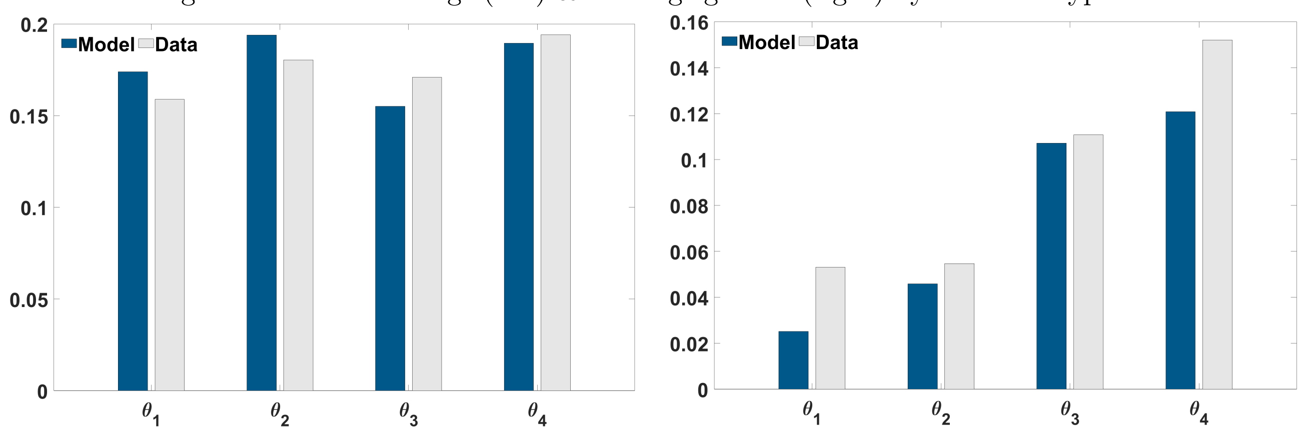
Ambition types, ladders, and earnings. Our estimates of the firm's technology and of market human capital—shown in Panels A and B of Table 2, respectively—imply that our model reflects well the observed ambition types and career ladders.

First, the estimated production function parameters imply that the marginal productivity of skills in both producer and manager jobs are higher in ladder L_2 than in ladder L_1. Therefore, we interpret L_1 as the *flat ladder* and L_2 as the *steep ladder*. Moreover, within each ladder, manager productivity increases more with skills, implying a steeper earnings profile for managers in both career paths.

Second, the market human capital accumulation and depreciation rates align with this interpretation of the two ladders. In effect, L_2 features (i) higher gains in human capital due to full-time work on average—which is also true separately for all ambition types except θ_4—, (ii) a bigger loss of human capital when working part-time, and (iii) a slightly bigger super-full-time premium than L_1.

Third, our estimates of the mean of initial human capital and the accumulation rates by ambition type imply a clear interpretation of the four ambition types that is consistent with how we distinguish them in the data. Indeed, θ_1 and θ_3—the low starting-wage types—show the lowest draws of initial human capital. The opposite is true for θ_2 and θ_4—the high starting-wage types. θ_1 and θ_2—the low wage-growth types—feature the lowest accumulation rates, while θ_3 and θ_4—the high-wage growth types—feature the highest accumulation rates.

Figure 4: Initial earnings (left) & earnings growth (right) by ambition type



Notes: The horizontal axis are *ambition types* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

All in all, our estimates imply that the ambition type of workers is consistent with our classification based on starting wages and wage growth. This can be seen in detail in Panel A of Appendix Table A.6, in which we show that our estimated model reflects very well the various targeted moments of earnings by ambition type and ladder. We also provide an illustrative summary in Figure 4, which shows period-1 earnings by ambition type (left panel) and earnings growth by ambition type (right panel). Darker blue bars correspond to the model-generated moments and lighter gray bars to their data counterpart. Like in the data, workers of ambition types θ_1 and θ_3 start their careers with lower earnings than ambition types θ_2 and θ_4. Moreover, our model captures the growth patterns of the ambition types whereby types θ_3 and θ_4 experience significantly higher earnings growth than types θ_1 and θ_2.

Marriage and fertility patterns. We present our estimates of the marriage and fertility processes in Panel C of Table 2. Our estimated values imply that we successfully capture the targeted observed fractions of each type of household and the timing of fertility—some of which are illustrated in Figure 5 and all of which are outlined in Panels B and C of Table A.6.

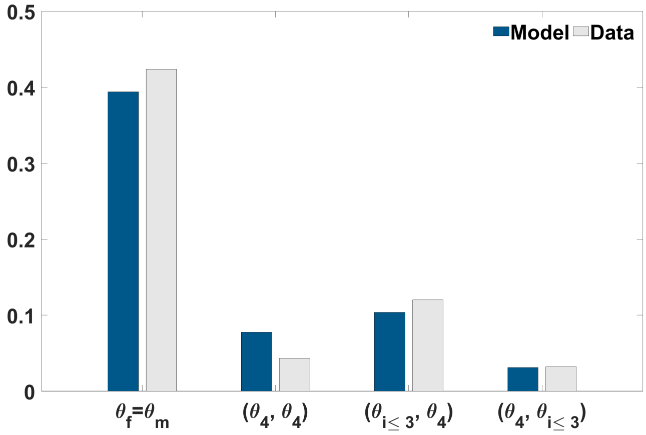
The presence of complementarities in spousal earnings within the household is the model feature that allows us to rationalize three observed facts: (a) that same-ambition couples are frequent (Figure 5, left panel); (b) that there is a positive relationship between own career achievement and spousal ambition (as shown in Section 2.3, Table 1); (c) and that there is a positive correlation in spouses' labor supply (as evidenced, for example, by a low participation gap in homogamous couples which we target—see Panel D of Table A.6).[26] However, this feature alone would tend to overstate the share of married individuals and the degree of positive assortative matching on ambition in the model, relative to those moments in the data. Our estimates of the marital preference shock and of the noneconomic value of singlehood provide a counterbalance to this mechanism and allow our model to match those moments very well as seen in Table A.6, Panel B. Note that the gains from marriage brought by the complementarities in earnings within the household are stronger for ambition types with higher earnings growth and, as a result, we need a higher noneconomic value of singlehood to induce those types to remain unmarried at the rates they do in the data.

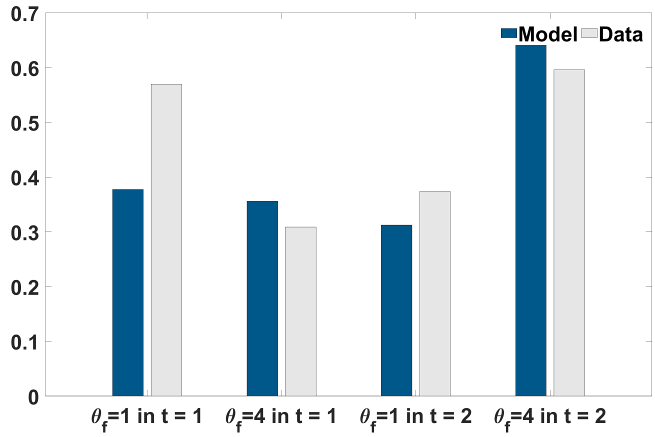
Furthermore, the rate and timing of fertility (Figure 5, right panel) are matched thanks to the utility boost from children and the public good production floor in the presence of children. Note that the consumption value from children is complementary to private and public consumption in the utility function, which provides a force towards higher fertility among the highest ambition types. As a result, our estimate of the utility from children enables us to match well the fertility rates of θ_4-women in periods 1 and 2, but we underestimate the fertility of θ_1-women in period 1. In turn, the technological restriction when having children allows us to match the fact that high-ambition type women delay fertility while low-ambition type women have children earlier.

Gender gaps in labor outcomes over the life cycle. Finally, we present our estimates of the family human capital process in Panel D of Table 2 and discuss how they relate to the targeted labor

[26]Further, pooling observations across all three life cycle periods, we find that 80.3% of married spouses have the same labor supply status (grouping full-time and super-full-time). If we only consider individuals participating in the labor market, the share increases to 89.1%. This suggests a high correlation in spouses' labor supply. This is further confirmed by a Spearman's rank correlation coefficient of 0.23. The null hypothesis that spouses' labor supply are independent is rejected by the data.

Figure 5: Frequency of couple type (θ_f, θ_m) (left) & Timing of fertility (right)

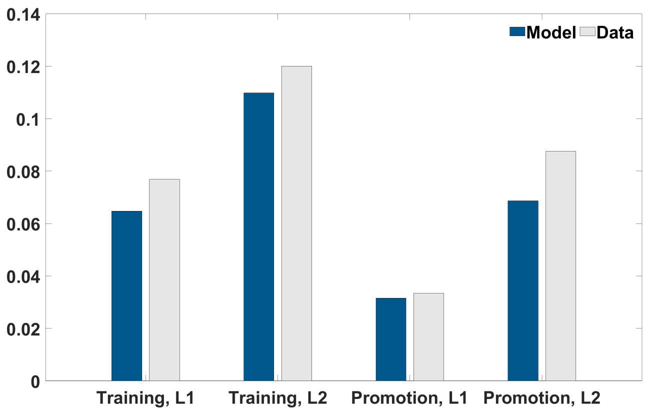




Notes: The horizontal axis in the left panel corresponds to four types of couples, introduced in Section 2.3. The horizontal axis in the right panel captures the group of women of type θ_f observed in period t.

supply patterns shown in Panel D of Table A.6. First, the estimated mean and variance of the initial family shock—which is common to both spouses—implies that we reflect well the patterns of male participation by ambition type whereby type θ_1 exhibits the lowest and type θ_4 the highest participation. However, we underestimate period-1 participation for the lowest ambition types. Moreover, the estimated returns to experience at home mean that only 68.12% of past family human capital passes on to the next period and implies that we match well the likelihood that women who do not participate in one period re-enter the labor force in the next.

Figure 6: Firm-side investments



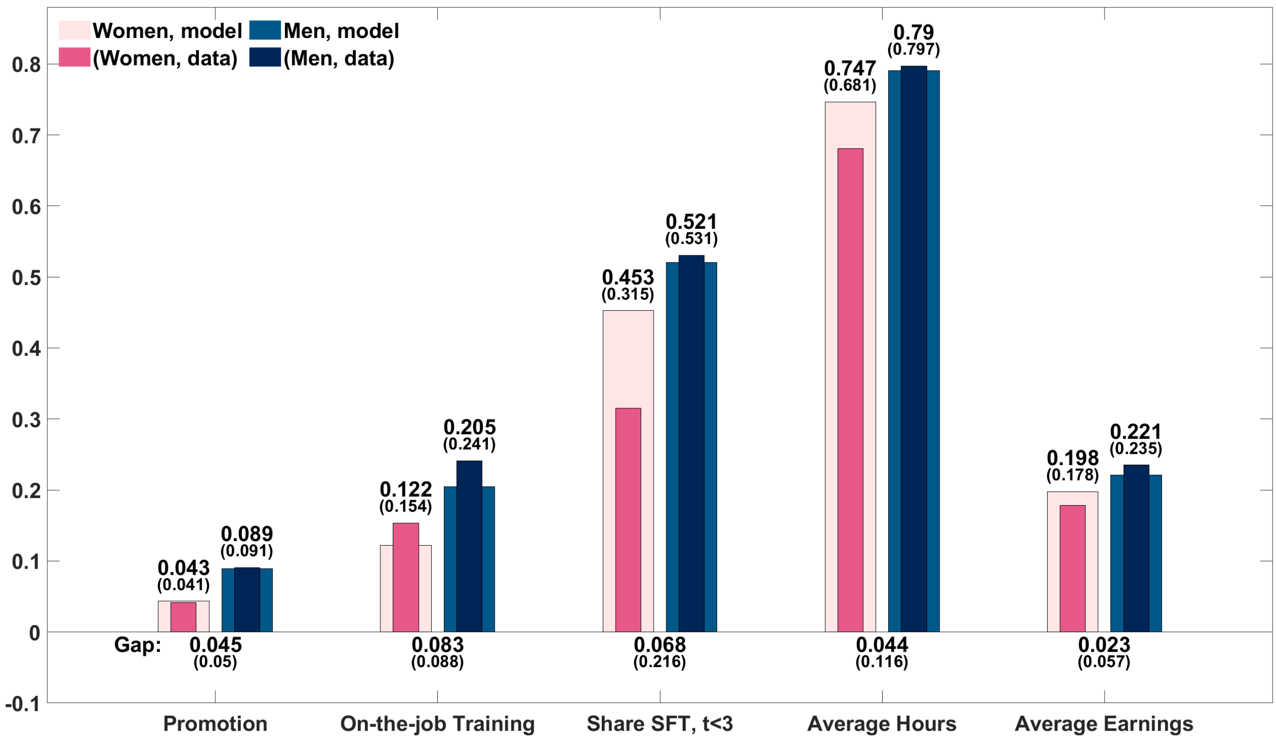
Notes: L1 refers to the flat ladder and L2 to the steep ladder.

Secondly, we emphasize the mechanisms through which our model generates the observed gender differences in labor market outcomes. The estimated value of the female home productivity advantage, κ, means that women's initial family human capital is 16.13% higher than that of men. This relatively small advantage of women in home production implies that we match well the observed initial gender differences in participation in homogamous couples, which is 5 percentage points in the model and 4 percentage points in the data. Interestingly, these small initial gaps amplify as men and women develop their careers, even though the structure of our model features no further gender differences on the family side. The combination of different labor supply choices by men and women and firm-side policies being dependent on the history of human capital and gender imply that both in our estimated model and the data women are less likely to be trained and promoted in both ladders. As illustrated in Figure 6 and detailed in Panel E of Table A.6, our model reflects very well the magnitudes of the observed gender gaps in career achievement. Capturing these gaps well is important for our interpretation of the policy counterfactuals in Section 5.

4.2 Implications for (untargeted) gender gaps in families' and firms' investments

Our estimated baseline model reflects well the sizable gender gaps in career achievement and investments in human capital made by both firms and families, which—notably—were not targeted in estimation. Figure 7 shows five key outcomes for women in lighter pink bars and for men in darker blue bars. Wider bars—and the big-font numbers on top—represent the value of the outcome in our model. The narrower darker bars—and the small-font numbers within parenthesis on top—represent the same moment in the data. The numbers below the bars labeled *Gap* (in big font for model and within parentheses for data) represent the value of men minus that of women, that is, the gender gap of the outcome in levels. The first outcome is the share promoted to manager; the second is the shared trained by the firm; the third set of bars represents the average fraction of individuals working super-full-time in the investment periods $t = \{t_1, t_2\}$; the fourth outcome is the average fraction of time working over the life cycle; and the last outcome is the average annual earnings over the life cycle in millions of Danish Kroner (DKK). In our baseline model (data), the firm invests relatively more

Figure 7: Untargeted gender gaps in baseline model (and data)



Notes: SFT stands for super-full-time. Average hours are calculated given our parametrrization $I = \{0, \frac{1}{3}, \frac{1}{2}, 1\}$. Average earnings are measured in millions of Danish Kroner. All other outcomes reflect fractions of workers.

in men than in women, training 20.5% (24.1%) of men and only 12.2% (15.4%) of women on-the-job. Efficient collective behavior by families implies, in turn, that women supply fewer hours of work to the market relative to men, with men devoting 4.4% (11.6%) more of their total lifetime to their careers.

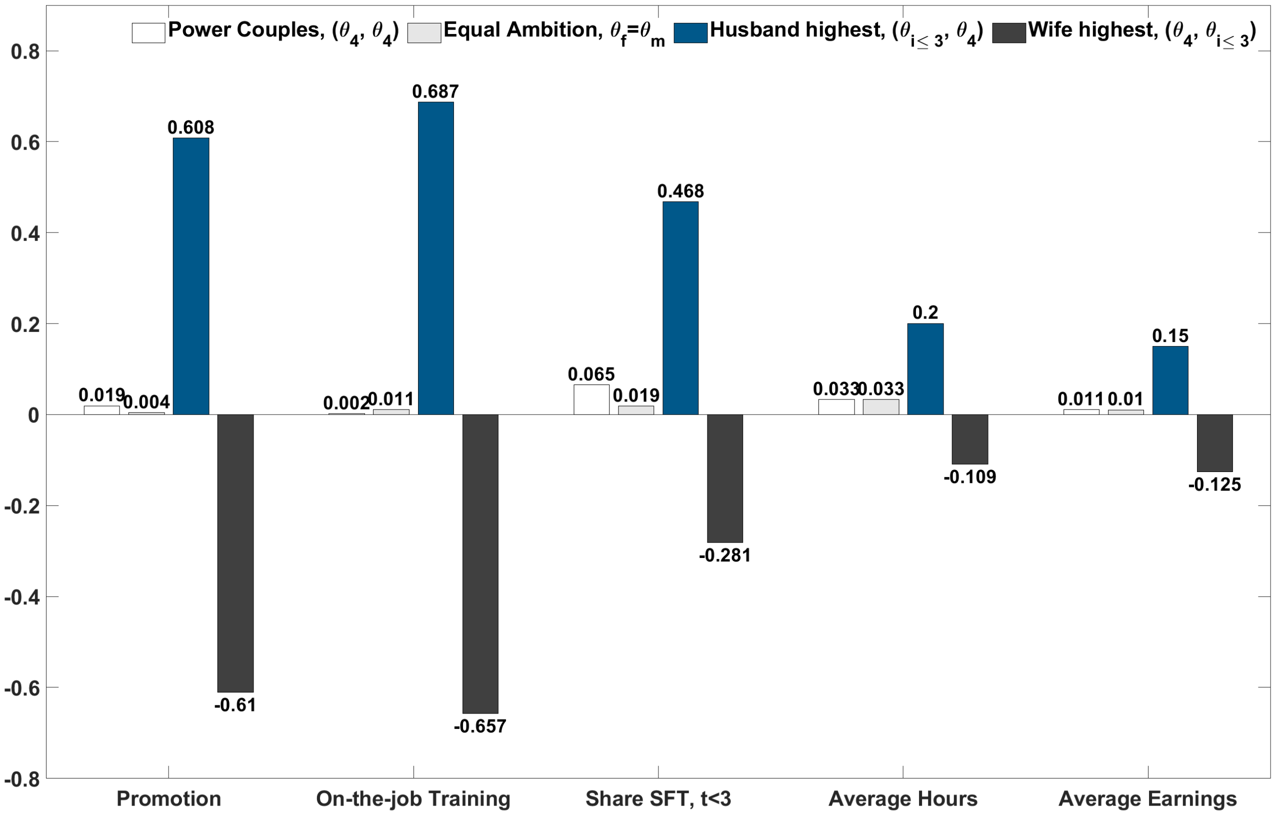
As a consequence, women earn 23,000 DKK (57,000 DKK) less than men per year, representing a gap of 11.6% (32%) relative to women's annual earnings.

The combination of less investments in women's human capital by firms and families implies a sizable gender gap in the fraction in promotions to manager, with women reaching managerial positions with 4.5 (5.0) percentage points less probability than men, meaning that women are—as in the data—half as likely as men to be promoted to managers.

4.3 The role of the family

Consistent with the empirical evidence presented in Section 2, our model reflects well the heterogeneity in training and promotion gender gaps by type of couple (Figure 1, right panel). Figure 8 shows the same outcomes as in Figure 7 but by type of selected couples. White bars represent couples in which both spouses are of the high-starting-wage and high-wage-growth type (θ_4); darker gray bars, couples in which both spouses are of the same ambition type ($\theta_f = \theta_m$); darker blue bars, couples in which the husband is of the highest type and the wife of a lower type ($\theta_{i\leq 3}, \theta_4$); and the darkest bars represent couples with the wife being of the highest type and husband of a lower type ($\theta_4, \theta_{i\leq 3},$).

Figure 8: Gender gaps in baseline model by type of couple (men - women)



Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$. SFT stands for super-full-time. Average hours are calculated given our parametrrization $I = \{0, \frac{1}{3}, \frac{1}{2}, 1\}$. Average earnings are measured in millions of Danish Kroner. All other outcomes reflect fractions of workers.

Interestingly, promotion gender gaps are highest within those households that exhibit the highest gender gaps in labor supply and on-the-job managerial training. Not surprisingly, all those gaps are higher in couples in which the husband is of the highest type and the wife of different ambition.

However, we also see positive gender gaps in firm-side and family-side investments among equal-ambition couples and, particularly, among power couples. This suggests that policies that induce more equal investments between men and women within families or within firms have the potential to reduce economy-wide gender gaps in access to managerial positions—something we study in Section 5.

4.4 The role of information and gender asymmetries

Two salient features of our model are that firms do not observe their workers' *family* type and the specification of two sources of gender inequalities—women's productivity advantage in home production and the firm's setting training and promotion policies differentially by gender. To assess the role each plays in our model, we keep the model's parameters at the estimated values and shut down each of the elements, one at a time, and evaluate changes in our main outcomes relative to our baseline. We therefore consider three alternative specifications of our model: firms have full information, there is no married women advantage at home ($\kappa = 1$), and the firm sets policies excluding the gender of the worker type.

In Appendix Table A.7, we show the value of each of the five main outcomes (those analyzed in Sections 4.2) for the three alternative specifications and their difference relative to the baseline values.

These re-specifications of the model induce equilibrium responses in marriage and fertility that are useful to highlight to interpret our results. To this end, we show in Table A.8 the fraction of households characterized by each of the marriage and fertility patterns indicated by the labels in its column (1). In turn, firms optimally react to the changing family environment by changing the composition of their managers. To illustrate this, we show in Table A.9 the fraction of all managers belonging to the demographic group listed in its column (1).[27] In both tables, column (2) shows the corresponding baseline shares and columns (3) to (5) report the changes relative to the baseline associated with each shut-down exercise, which we describe next.

Full information. In order to assess the role of asymmetric information between workers and firms, we consider a counterfactual scenario in which the firm has full information about the *family type* of their workers. Compared to the baseline model, the firm now directly observes the following additional characteristics of their workers: marital status, household's family human capital, and the ambition type, market human capital, and employment history of the spouse (if any). The firm's training and promotion policies are now a function of the full *family type*, and households anticipate these policies when making marriage and investment choices over the life cycle.

We find that allowing firms to observe the *family type* of their workers has limited equilibrium effects on gender gaps both on average and by couple type. Gender gaps in training and promotion increase slightly. As seen in Panel A of Appendix Table A.7, on aggregate, men receive 0.316% more on-the-job training and are 1.781% more likely to become managers relative to baseline, while women

[27] Because in Panel B we see that almost all managers in the model are of the ambition types representing education programs with high wage growth (θ_3 and θ_4), in Panels C and D, we focus on the share of all managers in the families of individuals of those ambition types by gender.

are less trained and less promoted.

One reason for these small changes is that the firm in our baseline model is extremely sophisticated and forms beliefs about its workers' families with perfect knowledge about the marriage market equilibrium frequencies, distribution of family shocks, and optimal household choices in different couple types. Hence, the observed employment history of a worker allows the firm in our model to infer substantial information about their family type and helps to identify workers with the highest expected return on training and promotion.

Another reason is that individuals react to the full information setting by changing their marital and fertility choices in order to become of the family type that firms prefer for training and promotions. Interestingly, column (3) of Table A.8 shows that in the new marriage market equilibrium under full information, there are more couples in which the husband is of the highest type and the wife of lower ambition—which are the families that allow men to signal their career commitment. And indeed, in Panel C of Table A.9, column (3), we see that firms now select many more male managers who are in these asymmetric couples (the fraction of male managers from couple type $(\theta_{i\leq 3}, \theta_4)$ increases by 0.116 points), and in our analysis of the main outcomes by type of couple—available in Online Appendix Figure OA.2—we find that men in these couples are more likely to be trained while their wives are less likely. On the contrary, there are fewer couples in which the wife is of the highest ambition type and the fraction of type-4 female managers who marry down decreases by 0.018 points.

Symmetric home production productivity, $\kappa = 1$. We inspect the role of gender asymmetries firstly by eliminating the initial female advantage in family human capital. That is, we set the parameter $\kappa = 1$ while keeping all other parameters at the estimated levels, and we solve the model. The main effect is that the initial gender gap in participation in same-ambition couples (which we target in the estimation and match well) completely disappears.[28] Moreover, the gender gaps in training and promotion decrease, but they are not eliminated and continue to be sizable, as shown in Panel B of Table A.7. The result whereby gender gaps in family lifetime labor supply, training, and promotion exist even with equal productivity at home emphasizes the role of the marriage market in shaping the overall equilibrium effects, as gaps are more pronounced in families in which men are of higher ambition than women. When $\kappa = 1$, marriage may either become less attractive because productivity in the household decreases, or more attractive for households hoping to invest in the careers of both spouses. Indeed, in column (4) of Appendix Table A.8, we see that the share of singles increases by 0.178 points relative to baseline and that all types of couples considered are less frequent. However, for individuals of the highest ambition type, we reassuringly find a relative shift from asymmetric couples to power couples. Importantly, training and promotion gaps almost disappear for couples of equal ambition, in particular for power couples (Online Appendix Figure OA.2). The remaining sizable aggregate gaps are explained by different representations of men and women among the highest ambition group, which influences the share of singles and the frequency of families in which spouses

[28]To save space we do not report the value of targeted moments for the alternative specifications but they are available upon request.

are of different ambition types. Indeed, column (4) of Appendix Table A.9 shows that more male singles become managers while promotions of men marrying down decline the most and drive the overall reduction in promotions for men. For women, we find opposite patterns, with fewer female singles being promoted and instead the overall increase in female promotions driven by women with children, especially in power couples.

History-based firm policies. Finally, we solve for a specification of the model in which the firm does not condition on gender when setting training and promotion policies, and families internalize that. Interestingly, this modification leads to meager changes (Panel C of Table A.7), the most salient of which are that women are 3.9% more likely to be trained but also 3.9% less likely to be promoted. Similar to the "Full Information" scenario, one reason why we here also see small effects is that in our baseline model firms condition not only on gender but also on observed employment history, which strongly correlates with gender and household type. Not allowing the firm to condition on gender has minimal effects given the sophistication of our firm that forms expectations based on knowledge of marriage market frequencies and optimal choices of different types of households. Specifically, we find almost no changes in gender gaps for power couples (Online Appendix Figure OA.2), which suggests that the firm can continue to identify these highly ambitious workers. At the same time, we document moderate gender convergence in unequal couples, suggesting that highly ambitious spouses in these couples lose some of their ability to signal their ambition to the firm, while their less ambitious spouses can now increase their chances of training. Consistent with firm policies and household choices remaining similar, we do not find significant changes in marriage market frequencies in this scenario.

In summary, we show that our modeling choices on gender asymmetries and the information structure do not drive our main results or the gender gaps in promotion, training, and family labor supply.

5 Effective policies to narrow gender gaps in career achievement

Our estimated model reflects well both targeted and untargeted key features of the data. In particular, our analysis of gender gaps in managerial promotions by couple type suggests that the interaction between family-side investments and on-the-job managerial training is an important mechanism behind those gaps. This makes our model a suitable framework to evaluate which policies are effective in narrowing gender gaps by inducing both families and firms to invest more into the careers of women. In this section, we start from the estimated baseline model, and shock workers with changes in their policy environment *before* they enter the marriage and the labor markets.

5.1 Implementation

We consider three policy counterfactuals that are inspired by ongoing public debates. Specifically, we consider two alternative parental leave designs and a managerial quota.

Parental leave policies are by now established as a prominent feature of labor markets in developed countries (Olivetti and Petrongolo, 2017), but the debate about optimal policy design continues. For example, within the EU there is an ongoing debate about parental leave rules regarding duration of leave, split between the parents, and generosity. In 2019, the EU enacted the Work-Life Balance Directive (EU 2019/1158), which, among other changes, mandated all countries to establish two months of parental leave earmarked for fathers. This has prompted, e.g., the introduction of 9 weeks of earmarked paternity leave in Denmark in 2023.

Our analysis of a managerial quota is motivated by the introduction of corporate gender quotas in many countries (Bertrand, 2018),[29] as well as various examples of voluntary corporate policies to increase the number of women in leadership roles (McKinsey & Company, 2024). These policies are often based on the gender proportionality principle (GPP) for promotions to maintain similar female representation across the hierarchy and/or set specific targets of minimum representation.[30]

Paid parental leave for mothers. First, we consider an extension of paid parental leave for mothers. To this end, we modify the problem of families and the problem of firms in the model. On the family side, those who have children must take parental leave (PL) during 10% of the period when their child is born, at a replacement rate corresponding to part-time earnings, while choosing their labor supply for the remaining 90% of the period. Since the average length of an early or mid-career period in our sample is 6.8 years, this policy corresponds to approximately 8 months of leave. During the time on leave, skills depreciate at the rate of non-participation. During the time in the labor force, skills continue to evolve as in the baseline model. Firms observe leave-taking and incorporate the corresponding skill depreciation when setting training and promotion policies. Households anticipate how their fertility choices and parental leave will affect their lifetime earnings by changing their skill accumulation and probability of training and promotion.

Parental leave for mothers and fathers. This policy counterfactual is similar to the parental leave for mothers but it imposes the same leave duration of 10% of a career period also on fathers, at the same replacement rate (part-time earnings). Human capital depreciation and what the firm observes are modeled as in the case of PL for mothers only.

Managerial quota. In this counterfactual, we impose that firms must promote at least an equal share of women to managers. The firm can choose to promote fewer individuals in total in order to avoid promoting unprofitable women at the margin, but this comes at the cost of promoting fewer men. To implement this counterfactual, we modify the firm's problem to first determine a rank order of women by expected returns from training and promotion. We then consider all possible choices

[29]The list of countries with a gender quota for corporate boards of directors includes Norway, Belgium, France, Germany, Iceland, India, Israel, Italy, and Spain. In addition, the EU recently enacted the Gender Balance on Corporate Boards Directive (EU 2022/2381), which requires companies to meet a target of 33% of the underrepresented sex among all directors by June 30, 2026.

[30]One example of a company that introduced the GPP is Unilever (Chilazi et al., 2021). The Boston Consulting Group combines GPP with a targeted share of women at all stages of the company, including senior leadership, of 40% (The Boston Consulting Group, 2016).

of total managers by ladder and calculate expected profits associated with each choice based on first training and promoting the highest ranked women for 50% of the managerial positions, and then filling the remaining slots with the most profitable male or female candidates remaining.[31] Finally, we select the number of managers on each ladder that maximizes total expected profits.

5.2 The equilibrium impacts of the paid parental leave and quota policies

Table 3 shows the levels (normal font) and percentage changes relative to baseline (italics) of the main outcomes for the three counterfactual policies considered. In addition, columns (6) to (8) of Tables A.8 and A.9 show, respectively, the equilibrium differences in terms of marital and fertility patterns and in terms of the demographic composition of managers for each of the policies relative to baseline.

Paid parental leave for mothers. Panel A of Table 3 shows that a more generous leave policy for mothers significantly increases the gender promotion gap by 10.671% relative to the baseline. This widening stems from a reduction by 0.2 percentage points in the share of women promoted (an effect of 5.534% relative to women's baseline level) together with a 2.756% increase in the share of promoted men. In this environment, women reduce their labor supply (outside of the leave period), and especially in the first two career stages, which are crucial for career progress. This is reflected by a 1.347% reduction in the share of women working super-full-time. In contrast, men increase their labor supply, consistent with recognizing better promotion opportunities. Firms react to the policy change by supporting more men and fewer women with on-the-job managerial training. This increases the training gender gaps by 3.655% of the baseline gap and reinforces household specialization.

Our results on extending paid leave for mothers are consistent with recent empirical findings of negative effects on first-time mothers in the U.S. (Bailey et al., Forthcoming) and comprehensive evidence from other contexts, documenting negative effects of long parental leave duration on women's careers and an increase in the gender earnings gap for highly skilled women (Olivetti and Petrongolo, 2017). Our model suggests that family labor supply and on-the-job training are the mechanisms through which the negative effects of paid parental leave materialize if the policy is targeted exclusively at women. Therefore, we next consider a policy change that induces more equal investments in the careers of men and women by both families and firms.

Paid parental leave for mothers and fathers. Panel B shows that a parental leave policy that equalizes the time on leave for mothers and fathers reduces all gender gaps relative to baseline. The managerial promotion gap is reduced by 1.627% due to an increase in the fraction of women promoted and a decrease in that of men. Interestingly, when firms expect both men and women to take a similar amount of time off due to parental duties, they train 2.575% more women and 1.036% fewer men relative to baseline. This implies a reduction in the gender training gap by 6.359%. On the family

[31] For each total number of managers, our solution algorithm is an extension of the equitable planner's problem in Kleinberg et al. (2018). In our case, the firm faces an additional tradeoff in filling remaining manager slots because training without promotion can sometimes be more profitable, especially when facing a managerial quota.

Table 3: Gender gaps in counterfactual policies and their *% change relative to baseline*

	Promotion		On-the-job Training		Share SFT, $t < 3$		Average Hours		Average Earnings	
	Value	*%Change*	Value	*%Change*	Value	*%Change*	Value	*%Change*	Value	*%Change*
Panel A. Paid parental leave for mothers										
Men	0.091	*2.756*	0.207	*1.345*	0.536	*2.853*	0.794	*0.413*	0.215	*-2.668*
Women	0.041	*-5.534*	0.122	*-0.221*	0.446	*-1.347*	0.745	*-0.204*	0.191	*-3.296*
Gap	0.050	*10.671*	0.086	*3.655*	0.089	*30.702*	0.049	*10.947*	0.024	*2.669*
Panel B. Paid parental leave for mothers and fathers										
Men	0.089	*-0.412*	0.203	*-1.036*	0.528	*1.367*	0.791	*0.087*	0.214	*-3.130*
Women	0.044	*0.860*	0.125	*2.575*	0.472	*4.313*	0.752	*0.704*	0.193	*-2.256*
Gap	0.045	*-1.627*	0.077	*-6.359*	0.056	*-18.166*	0.039	*-10.437*	0.021	*-10.563*
Panel C. Managerial quota										
Men	0.067	*-24.384*	0.207	*1.108*	0.523	*0.459*	0.790	*-0.049*	0.220	*-0.305*
Women	0.065	*50.004*	0.125	*2.370*	0.451	*-0.264*	0.746	*-0.085*	0.198	*0.171*
Gap	0.002	*-95.419*	0.082	*-0.754*	0.072	*5.250*	0.044	*0.555*	0.022	*-4.355*

side, this policy increases the share of singles and reduces fertility (see column (7) of Table A.8) and, as a result, the average hours worked increase for both men and women. However, men and women now work more similar hours, which results in a reduction in the gender gap in earnings (even though earnings levels decline slightly because of foregone income and skill depreciation during the mandatory leave period).

We further investigate the role that the family plays in the overall policy effects. First, fertility rates decrease for couples with highly ambitious spouses because childless workers can signal more commitment to their careers by opting out of the leave program. Indeed, women without children account for a higher share of manager promotions in this scenario (see column (7) of Table A.9, Panel C). Second, Panel B of Figure A.4 documents interesting heterogeneity in the reduction of gender gaps by the ambition-type composition of couples. While a policy that mandates leave for both spouses reduces gender gaps in same-ambition couples—including the promotion gap for power couples—, we find that promotion and training gaps are exacerbated for unequal couples, in particular for couples in which women marry up to the most ambitious men. Interestingly, these families reduce their fertility substantially, supporting the signaling mechanism in favor of men. In the counterfactual equilibrium, the share of power couples remains stable, but the fraction of asymmetric couples with one spouse of the highest ambition declines. This compositional change further contributes to the economy-wide narrowing of gender gaps in career investment.

These adjustments in marriage patterns and household investments explain why we find that gender convergence in promotions is mitigated in partial equilibrium when we do not allow families to change their partner in response to the reform (see Table A.10). Interestingly, fixing the marriage market amplifies family-side investment gaps because there is less reduction in hours gaps at the baseline marital choices, but firms' reactions vary by gender. They increase women's training, but less than in the scenario in which the marriage market adjusts, and reduce men's training more strongly. On net, gender convergence in training is slightly higher at the baseline marriage market equilibrium.[32]

[32]Similarly, when individuals are not allowed to change their partner in response to a leave-for-mothers only policy, firms are even less likely to train women relative to the full equilibrium in which the composition of couples changes, but training for men also increases less. Here, in contrast to the case of leave for both, the training gap is on net exacerbated if the marriage market is fixed.

Taken together, our results for equal leave of mothers and fathers are consistent with some empirical evidence that earmarked parental leave for fathers increases mothers' labor supply (Patnaik, 2019) and lowers fertility (Farré and González, 2019). Dahl et al. (2014) show peer effects in parental leave take-up among coworkers, especially if the peer is a senior manager. This is consistent with an impact of firm policies on household decisions. Yet, many studies find negligible effects of paternity leave, possibly because of limited take-up and leave duration (see Bertrand (2018) and references therein). Our scenario documents larger gender convergence for a more radical reform that mandates longer leave for both parents. In addition, existing empirical results are not able to fully capture the long-run adjustments in the marriage market, which we find to play an important role in the overall reduction of the gender promotion gap.

Managerial quota. Finally, we investigate a *demand-side* policy that has the potential to induce more equal investments in women and men, namely, a managerial quota. When the firm is required to promote equal shares of men and women, it reacts by investing more in all workers.[33] It increases the share of trained women by 2.37% and that of men by 1.108%, resulting in a small reduction of the gender gap in training. In the aggregate, men slightly increase their labor supply in the early career stages while women reduce theirs marginally.

However, these small differences in family- and firm-side investments mask interesting heterogeneity by type of household (Figure A.4, Panel C). First, gender promotion shares increase for women and decrease for men in all couple types. The largest change in favor of women happens in power couples because the highest ambition types are the most likely to be promoted. Individuals in power couples continue to receive the highest investments by firms, and they react to the policy by increasing the labor supply of the husband and reducing that of the wife, implying a widening of the gender gap in hours worked. This is consistent with men in these families facing the highest increase in competition for a managerial position and with women not needing to increase their human capital while being protected by the quota. In addition, men increase their labor supply in response to the higher promotion chances of their wives due to income complementarities.

Another interesting type of couple are those in which lower ambition women marry the most ambitious men. Women in these couples receive more investments from both firms and families while their husbands reduce their labor supply and do not receive more training by firms. For men in these couples, the chances of promotion go down. However, the wives' chances go up, and the family reacts by shifting investments from husbands to wives. This adjustment differs from power couples because the training chances of women with lower ambition depend on high career investments (ladder, labor supply) and their promotion chances are lower than for high-ambition women. Therefore, the income-complementarity forces are less pronounced in asymmetric couples than in power couples. Interestingly, column (8) of Table A.8 shows that in the new marriage market equilibrium there are fewer households in which men marry down.

[33] In practice, promotion rates are not perfectly equalized across gender in our simulation results because of finite sample bias for this low-frequency outcome.

Finally, we consider the effects of the managerial quota at the baseline marriage market equilibrium. We find that the gender gap in managerial training declines further (compared to the results with marriage market adjustment), but that gender gaps in labor supply increase. These opposite effects arise because the fixed marriage market prevents the reduction in couples in which men marry down.

Overall, our results on the managerial quota show relatively small effects on average household labor supply, which is consistent with the negligible impact of a board quota in Norway on the decisions of young women at large (Bertrand et al., 2018). Yet, our analysis reveals substantial heterogeneity in the changes in career investments across couple types. We note that, in contrast to a narrow board quota, our intervention results in career investments for many more ambitious women directly through changes in the firm's training policy. While the quota incentivizes additional investment in women in many households, it ensures career success and reduces household investments for the most ambitious women, consistent with the "patronizing equilibrium" described by Coate and Loury (1993).

The welfare effects of narrowing the gender gaps in career achievement. To assess welfare effects, we first measure average household utility in each scenario and how it changes relative to the baseline.[34] Perhaps at first glance surprisingly, we find that aggregate welfare decreases under the leave policies, indicating that additional income for households during parental leave does not compensate for the career costs of leave-induced skill depreciation. Specifically, the additional leave for mothers reduces average household utility by 1.052% and leave for both parents reduces welfare by 3.387%. In contrast, household utility increases marginally by 0.011% on average as a result of the female manager quota compared to the baseline. The last two columns of Table 3—which show average lifetime earnings by gender—suggest that part of the reduction in utility under the two leave programs is reflected in lower household income relative to baseline, as in these scenarios both men and women have lower average earnings.

However, a closer look at the behavioral responses induced by each scenario renders the welfare effects reasonable. First, when additional parental leave is imposed on mothers, households respond by increasing career investments for husbands. And when parental leave is mandatory for both spouses, individuals react by increasing their career investments. The latter policy achieves more equal hours on the family side but induces less firm-side investments in men. On top of this, the parental leave policies distort marital and fertility choices. Together, these equilibrium reactions on the family side directly affect utility and imply that parental leave policies, even when some version of them may reduce gender gaps, also reduce aggregate welfare. Households lose more on average when the policy imposes labor supply constraints on both spouses.

[34] Specifically, we compute the indirect life-time utility for each type of household $(\theta_f, \theta_m) \in \Theta^2 \setminus (\emptyset, \emptyset)$ by evaluating (HP_h) at the household's optimal choices in each counterfactual scenario, given the firm's equilibrium policies. This utility also includes the expected value of leave benefits if applicable. Let us denote the indirect value of each household type under any scenario by $V^{\theta_f \theta_m}_{scenario}$. We then aggregate over single and married households using their equilibrium matching frequencies $\Gamma(\theta_f, \theta_m)$ in the respective counterfactual scenario to measure average household utility:

$$\text{Total Welfare of scenario} = \sum_{(\theta_f, \theta_m)} V^{\theta_f \theta_m}_{scenario} \times \Gamma(\theta_f, \theta_m).$$

In contrast, the managerial quota operates mostly through the firm-side, so household utility is less affected. Firms must increase their efforts to find suitable women for management and respond to the widening of the gender gap in hours by training both women and men to a larger extent. This leads to an overall welfare gain, although this average hides substantial heterogeneity across couple types.

To shed light on these differences across couples, we implement a consumption-equivalence approach, determining what additional percent of per-period total household consumption a family would need to receive or give up at baseline to be indifferent between the policy and the baseline. A negative percent unit means that the policy harms the couple relative to the status quo. We report the results in Table 4. In Panel A, we further distinguish welfare effects between couples with either high or low initial family human capital $\bar{\phi}$. Panel B, in turn, distinguishes couples based on their vector of initial market human capital, $\eta_{1h} = (\eta_{1f}, \eta_{1m})$. We analyze welfare for couples in which both spouses have low initial market human capital (which we denote by *low* η_{1h}) and for couples in which at least one spouse has a high draw (which we denote by *high* η_{1h}).[35]

For leave benefits, Panel A of Table 4 shows that households with low family human capital $\bar{\phi}$ are worse off, consistent with these couples making higher career investments and, thus, experiencing larger dynamic career costs of leave taking, while benefiting less from time at home during leave. For each couple type, these differences by realized family human capital are exacerbated when leave is mandatory for both spouses because career costs are imposed on both parents.

In Panel B, we typically find much larger negative welfare effects from leave policies for couples with high initial market human capital η_{1h}, consistent with them making higher career investments and hence facing higher career costs of mandatory parental leave. One exception are power couples who face similar losses irrespective of η_{1h}, suggesting that these couples always make high career investments. Notably, other couples of equal but lower ambition type gain from the leave policy if both spouses have low market human capital. These couples are less attached to the labor market at baseline and receive a windfall of benefits for their time at home through parental leave policies.

Turning to welfare effects of a female manager quota in the last two columns of Table 4, we find increasing welfare for couples with women of the highest ambition type, whereas couples with highest-ambition husbands and less ambitious wives lose. Panel A shows that these welfare gains or losses are systematically more pronounced for households with low family human capital $\bar{\phi}$ because these couples are making higher career investments and either benefit or suffer from changes in firm policies for training and promotion. The exception are women of the highest ambition type matched with a less ambitious husband—here, the welfare gains are larger if family human capital $\bar{\phi}$ is high. This

[35] To be precise, we define low $\bar{\phi}$ as draws of the lowest value in the $\bar{\phi}$ grid, and low η_{1h} as realizations in which both spouses draw the lowest two values on the grid for initial market human capital η_1. In both cases, based on the estimated distributions of $\bar{\phi}$ and η_1, the probability of a low draw for an individual is approximately one third. Then, we consider the indirect utility of total (private and public) consumption under each scenario (baseline or counterfactual policies) for household h, of type (θ_f, θ_m), with initial draws $\bar{\phi}$ and η_{1h}, and denote this indirect value by $V^{\theta_f \theta_m}_{scenario}(c|\bar{\phi}, \eta_{1h})$. Finally, we solve for the value of ε that equalizes the indirect household utility of per-period consumption between each policy and the baseline, and report $100 \times \varepsilon$ in Table 4,

$$V^{\theta_f \theta_m}_{baseline}\big((1+\varepsilon)c|\bar{\phi}, \eta_{1h}\big) = V^{\theta_f \theta_m}_{policy}\big(c|\bar{\phi}, \eta_{1h}\big),$$

Table 4: Consumption equivalence by type of couple and skills

	Paid parental leave mothers		Paid parental leave mothers and fathers		Managerial quota	
Panel A. Family human capital:	low $\bar{\phi}$	high $\bar{\phi}$	low $\bar{\phi}$	high $\bar{\phi}$	low $\bar{\phi}$	high $\bar{\phi}$
Power Couples, (θ_4, θ_4)	-9.628	-7.529	-10.459	-7.218	0.773	0.198
Equal Ambition, $\theta_f = \theta_m$	-7.850	-4.757	-8.419	-4.418	0.265	0.093
Husband highest, $(\theta_{i\leq 3}, \theta_4)$	-11.928	-7.581	-11.680	-6.982	-1.505	-1.232
Wife highest, $(\theta_4, \theta_{i\leq 3})$	-10.114	-7.491	-11.623	-6.746	0.157	1.362
Panel B. Market human capital:	low η_{1h}	high η_{1h}	low η_{1h}	high η_{1h}	low η_{1h}	high η_{1h}
Power Couples, (θ_4, θ_4)	-8.708	-8.152	-7.992	-8.325	1.029	0.345
Equal Ambition, $\theta_f = \theta_m$	0.034	-6.177	0.622	-6.163	0.871	0.073
Husband highest, $(\theta_{i\leq 3}, \theta_4)$	-5.804	-9.224	-5.193	-8.914	0.348	-1.442
Wife highest, $(\theta_4, \theta_{i\leq 3})$	-4.740	-8.722	-4.389	-8.836	1.224	0.993

Notes: Low (high) $\bar{\phi}$ denotes the group of couples in which the initial draw of family human capital is the lowest value on the $\bar{\phi}$ grid. Low (high) η_{1h} denotes the group of couples in which both spouses (at least one spouse) draw(s) a value of initial market human capital equal to (higher than) the lowest two values on the η_1 grid. In both cases, based on the estimated distributions of $\bar{\phi}$ and η_1, the probability of a low draw is approximately one third.

is consistent with these women focusing on household production at baseline but receiving a path to management despite initially taking advantage of their high $\bar{\phi}$ by choosing low labor supply. In other words, the quota allows these women to "have it all". Perhaps surprisingly, the welfare effects are also larger for couples where both spouses have low initial market human capital, see Panel B of Table 4. This result is consistent with low human capital endowments making these couples unattractive for firm training and promotions at baseline. But these couples receive a disproportionate increase in firm investment under the quota regime.

To sum up, our welfare analysis highlights that policies that reduce gender differences in career outcomes may come (at least in the short run) at a cost for families if these policies constrain workers' choices. Our equilibrium model of the interplay between families' and firms' investments in workers' human capital suggests that policies that incentivize more equal on-the-job investments across the genders achieve both higher gender equality and welfare gains for families. We note that this assessment only speaks to the investment welfare value for workers and does not account for effects on firms. We further acknowledge that there may be impacts on children from leave programs and longer-term welfare effects if gender equality changes other markets in the economy or the political arena.

6 Conclusion

In this paper, we show how families and firms—two uncoordinated groups—interact in shaping gender inequality in labor market outcomes, in particular in reaching management positions. Using high-quality register data from Denmark, we document large gender gaps in managerial training and promotions. Importantly, we show that these gaps arise from a combination of family-side and firm-side choices: joint career investment decisions of spouses vary across types of couples. Moreover, differences in household characteristics influence firm-side investments in managerial training and promotion decisions. We then develop and estimate a quantitative life cycle model that captures these family-firm interactions in a joint equilibrium of the marriage and the labor market. This novel framework captures the interplay between families and firms through marriage, families' joint career and fertility decisions, and an active role of the firm in making managerial training and promotion decisions.

Model estimates indicate that initial gender differences in the household are small but amplify over the life cycle through reinforcing gender gaps in family and firm investments. We find that policies that incentivize more equal career investments across genders—e.g., equal parental leave for mothers and fathers, or a managerial quota by gender—can achieve higher gender equality. However, they come with different caveats and distributional consequences. Parental leave extensions, including earmarked leave for fathers, reduce welfare for most households due to the career costs of leave-induced skill depreciation. While a quota achieves both a reduction in gender inequality and average welfare gains, it disincentivizes career investments of highly ambitious women who are protected by the quota. Moreover, the quota reduces welfare for households of highly ambitious men and less ambitious women. Changes in the marriage market equilibrium that would materialize in the long run can mitigate these welfare losses because fewer such couples form under this policy.

We believe our unified framework can serve as a building block for future work that analyzes the equilibrium effects of alternative policy designs. Two alternative types of policy interventions seem particularly interesting for further investigation.

First, recent empirical evidence on the effects of family-side childcare policies on parental earnings is encouraging (Humphries, Neilson, Ye, and Zimmerman, 2024), and it would be valuable to understand their full distributional effects in equilibrium. By adding a production function for child development and household choices for the type of childcare, our model could be used to investigate how such policies change the tradeoffs of spousal time allocation between the household and the workplace, and which policy designs may incentivize more equal career investments of the spouses.

Second, family-friendly firm policies, especially remote work and scheduling flexibility, have gained attention in the public debate recently. In addition, evidence of persistent gender gaps in leadership roles points to the lack of flexible reentry and adjustable career progression in corporate hierarchies (Bertrand, Goldin, and Katz, 2010). While some evidence suggests that remote work options increase employment of mothers after childbirth (Harrington and Kahn, 2023), these benefits may come at the cost of larger earnings gaps (Blau and Kahn, 2013). To analyze the overall effectiveness of such firm-side policies in reducing gender inequality, one could consider a model extension with heterogeneous firms that differ in flexibility and skill accumulation. Firms may also offer varying degrees of training quality and career progression opportunities. This analysis could shed light on the changing sorting patterns of workers across jobs, accounting for adjustments in marital sorting patterns, household choices, and firm policies in equilibrium. Taking this analysis one step further, one could endogenize firms' choices regarding flexibility and skill accumulation as they compete in the market for talent.

Our empirical evidence and quantitative analysis highlight the relevance of the reinforcing interaction between families and firms in shaping gender gaps in career achievement. Our key insight is that accounting for these interdependencies can be instrumental in designing policies and organizational practices that foster more equitable and efficient outcomes in the labor market.

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Appendix A Additional Empirical Evidence

Table A.1 estimates the following regression model

$$\mathcal{O}_{ilt} = \beta_0 + \beta_1 \cdot female_i + \delta_l + \delta_\theta + \delta_{\{I_i\}_t} + \epsilon$$

where $\mathcal{O}_{ilt}$ is an indicator for worker i at firm-ladder l having received training or promotion by year t, and the key regressor of interest is the indicator for female, $female_i$. We include different types of controls, including firm-by-ladder fixed effects δ_l, ambition-type fixed effects δ_θ, and controls for the worker's employment history, $\delta_{\{I_i\}_t}$. We find a raw gender training gap of 8.8 percentage points and a raw gender promotion gap of 2 percentage points (see columns (1) and (4), respectively). After including firm-by-ladder fixed effects, the gender training gap declines slightly to 6.4 percentage points (column (2)) and the gender manager gap remains stable (column (5)). These gaps shrink by 67% and 53% respectively but remain statistically significant after controlling for firm-ladder, worker type, and worker employment history, see columns (3) and (6).

Table A.1: Training and Promotions

	(1)	(2)	(3)	(4)	(5)	(6)
		Training			Manager Promotion	
Female	-0.0881***	-0.0639***	-0.0290***	-0.0196***	-0.0197***	-0.0092***
	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.001)
Firm-Ladder FE	No	Yes	Yes	No	Yes	Yes
Worker Ambition FE	No	No	Yes	No	No	Yes
Worker Exp FE	No	No	Yes	No	No	Yes
Observations	1,860,063	1,860,063	1,827,942	1,860,063	1,860,063	1,827,942
R-squared	0.011	0.359	0.430	0.003	0.213	0.245

Notes: Standard errors clustered at the worker level in parentheses. *** Significant at the 1% level.

Table A.2 provides robustness evidence related to Table 1 in the main text. Specifically, we separate high-ambition types with steep career wage growth into two groups, either with low or high starting wages, denoted θ_3 and θ_4 respectively.

As expected, we find that a higher ambition type of a worker is associated with higher training and promotion chances. Spouses with high career growth are related to an additional increase in training and promotion frequencies, with a somewhat larger coefficient estimate for θ_4 than for θ_3. These relationships are substantially larger for men than for women, even though we still find statistically significant positive estimates for career outcomes of women whose husbands are of high ambition type, see the total effects in the last rows of the table.

Table A.2: Training and promotions vary with worker's family type

	(1)	(2)	(3)	(4)
	Training		Manager Promotion	
female	-0.0286***	-0.0126***	-0.0090***	-0.0038***
	(0.002)	(0.003)	(0.001)	(0.001)
high-ambition θ_3	0.3046***	0.2139***	0.0306***	0.0238***
	(0.007)	(0.007)	(0.002)	(0.002)
high-ambition θ_3 * female	-0.0213**	-0.0394***	-0.0167***	-0.0160***
	(0.010)	(0.009)	(0.003)	(0.003)
high-ambition θ_4	0.5116***	0.3647***	0.0578***	0.0528***
	(0.006)	(0.006)	(0.002)	(0.002)
high-ambition θ_4 * female	-0.0862***	-0.0658***	-0.0072**	-0.0080**
	(0.010)	(0.010)	(0.004)	(0.004)
high-ambition spouse θ_3	0.1065***	0.0587***	0.0304***	0.0258***
	(0.009)	(0.009)	(0.003)	(0.003)
high-ambition spouse θ_4	0.1355***	0.0928***	0.0443***	0.0361***
	(0.010)	(0.009)	(0.004)	(0.004)
high-ambition spouse θ_3 * female	-0.0670***	-0.0409***	-0.0262***	-0.0178***
	(0.011)	(0.011)	(0.004)	(0.004)
high-ambition spouse θ_4 * female	-0.0625***	-0.0376***	-0.0397***	-0.0322***
	(0.011)	(0.011)	(0.005)	(0.004)
FE for Firm-Ladder, Age, LS History	No	Yes	No	Yes
Observations	1,860,063	1,827,942	1,860,063	1,827,942
R-squared	0.211	0.432	0.022	0.246
Total Effect, θ_3 spouse for female	0.0395	0.0178	0.00420	0.00795
P-Value	9.00e-09	0.00418	0.0189	0.0165
Total Effect, θ_4 spouse for female	0.0731	0.0552	0.00463	0.00391
P-Value	<0.0001	<0.0001	0.0051	<0.0001

Notes: Standard errors clustered at the worker level in parentheses. *** Significant at the 1% level. ** Significant at the 5% level.

The sample in both Tables 1 and A.2 includes individuals who are singles. These results are, if anything, somewhat more pronounced when excluding singles and focusing only on cohabiting and married individuals, as evidenced in Table A.3.

Table A.3: Training and promotions vary with worker's family type

	(1)	(2)	(3)	(4)
	Training		Manager Promotion	
female	-0.0610***	-0.0292***	-0.0166***	-0.0061***
	(0.003)	(0.004)	(0.001)	(0.001)
high-ambition	0.4427***	0.3136***	0.0581***	0.0487***
	(0.006)	(0.007)	(0.002)	(0.002)
high-ambition * female	-0.0933***	-0.0835***	-0.0278***	-0.0229***
	(0.009)	(0.009)	(0.003)	(0.003)
high-ambition spouse	0.1063***	0.0694***	0.0259***	0.0225***
	(0.007)	(0.007)	(0.003)	(0.003)
high-ambition spouse * female	-0.0235***	-0.0159*	-0.0182***	-0.0183***
	(0.009)	(0.008)	(0.003)	(0.003)
FE for Firm-Ladder, Age, LS History	No	Yes	No	Yes
Observations	1,209,864	1,188,979	1,209,864	1,188,979
R-squared	0.214	0.462	0.024	0.283
Total Effect, high-amb spouse for female	0.0828	0.0535	0.00767	0.00416
P-Value	<0.0001	<0.0001	<0.0001	0.0013

Notes: Standard errors clustered at the worker level in parentheses. *** Significant at the 1% level. * Significant at the 10% level.

Finally, we refine the analysis of spousal types and distinguish couples whose degrees fall into the same field of study in Table A.4. On average, individuals receive more training and promotions when their spouse holds a degree in the same field. This relationship is statistically significant and sizable for men, but much less pronounced for women. However, if the spouse is highly ambitious and in the same field, then we find a substantially higher training rate (but unchanged promotion rate) for women, but not vice versa for men.

Table A.4: Training and promotions vary with worker's family type

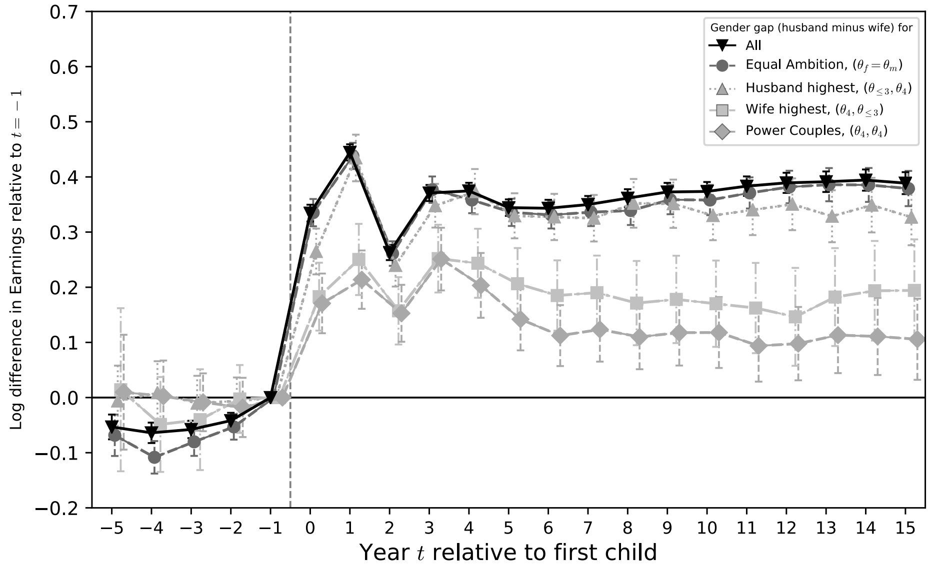
	(1)	(2)	(3)	(4)
	Training		Manager Promotion	
female	-0.0242***	-0.0114***	-0.0066***	-0.0021**
	(0.003)	(0.003)	(0.001)	(0.001)
high-ambition	0.4305***	0.3009***	0.0476***	0.0406***
	(0.005)	(0.005)	(0.002)	(0.002)
high-ambition * female	-0.0793***	-0.0666***	-0.0154***	-0.0146***
	(0.007)	(0.007)	(0.002)	(0.002)
high-ambition spouse	0.1259***	0.0837***	0.0352***	0.0306***
	(0.008)	(0.007)	(0.003)	(0.003)
high-ambition spouse * female	-0.0752***	-0.0458***	-0.0301***	-0.0243***
	(0.009)	(0.009)	(0.003)	(0.003)
same field	0.0409***	0.0344***	0.0222***	0.0169***
	(0.006)	(0.006)	(0.002)	(0.002)
same field * female	-0.0319***	-0.0220***	-0.0210***	-0.0157***
	(0.008)	(0.008)	(0.003)	(0.003)
high-ambition spouse * same field	-0.0010	-0.0244	0.0001	-0.0034
	(0.016)	(0.015)	(0.007)	(0.006)
high-ambition spouse * same field * female	0.0761***	0.0563***	0.0040	0.0047
	(0.021)	(0.020)	(0.008)	(0.007)
FE for Firm-Ladder, Age, LS History	No	Yes	No	Yes
Observations	1,860,063	1,827,942	1,860,063	1,827,942
R-squared	0.200	0.428	0.021	0.245

Notes: Standard errors clustered at the worker level in parentheses. *** Significant at the 1% level. ** Significant at the 5% level.

Figure A.1 complements Figure 2 in the main text by showing the gender gap in log earnings. We again report estimates of event-time dummies relative to the year before the birth of the first child after controlling for fixed effects for calendar year, age of the mother, and age difference between the spouses. The sample includes observations of couples where both spouses have completed their formal education.

In Figure A.1, we also find a large and persistent "child penalty" in earnings across all couple types (dark down-pointing triangle plot). This penalty is the smallest—but still sizable—for highly-ambitious women who are either in power couples (diamond plot) or married to husbands of a lower type (square plot). These results emphasize that families in which the mother is of the highest ambition type, θ_4, experience a significantly lower child penalty than other households, consistent with better chances of being trained and promoted to manager for these ambitious women.

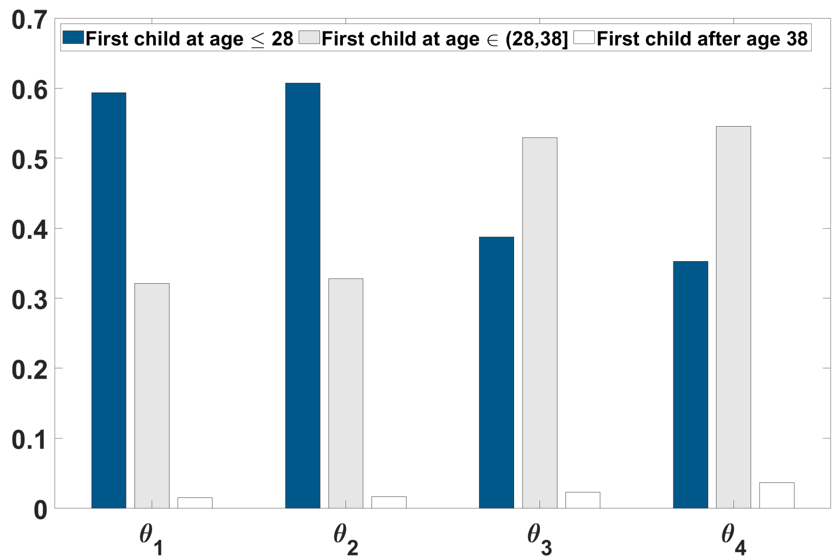
Figure A.1: Heterogeneous child penalty in earnings by couple type



Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

Finally, we analyze the timing of fertility for different types of women. Figure A.2 distinguishes women by the four ambition types and plots the share of women who have their first childbirth in one of three age bins: age 28 or younger (blue), age 29-38 (gray), and after age 38 (white). These age bins represent different career stages, which correspond to periods in our model as explained below. Interestingly, less ambitious women are more likely to have children early in their careers, while more ambitious women tend to delay fertility. One plausible explanation might be that ambition type-θ_3 and θ_4 women finish their education later. However, the fact that these type of women modify their labor supply the least relative to men in these households might also indicate that the timing of fertility has to do with the likelihood of receiving training and a managerial promotion.

Figure A.2: More ambitious women delay fertility



Notes: The height of bars represents the fraction of married women having a first child. The horizontal axis is the married woman's *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

Appendix B Model

Training Costs In this section we provide details about the training problem. To arrive at the training cost function for the representative firm, we start from the cost of training for one manager and their team,

$$c^{OJT}\left(n^{tr}\right)=\hat{\zeta}\left(n^{tr}\right)^{2},$$

where n^{tr} denotes the number of trainees per manager.

Assume that the total number of trainees N_{tr} are distributed equally across all managers in the market, N_{mg}, such that

$$n^{tr}=\frac{N_{tr}}{N_{mg}}.$$

Then the total cost of training across firms in the market is given by

$$C^{OJT}=\sum_{j}n_{j}^{mg}\cdot\hat{\zeta}\left(n^{tr}\right)^{2}.$$

where n_j^{mg} denotes the number of managers at firm j. This condition can be rewritten as

$$C^{OJT}=N_{mg}\cdot\hat{\zeta}\left(\frac{N_{tr}}{N_{mg}}\right)^{2}.$$

Based on this equation, we can see that this cost function maps into the representative firm's problem with cost parameter ζ if the team-specific cost parameter $\hat{\zeta}$ is given by

$$\hat{\zeta}=\zeta N_{mg}.$$

Appendix C Identification

C.1 Human capital and production technology

We need to separately identify (i) production output *per unit of time* in job $J=\{p,mg\}$, parameterized as follows:

$$y_{L,J}(\eta)=a_{L,J}+b_{L,J}\cdot\eta,$$

$a_{L,J}$: baseline output per unit of time for job J in ladder L

$b_{L,J}$: productivity of human capital in job J of ladder L

and (ii) the human capital process, which parameterizes the *beginning-of-t* η_t as a function of labor supply last period,

$$\eta_{it}=\left[\eta_{it-1}+\alpha_{L_i,\theta_i}+\delta_{L_i}^{S}\mathbb{1}_{\{I_{it-1}=SFT\}}-\delta_{L_i}^{P}\mathbb{1}_{\{I_{it-1}=PT\}}-\delta_{L_i}^{N}\mathbb{1}_{\{I_{it-1}=NT\}}\right]\cdot\tau.$$

$\alpha_{L,\theta}$: skill accumulation in ladder L for ambition type θ per unit of labor supply when working full-time

δ_L^S: boost in skill accumulation in ladder L when working super-full-time rather than full-time

δ^P_L: penalty in skill accumulation in ladder L when working part-time rather than full-time

δ^N_L: penalty in skill accumulation in ladder L when not working

τ: boost in skill accumulation after completed training

and the initial distribution of skills is parameterized by ambition type,

$$\eta_1(\theta) \sim F(\mu_\theta, \sigma) \quad \forall \theta \in \Theta.$$

Returns to Skills Notice that the evolution of human capital is allowed to differ across ladders, but is common across jobs within ladder. In addition, gender does not influence human capital accumulation or returns to skills. Initial human capital varies by ambition type, and we assume stable differences in accumulation rates, captured by type-specific $\alpha_{L,\theta}$ but type-invariant penalties δ^N and δ^P or boosts δ^S and τ. This suggests that we can analyze earnings changes of specific family types over time in order to estimate the return on human capital in each ladder and job.

Take as an example a worker type $\omega_{i'} = (\cdot, \theta_2, L_{11}, L_{12}, FT_1, FT_2)$. Here we consider both men and women in the first argument because gender does not affect productivity or the HC process. Further note that all workers are employed as producers in the first two periods. This worker type remains on the same ladder and producer job at full-time work in the first two periods. Conditional on the extent of human capital accumulation over time, the change in earnings between period 1 and 2 identifies $b_{L_1,p}$. Since we focus on earnings growth and the production technology is linear, the average initial endowment of human capital for this worker type will cancel out. In practice, we use average earnings growth of all full-time workers between periods 1 and 2 by ladder as targeted moments (EP9 and EP10 in Table A.6).

HC Accumulation Similarly we could have identified the parameter $b_{L_1,p}$ using a different worker type, for example workers of same ambition type and different labor supply, $\omega_{i''} = (\cdot, \theta_2, L_{11}, L_{12}, NP_1, FT_2)$. More generally, all workers in the same ladder and job receive the same return $b_{L,J}$ to human capital.

There are two reasons why earnings may differ across worker types conditional on the same ladder and job: First, selection into employment histories can be based on differences in initial skill endowments. This selection will imply that worker types i' and i'' may have different average salaries in the same ladder and job. Second, earnings may differ because of human capital accumulation. Here, the difference between worker types $\omega_{i'}$ and $\omega_{i''}$ is full-time versus non-participation in period 1, for example. The latter leads to skill depreciation and, all else equal, lower earnings in the future.

Our theoretical framework allows us to model the initial selection in order to match initial earnings levels in the data. Specifically, we target mean earnings in period 1 separately for each ambition type and ladder (EP1–EP8 in Table A.6). This implies that earnings differences in period 2 conditional on initial sorting across labor supply choices are due to differences in HC accumulation. These targeted moments for each ladder (EP19–EP20 in Table A.6) will help us identify the respective non-participation penalty in HC accumulation, δ^N_L.

Analogous to the previous argument, we can also identify the super-full-time boost in HC accumulation, δ^S_L by comparing earnings in period 2 for worker types who only differ based on their labor supply choices in period 1 — one group is observed in super-full-time work, and the other in full-time work (EP27–EP28 in Table A.6).

A similar argument also applies for the part-time penalty in HC accumulation, δ_L^P. However, the share of part-time workers in the data is small and we want to preserve flexibility for the solver in estimation to select parameter combinations that yield (close to) no part-time work in equilibrium. As a result, we target the share of part-time workers on each ladder rather than these workers' subsequent earnings profiles to identify δ_L^P (LS9–LS10 in Table A.6).

Finally, to identify the type-specific component of human capital accumulation, we note that conditional on ladder, job, labor supply choices, and after accounting for selection, variation in earnings changes across ambition types can only be explained by differences in HC accumulation, $\alpha_{L,\theta}$. This is why we include earnings growth between period 1 and 2 for continuously full-time employed workers as targeted moments, separately for each ambition type and ladder (EP11–EP18 in Table A.6).

Skill Endowments and Baseline Output We note that we cannot separately identify baseline output $a_{L,J}$ and initial skill endowments μ_θ^η for all production lines and ambition types. Instead, we make one normalization and set $a_{L1,p} = 0$. Then, differences in mean earnings in period 1 across ambition types on the same ladder help identify the differences in initial skill endowments, μ_θ, while differences in starting salaries conditional on ambition type across ladders relate to differences in baseline productivity (EP1–EP8 in Table A.6). The initial skill dispersion σ^η is related to initial participation rates and the variance of labor supply, which we target by using participation rates of men by ambition type in period 1, as well as the variance of men's initial labor supply choices (coded as $NP = 0, PT = 0.33, FT = 0.66, SFT = 1$), see moments LS1–LS5 in Table A.6.

Training The human capital boost from training relates closely to differential changes for workers with and without training comparing earnings before and after the training period. To rule out differential human capital accumulation conditional on ambition type, we restrict attention to workers that are continuously full-time employed from period 1 to period 3.

Since the training boost is a common factor τ irrespective of worker type, we target pooled average earnings growth between period 2 and period 3 for all full-time workers (i) with and (ii) without training (moments EP21–EP22 in Table A.6).

Managers The remaining component is the production technology of managers. Here, the challenge is that we only observe workers moving into management positions later in their careers. Yet, based on the previous arguments, we can identify the production process for producers, human capital accumulation across ambition types, and training boost. We also get a good sense of the composition of future trainee groups through their choices and earnings in periods 1 and 2.

Hence, we can predict through the lens of the model what skill level these promoted workers have reached after the training and what their counterfactual output as producers would have been. This information implies that we can use their managerial earnings to back out the output they generate as managers for any given value of rent sharing λ. The level of pay in manager jobs helps identify the baseline productivity, $a_{L,mg}$ relative to the normalized baseline as a producer. Comparing manager earnings in the cross-section of period 3 for promoted workers of different ambition types (with different skill levels) helps us to identify the return to skills among managers, $b_{L,mg}$. Specifically, we target mean earnings of managers in each ladder separately for ambition types 3 and 4 (EP23–EP26 in Table A.6).

C.2 Family human capital

Having pined down the parameters of market human capital and the production technology, which determine wages by ambition type, we use moments of labor supply to discipline the process of family human capital. First, conditional on starting earnings and human capital, the reason we would observe variation in participation or in labor supply within gender and ambition type in our model is the draw of the importance of time in home production. Hence, we overidentify the mean and variance of the initial family shock, μ^{ϕ} and σ^{ϕ}, by the participation rate of men by ambition type and the variance in male's labor supply measured in period 1 (LS1–LS5 in Table A.6). After the baseline parameters of family human capital have been identified, any differences in participation between men and women of the same ambition type pin down the female advantage in home production, κ. Hence, we use the participation gap in same-ambition couples to identify κ (LS6 in Table A.6). Finally, given the process of market human capital has already been identified, variation in re-entry to the labor market after a period of nonparticipation disciplines the depreciation in family human capital γ. Here, we target reentry of women separately in period 1 and period 2 (LS7–LS8 in Table A.6).

C.3 Marriage and fertility patterns

Moreover, having identified all the elements that define households' budget constraints and home production, the reason why women of the same type would differ in their fertility choices is explained by the common utility value from children χ^u and the home production floor when having children χ^Q. Hence, we target the fraction having children by ambition type and period to pin down these parameters. Specifically, we target fertility rates of ambition types θ_1 and θ_4 separately in period 1 and 2 (moments FP1–FP4 in Table A.6).

Finally, we exploit the properties of the Logistic distribution to argue that the scale of the marital preference shock σ_β and the noneconomic value from singlehood for low-growth types $\chi^{\emptyset}_{1,2}$ and high-growth types $\chi^{\emptyset}_{3,4}$ are identified from the fraction of couples of a certain type (θ_f, θ_m) and from the fraction of singles by ambition type. Specifically, we target the fractions of couples with the highest-ambition types, as well as the share of all equal couples (MM1–MM4 in Table A.6) and the share of male singles by ambition type (MM5–MM8).

C.4 Training and promotion policies

To identify the firm's training cost parameter ζ, we measure the share of workers on each ladder who receive training. Specifically, we target the share of trained workers on each ladder by gender (FI1–FI4 of Table A.6), which further relates to marriage market fundamentals and to gender differences in family human capital that jointly drive the distribution and investment decisions of households. As an additional overidentifying restriction related to these fundamental parameters, we also target the gender promotion gap on each ladder (FI5–FI6).

Appendix D Estimation details

D.1 Periods in the data

In this subsection, we describe how we operationalize the four periods, e.g., t_0,t_1,t_2, and t_3, in the data. Individuals enter the model upon graduation from their highest completed educational program (see OA.3). However, age at graduation generally differs across ambition types with θ_1 on average graduating much earlier than θ_4. Hence, we adopt different age thresholds across ambition types when assigning periods in the data.

We define the age threshold between periods t_0, t_1 and t_2, e.g., $\overline{a_{12}}_i$, for individual i in the data as the maximum of: the age at three years upon entry and age 28. Age 28 is chosen as the 75th percentile of the age at graduation for individuals θ_4 who are the ones who graduate the latest. However, some individuals will not have entered the inflow sample by age 28. Thus, we set their age threshold $\overline{a_{12}}_i$ such that we observe their early career choices for three years. The age threshold between periods t_2 and t_3, e.g., $\overline{a_{23}}_i$, for individual i in the data is the maximum of: the age of i three years after entering t_2 and the minimum of the age at promotion or age 38. Age 38 is chosen as the 25th percentile of the age of promotion and the three-year requirement ensures that we observe everybody for at least three years in t_2. In sum, these fuzzy age thresholds ensure that we for all individuals in the inflow sample can distinguish their early career decisions from their period of potential training, and further, from their period of potential promotion.

Figure A.3: Periods in the model and life cycle of individuals in the data

Periods in model:	t_0*: Matching &* t_1*: Early Career*	t_2*: Training*	t_3*: Career Progress*
Age cutoffs in data:	$\overline{a_{12}}_i = max\{entry_i + 3, 28\}$	$\overline{a_{23}}_i = max\{\overline{a_{12}}_i + 3, min\{pro_i, 38\}\}$	
Key outcomes:	85% young couples → marry 49% females → first child	35% females → first child 76% trainees → trained	99.9% managers → promoted

Figure A.3 provides a graphical overview of the life-cycle periods in the model and the data. Consistent with the model, 85% of couples have married by the end of t_1, 76% of trainees receives training in t_2, and all managers are promoted in t_3. As in the model, individuals have children in both

t_1 and t_2, e.g., 49% of females in t_1 and 35% in t_2.

D.2 Ambition types distributions in the data

Table A.5: Ambition types distributions by gender

Ambition type, (w_0, g)	$\theta_1 = (low, low)$	$\theta_2 = (high, low)$	$\theta_3 = (low, high)$	$\theta_4 = (high, high)$
Men	0.330	0.407	0.109	0.154
Women	0.431	0.415	0.082	0.072

D.3 Targeted moments

Table A.6: The 56 moments targeted in estimation.

Label	Description	Model	Data
Panel A. Earnings Process (EP)			
EP1	**Mean earnings of** $\theta_i = 1$ **in** L_1 **and** t_1	0.15	0.16
EP2	**Mean earnings of** $\theta_i = 2$ **in** L_1 **and** t_1	0.16	0.18
EP3	**Mean earnings of** $\theta_i = 3$ **in** L_1 **and** t_1	0.15	0.16
EP4	**Mean earnings of** $\theta_i = 4$ **in** L_1 **and** t_1	0.18	0.19
EP5	**Mean earnings of** $\theta_i = 1$ **in** L_2 **and** t_1	0.23	0.16
EP6	**Mean earnings of** $\theta_i = 2$ **in** L_2 **and** t_1	0.25	0.18
EP7	**Mean earnings of** $\theta_i = 3$ **in** L_2 **and** t_1	0.16	0.18
EP8	**Mean earnings of** $\theta_i = 4$ **in** L_2 **and** t_1	0.21	0.20
EP9	**Earnings growth of FT workers in** L_1 in period t_2	0.05	0.06
EP10	**Earnings growth of FT workers in** L_2 in period t_2	0.07	0.13
EP11	**Earnings growth of** $\theta_i = 1$ **FT workers in** L_1 **in** t_2	0.02	0.05
EP12	**Earnings growth of** $\theta_i = 2$ **FT workers in** L_1 **in** t_2	0.04	0.05
EP13	**Earnings growth of** $\theta_i = 3$ **FT workers in** L_1 **in** t_2	0.09	0.09
EP14	**Earnings growth of** $\theta_i = 4$ **FT workers in** L_1 **in** t_2	0.12	0.13
EP15	**Earnings growth of** $\theta_i = 1$ **FT workers in** L_2 **in** t_2	0.03	0.07
EP16	**Earnings growth of** $\theta_i = 2$ **FT workers in** L_2 **in** t_2	0.06	0.08
EP17	**Earnings growth of** $\theta_i = 3$ **FT workers in** L_2 **in** t_2	0.13	0.13
EP18	**Earnings growth of** $\theta_i = 4$ **FT workers in** L_2 **in** t_2	0.12	0.17
EP19	**Earnings difference FT vs NT at** t_2 **in** L_1	0.06	0.08
EP20	**Earnings difference FT vs NT at** t_2 **in** L_2	0.10	0.11
EP21	**Earnings growth at** t_3 **without training**	0.07	0.05
EP22	**Earnings growth at** t_3 **with training**	0.16	0.09
EP23	**Mean earnings of** $\theta_i = 3$ **managers in** L_1	0.54	0.42
EP24	**Mean earnings of** $\theta_i = 4$ **managers in** L_1	0.57	0.56
EP25	**Mean earnings of** $\theta_i = 3$ **managers in** L_2	0.59	0.62
EP26	**Mean earnings of** $\theta_i = 4$ **managers in** L_2	0.64	0.74
EP27	**Earnings difference SFT vs FT at** t_2 **in** L_1	0.07	0.04
EP28	**Earnings difference SFT vs FT at** t_2 **in** L_2	0.09	0.06
Panel B. Marriage Patterns (MM)			
MM1	**Fraction** $\theta_f = \theta_m$	0.39	0.42
MM2	**Fraction** $(\theta_f = 4, \theta_m = 4)$	0.08	0.04
MM3	**Fraction** $(\theta_f \leq 3, \theta_m = 4)$	0.10	0.12
MM4	**Fraction** $(\theta_f = 4, \theta_m \leq 3)$	0.03	0.03
MM5	**Fraction single men** $\theta_m = 1$	0.37	0.41
MM6	**Fraction single men** $\theta_m = 2$	0.37	0.27

Continuation of Table A.6			
Label	**Description**	**Model**	**Data**
MM7	**Fraction single men** $\theta_m = 3$	0.40	0.36
MM8	**Fraction single men** $\theta_m = 4$	0.26	0.27
Panel C. Fertility Patterns (FP)			
FP1	**Fraction** $\theta_f = 1$ **having first child in** t_1	0.38	0.57
FP2	**Fraction** $\theta_f = 4$ **having first child in** t_1	0.36	0.31
FP3	**Fraction** $\theta_f = 1$ **having first child in** t_2	0.31	0.37
FP4	**Fraction** $\theta_f = 4$ **having first child in** t_2	0.64	0.60
Panel D. Labor Supply (LS)			
LS1	**Participation rate of men** $\theta_m = 1$ **and** t_1	0.74	0.85
LS2	**Participation rate of men** $\theta_m = 2$ **and** t_1	0.81	0.91
LS3	**Participation rate of men** $\theta_m = 3$ **and** t_1	0.76	0.91
LS4	**Participation rate of men** $\theta_m = 4$ **and** t_1	0.96	0.98
LS5	**Variance of men's labor supply in** t_1	0.15	0.10
LS6	**Participation gap in homogamous couples in** t_1	0.05	0.04
LS7	**Women's probability of re-entry (**t_1 **to** t_2**)**	0.61	0.49
LS8	**Women's probability of re-entry (**t_2 **to** t_3**)**	0.38	0.37
LS9	**Share working PT in** t_1 **and** t_2 **in** L_1	0.09	0.04
LS10	**Share working PT in** t_1 **and** t_2 **in** L_2	0.11	0.02
Panel E. Firm's Investments (FI)			
FI1	**Share of men trained in** L_1	0.16	0.19
FI2	**Share of women trained in** L_1	0.09	0.11
FI3	**Share of men trained in** L_2	0.32	0.47
FI4	**Share of women trained in** L_2	0.21	0.35
FI5	**Promotion gender gap in** L_1	0.03	0.03
FI6	**Promotion gender gap in** L_2	0.07	0.09

Appendix E Supporting figures and tables

Table A.7: Gender gaps in alternative model specifications and their *% change relative to baseline*

	Promotion		On-the-job Training		Share SFT, $t < 3$		Average Hours		Average Earnings	
	Value	*%Change*	Value	*%Change*	Value	*%Change*	Value	*%Change*	Value	*%Change*
Panel A. Full information										
Men	0.090	*1.781*	0.205	*0.316*	0.479	*-8.015*	0.781	*-1.237*	0.219	*-0.987*
Women	0.042	*-2.909*	0.120	*-1.482*	0.383	*-15.359*	0.726	*-2.809*	0.193	*-2.601*
Gap	0.048	*6.259*	0.085	*2.966*	0.096	*40.677*	0.055	*25.575*	0.026	*12.741*
Panel B. $\kappa = 1$										
Men	0.088	*-1.518*	0.203	*-0.990*	0.581	*11.602*	0.806	*2.040*	0.225	*1.619*
Women	0.045	*3.738*	0.123	*0.765*	0.535	*18.239*	0.785	*5.173*	0.208	*4.968*
Gap	0.043	*-6.538*	0.080	*-3.578*	0.046	*-32.406*	0.021	*-51.390*	0.017	*-26.853*
Panel C. History based										
Men	0.090	*1.260*	0.208	*1.643*	0.530	*1.800*	0.793	*0.330*	0.222	*0.289*
Women	0.042	*-3.914*	0.127	*3.920*	0.458	*1.118*	0.748	*0.246*	0.198	*0.283*
Gap	0.048	*6.201*	0.081	*-1.713*	0.073	*6.318*	0.045	*1.761*	0.023	*0.338*

Table A.8: Equilibrium marriage and fertility patterns by model specification

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Share at baseline	Absolute change relative to baseline under					
		Full information	$\kappa = 1$	History based	Parental leave mothers	both	Managerial quota
Panel A. Marriage market							
Power Couples, (θ_4, θ_4)	0.037	0.010	-0.002	0.000	0.000	-0.001	0.000
Equal Ambition, $\theta_f = \theta_m$	0.187	0.071	-0.066	0.000	-0.002	-0.016	0.000
Husband highest, $(\theta_{i\leq 3}, \theta_4)$	0.049	0.031	-0.017	-0.003	-0.005	-0.009	-0.007
Wife highest, $(\theta_4, \theta_{i\leq 3})$	0.015	-0.001	-0.004	0.000	0.000	-0.003	0.002
Singles	0.526	-0.188	0.178	0.004	0.014	0.049	0.004
Panel B. Fertility							
No children	0.489	-0.134	0.140	0.003	0.047	0.133	0.004
Children in t_1	0.240	0.058	-0.074	-0.002	0.045	0.072	0.001
Children in t_2	0.271	0.076	-0.066	-0.001	-0.092	-0.206	-0.005
Panel C. Fertility by couple type							
(θ_4, θ_4) **with Children**	1.000	0.000	0.000	0.000	-0.105	-0.005	0.000
$\theta_f = \theta_m$ **with Children**	0.758	0.019	0.029	0.000	-0.039	-0.087	0.000
$(\theta_{i\leq 3}, \theta_4)$ **with Children**	0.992	0.006	0.003	0.000	-0.341	-0.158	0.000
$(\theta_4, \theta_{i\leq 3})$ **with Children**	0.989	0.002	0.002	0.001	-0.173	-0.169	0.001

Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

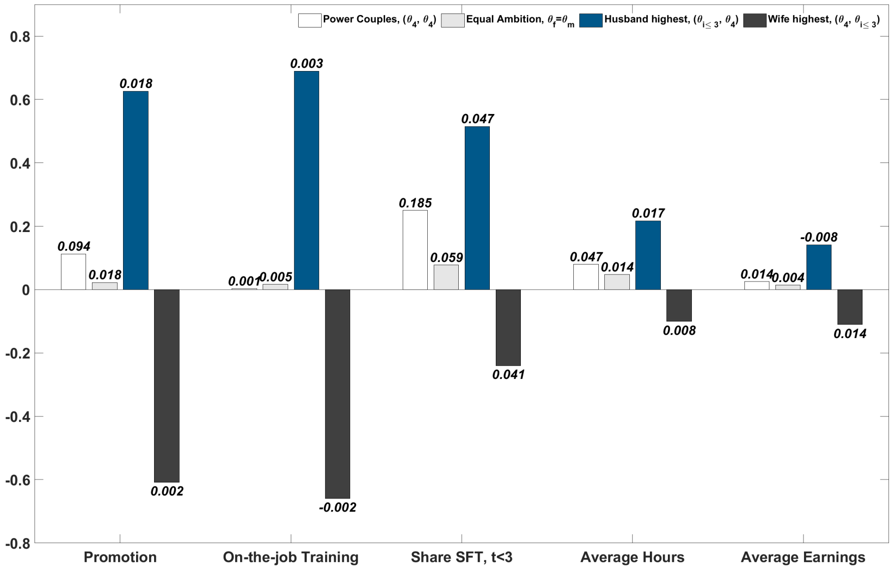
Table A.9: Fraction of all managers represented by ambition type, gender, fertility, and type of household

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Share at	Change relative to baseline under					
	baseline	Full Information	$\kappa = 1$	History based	Parental leave mothers	both	Managerial quota
Panel A. By Gender							
Women	0.328	-0.010	0.012	-0.011	-0.018	0.003	0.164
Men	0.672	0.010	-0.012	0.011	0.018	-0.003	-0.164
Panel B. By Ambition Type							
Ambition Type 1	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ambition Type 2	0.004	0.000	0.000	0.001	0.000	0.001	0.003
Ambition Type 3	0.007	0.000	0.001	0.001	0.001	0.001	0.112
Ambition Type 4	0.988	-0.001	0.000	-0.002	-0.001	-0.002	-0.115
Panel C. Ambition Type 4							
Women	0.323	-0.010	0.011	-0.013	-0.019	0.002	0.051
singles	0.013	-0.013	-0.002	0.000	0.000	0.014	-0.013
with children	0.310	0.002	0.013	-0.013	-0.040	-0.019	0.064
without children	0.013	-0.013	-0.002	0.000	0.021	0.021	-0.013
in Power Couples, (θ_4, θ_4)	0.218	0.020	0.029	-0.004	-0.019	0.002	0.040
marrying down, $(\theta_4, \theta_{i\leq 3})$	0.093	-0.018	-0.016	-0.009	0.000	-0.014	0.024
Men	0.665	0.010	-0.012	0.011	0.018	-0.004	-0.166
singles	0.133	-0.127	0.041	0.033	0.023	0.036	-0.012
with children	0.532	0.137	-0.053	-0.022	-0.098	-0.063	-0.154
without children	0.133	-0.127	0.041	0.033	0.115	0.059	-0.012
in Power Couples, (θ_4, θ_4)	0.225	0.021	0.022	-0.004	0.017	0.002	-0.036
marrying down, $(\theta_{i\leq 3}, \theta_4)$	0.307	0.116	-0.075	-0.018	-0.022	-0.042	-0.118
Panel D. Ambition Type 3							
Women	0.003	0.000	0.000	0.001	0.000	0.000	0.112
singles	0.001	0.000	0.001	0.000	0.000	0.000	0.019
with children	0.002	0.001	-0.001	0.001	0.000	0.000	0.093
without children	0.001	0.000	0.001	0.000	0.000	0.001	0.019
in Equal Couples, (θ_3, θ_3)	0.000	0.000	0.000	0.000	0.000	0.000	0.004
marrying up, (θ_3, θ_4)	0.001	0.000	0.000	0.000	0.000	0.000	0.027
Men	0.004	0.000	0.001	0.000	0.000	0.001	0.000
singles	0.001	-0.001	0.001	0.000	0.000	0.001	0.000
with children	0.003	0.001	-0.001	0.000	0.000	-0.001	0.000
without children	0.001	-0.001	0.001	0.000	0.000	0.002	0.000
in Equal Couples, (θ_3, θ_3)	0.000	0.000	0.000	0.000	0.000	0.000	0.000
marrying up, (θ_4, θ_3)	0.000	0.000	0.000	0.000	0.000	0.000	0.000

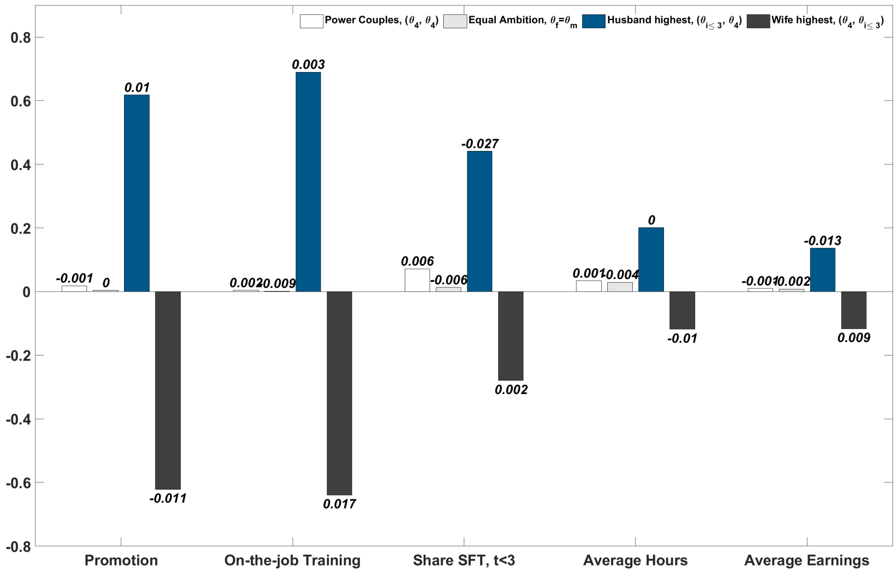
Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

Figure A.4: Gender gaps in counterfactual policies by couple type

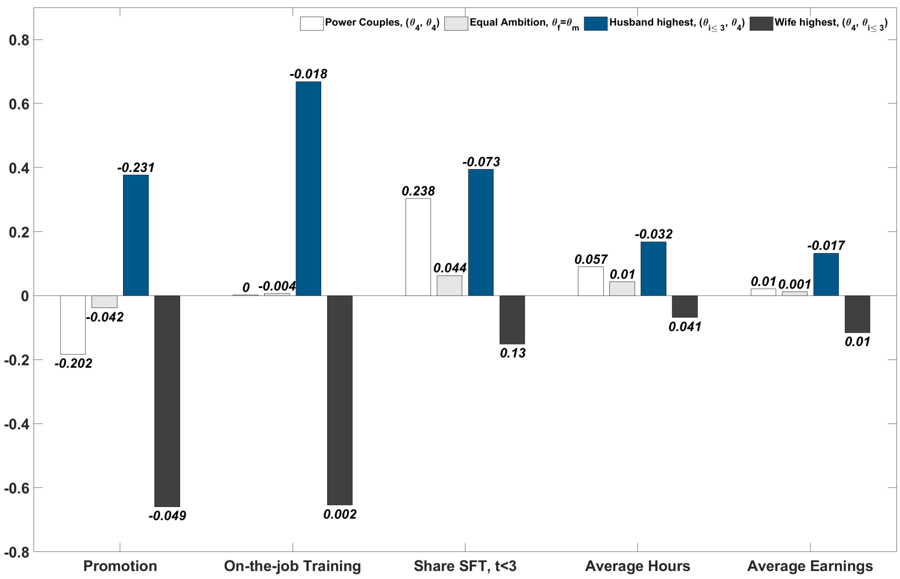
Panel A: Paid parental leave for mothers



Panel B: Paid parental leave for both mothers and fathers



Panel C: Managerial quota



Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$. SFT stands for super-full-time. Average hours are calculated given our parametrrization $I = \{0, \frac{1}{3}, \frac{1}{2}, 1\}$. Average earnings are measured in millions of Danish Kroner. All other outcomes reflect fractions of workers. Bar heights plot gender gaps in levels: husband minus wife. Numbers in italics represent the change relative to baseline.

Table A.10: Gender gaps in counterfactual policies and their *% change relative to baseline* at the *baseline* marriage market equilibrium

	Promotion		On-the-job Training		Share SFT, $t < 3$		Average Hours		Average Earnings	
	Value	*%Change*	Value	*%Change*	Value	*%Change*	Value	*%Change*	Value	*%Change*
Panel A. Paid parental leave for mothers										
Men	0.092	*3.884*	0.206	*0.591*	0.533	*2.397*	0.793	*0.334*	0.215	*-2.832*
Women	0.040	*-7.453*	0.120	*-1.843*	0.441	*-2.579*	0.743	*-0.520*	0.191	*-3.692*
Gap	0.052	*14.710*	0.086	*4.179*	0.092	*35.388*	0.050	*14.894*	0.024	*4.480*
Panel B. Paid parental leave for mothers and fathers										
Men	0.089	*0.064*	0.202	*-1.406*	0.515	*-1.114*	0.788	*-0.341*	0.213	*-3.750*
Women	0.043	*-0.077*	0.125	*2.460*	0.451	*-0.425*	0.745	*-0.161*	0.191	*-3.248*
Gap	0.046	*0.198*	0.077	*-7.104*	0.064	*-5.688*	0.042	*-3.412*	0.021	*-8.018*
Panel C. Managerial quota										
Men	0.067	*-24.812*	0.207	*0.954*	0.523	*0.483*	0.790	*-0.092*	0.220	*-0.356*
Women	0.065	*50.403*	0.126	*2.939*	0.449	*-0.729*	0.746	*-0.140*	0.198	*0.137*
Gap	0.002	*-96.636*	0.081	*-1.971*	0.074	*8.518*	0.044	*0.729*	0.022	*-4.547*

Online Appendix

Families' Career Investments and Firms' Promotion Decisions

Frederik Almar, Benjamin Friedrich, Ana Reynoso, Bastian Schulz and Rune Vejlin

Appendix OA Data

This Appendix provides a comprehensive overview of the data sources used in our paper and it documents how we process them. Section OA.1 provides an overview of the data sources we use. We provide details on sample selection in Section OA.2. The key variables are discussed in Section OA.3.

OA.1 Data Sources

We use administrative data provided by Statistics Denmark, which records information for the universe of Danish residents. We use a selection of registers from this comprehensive source.

Our starting point is to create a *baseline sample* with yearly observations for the entire Danish population in the years 1980–2018 aged 19 to 55. This data set is created from two registers provided by Statistics Denmark: the register PERSONER covers the years 1980–1984, and the register BEF covers the remaining years from 1985–2018. Both registers cover all individuals living in Denmark at the end of each calendar year.[1] From these sources, we obtain basic individual characteristics such as age, gender, identifiers for parents, identifiers for cohabiting partners and spouses, identifiers for children, municipality of residence, and country of origin.

We add educational variables from the registers UDDA and VEUV. UDDA is a panel data set with yearly observations.[2] This gives us detailed information on completed educational programs across all levels and fields and the date of graduation. VEUV contains information on postgraduate courses/continuing education, which we use to characterize some forms of management training, see Section OA.3.

Next, we sequentially add labor market variables from a range of different registers provided by Statistics Denmark. First, we add variables on labor market income from either employment or self-employment from the register IND. Second, we add hourly wages, hours worked, and employer- and industry identifiers from the register IDAN. IDAN contains information on multiple employment relationships of different types undertaken by an individual during a year. Individuals may have more than one employment record per year. In our case, we want to identify the most "relevant" employment relationship for each individual. To do this, we rank the different types of employment and keep the variables from the highest-ranked employment relationship. The ranking is as follows:

[1] In the data provided by Statistics Denmark, all variables in PERSONER and BEF are measured at the start of the calendar year. Because most other data sets refer to the end of the calendar year or the second half of the year, we adjust these data sets by lagging the year variable by one. This ensures that all variables are measured as close in time to each other as possible.

[2] From 1980–2007, the information is measured by the end of the year, while from 2008 and forward it is measured in October.

1. Main job in November
2. Most important non-November job
3. Self-employed worker with employees in November
4. Self-employed workers without employees in November
5. Helping spouse,

where 1 is the highest rank. We drop employment observations (but not individuals) that do not belong to any of these five ranked categories. Accordingly, the individuals are recorded as non-employed (zero labor supply).

Third, we add variables on labor supply and the accumulation of human capital. We extract the yearly labor supply status in the main job from the register RAS. This is used directly in the model estimation, see more details on the variables in Section OA.3. Furthermore, we extract labor market experience. We use it to deflate wages properly, taking into account the aging population and, relatedly, increasing productivity and wages due to more experienced workers. To measure experience, we combine the registers EXPYEAR and IDAP.[3] We accumulate all individual work experience gained between 1964–1979 from EXPYEAR.[4] This information is then merged with individuals observed in IDAP in 1980 to ensure that we include their work experience prior to 1980. Next, we accumulate experience up to 2018 to get the accumulated work experience for each individual at the end of a particular year. We correct for breaks in experience spells by writing experience forward.[5]

Fourth, to identify the occupation in which an individual is employed, we use the register AKM. The 6-digit occupation variable only covers the 1991–2018 period. After fixing some missing data problems in the original variable,[6] we create a new aggregate variable containing the first 3 digits of the 6-digit code, resulting in 149 occupational categories. Moreover, we fix the break in 2009–2010 for the 3-digit code by mapping forward.[7]

Finally, we use the dataset AKU, which contains the Danish part of the European Labor Force Survey (LFS). AKU is a comprehensive survey covering the 2000–2018 period providing detailed responses on hours worked and other characteristics of the respondents' working conditions. The survey respondents can be linked to the administrative registers. We use AKU to cross-check the

[3]Labor supply variables from Statistics Denmark prior to 2008 are based on data from a mandatory Danish pension fund (ATP). In the ATP data, full-time work corresponds to around 30 hours or more per week, i.e., we cannot distinguish between an individual working 35 or 50 hours per week prior to 2008. After 2008, we have contractual work hours.

[4]EXPYEAR builds on mandatory pension payments and transforms the pension payments into hours worked using an algorithm developed by Statistics Denmark.

[5]These breaks occur when individuals are missing in the data for some years, which is usually due to stays abroad. Without this correction, their accumulated experience would be set to zero upon their return.

[6]One of the original variables already covers the entire 1991–2018 span, but we have detected some cases with missing coverage. We fix this as much as possible by using two other occupation variables covering the 1993–2009 and 2010–2013 time spans, respectively (reflecting a break in the nomenclature between 2009–2010). In cases where the 1991–2018 variable is missing, we substitute for one of the other variables, depending on the particular year, if these have non-missing observations instead. We remove duplicates to ensure that we have one observation per individual per year.

[7]This means that we compare codes of individuals just before and after the break working in the same establishment, and then we use the most prevalent changes in codes to update the pre-2010 codes. For example, if a group of individuals has the occupation code '123' in 2009, and a majority of them have changed to '321' in 2010, then we map forward by changing all pre-2010 '123' codes to '321'.

variables on labor supply from RAS and to create additional moments for an especially demanding labor supply status, super-full-time work, which we define in Section OA.3.6.

OA.2 Samples

We start from the *baseline sample* described above. It consists of the entire population of Danish residents in the age range of 19 to 55 in the period 1980–2018.

For each individual, we keep all observations from the first time they appear in the baseline sample until 2018 and select the cohort who graduated from their highest educational program in the years 1991–1995. We call this our *inflow sample*. To follow the career of these individuals over time, we apply the following steps to get from *baseline sample* to *inflow sample*:

1. We use the register UDDA to compute the *highest education* achieved by individuals over the period they are observed. Moreover, we use the register IND to compute whether an individual is ever self-employed.
2. We create an *interim sample* by selecting those individuals who achieved their highest education between 1991 and 2008 and who have never been self-employed. This interim sample is used solely to estimate the ambition types and career ladders, see Section OA.3.
3. We use the registers PERSONER and BEF to match each individual to the identity of their *decisive domestic partner*, see Section OA.3.2 for the definition. 46% of individuals in the interim sample are matched to such a partner. 40% of these individuals are married to a decisive domestic partner who is in the baseline sample but did not graduate between 1991 and 2008. In this case, we reintroduce the observations of these partners.
4. Finally, we exclude individuals who graduated between 1996 and 2008 and their partners. We also exclude individuals who are neither married according to our definition of partnership nor single. We further exclude individuals who had their first child before graduating from their highest education and who cannot be assigned an ambition type, career ladder, training- or manager status. The remaining sample is our *inflow sample*. Taken together, the inflow sample contains individuals who graduated between 1991 and 1995 and their identified partners. 60% of individuals who graduated between 1991 and 1995 are matched to a partner of whom 74% are in the baseline sample but did not graduate between 1991 and 1995.

Because we use administrative population records, attrition is very infrequent: 84% of individuals are observed for at least 20 years. Our inflow sample is an unbalanced panel of 152,390 individuals who achieved their highest education between 1991 and 1995 and their partners. The reason for starting in 1991 is that we do not observe occupation before 1991—a key variable in our analysis. Moreover, we decide to end our sample in 1995 to focus our analysis on a single cohort of individuals who we observe for at least 24 years. 83% of individuals in our sample are born between 1963 and 1976. In total, our data consists of 52,231 couples, 23,024 single women, and 24,905 single men.

OA.3 Key variables

OA.3.1 Educational programs

We assign individuals the highest educational program (excluding programs related to on-the-job training, see below) achieved by the age of 35. We define an educational program as the four-digit education code (variable HFAUDD from the UDDA register). There are 1,108 unique codes in the inflow sample. In the case of compulsory schooling, we further divide by region of graduation to capture variation in the value of a compulsory education across rural and urban areas. A program within secondary education would be carpenter or care worker (both vocational training programs). At the bachelor level, examples of programs include teacher and nurse. Finally, at the graduate level, business with focus on marketing is an example of a program.

OA.3.2 Partnership and marital status

We start from all individuals in the *baseline sample* to identify each individual's *decisive domestic partner*. We consider both legally married and cohabiting couples.[8] The idea is to identify the person with whom an individual makes joint decisions about fertility and career investments, which are the key choices in our model. To operationalize this idea in the data, we consider partners who are attached to the individual for at least five consecutive years, with the spell starting when the wife is 28 years old or younger and the husband 32 years old or younger. In cases where multiple partners meet these criteria, we keep the partner from the longest-lasting relationship. The asymmetric age thresholds for men and women are chosen to balance the single shares by sex. Age 28 for women is set equal to the age threshold between periods t_1 and t_2 in the model (see Appendix D.1), while age 32 for men is chosen by targeting the single share of women who got together with their decisive domestic partner partner at age 28 or earlier. These different age thresholds reflect that women, on average, get married at an earlier age and to a slightly older partner.

Our definition of marriage is having a *decisive domestic partner*. The non-married group in the data consists of two groups of individuals: 1) some who will not make joint decisions about fertility and career investments with a partner, see our definition above, and 2) some who would be classified as married had the age thresholds been slightly higher. Through the lens of our model, this second group of individuals who marry "too late" can be viewed as neither married nor single. Thus, we define a four-year "buffer zone" of couples who are not classified as married but would have been classified as married had the age thresholds been 32 for women and 36 for men. Subsequently, we classify individuals as *single* if they are neither married nor in the buffer zone. In our inflow sample, 69% of individuals have a decisive domestic partner. For comparison, in the baseline sample 64% of individuals who are married by the age 30 have a partner for at least 5 years. By the age of 50, this share is 81%.[9]

[8] Cohabiting couples are defined by Statistics Denmark as two opposite-sex individuals who share the same address, exhibit an age difference of less than 15 years, have no family relationship, and do not share housing with adults other than their partner. Our data do not allow us to identify cohabiting same-sex couples.

[9] Here we only include individuals who turn 19 between 1980-1995 to make the numbers more comparable to the

OA.3.3 Hourly wages, wage growth, and starting wages

To define our ambition types and career ladders, we have to define starting wages and wage growth. We base these calculations on the (larger) *interim sample* to get precise estimates of starting wages and wage growth at the levels of educational programs and occupation-firm combinations.

Our starting point is the *hourly wage* as provided by Statistics Denmark in its narrow definition, which excludes benefits and various contributions. This narrow definition is most reflective of marginal productivity. We deflate hourly wages by running a regression of log wages on year dummies with 2000 as the base year. We further control for differences in wage-experience profiles by education by including interactions of educational levels and accumulated labor market experience, see Section OA.1. This ensures that log hourly wages are comparable over time even as the education and experience composition of the sample changes. We then subtract the year dummies from the hourly wage, thereby constructing an hourly wage measure that controls for wage inflation and aggregate changes in education and experience.

We define individual *wage growth* in our *interim sample* as the difference between an individual's average hourly wage in years 1 to 5 in the sample, which we define as their *starting wage*, and their average hourly wage in years 9 to 11 in the sample. We focus on wage growth in the early career because wage growth is highest during that period and, thus, most indicative of human capital investments. Whenever we aggregate individual wage growth, e.g., at the educational program or occupation-firm level, we trim the top 1% of individual wage growth to exclude outliers.

OA.3.4 Ambition types

First, we aggregate the individual observations of starting wages and wage growth in the interim sample to the average at the educational program level. Second, we standardize the educational program averages of starting wages and wage growth. Third, we use k-means clustering, with the standardized variables as inputs, to assign educational programs (and thereby their graduates) to four clusters, which we label *ambition types*. Intuitively, the k-means clustering algorithm (Steinley, 2006) minimizes the within-cluster variation in standardized averages of starting wages and wage growth across the four categories. In other words, each of the four categories is internally homogeneous in terms of the starting wages and wage growth obtained by the graduates of the educational programs within that category. In Almar et al. (2024) we study marital sorting based on ambition and how it relates to changes in household inequality.

OA.3.5 Career Ladders

To define career ladders in the data, we first aggregate the individual observations of wage growth in the interim sample to the average of coworkers who begin their careers in the same occupation-firm combination. Second, we define occupation-firm combinations that exhibit hourly wage growth at or above the 80th percentile as *steep ladders* and *flat ladders* otherwise.

inflow sample.

We assign individuals to their most frequent occupation-firm combination over the first five years in the interim sample. We condition the cell size of these combinations to consist of at least five coworkers to smooth out individual contributions. The flip side of this restriction is that firm-occupation combinations defined by very fine occupational codes might not include at least five coworkers. Thus, we proceed in five iterations starting with finely defined occupational codes and then gradually moving to coarser versions. If an individual is not assigned to an occupation-firm combination in one iteration, we try to assign them in the next iteration. First, we assign occupation-firm combinations based on three-digit occupational codes and firm ID. Second, we use two-digit occupational codes and firm ID. Third, we use detailed four-digit educational program codes and firm ID. Fourth, we use a coarse educational level code (with four levels) and firm ID. Fifth, we only use firm ID.

The assignment of either steep or flat ladders has so far concerned the first period t_1 in the model (see Section D.1). For the subsequent periods t_2 and t_3, the assignment of occupation-firm combinations to ladders is based on the initial t_1 assignments, i.e., an occupation-firm combination that is defined as steep in t_1 is similarly defined as steep in t_2 and t_3. In cases where an individual has both flat and steep ladder positions within periods t_2 and t_3, we use the most prevalent type to characterize the entire period.

20% of women and 27% of men sort into the steep ladder in t_1. There is significant variation across workers within ambition types (Appendix OA.3.10). For example, approximately 25% and 50% of individuals of ambition type θ_1 and θ_4, respectively, sort into the steep ladder (Figure OA.1). Our interpretation is that a worker's ambition type reflects the expected career path of the worker based on the educational degree from an ex ante perspective. In contrast, being on a step or a flat ladder is a choice (made jointly with the spouse if married according to our definition) that is part of a particular career-life balance plan. For example, a law graduate can be on a steep career ladder at a private law firm or a flat ladder in the public sector.

OA.3.6 Labor supply

We use the RAS register and the AKU survey dataset to construct four labor supply states: non-participation, part-time, full-time, and super-full-time work. The three former states are directly determined by the part-time/full-time variable available in RAS. Non-participation refers to not being employed, e.g., not having a highest-ranked employment relationship, by the end of November in a given year. Those who have a highest-ranked employment relationship in a given year are characterized as either part-time or full-time employed depending on the hours worked per week. The threshold between part-time and full-time is 30 hours (1980-1992), 27 hours (1993-2007), or 32 hours (2008-2018) (Lund and Vejlin, 2016).[10]

We assign the mode of non-participation, part-time, and full-time within a period (see Section D.1) as the labor supply status of the corresponding period. In cases where more than one mode exists, we always assign part-time as the labor supply status for this given period.

[10]Please see Section OA.1 for a detailed description of how weekly working hours are measured.

Prior to 2008, we do not observe contracted hours in RAS, i.e., we do not observe to what extent individuals work above the part-time/full-time threshold, e.g., 35 hours or 50 hours. However, with detailed information on hours worked and time worked during a week from AKU, we can identify surveyed individuals working *super-full-time*. Hence, this labor supply status is only observed for a subset of those working full-time. For comparability, we apply the same definition based on AKU throughout our sample period, e.g., both prior to and after 2008. We define super-full-time relative to the Danish standard full-time working week corresponding to 37 hours. Hence, a surveyed individual works super-full-time in two cases: 1) the individual reports that a usual working week is 38 hours or more; 2) the individual reports that a usual working week is 37 hours, and they either report sometimes working in the evening, at home, on Saturdays, on Sundays, at night, or sometimes working overtime.[11]

OA.3.7 Managers and promotions

We consider workers to be promoted if they are observed holding a managerial occupational code for at least two consecutive years. That is, the first digit in the (D)ISCO code as provided by Statistics Denmark has to be equal to 1 for two consecutive years.

OA.3.8 On-the-job management training

Our strategy to measure whether a worker has received on-the-job management training is based on two data sources: (i) information on continuing education programs—MBA degrees and, specific to Denmark, HD degrees[12]—from the education register VEUV; (ii) information on worker transitions between different occupational codes, which we observe in the AKM register, see Section OA.1.

The idea is that firms can use both external education programs (HD, MBA) and specific roles within the firm (reflected in occupational codes) to train workers and prepare them for managerial positions. Our data show a clear negative relationship between firm size and the likelihood of workers completing external continuing education degrees. This suggests that larger firms tend to train their workers internally, which we seek to identify by tracking workers' progression through different occupational codes before being promoted into management (see, e.g., Frederiksen and Kato, 2017).

We combine information on external education programs and internal management training to predict the probability of becoming a manager.

Let ET_i be a dummy for whether a worker i has completed such an external training program. Let IT_{it}^k be a dummy for whether worker i is observed with a specific non-managerial (2-digit) occupational code k during period t. These periods refer to career stages in line with our model (e.g., early career), see Sections 3.6 and D.1.

[11] Importantly, we disregard shift workers who report working in the evening, at night, on Saturdays, or on Sundays as these are standard working conditions for shift workers and not a sign of extraordinarily high labor supply.

[12] HD stands for "Handelshøjskolens Diplomuddannelse", i.e., business school graduate education. HD programs are flexible, business economics diploma programs for employed individuals seeking management training. HD programs are offered both as individual courses and as a complete education program at two levels. HD1: Basic education in business economics. HD2: Specialized education dedicated to a specific subject area within business economics. HD1 and HD2 together are typically a four-year part-time educational program that corresponds to a bachelor's level education.

To capture how both external and internal on-the-job management training contribute to the probability of being promoted into management, we estimate the following binary response model. The outcome mg_i is a dummy for whether individual i is ever observed as a manager:

$$P(mg_i = 1) = G\left(\alpha + \sum_k \sum_t \alpha_{kt}^{IT} IT_{it}^k + \alpha^{ET} ET_i\right) \tag{2}$$

where α is a constant, α_{kt}^{IT} is the effect of a specific occupation k in career stage t, α^{ET} is the effect of having completed external management training, and G is the logistic CDF. Estimating this model reveals that external management training is quantitatively the most important predictor of a promotion into management because α^{ET} is an order of magnitude larger than the positive and significant occupation-specific effects α_{kt}^{IT}.

We use a "receiver operating characteristic" (ROC) curve to assess the predictive power of model (2). The ROC curve is a graphical tool that illustrates the performance of a binary classifier model at varying threshold values. At a probability threshold of 0.05, our estimated model correctly classifies 85.98% of individuals in our data (managers with training and non-managers without training). 50.80% of managers previously received training.

Finally, to construct the training variable, we select the occupation $\times$ career stage interactions IT_{it}^k that are positive and statistically significant predictors of becoming a manager. Moreover, we include ET_i, the dummy for having completed a continuing education managerial training program.

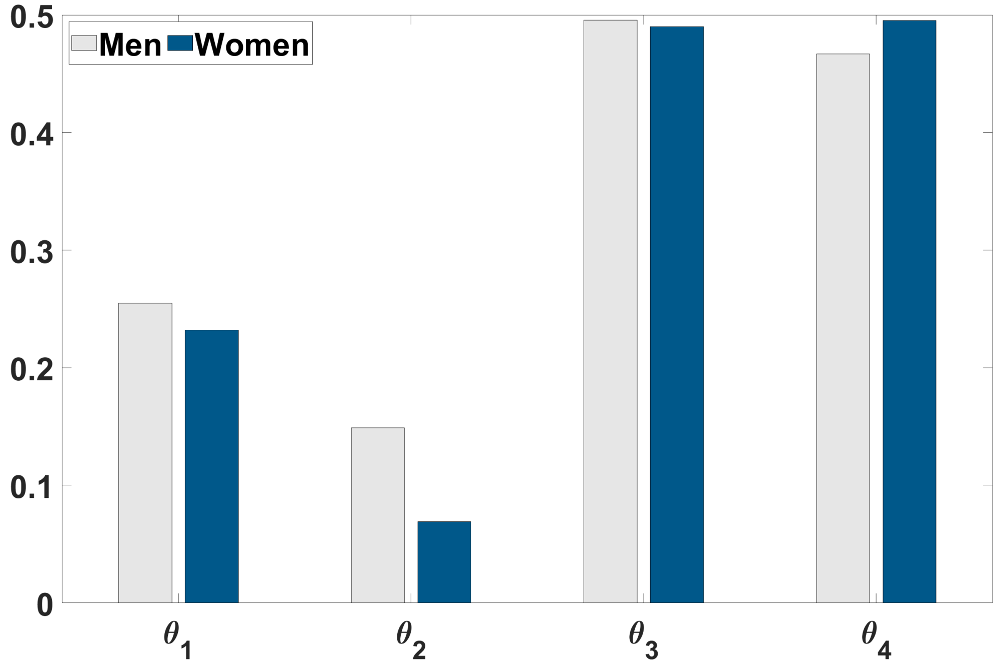
OA.3.9 Annual earnings

To construct moments of earnings, we use the annual taxable salary variable from the IND register. We first deflate earnings by subtracting the year effects described above for hourly wages. Then, we assign earnings by period in the model (see Section D.1) as the by-period mean of deflated earnings.

OA.3.10 Descriptive statistics

Table OA.1 provides a summary of descriptive statistics for the inflow sample, which is the sample we use for estimating the model. The total number of observations is 3,491,849 covering the years 1991 to 2018, yielding an average of 124,709 observations per year. The average birth year is 1970 and the average share of females is 48% across years. 90% of the sample consists of native Danes. The most common level of education across ages and years (not the highest degree ever achieved) is secondary education (54%) followed by an equal share with primary education or a bachelor's degree (19%). Master's and Ph.D. degrees make up the lowest share (8%). In Panel B, we show variables relevant to the marriage market part of the model. 71% are married, where married means married with a decisive domestic partner according to our definition above. The average age of marriage is less than 25 years. Recall that our definition of marriage includes both legal marriage and cohabitation. 63% of individuals have children and the average age of having the first child is just above 29 years. Panel C on labor market outcomes shows sizable gender gaps. Men generally work more hours in the

Figure OA.1: Fraction of men and women in the steep ladder by ambition type



Notes: θ refers to the *ambition type* defined and constructed as explained in Section 2.2: $\theta_1 = (low, low)$, $\theta_2 = (high, low)$, $\theta_3 = (low, high)$, and $\theta_4 = (high, high)$.

labor market, are more likely to be on a steep career ladder, and are more likely to receive on-the-job managerial training and promotions.

Moreover, Figure OA.1 shows that the fraction of men and women in the steep ladder varies by ambition type. Specifically, individuals or higher ambition are more likely but sort into more demanding career paths, but not all graduates from the same types of programs select into the same type of ladder. For example, only about 50% of individuals of the highest ambition types θ_3 and θ_4 sort into the steep ladder.

Table OA.1: Descriptive Statistics for the inflow sample

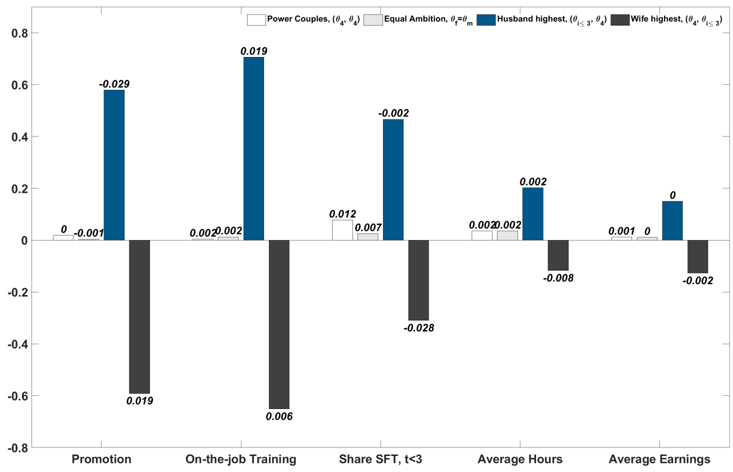
Sample	Inflow	
Total observations	3,491,849	
Observations per year	124,709	
	Mean	SD
Year	2,005	8.23
Panel A: Demographics		
Birth year	1,970	4.81
Sex ratio	0.48	0.03
Immigrant status	0.10	0.30
Primary school	0.19	0.39
Secondary school	0.54	0.50
Bachelor	0.19	0.39
Master & Ph.D.	0.08	0.27
Panel B: Marriage market and family		
Married	0.71	0.45
Age at marriage	24.62	3.35
Has children	0.63	0.48
Age at first child	29.27	4.77
Panel C: Labor market outcomes		
Women		
Non-participation	0.15	0.36
Part-time work	0.10	0.30
Full-time work (incl. super full-time)	0.74	0.44
Initial ladder steep	0.20	0.40
On-the-job training	0.16	0.37
Promotion	0.04	0.21
Men		
Non-participation	0.13	0.33
Part-time work	0.05	0.22
Full-time work (incl. super full-time)	0.82	0.39
Initial ladder steep	0.27	0.44
On-the-job training	0.25	0.43
Promotion	0.10	0.30

Notes: based on the inflow sample (see OA.2). The sex ratio denotes the number of women to men in a given year. The variable immigrant status takes on value 1 if an individual is either an immigrant or child of an immigrant and 0 otherwise. Marriage is defined as in OA.3 (here we pool all observations across years). All variables in panel C are defined in OA.3.

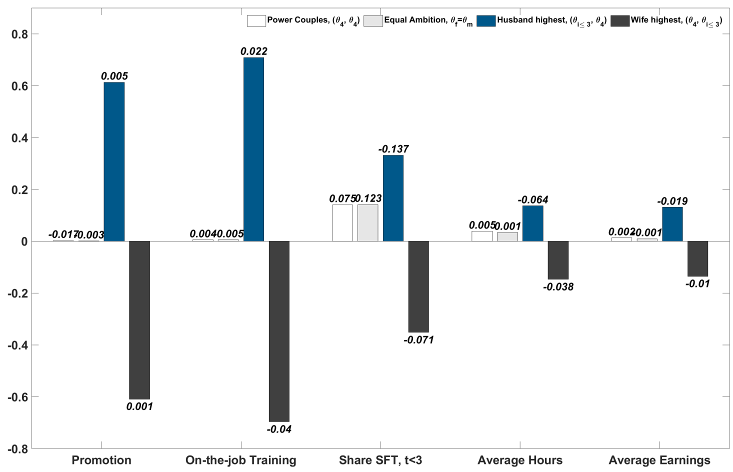
Appendix OB Supporting figures for alternative specifications

Figure OA.2: Gender gaps in model with alternative specifications by couple type

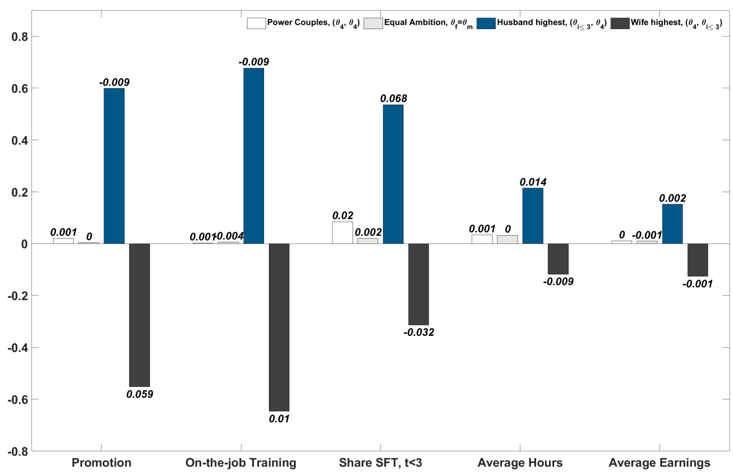
Panel A: Full information



Panel B: $\kappa = 1$



Panel C: History based



Note: numbers in italics represent the change relative to baseline.