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ABSTRACT

The Educated Class and the Fragility of Consumer Society

We analyze the importance of the educated class for the persistence of mass consumption societies in an economy with a hierarchy of needs.Through the demand for managerial talent (which is needed to operate advanced industrial technologies), the latter generate their own demand for skills. In turn, high wages for skilled labor raise the demand for a broad range of industrial products. Thus, mass consumption society is self-sustaining but may also collapse. An increase in the managerial labor requirement, while a form of technical regress, may sustain a high skilled wage, high industrialization, equilibrium. In the dynamic analysis, a collapse of mass consumption society may be triggered after the economy has accumulated a critically high level of human capital. Following a collapse, the educated class disappears but gradually recovers as its own scarcity ignites a positive feedback loop between the demand for skills and the income of skilled workers. But collapses may happen again, and the economy may experience cycles.

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1 Introduction

The post world-ward II era has witnessed the rise of several trends. First, growth was supported by the rise of mass consumption societies. A growing number of people could purchase an increasing range of products, which in turn supported a regime where those goods could be mass produced. Second, a middle class of clerical workers and managers emerged, (sometimes called "cadres" in French). These people where characterized, arguably, by their relatively high education level, the fact that their occupation was not directly involved in production tasks, and a relatively hedonistic lifestyle, involving the purchase of a variety of branded products.[1]

This paper analyzes how these trends are related. The existence of a large middle class of educated workers generates a sizeable demand for a broad range of goods, which makes it profitable to elect industrial mass production technologies for these goods. This link is well understood by the literature. Conversely, mass production technologies are based upon a high degree of division of labor, which raises the demand for coordination and therefore for educated workers (lawyers, accountants, managers, engineers, human resource specialists, marketers...) who perform the overhead tasks needed for a complex business to operate.

It is this additional link which is studied in this paper, where I show that mutiple equilibria may arise. The economy may be at a high equilibrium with a large middle class, a high skill premium and a large number of mass produced goods, or at a low equilbrium with a small middle class and a small number of mass produced goods. A large number of "cadres" broadens the range of goods for which market size is large enough to trigger industrialization; conversely, industrialization raises the demand for "cadres" due to the coordination tasks that are needed to operate mass production technologies.

[1] See e.g. Dubois (1969).

In the low equilibrium, only a small range of products are mass produced to address the needs of a large but poor proletariat, while a minority of firms owners consume a broad range of goods that are produced using traditionial techniques. The demand for cadres is low because craft technologies use few managerial tasks. In the high equilibrium, the large "cadre" population both fulfills the needs for skilled labor in mass produced sectors, and sustains the large number of such sectors by spending on a broad range of goods.

The owners of the industrial firms that have the larger market share are worse-off in the "high", mass consumption, equilibrium. They specilize in basic goods that would also have a high market share in the low equilibrium, but have to pay higher wages to skilled workers in the high equilbrium. Owners of mass production technologies in relatively sophisticated goods are better-off in the high equilibrium, since the mass production technology would not be profitable for them in the low equilibrium, because the market size for their good would be too small.

Intringuingly, regulations that raise the size of the bureaucracy, such as increased standards for reporting and conformity, while reducing efficiency in partial equilibrium, may be beneficial in general equilibrium. This is because by raising the demand for skilled workers such rigidities make the high equilibrium more likely and the low equilibrium less likely. Conversely, improvements in the efficiency of management such as the "M form" and the slashing of intermediate layers may destroy the middle class in general equilibrium and trigger a brutal transition toward the low equilibrium.

A dynamic model is then studied, where people differ in innate ability and transmit human capital to their offspring. Industrialized equilibria with a large educated middle class may be fragile because they co-exist with a low equilibrium. If the economy collapses to the low equilibrium the returns to skilled labor fall, due to lack of demand for those skills. However the economy eventually recovers from the collapse, because skills become scarce again, which pushes up the skilled wage and therefore the demand for industrial products. Hence, cyclical trajectories may arise, similar to Shleifer (1986), although the mechanism is quite different here.

This paper builds on Murphy et al. (1991). It shares two important features with their model. First, each sector may use one of two technologies, a constant return "traditional" technology and an increasing returns technology with a fixed cost but a lower marginal cost than the old technology. Second, a demand structure based on a hierarchy of needs, which delivers a link between the distribution of income and the market size for each good, and therefore the number of goods that are industrialized.

The novelty here is that the distribution of income is made endogenous, by distinguishing between skilled and unskilled workers. My key assumption is that skilled workers play an important role in performing the overhead tasks associated with operating the new technology. Industrialization raises the demand for skilled workers, which leads to the emergence of a middle class. In turn, this makes it profitable for a broader range of goods to industrialize – thus generating a positive feedback loop between the size of the middle class and the degree of industrialization. This feedback loop is then shown to be conducive to multiple equilibria under some circumstances.

Related theoretical literature also includes Matsuyama (2002), Mani (2001), Foellmi and Zweimüller (2008), Foellmi et. al (2014), Desdoigts and Jaramillo (2020). These contributions differ in many ways from the present one but share with it an important role for non homothetic preferences. Mani (2001), shares a number of features with this paper. In particular, there is path dependence in inequality because the demand for skills depends on the structure of the demand for goods, which itself depends on income distribution through non homothetic preferences. However, the assumptions and economic mechanisms analyzed by Mani, as well as her results, largely differ from this paper. Recent empirical literature includes Beaudry et al. (2013), Argan and Gary-

Bobo(2019), Comin et al. (2020), Chai et al. (2022). Beaudry et al. and Argan and Gary-Bobo, in particular, document a fall in the observed returns to skills after 2000, which may be consistent with a transition from a high equilibrium to a low equilibrium. An implication of my model is that such a transition is likely to increase product market concentration: As shown below, the range of goods consumed by workers is lower in the low equilibrium, and the proportion of industrialized goods among those goods is higher (despite that their absolute number is lower). These two forces should lead to an increase in measures of market concentration. Indeed, this is documented by a substantial recent literature (See e.g. Hall (2018), Van Reenen (2018), Grullon et al. (2019), De Loecker et al. (2020), Lanier et al. (2021)).

2 Static model

In this section I spell out the model's basic assumptions and derive some key properties of equilibrium. There is a continuum of goods indexed by i. In each industry i, there are two technologies, the old one and the new one. There are two factors of production, raw labor and skilled labor. The old (craft) technology has constant returns and uses raw labor only. It requires c_O units of raw labor per unit of the final good.The new (mass production) technology requires $c_N < c_O$ units of raw labor per unit of the final good, plus a fixed amount m of skilled labor. Skilled workers perform the overhead tasks.

The old technology can be freely used by any firm and there is free entry of firms. The new technology, in each sector, is owned by an incumbent firm.

The wage of raw labor is normalized to 1. The wage of skilled labor is denoted by ω.

These assumptions already allow us to derive some properties of equilibrium. First, clearly, the competitive fringe sells its output at price c_O.

The incumbent can charge any price $\leq c_O$ and get the whole market. In this model, the own price elasticity of demand for any good is negligible.[2] Therefore, the incumbent, if active will charge a price equal to p_O, which also prevails if the incumbent is inactive.

Consequently, in any market i with output $y(i)$, two possibilities arise:

- If $m\omega > (c_O - c_N)y(i)$, it is not profitable for the incumbent to operate because fixed costs are too high given market size. The old technology is in use. The sector only employs raw labor and does not generate any profit.

- If $m\omega < (c_O - c_N)y(i)$, the incumbent gets the whole market. The new technology is in use. The sector employs $c_N y$ units of raw labor, paid the same amount $c_N y$, m units of skilled labor, paid $m\omega$, and pays a profit equal to

$$\pi(i) = (c_O - c_N)y(i) - \omega m$$

  to the owners of the incumbent firm.

By construction, due to the hierarchy of needs structure of preferences, $y(i)$ falls with i. Therefore, there is a cutoff value of i, i^*, defined by

$$y(i^*) = \frac{\omega m}{c_O - c_N},$$

such that the modern technology is in use iff $i < i^*$.

[2] To see this, take a discrete approximation of the hierarchy of needs model where the number of goods is finite and one consumes an amount δ of each good. Consider good k, assuming the lower ranked goods are all priced at p^*. Let F be the c.d.f. of income. Let p be the price charged by the incumbent of good k and c the incumbent's unit cost. Then the demand for good k is $1 - F((k-1)p^*\delta + p\delta)$ and profits are $\pi = (p-c)\left(1 - F((k-1)p^*\delta + p\delta)\right)$. Hence $d\pi/dp = 1 - F + \delta(p-c)f$. The continuum of goods model is the limit of this model when the number of goods N goes to infinity and δ falls to zero inversely proportionately. Say $\delta = 1/N$. For a good of relative position x, $k = x/\delta$. For any such x, $d\pi/dp \to 1 - F > 0$.

I now turn to the description of the consumer side. There is a continuum of consumers-workers indexed by j. This continuum has a total mass equal to 1 and j is uniformly distributed over $[0,1]$. Preferences are based on a hierarchy of needs structure. People first consume the lower ranked goods, and they need one unit of each good. They are endowed with a fixed supply l of raw labor. Consumer j's utility is

$$u(n,s,j) = n - \gamma j^{-\frac{1}{\lambda}} s^{\frac{1}{\lambda}+1},$$

where n is the mass of goods being consumed – the hierarchy of needs structure implies that these are the goods in $[0,n]$ – and s is the supply of skilled labor.

The owners of the firms are a separate capitalist class of arbitrarily small measure. Therefore they consume an arbitrarily large variety of goods, but their contribution to the demand for each good is infinitesimal.

Again, these assumptions allow us to derive some important properties of equilibrium. Let z be the income of a consumer. Clearly, $n = z/c_O$ and $z = \omega s + l$. The consumer's optimal labor supply, therefore, is given by

$$s(j,\omega) = hj\omega^{\lambda}, \tag{1}$$

where

$$h = \left(\frac{\lambda}{\gamma c_O(1+\lambda)}\right)^{\lambda}. \tag{2}$$

Therefore, in equilibrium the income of consumer j is

$$z(j,\omega) = hj\omega^{1+\lambda} + l. \tag{3}$$

and the breadth of goods he consumes is

$$n(j,\omega) = z(j,\omega)/c_O.$$

From there we can compute the market size for each good – recall that the contribution of the capitalists is negligible. Clearly any good i is bought by all consumers such that $z(j,\omega)/c_O > i$. In particular, the poorest worker has an income equal to l, implying that all goods such that $i \leq l/c_O$ are consumed by all agents, i.e. $y(i) = 1$. For $i > l/c_O$, market size is given by

$$y(i,\omega) = 1 - j(c_O i,\omega),$$

where $j(z,\omega)$ is the inverse function of z, i.e. $z(j(z,\omega),\omega) \equiv z$.

3 Equilibrium determination

There are two types of equilibria. In a type I equilibrium, which is the most natural configuration, all goods which serve all customers are industrialized. One has $i^* > l/c_O$ and therefore $\omega < \frac{c_O - c_N}{m}$, since $y(i^*) < 1$. In a type II equilibrium, $i^* < l/c_O$. There are fewer industrialized goods than goods that serve all customers. since these goods are all identical, firms with a full market size must be indifferent between using the new technology and the old technology, implying that $\omega = \frac{c_O - c_N}{m}$.

3.1 Type I equilibrium

I now characterize a type I equilibrium, which is the most relevant for analyzing multiple equilibria. Let us denote the total supply of skilled labor by $S(\omega)$. Then the equilibrium values of i^* and ω are determined by

$$mi^* = S(\omega) \quad (4)$$

$$1 - j(c_O i^*,\omega) = \frac{\omega m}{c_O - c_N}. \quad (5)$$

The first relationship, (4) describes the equilibrium on the market for skilled labor. The LHS is the demand for skilled labor while the RHS is its supply, denoted as a function of its wage $\omega, S(\omega)$. The second relationship (5) is the zero profit condition for the marginally industrialized good i^*.

With our functional forms, the inverse income function is

$$j(z,\omega) = (\frac{z-l}{h})\omega^{-\lambda-1}.$$

The aggregate supply for skilled labor is

$$S(\omega) = \int_0^1 hj\omega^\lambda dj = h\frac{\omega^\lambda}{2}.$$

Substituting, the equilibrium system can be reexpressed as

$$mi^* = h\frac{\omega^\lambda}{2}, \tag{6}$$

$$1 - (\frac{c_O i^* - l}{h})\omega^{-\lambda-1} = \frac{\omega m}{c_O - c_N}. \tag{7}$$

This system defines a bona fide type I equilibrium if its solution is such that $i^* > l/c_O$, from which it follows naturally from (7) that $\omega < \frac{c_O - c_N}{m}$.

3.2 Type II equilibrium

In a type II equilibrium, the wage of skilled labor is necessarily equal to

$$\omega = \frac{c_O - c_N}{m}. \tag{8}$$

That is, skilled worker appropriate all the monopoly rents from an incumbent with full market size. Since some full market size goods are industralized, while others are not, profits must be zero for an incumbent with full market size. The equilibrium i^* is then obtained from the equilibrium condition on the skilled labor market (6), where (8) has been used:

$$i^* = \frac{h(c_O - c_N)^\lambda}{2m^{1+\lambda}}.$$

This equilibrium is indeed of type II provided the resulting i^* is smaller than l/c_O, i.e.

$$\frac{c_O(c_O - c_N)^\lambda}{2m^{1+\lambda}} < \frac{l}{h}. \tag{9}$$

3.3 Equilibria: Existence, type and multiplicity

In this section I characterize the set of equilibria as a function of parameter values. It will be useful to distinguish between "high" and "low" equilibria even when there is a unique equilibrium. For this purpose I introduce the following definition:

Definition 1 – An equilibrium is high (resp. low) if

$$\omega m > \ (\textit{resp.} \ <) \ \frac{\lambda + 1}{2(\lambda + 2)}(c_O - c_N).$$

Therefore, high equilibria have higher wages for skilled workers, and higher overhead costs, relative to the profits per unit sold, than low equilibria. Since $\frac{\lambda+1}{2(\lambda+2)} < 1$, type II equilibria are always high.

The following Proposition fully characterizes the equilibrium set:[3]

Proposition 1 – A. There exists an equilibrium.

B. If $\lambda(\lambda + 2) \geq \frac{c_O - c_N}{c_O + c_N}$ *the equilibrium is unique. The equilibrium is of type I if* $\frac{c_O(c_O - c_N)^\lambda}{2m^{1+\lambda}} > \frac{l}{h}$ *and of type II if* $\frac{c_O(c_O - c_N)^\lambda}{2m^{1+\lambda}} < \frac{l}{h}$.

C. If $\lambda(\lambda + 2) < \frac{c_O - c_N}{c_O + c_N}$*, there exists a nonempty interval* $[l^-, l^+]$*, such that for* $l \in (l^-, l^+)$ *there are three equilibria of type I. The equilibrium with the highest* ω *is high and the one with the lowest* ω *is low.*

D. If $\lambda(\lambda + 2) < \frac{c_O - c_N}{c_O + c_N}$*, and* $l \notin (l^-, l^+)$ *there exists a unique equilibrium. The equilibrium is of type I (resp. type II) if and only if (9) is violated (resp. holds).*

Multiple equilibria arise from a strategic complementarity between skilled wages and the demand for product variety. When skilled wages go up, workers are richer and they consume a broader range of goods. This raises the market size for some traditional goods in such a way that it is profitable for

[3] All proofs are in the Appendix.

the modern technology to operate. This in turn raises the need for overhead and the global demand for skilled labor, thus validating the initial increase in wages. At the same time, there is a countervailing effect: The greater value of ω raises overhead costs, which tends to reduce the number of goods using the modern technology. If the first effect dominates over some range, the zero profit schedule (7) has upward sloping portions in the (ω, i^*) plane, which opens up the possibility of multiple equilibria.

The multiple equilibria case is illustrated on Figure 1, which was drawn for a set of parameters that deliver multiple equilibria. These values are $c_O = 1, c_N = 0.1, \lambda = 0.2, m = 0.15, \gamma = 1, l = 1.7$.

The middle equilibrium is unstable in the following sense. Consider a small positive perturbation to ω. Locally, this raises the supply of skilled labor through the RHS of (6). However, through the market size effect it is profitable to industrialize even more sectors than allowed by the rise in labor supply, as captured that the fact that zero profit condition is steeper than the labor market equilibrium condition. Therefore, the demand for skilled labor goes up by even more than the supply of skilled labor, which tends to further raise ω, thus moving the economy further away from the initial equilibrium. Therefore, we will ignore this central equilibrium and compare two equilibria, the high ω one (called high) and the low ω one (called low).

As is always the case unless nonlinearities are very strong, the range of values $[l^-, l^+]$ that generate multiplicity is not very large. But it is not negligible either. In the example of Figure 1, $l^- = 1.57$ and $l^+ = 1.79$. Also, even absent multiple equilibria such nonlinearities may generate very strong response to shocks. This may happen when a shock shifts the equilibrium relationships so that the high equilibrium disappears and is replaced by a low equilibrium, or vice-versa.

4 Comparing equilibria

In this section I compare the high equilibrium with the low equilibrium. Proposition 2 tells us that inequality among workers is greater in the high equilibrium. This is because the returns to skill are greater. To the extent that it is driven by greater returns to education, a larger middle class means greater wage inequality. Arguably, in the early industrial revolution, inequality between labor and capital was high but inequality between workers was low as there were much fewer managerial positions.

Despite that, all workers benefit from being in the high equilibrium, including the low skilled ones, even though the productivity gains from industrialization, in a given sector, are appropriated by the incumbent. This is because the purchasing power of unskilled labor is unchanged, while that of skilled labor had gone up. If, say, industrialization led to some complementary input to be reallocated from raw labor to managerial skilled labor, the former's real wage might fall, and unskilled workers could potentially be worse-off in the high equilibrium.[4]

Proposition 2 – A. Wage inequality is higher in the high equilibrium, compared to the low equilibrium

B. All workers are better-off in the high equilibrium

C. Profits are either higher or lower in the high equilibrium

D. If profits are higher, the high equilibrium Pareto dominates the low equilibrium, and has greater overall inequality, provided the capitalist class is small enough.

E. The total number of goods consumed by workers is higher in the high equilibrium

F. The ***proportion*** *of industrialized goods among the total number of goods consumed by workers is* ***higher*** *in the* **low equilibrium**

[4] See Krusell et al. (2000), Zeira (1998), Caselli (1999), Beaudry and Green (2003)

It is also interesting to compare profits between the high and low equilibria for a given firm.

Proposition 3 – Assume there are several equilibria. Let i_L^* (resp. i_H^*) be the value of i^* in the low (resp. high) equilibrium. Then there exists a critical $\tilde{\imath} \in (l/c_O, i_L^*)$ such that firm i has higher (resp. lower) profits in the high equilibrium than in the low equilibrium iff $\tilde{\imath} < i \leq i_H^*$ (resp $i < \tilde{\imath}$).

*

Firms are better-off in the high equilibrium if the rise in their market size induced by the greater ω exceeds the increase in overhead costs. Market size goes up by more, the more highly ranked the good in the hierarchy of needs. For example, for goods with a low enough rank to cover the whole market, market size cannot be higher in the high equilibrium, and those firms must have lower profits in the high equilibrium due to a higher skilled wage. Hence, the mùarket size effect is more likely to dominate the overhead cost effect, the more highly ranked the good. Therefore, the more highly ranked goods are more profitable in the high equilibrium than in the low equilibrium, while the converse holds for the low ranked goods that are consumed by many people and only have a small margin for increasing their market size.

The significance of Proposition 3 depends on features of the economy that are not modelled here. If capitalists hold a diversified portfolio, the distribution of profits across goods does not matter. What matters for this class is total profits, given by

$$\begin{aligned}\Pi &= (c_O - c_N - \omega m)\frac{l}{\omega} + \int_{l/c_O}^{i^*} \left((c_O - c_N) \left[1 - (\frac{c_O i - l}{h})\omega^{-\lambda-1} \right] - \omega m \right) di \\ &= (c_O - c_N - \omega m) i^* - (c_O - c_N)\frac{c_O}{h}\omega^{-\lambda-1}(i^* - l/c_O)^2/2.\end{aligned}$$

Numerical simulations (available from the author upon request) suggest that aggregate profits are lower in the high equilibrium; it has not proved possible to come up with a numerical example where the opposite holds. This suggests that the effect of a higher ω on inframarginal goods with a large market share dominates, and that because it makes the capitalist class worse-off, the high equilibrium will not Pareto-dominate the low equilibrium.

If capitalists do not hold a diversified portfolio, there is a conflict of interest over the extent of industrialization and mass consumption between owners of the new technology in old consumer goods, who prefer a lower demand for managerial workers in the low equilibrium, and owners of the new technology in new consumer goods, who prefer a large customer base in the high equilibrium.

For illustrative purpose, we can compare the two equilibria of interest in our example of Figure 1. The properties of these equilibria are summarized in the following Table

	Low eq.	High eq.
i^*	1.9	3.0
ω	0.36	3.6
GDP	3.32	4.17
Labor income/GDP %	51.2	40.8
Skilled income/GDP	3.1	39.0
Profits/GDP	45.7	20.2
% of industrial goods	99.7	60.8
% of goods consumed by all workers	89.2	34.3

Table 1 – Comparison between the two equilibria in the example of figure 1.

We note that the low equilibrium has a much lower skilled wage, and therefore much lower wage inequality, than the high equilibrium. As a result, even though it has fewer industrial goods, the *proportion* of industrial goods is much higher than in the high equilibrium. As shown in Proposition 2.F., this feature is general and does not rest on specific parameter values. The demand for artisanal goods, which comes from comparatively richer workers, is much lower in the low equilibrium than in the high equilibrium. In the low equilibrium, fixed operating costs are quite small due to the low cost of skilled labor. At the same time, due to the small size of the bourgeoisie, market size falls very fast as the ranking of a good in the hierarchy of needs goes up (in a perfectly egalitarian society, it would collapse from 100 % to zero after a certain threshold). For these reasons, the highest ranked industrial good $i^* = 1.9$ is *both* close to the highest ranked good consumed by all workers $l/c_O = 1.7$ *and* it has a very low market size (equal to $\omega m/(c_O - c_N) = 6\%$, versus 60% in the high equilibrium).

5 Comparative statics

In this section I perform some comparative statics exercises. The key parameter of interest is m, the amount of overhead skilled labor which is necessary to operate the new technology.

Proposition 4 – A. An increase in m unambiguously reduces i^*.

B. An increase in m raises ω (resp. reduces ω) if in equilibrium

$$\omega < (\text{resp.} >)\ \omega_c = \frac{1}{m}\sqrt{\frac{c_O(c_O - c_N)}{2}}$$

C. If there are multiple equilibria, $\omega_L < \omega_c$. Consequently, an increase in m raises ω in the low equilibrium.

D. As m goes up there exists a critical m such that the low equilibrium ceases to exist as m raises above that level.

E. As m goes down there exists a critical m such that the high equilibrium ceases to exist as m falls below that level.

An increase in m is a form of technical regress. It may arise from government-imposed regulations that force each firm to higher more bureaucrats to operate their business. But, by raising the demand for skilled workers, such a change may trigger a shift from the low equilibrium to the high equilibrium. The rise of the bureaucratic class raises the demand for a whole new range of goods, thus triggering a wave of industrialization. Conversely, trimming the administration makes an individual firm more profitable, but in general equilibrium it may destroy the middle class and lead to de-industrialization because a number of goods fail to have a sufficient market size.[5]

In the next proposition, I study the effect of other parameters such as the supply of skilled and unskilled workers and the variable cost in the new technology:

Proposition 5 – A fall in c_N *uambiguously raises* ω *and* i^**. Furthermore, a fall in* c_N *makes it less likely that the low equilibrium exists.*

A fall in γ *unambiguously reduces* ω; *the number of industrial goods* i^* *goes up (resp. falls) if*

$$\omega < \text{(resp. >)}\ \frac{c_O - c_N}{2m};$$

if there are multiple equilibria, i^* *goes up in the low equilibrium; the high equilibrium is less likely to exist.*

An increase in l *has the opposite effect of a fall in* γ*.*

A reduction in c_N means that the new technology is more productive. Incentives for industrialization are greater. As a result more sectors get industrialized and the demand for skilled labor goes up. This induces an increase in ω which further raises the market size for industrial products. This process may eliminate the low equilibrium, triggering a transition to the consumer society.

[5] Indeed, the trends since 2000 toward a lower return to skill and greater concentration, coincide with the trimming of managerial layers and the rise of the celebrated "M-form" of corporate organization. See Hamel (2011).

A reduction in γ is associated with an increase in the supply of skilled workers. This tends to make it more profitable to industrialize. However the resulting fall in the wage of skilled workers also reduces the market size for industrial products. While industrialization, i.e. i^*, goes up at the low equilibrium, one cannot rule out that on net the latter effect dominates at the high equilibrium. Finally, the increase in the supply of skilled labor may trigger a collapse of the high equilibrium, in which case the size of the industrial sector and the income of the middle class dramatically fall; in other words "too much middle class kills the middle class".

6 Aggregate welfare

This section provides additional results regarding aggregate welfare, defined in a utilitarian way. The two key results are that under multiple equilibria, aggregate welfare is higher in the high equilibrium, and that whenever a type II equilibrium exists, it is efficient.

Aggregate utilitarian welfare is equal to

$$W = N - \int_0^1 \gamma j^{-\frac{1}{\lambda}} s(j,\omega)^{\frac{1+\lambda}{\lambda}} dj, \tag{10}$$

where N is the total number of units of goods consumed by the people.

Iny any decentralized economy, we have that

$$N = N_w + N_c,$$

where

$$N_w = \int_0^1 n(j,\omega) dj$$

is the number of goods consumed by workers, and

$$N_c = \int_0^{i^*} \frac{\pi(i,\omega)}{c_O} di = \frac{\Pi}{c_O}$$

is the number of goods consumed by the capitalists.[6]

Proposition 6 – A. An allocation maximizes total welfare W if and only if

(i) All industrialized goods have a full market share,

(ii) Each individual j consumes an interval $[0, n(j)]$ of goods

(iii) Each individual supplies

$$s(j) = hj\left(\frac{c_O - c_N}{m}\right)^{\lambda}$$

units of skilled labor.

B. In a type II equilibrium, the allocation maximizes total welfare.

The social planner can always allocate additional units of any good to somebody who needs it, i.e. is not already consuming this particular good. Assume there is a strictly positive measure of industrial goods with a less than full market share. Then with the same raw labor input one can produce the same aggregate number of units of goods, but rearrange production so as to have a narrower range of goods, each with a bigger market share than in the original allocation. Clearly, this leaves N unchanged while saving on overhead costs, i.e. reducing the skilled labor input and its associated disutility, as captured by the second term in (10). The new units of the goods whose market share has gone up can always be allocated to people who need it, thus exactly offsetting the utility loss of those consumers who consume fewer goods as a result of the reallocation. Consequently, all industrial goods must be consumed by everybody in an allocation which is optimal from a utilitarian viewpoint.

[6]The *mass* of capitalists is infinitely small, the *range* of goods consumed by a given capitalist is infinitely large. However altogether the aggregate income of capitalists is finite and so is the total units of goods they consume.

Part A,iii and claim B of Proposition 6 imply that a type II equilibrium is optimal from a utilitarian viewpoint. To understand why, consider the following. The social value of skilled labor is determined as follows: One extra unit of skilled labor supplied to the market allows to industrialize $1/m$ goods. It is efficient to allocate enough raw labor to those newly industrialized goods so as to cover the whole market. Therefore, since the mass of consumers is normalized to 1, c_N/m units of raw labor are needed. By reallocating those workers away from the traditional sectors into the newly industrial sectors, one foregoes $c_N/(mc_O)$ units of traditional goods. Therefore the net social gain in terms of the aggregate units of goods consumed is equal to $\frac{1}{m}\left(1 - \frac{c_N}{c_O}\right)$. Expressed in terms of raw labor, which is the numéraire, this gain is equal to $\frac{c_O - c_N}{m}$. Consequently, one could decentralize the utilitarian optimum by having a skilled wage equal to $\omega = \frac{c_O - c_N}{m}$. But this is precisely what the skilled wage is in a type II equilibrium. Why? In a type II equilibrium industrialized firms earn no rents because they compete with traditional firms in non industrial sectors which also cover the whole market and where profits are zero. At the same time incumbent firms in those industrialized sectors appropriate all the variable cost reductions associated with industrialization, due to limit pricing. These gains are dissipated in fixed costs, i.e. they are transferred to skilled labor as industrial firms outcompete each other to attract skilled workers. Therefore, as a result of this competitive process, the skilled wage exactly reflects the total savings in raw labor associated with industrializing a sector which has a full market share — that is, it coincides with the social value of skilled labor expressed in terms of the numéraire. In short, a type II equilibrium is a utilitarian optimum because in such an equilibrium all industrial firms have a full market share and the skilled wage coincides with the social value of skilled labor.

It follows from this reasoning that whenever $\omega < \frac{c_O - c_N}{m}$, i.e. in a type I equilibrium, the social value of skilled labor is higher than its wage, implying that its supply is inefficiently low. An equilibrium with a higher value of ω is therefore preferable, hence:

Proposition 7 – If there are multiple equilibria then aggregate welfare W is higher in the high equilibrium.

7 Dynamics: Skill acquisition and the collapse of the educated class

In this section I make the model dynamic and endogenize the dynamics of human capital accumulation. To do so, I modify the model as follows: I assume that skilled workers are members of dynasties, each with a specific, heritable ability level indexed by i. Each generation decides how much to consume and how much to invest in the human capital of their offspring, which in turn determines the latter's skill level. Therefore, the supply of skills at each date t is now inelastic in the sense that it is determined by decisions at date $t-1$.[7]

Consider a skilled worker with ability j and income z.He invests an amount x into his offspring's human capital, which is a disutility cost expressed in terms of number of varieties. That is, increasing x by dx has the same direct effect on utility as reducing the measure of goods that are consumed by dx. Human capital is equal to the endowment of skilled labor, which enters the production of industrialized goods the same way as before. The production function for that human capital is

$$h' = \beta x + \alpha j. \tag{11}$$

All else equal, higher ability people will have more human capital. People value their consumption as well as their offspring's human capital. The

[7] That is, the intra-period determination of equilibrium is similar to the above model with $\lambda = 0$.

individual's utility function is

$$U = \ln(n - x) + \theta \ln h', \tag{12}$$

where

$$n = z/c_O \tag{13}$$

is the number of varieties consumed by the individual.

Maximizing (12) with respect to x, and using (11) yields the optimal level of the offspring's human capital

$$h' = \frac{\theta}{1+\theta}\left(\frac{\beta}{c_O}z + \alpha j\right). \tag{14}$$

In order to limit the discussion to the most relevant cases, I impose the following restriction on α and β

ASSUMPTION 1 –

$$\mu = \frac{\beta l}{\alpha c_O} < 1.$$

The law of motion for the human capital of any dynasty j is then characterized by the following lemma:

LEMMA 1 – Assume there exists some initial date t_0 when $h = 0$ for all j. Then at any given date subsequent date the human capital of individual j is

$$h(j) = k(j + \mu),$$

where the law of motion for k is

$$k' = \frac{\theta}{1+\theta}(\alpha + \frac{\beta}{c_0}\omega k) \tag{15}$$

and at $t = t_0 + 1$ the initial value of k is

$$k_0 = \frac{\theta\alpha}{1+\theta}.$$

At any date t, then the distribution of skills is determined by a single parameter, k, which is inherited from the past behavior of preceding generations in transmitting human capital to their offsprings. Given k, intra-period equilibrium determination is similar to the static analysis of the preceding section. Since the supply of skills is fixed and given by

$$S = \int_0^1 k(j+\mu)dj = k(\mu + 1/2),$$

the number of industrialized sectors is

$$i^* = \frac{S}{m} = \frac{k(\mu + 1/2)}{m}.$$

The income of individual j is

$$z(j, \omega, k) = \omega k(j+\mu) + l.$$

Consequently, the lowest ranked individual to consume good i is

$$j(i, \omega, k) = \max(\min(\frac{c_0 i - l}{k\omega} - \mu, 1), 0).$$

The zero profit condition for the marginal industrialized sector i^* is

$$m\omega = (c_O - c_N)(1 - j(i^*, \omega, k)). \tag{16}$$

There are three possible types of equilibria

1. An equilibrium with $\omega = 0$, which is a special case of the low equilibrium studied in the preceding section, in the case where $\lambda = 0$. In such a situation, there is an oversupply of skills and the marginal industrialized good has a zero market share, otherwise it would have strictly positive profits. It must then be that $j(i^*, \omega, k) = 1$. Since $\lim_{\omega \to 0} \frac{c_0 i - l}{k\omega} - \mu = sgn(c_0 i^* - l) \times \infty$, such an equilibrium exists if and only if $c_0 i^* > l$, i.e.

$$kc_0(\mu + 1/2) > ml.$$

2. An equilibrium such that everybody buys the critical good, similar to a type II equilibrium in the preceding section. In such a situation $j(i^*, \omega, k) = 0$, and $\omega = \frac{c_O - c_N}{m}$. Such an equilibrium exists provided $\frac{c_0 i - l}{k\omega} - \mu < 0$ for $i = i^*$ and $\omega = \frac{c_O - c_N}{m}$. This is equivalent to

$$ml > k(c_O/2 + \mu c_N).$$

3. An equilibrium such that $0 < \omega < \frac{c_O - c_N}{m}$, similar to a type I high equilibrium. In such a situation $j(i, \omega, k) = \frac{c_0 i - l}{k\omega} - \mu$, implying from (16) that ω is solution to

$$\frac{mk}{c_O - c_N}\omega^2 - k(1+\mu)\omega + c_O i^* - l = 0.$$

Such an equilibrium exists provided the above equation has at least one real positive root in ω such that $\omega < \frac{c_O - c_N}{m}$.

The following Proposition summarizes the intra-period equilibrium determination, conditional on k:

Proposition 8 – Let

$$\begin{aligned}
\psi_1 &= \frac{c_O + c_N}{4} + \mu\left(\frac{c_N + c_O}{2}\right) - \frac{\mu^2}{4}(c_O - c_N), \\
\psi_2 &= c_O/2 + \mu c_N > \psi_1, \\
\psi_3 &= c_O(\mu + 1/2) > \psi_2.
\end{aligned}$$

Then

(i) If $k > \frac{ml}{\psi_1}$ *the only equilibrium is* $\omega = 0$

(ii) If $\frac{ml}{\psi_2} < k < \frac{ml}{\psi_1}$ *there are two equilibria: a low one such that* $\omega = 0$ *and a high (type I) one such that* $0 < \omega < \frac{c_O - c_N}{m}$

(iii) If $\frac{ml}{\psi_3} < k < \frac{ml}{\psi_2}$ *there are two equilibria: a low one such that* $\omega = 0$ *and a high (type II) one such that* $\omega = \frac{c_O - c_N}{m}$

(iv) If $k < \frac{ml}{\psi_3}$ *the only equilibrium is* $\omega = \frac{c_O - c_N}{m}$.

Remark: All the ψ_is are increasing in c_O and c_N. An adverse productivity shock affecting variable costs in either technology makes low equilibria more likely and high equilibria less likely. The converse holds for adverse productivity shocks affecting fixed costs. As already discussed, this is because such shocks raise the demand for skilled labor.

As the economy grows over time, so does k as people accumulate more human capital. This makes it more likely that a low equilibrium such that $\omega = 0$ exists, less likely that a high equilibrium exists, and more likely that the high equilibrium is of type I instead of type II.

Proposition 8 characterizes equilibria at a point in time for a given k, i.e. for a given inherited level of human capital. In the next subsection, I study how that level is determined in steady state.

7.1 Steady states

I now characterize the steady states of the economy:

Proposition 9 – Let

$$\begin{aligned} m_0 &= \frac{\theta}{1+\theta}\frac{\alpha c_O}{l}\left(\mu + 1/2\right), \\ m_1 &= \frac{\theta}{1+\theta}\frac{\alpha}{l}\left(c_0\left(\mu + 1/2\right) - \frac{c_0 - c_N}{4}(1-\mu^2)\right) < m_0. \end{aligned}$$

Then (i) There exists a steady state such that $\omega = \frac{c_O - c_N}{m}$ *if and only if* $m > m_0$

(ii) There exists a steady state such that $\omega = 0$ *if and only if* $m < m_0$

(iii) There exists a steady state such that $0 < \omega < \frac{c_O - c_N}{m}$ *if and only if* $m_1 < m < m_0$.

While in the short run, a type II equilibrium can co-exist with a zero wage equilibrium, in steady state that is not possible. There are multiple steady states in the parameter zone defined by $m_1 < m < m_0$, but the high steady state is then of type I, not type II.

7.2 Surges and collapses

In any steady state, there may be an equilibrium which co-exists with the one that perpetuates the steady state. That is, a low steady state can "surge" if there also exists an intra-period equilibrium such that $\omega > 0$. A high steady state can collapse if there also exists an intra-period equilibrium such that $\omega = 0$. The following proposition characterizes the set of parameter values for which there are multiple equilibria in a steady state (whereas Prop. 8 characterizes equilibria for a given value of k, irrespective of whether such value is consistent with steady state).

Proposition 10 – (i) Let

$$m_2 = \frac{\theta}{1+\theta}\frac{\alpha}{l}\left(\frac{c_O + c_N}{4} + \mu\left(\frac{c_N + c_O}{2}\right) - \frac{\mu^2}{4}\left(c_O - c_N\right)\right) < m_1.$$

Then if $m_2 < m < m_0$ *the low steady state can surge.*

(ii) Let

$$m_3 = \frac{\theta}{1+\theta}\frac{\alpha}{l}\left(c_O\left(2\mu+1\right) - \mu c_N\right) > m_0$$

Then if $m_0 < m < m_3$ *the high steady state can collapse.*

(iii) If $m_1 < m < m_0$ *the type I high steady state can collapse.*

In the next two subsections, I analyze some transitional dynamics following a surge and a collapse.

7.3 Dynamics following a surge

Proposition 11 – There exists $m_4 \in (m_1, m_0)$ *such that if* $m_4 < m$ *and* $k < \frac{ml}{\psi_1}$ *then there exists a subsequent trajectory such that* $\omega > 0$.

Corollary – If $m_4 < m < m_0$ *then a surge may be permanent.*

Under the conditions of Proposition 11, the economy may permanently escape the low equilibrium, in that following such a surge, there exists a trajectory such that the economy is at a high equilibrium forever. This, however, does not prevent a subsequent collapse, i.e. a return to the low equilibrium. Proposition 11 only shows that a permanent surge is a possibility. Also, since $m_4 > m_1$ the condition $m_4 < m < m_0$ is more stringent than for a type I steady state to exist, implying that if it does converging to that steady state after a surge is not guaranteed.

7.4 Dynamics following a collapse

Proposition 12 – (i) If $m_1 < m < m_0$ *then a collapse may be permanent*

(ii) if $m_0 < m < m_3$ *then the collapse lasts one period, after which the only equilibrium is of type II. Then there exists a trajectory which monotonically converges to the steady state and such that the equilibrium is always of type II. Furthermore, there exists a critical date after which, along that trajectory, a collapse may happen again.*

Proposition 12 opens up the possibility of long cycles in the dynamics of income distribution and industrialization. Following a surge the economy gradually accumulates human capital; new generations of people are more educated than their parents. After a while this "skill glut" takes the economy to a danger zone where the middle class/consumer society equilibrium may collapse to a low equilibrium where $\omega = 0$, which we may interpret as the disappearance of the middle class. The supply of skills is high enough for market participants to be able to rationally coordinate on an equilibrium where the price of skills has fallen to zero, as the share of industrialized sectors has shrunk in such a way that the demand for skilled overhead labor is always below the supply, which in turn reduces the demand for some industrialized goods below a critical threshold which makes it profitable to return to the traditional technology.

If $m > m_0$, there does not exist a low steady state. However, if $m_0 < m < m_3$, there does exist a low temporary equilibrium, which may collapse the high steady state any time. The collapse leads to a reduction in the stock of human capital, due to the disappearance of the middle class. However this eliminates the "skill glut", and the skilled wage starts going up again. Human capital is being gradually reaccumulated–the collapse cannot be permanent. The economy converges to a high equilibrium unless it collapses again, which is possible after having accumulated enough human capital to end up again in the "danger zone".

8 Conclusion

This paper has analyzed the importance of the educated class for the persistence of mass consumption societies. It has pointed out that, through the demand for managerial talent, the latter generate their own demand for skills, implying that mass consumption society is self-sustaining but may also collapse. In the dynamic analysis, it has been shown that this collapse may be triggered after the economy has accumulated human capital sufficiently for a low equilibrium to exist. Following a collapse, the educated class disappears but gradually recovers as its own scarcity ignites a positive feedback loop between the demand for skills and the income of skilled workers. Since the zone where a new collapse is possible is eventually reached, the economy may experience cycles.

9 Appendix

9.1 Proof of Proposition 1

Eliminate i^* between (6) and (7) to get

$$f(\omega) \equiv \omega^{1+\lambda} - \frac{m}{c_O - c_N}\omega^{2+\lambda} - \frac{c_O}{2m}\omega^{\lambda} = -\frac{l}{h}. \quad (17)$$

A type I equilibrium exists if and only if we can find ω such that (17) holds and $\omega < \frac{c_O - c_N}{m}$, from which the condition $i^* > l/c_O$ will automatically follow from (7).

We note that $f(0) = 0$ and that $f(\frac{c_O - c_N}{m}) = -\frac{c_O(c_O - c_N)^{\lambda}}{2m^{1+\lambda}}$.

Assume $-\frac{c_O(c_O - c_N)^{\lambda}}{2m^{1+\lambda}} < -l/h$ Then by continuity, $\exists \omega \in (0, \frac{c_O - c_N}{m})$ s.t. $f(\omega) = -\frac{l}{h}$. Clearly this satisfies all the conditions for a type I equilibrium. Now if $-\frac{c_O(c_O - c_N)^{\lambda}}{2m^{1+\lambda}} > -l/h$, condition (9) holds, implying that $\omega = \frac{c_O - c_N}{m}$, $i^* = \frac{h(c_O - c_N)^{\lambda}}{2m^{1+\lambda}}$ is a type II equilibrium. Therefore, there always exists an equilibrium. This proves claim A.

Let

$$\begin{aligned} a &= \frac{m}{c_O - c_N}, \\ b &= \frac{c_O}{2m}. \end{aligned}$$

Next, compute f' and note that

$$f' \propto (\lambda + 1)\omega - (\lambda + 2)a\omega^2 - \lambda b. \quad (18)$$

Clearly, if

$$(\lambda + 1)^2 - 4\lambda(\lambda + 2)ab \leq 0, \quad (19)$$

then $f' < 0$ throughout, implying uniqueness, while the condition which determines the equilibrium type follows straightforwardly from the preceding discussion. Condition (19) is equivalent to

$$\frac{(\lambda + 1)^2}{\lambda(\lambda + 2)} = 1 + \frac{1}{\lambda(\lambda + 2)} \leq 4ab = 2\frac{c_O}{c_O - c_N},$$

which is clearly equivalent to

$$\lambda(\lambda+2) \geq \frac{c_O - c_N}{c_O + c_N}.$$

This proves claim B.

Now assume that

$$\lambda(\lambda+2) < \frac{c_O - c_N}{c_O + c_N}.$$

Then f' reaches a positive maximum at

$$\omega = \frac{\lambda+1}{2a(\lambda+2)} = \tilde{\omega}.$$

It equates zero at

$$\omega = \frac{\lambda + 1 \pm \sqrt{(\lambda+1)^2 - 4\lambda(\lambda+2)ab}}{2a(\lambda+2)}.$$

Calling these roots $\omega^- < \omega^+$, we clearly have $f' > 0$ inside $[\omega^-, \omega^+]$ and $f' < 0$ outside of that interval. Therefore, $f(\omega^-) < f(\omega^+)$. If there exists $\omega \in (\omega^-, \omega^+)$ such that $f(\omega) = -l/h$, then by continuity, there exists $\omega \in (0, \omega^-)$ such that $f(\omega) = -l/h$, since $f(0) = 0$ and $f(\omega^-) < -l/h$; similarly there exists $\omega > \omega^+$ such that $f(\omega) = -l/h$, since $\lim_{\omega\to\infty} f(\omega) = -\infty$ and $f(\omega^+) > -l/h$. Therefore for $l/h \in (-f(\omega^+), -f(\omega^-))$ we have constructed three solutions to (17), and there cannot be more given the shape of f. To prove that these solutions are type I equilibria, it remains to be shown that they are such that $\omega < \frac{c_O - c_N}{m}$.

Let $\omega_H > \omega^+$ be the largest solution to $f(\omega) = -l/h$. Over $[0, \omega_H]$, f has two local minima at ω^- and ω_H. Since $f(\omega_H) = -l/h > f(\omega^-)$, the latter is also a global minimum of f over $[0, \omega_H]$. We now prove that $f(\omega^-) > f(\frac{c_O - c_N}{m})$, from which it must follow that $\omega_H < \frac{c_O - c_N}{m}$.

To see this, recall that $f(\frac{c_O - c_N}{m}) = -\frac{c_O(c_O - c_N)^\lambda}{2m^{1+\lambda}} = -a^{-\lambda}b$. We will prove that

$$-f(\omega^-) < a^{-\lambda}b. \tag{20}$$

Substituting the condition $f'(\omega^-) = 0$ from (18) into the definition of f, we get that

$$-f(\omega^-) = \left(\omega^-\right)^\lambda \frac{2b - \omega^-}{\lambda + 2}.$$

Since $f(\omega^-) < 0$, we know that $\omega^- < 2b$. therefore

$$-f(\omega^-) < \left(\omega^-\right)^\lambda \frac{2b^-}{\lambda + 2}.$$

Let $k = \frac{\lambda+1-\sqrt{(\lambda+1)^2-4\lambda(\lambda+2)ab}}{2(\lambda+2)}$. Then $\omega^- = k/a$. Furthermore, $k < 1$, implying that

$$k^\lambda < 1 < \frac{\lambda + 2}{2}.$$

Therefore

$$\left(\omega^-\right)^\lambda \frac{2b^-}{\lambda + 2} = \frac{2bk^\lambda}{\lambda + 2} a^{-\lambda} < a^{-\lambda} b.$$

The preceding inequalities clearly imply that (20) holds. Consequently all the solutions to $f(\omega) = -l/h$ are such that $\omega < \frac{c_O - c_N}{m}$, implying they define a type I equilibrium. This proves claim C.

Now assume there does not exist $\omega \in (\omega^-, \omega^+)$ such that $f(\omega) = -l/h$. There are two possibilities. First, one may have $f(\omega^-) > -l/h$. In this case, there exists a unique solution ω^{++} to (17), and it is such that $\omega^{++} > \omega^+$. If $\omega^{++} < \frac{c_O - c_N}{m}$, this defines a type I equilibrium. Furthermore, from the above discussion, we also have that $-l/h = f(\omega^{++}) > f(\frac{c_O - c_N}{m}) = -\frac{c_O(c_O - c_N)^\lambda}{2m^{1+\lambda}}$, implying that a type II equilibrium does not exist. Therefore, the only equilibrium is of type I. If $\omega^{++} > \frac{c_O - c_N}{m}$, then there is no solution to (17) such that $\omega \leq \frac{c_O - c_N}{m}$, therefore there is no type I equilibrium, however, now $-l/h = f(\omega^{++}) < f(\frac{c_O - c_N}{m}) = -\frac{c_O(c_O - c_N)^\lambda}{2m^{1+\lambda}}$, so a type II equilibrium exists and it is the only equilibrium. Second, one may have $f(\omega^+) < -l/h$. In this case, there exists a unique solution ω^{--} to (17), and it is such that $\omega^{--} < \omega^-$.From (20), necessarily, $\omega^- < \frac{c_O - c_N}{m}$. Therefore, ω^{--} defines a type I equilibrium. Furthermore, again from (**??**), $-l/h = f(\omega^{--}) > f(\omega^-) >$

$-a^{-\lambda}b = -\frac{c_O(c_O-c_N)^\lambda}{2m^{1+\lambda}}$. Hence there does not exist a type II equilibrium. This proves claim D.

To prove claim E, observe that the highest ranked good consumed by any worker corresponds to the richest worker $j = 1$. It is therefore give by $i_{\max} = \frac{h\omega^{1+\lambda}+l}{c_O}$. This is increasing in ω and therefore higher in the high equilibrium.

Now we have that, in any equilibrium

$$\begin{aligned}\frac{i_{\max}}{i^*} &\propto \omega + \frac{l}{h}\omega^{-\lambda} \\ &= \frac{\omega^2 m}{c_O - c_N} + \frac{c_O}{2m}.\end{aligned}$$

The latter expression comes from using (7). It is increasing in ω, thus its value is higher, and therefore $i^*/i_{\max}$ lower, in the high equilibrium. This proves claim F.

QED

9.2 Proof of Proposition 2

A. Wages are given by (3). Clearly, $z_{j\omega} > 0$. Thus absolute wage inequality goes up. Also, $\partial(z_j/z)/\partial\omega \propto z_{j\omega}z - z_j z_\omega = (1+\lambda)hl\omega^\lambda > 0$. Therefore, relative wage inequality goes up as well.

B. Clear by revealed preferences and the fact that ω goes up.

D. True under the assumption that profits are mutualized within the capitalist class.

9.3 Proof of Proposition 3

Observe that profits are independent of i for $i \leq l/c_O$, since $y(i) = 1$ for those goods. They are equal to

$$\pi(i, \omega) = c_O - c_N - \omega m,$$

which must be positive since (7) holds in equilibrium. For those goods, $\partial\pi/\partial\omega = -m < 0$. These firms serve the whole markets and their profits are lower in the high equilibrium than in the low equilibrium, since their overhead skilled workers cost more in the former. For firms such that $i > l/c_O$, profits are equal to

$$\pi(i,\omega) = (c_O - c_N)\left[1 - (\frac{c_O i - l}{h})\omega^{-\lambda-1}\right] - \omega m.$$

Clearly, $\partial\pi/\partial i < 0$, $\partial^2\pi/\partial i\partial\omega > 0$ and $\pi(i^*,\omega) = 0$ for the equilibrium values of i^* and ω. Denote by L (resp. H) the equilibrium values in the low (resp. high) equilibrium. We know that $\pi(l/c_0, \omega_H) < \pi(l/c_0, \omega_L)$. Furthermore, since $i_L^* < i_H^*$, $\pi(i_L^*, \omega_L) = 0 < \pi(i_L^*, \omega_H)$. By continuity and from the fact that $\pi_{12} > 0$, there exists a critical $\tilde{\imath} \in (l/c_O, i_L^*)$ such that firm i has higher (resp. lower) profits in the high equilibrium than in the low equilibrium if $\tilde{\imath} < i \leq i_H^*$ (resp $i < \tilde{\imath}$).

QED

9.4 Proof of Proposition 4

A. Both (6) and (7) define i^* as a function of ω, in such a way that i^* falls as m goes up. Furthermore, (6) defines an upward sloping relationship in the (ω, i^*) plane and (7) cuts it from below in any stable equilibrium. It is then obvious from Figure 1 that a higher m reduces i^*.

B. Going back to the proof of Proposition 1, observe that $\partial f/\partial\omega < 0$ around any stable equilibrium. Also

$$\frac{\partial f}{\partial m} = -\frac{1}{c_O - c_N}\omega^{2+\lambda} + \frac{c_O}{2m^2}\omega^{\lambda}.$$

This is positive iff $\omega < \omega_c$. Since $\frac{d\omega}{dm} = -\frac{\partial f/\partial m}{\partial f/\partial\omega}$, this completes the proof.

C. Using the notations of the proof of Prop. 1, we have that

$$\omega_c = \sqrt{b/a}.$$

We can check algebraically that

$$\omega^+ = \frac{\lambda + 1 + \sqrt{(\lambda+1)^2 - 4\lambda(\lambda+2)ab}}{2a(\lambda+2)} < \sqrt{b/a}.$$

This is equivalent to

$$\sqrt{(\lambda+1)^2 - 4\lambda(\lambda+2)ab} < 2(\lambda+2)\sqrt{ab} - (\lambda+1)\,,$$

which is >0 since

$$ab = \frac{c_O}{2(c_O - c_N)} > 1/2.$$

Hence taking squares on both sides, this is equivalent to

$$-4\lambda(\lambda+2)ab < 4(\lambda+2)^2 ab - 4(\lambda+1)(\lambda+2)\sqrt{ab},$$

or equivalently

$$1 > 2\sqrt{ab},$$

which is always true.

The rest follows from the observation that $\omega_L < \omega^+ < \omega_c$.

D. The low equilibrium ceases to exist as long as $f(\omega^-) > -l/h$. Note that

$$\frac{df(\omega^-)}{dm} = \frac{\partial f}{\partial m} + f'(\omega^-)\frac{\partial \omega^-}{\partial m} = \frac{\partial f}{\partial m} > 0,$$

since $\omega^- < \omega^+ < \omega_c$. Therefore, $f(\omega^-)$ raises monotonically with m. Observe that since a is proportional to m, and ab independent of m, $\lim_{m\to+\infty} \omega^{--} = 0$. Therefore

$$\lim_{m\to+\infty} f(\omega^-) = \lim_{m\to+\infty} -\frac{m}{c_O - c_N}\omega^{2+\lambda} \propto -m^{-1-\lambda} \to 0.$$

Therefore $f(\omega^-)$ converges monotonically to zero, implying that the low equilibrium no longer exists for m higher than a critical threshold.

E. The high equilibrium ceases to exist as long as $f(\omega^+) < -l/h$. Since $\omega^+ = \Omega/m = \omega^+(m)$, with Ω independent of m, clearly from (17) we have that for any m_0,

$$f(\omega^+(m), m) = f(\omega^+(m_0), m_0) \left(\frac{m}{m_0}\right)^{-1-\lambda},$$

where the dependence of f on m has been made explicit in the notation.

Furthermore, since at ω^+ $df/dm = \frac{\partial f}{\partial m} + f'(\omega^+)\frac{\partial \omega^-}{\partial m} = \frac{\partial f}{\partial m} > 0$, given that $\omega^+ < \omega_c$, the preceding expression must be increasing with m, implying that $f(\omega^+) < 0$, which can also be checked directly.[8] It follows that $f(\omega^+)$ is monotonically increasing with m and that $\lim_{m\to 0} f(\omega^+) = -\infty$, which proves the point.

9.5 Proof of Proposition 5

A fall in c_N unambiguous shifts the $f()$ function up. Since $\partial f/\partial\omega < 0$ locally, ω goes up. Then clearly i^* goes up by (6). If $f()$ shifts up by enough, clearly,

[8] Indeed, using (18) and noting that $f'(\omega^+) = 0$, we can reexpress $f(\omega^+)$ as

$$f(\omega^+) = \frac{(\omega^+)^{1+\lambda}}{\lambda+2} - 2\frac{\lambda+1}{\lambda+2}b\omega^\lambda.$$

This is negative iff

$$\omega^+ < 2(\lambda+1)b.$$

To check that this holds, note that this is equivalent to

$$\lambda + 1 + \sqrt{(\lambda+1)^2 - 4\lambda(\lambda+2)ab} < 4ab(\lambda+1)(\lambda+2),$$

or equivalently

$$(\lambda+1)^2 - 4\lambda(\lambda+2)ab < (\lambda+1)^2\left[1 + 16a^2b^2(\lambda+2)^2 - 8ab(\lambda+2)\right].$$

Rearranging, this is equivalent to

$$-\lambda < 2(\lambda+1)^2\left[2ab(\lambda+2) - 1\right],$$

which always holds since $ab > 1/2$.

the low equilibrium ceases to exist.

A fall in γ raises the value of h by virtue of (2). As a result the RHS of (17) goes up in algebraic terms, implying that ω falls since $f' < 0$ locally. Next, from (6) we have that

$$\frac{di^*}{i^*} = \frac{dh}{h} + \lambda \frac{d\omega}{\omega}.$$

From (17),

$$d\omega = \frac{1}{f'(\omega)} \frac{l}{h^2} dh < 0.$$

Therefore, since $dh > 0$,

$$\begin{aligned} di^* &\gtreqless 0 \Longleftrightarrow 1 + \frac{\lambda l}{\omega h f'(\omega)} \gtreqless 0 \\ &\Longleftrightarrow \omega f'(\omega) \lesseqgtr \lambda f(\omega). \end{aligned}$$

Observe that

$$\begin{aligned} \omega f'(\omega) &= (\lambda+1)\omega^{1+\lambda} - (\lambda+2)a\omega^{2+\lambda} - \lambda b \omega^{\lambda} \\ &= \lambda f(\omega) + \omega^{\lambda+1}(1 - 2a\omega). \end{aligned}$$

Therefore,

$$di^* \gtreqless 0 \Longleftrightarrow \omega \lesseqgtr \frac{1}{2a} = \frac{c_O - c_N}{2m}.$$

Clearly, as the RHS of (17) goes up, the high equilibrium is less likely to exist. Also, as $\omega_L < \omega^- < \tilde{\omega} < \frac{1}{2a}$, clearly $di^* > 0$ at the low equilibrium.

Finally, an increase in l is clearly equivalent to a fall in h of the same relative magnitude, i.e. has the same effect as an increase in γ.

9.6 Proof of Proposition 6

A. Let N_N the total units of industrial goods consumed and N_O the total number of traditional goods. Let S be the total supply of skilled labor. Then the number of distinct industrial goods is

$$n_N = \frac{S}{m}$$

Let us index those goods by $u \in [0, n_N]$ and denote by $y(u) \in [0, 1]$ the amount of good u produced, also equal to the fraction of consumers who buy it. Then

$$N_N = \int_0^{n_N} y(u)du$$

Employment of unskilled labor in industrial goods is

$$l_N = N_N c_N$$

Therefore

$$N_O = \frac{l - N_N c_N}{c_O}.$$

Since $N = l/c_O + N_N(1 - c_N/c_O)$, in an efficient allocation the distribution $y(u)$ solves the program

$$\begin{aligned} &\min_{\{y(u)\}} S \\ s.t.\ N_N &= \int_0^{S/m} y(u)du, \\ 0 &\leq y(u) \leq 1. \end{aligned}$$

Otherwise, one could get the same N while reducing S, which would raise total welfare W since by allocating the fall in S proportionally among workers, the disutility of labor would clearly fall.

Clearly then one must have $y(u) = 1$ except on a set of measure zero. This proves (i)

(ii) is straightforward from the structure of preferences

To prove (iii), note that one must have

$$N_N = S/m = n_N,$$

and therefore

$$N = \frac{l + (c_O - c_N)S/m}{c_O}.$$

The allocation of skilled labor which maximizes W solves

$$\max_{\{s(j)\}} \frac{c_O - c_N}{mc_O} \int_0^1 s(j)dj - \int_0^1 \gamma j^{-\frac{1}{\lambda}} s(j)^{\frac{1+\lambda}{\lambda}} dj. \tag{21}$$

The FOC is

$$\frac{c_O - c_N}{mc_O} = \frac{1+\lambda}{\lambda} \gamma j^{-\frac{1}{\lambda}} s(j)^{\frac{1}{\lambda}},$$

or equivalently

$$s(j) = hj \left(\frac{c_O - c_N}{m} \right)^{\lambda}.$$

This proves (iii).

Conversely, by concavity of the maximization program defined by (21), any allocation which satisfies (ii) and (iii) maximizes social welfare in the space of allocations such that industrial goods have a full market share. Since those allocations dominate the other ones, one cannot find another allocation with a higher value of W.

B. Since $i^* < l/c_O$, a type II equilibrium satisfies (i). Since $\omega = \frac{c_O - c_N}{m}$ in a type II equilibrium, by (1) it also satisfies (iii). Condition (ii) is trivially satisfied from the choice of consumers over which goods to pruchase. Consequently, a type II equilibrium is efficient in that the corresponding allocation maximizes W.

9.7 Proof of Proposition 7

Compute the following integrals

$$\begin{aligned} N_w &= \int_0^1 n(j,\omega)dj = \int_0^1 \frac{hj\omega^{1+\lambda} + l}{c_O} dj \\ &= \frac{l}{c_O} + \frac{h}{c_O} \frac{\omega^{1+\lambda}}{2}. \end{aligned}$$

$$D = \int_0^1 \gamma j^{-\frac{1}{\lambda}} s(j,\omega)^{\frac{1+\lambda}{\lambda}} dj = \frac{\gamma \omega^{1+\lambda} h^{\frac{1+\lambda}{\lambda}}}{2}.$$

Using (2) this is also equal to

$$D = \frac{h}{c_O}\frac{\omega^{1+\lambda}}{2}\frac{\lambda}{1+\lambda}.$$

Therefore

$$N_w - D = \frac{l}{c_O} + \frac{h}{c_O}\frac{\omega^{1+\lambda}}{2(1+\lambda)}.$$

Using (6) this is also equal to, in equilibrium

$$N_w - D = \frac{l}{c_O} + \frac{\omega m}{c_O}\frac{i^*}{(1+\lambda)} \tag{22}$$

Note that

$$\Pi = (c_O - c_N - \omega m)\, i^* - \frac{c_O - c_N}{2}(i^* - l/c_O)\left[\frac{c_O}{h}\omega^{-\lambda-1}(i^* - l/c_O)\right]$$

Using (7) this can be rewritten as

$$\begin{aligned}\Pi &= (c_O - c_N - \omega m)\, i^* - \frac{c_O - c_N - \omega m}{2}(i^* - l/c_O)\\ &= \frac{c_O - c_N - \omega m}{2}(i^* + l/c_O).\end{aligned}$$

Therefore

$$N_c = \frac{\Pi}{c_O} = \frac{c_O - c_N - \omega m}{2c_O}(i^* + l/c_O) \tag{23}$$

and from (22) and (23)

$$\begin{aligned}W &= N_w - D + N_c\\ &= \frac{l}{c_O}\frac{3c_O - c_N}{2c_O} + \frac{\omega m}{2c_O}\left(\frac{1-\lambda}{1+\lambda}i^* - \frac{l}{c_O}\right) + \frac{i^*}{c_O}\frac{c_O - c_N}{2}\end{aligned}$$

We now prove that this expression, for $i^* = h\frac{\omega^\lambda}{2m}$, is an increasing function of ω over $[\omega_L, \omega_H]$. From there it follows that W is larger in the high equilibrium than in the low equilibrium. Differentiating the RHS, we get

$$\begin{aligned}
\frac{dW}{d\omega} &= \frac{di^*}{d\omega}\left(\frac{c_O - c_N}{2c_O} + \frac{1-\lambda}{1+\lambda}\frac{\omega m}{2c_O}\right) + \frac{m}{2c_O}\left(\frac{1-\lambda}{1+\lambda}i^* - \frac{l}{c_O}\right) \\
&= \frac{\lambda i^*}{\omega}\left(\frac{c_O - c_N}{2c_O} + \frac{1-\lambda}{1+\lambda}\frac{\omega m}{2c_O}\right) + \frac{m}{2c_O}\left(\frac{1-\lambda}{1+\lambda}i^* - \frac{l}{c_O}\right) \\
&= \frac{\lambda i^*}{\omega}\frac{c_O - c_N}{2c_O} + \frac{m}{2c_O}(1-\lambda)i^* - \frac{m}{2c_O}\frac{l}{c_O} \\
&= \frac{m}{2c_O}\left[\lambda i^* \frac{c_O - c_N}{\omega m} + (1-\lambda)i^* - \frac{l}{c_O}\right].
\end{aligned}$$

Observe that $\omega_H < \frac{c_O - c_N}{m}$ since all equilibria are of type I when there are multiple equilibria. Therefore, $\frac{c_O - c_N}{\omega m} > 1$ over $[\omega_L, \omega_H]$. Hence from the preceding expression

$$\begin{aligned}
\frac{dW}{d\omega} &> \frac{m}{2c_O}\left[i^* - \frac{l}{c_O}\right] \\
&= .\frac{m}{2c_O}\left[h\frac{\omega^\lambda}{2m} - \frac{l}{c_O}\right].
\end{aligned}$$

The condition $i^* = h\frac{\omega^\lambda}{2m}$ is an equilibrium condition. Since the L equilibrium is of type I, it is such that $i^* - \frac{l}{c_O} > 0$. Therefore

$$h\frac{\omega_L^\lambda}{2m} > \frac{l}{c_O},$$

implying that, since the expression $h\frac{\omega^\lambda}{2m}$ is increasing in ω, $h\frac{\omega^\lambda}{2m} - \frac{l}{c_O} > 0$ over $[\omega_L, \omega_H]$.

Therefore, $dW/d\omega > 0$ over $[\omega_L, \omega_H]$, implying that W is higher at the high equilibrium than at the low equilibrium.

9.8 Proof of Lemma 1

First observe that if

$$h(j) = kj + \bar{k}, \tag{24}$$

then $z(j) = \omega(kj + \bar{k}) + l$. From (14) we then have that

$$h'(j) = \frac{\theta}{1+\theta}\left(\frac{\beta}{c_O}\omega kj + \frac{\beta}{c_O}l + \frac{\beta}{c_O}\omega\bar{k} + \alpha j\right).$$

It follows that if $h(j)$ is affine in j, so is $h'(j)$, and that the laws of motion for k and $\bar{k}$ are given by

$$k' = \frac{\theta}{1+\theta}\left(\frac{\beta}{c_O}\omega k + \alpha\right) \tag{25}$$

$$\bar{k}' = \frac{\frac{\beta}{c_O}\theta}{1+\theta}\left(l + \frac{\beta}{c_O}\omega\bar{k}\right). \tag{26}$$

Next, at date $t_0 + 1$, we clearly have that

$$\begin{aligned} h(j) &= \frac{\theta}{1+\theta}\left(\frac{\beta}{c_O}l + \alpha j\right) \\ &= \frac{\theta\alpha}{1+\theta}(\mu + j). \end{aligned}$$

This is consistent with Lemma 1 for $k = \frac{\theta\alpha}{1+\theta}$. Also, it is affine in j, so that by induction $h(j)$ will be affine in j at all subsequent dates. $<$

By induction, then, we conclude that at all dates $h(j)$ has a functional form given by (24).

Next, assume that $\bar{k} = \mu k$. From (26) we have that

$$\begin{aligned} \bar{k}' &= \frac{\frac{\beta}{c_O}\theta}{1+\theta}\left(l + \frac{\beta}{c_O}\omega\mu k\right) \\ &= \frac{\theta}{1+\theta}\frac{\frac{\beta}{c_O}l}{\alpha}\left(\alpha + \frac{\beta}{c_O}\omega k\right) = \mu k'. \end{aligned}$$

By induction, then the property that $h(j) = k(j+\mu)$ holds at all dates after t_0. This concludes the proof of the Lemma.

QED

9.9 Proof of Proposition 8

We already know that we can construct a type II equilibrium if and only if $ml > k\psi_2$, while a low equilibrium exists iff $ml < k\psi_1$. Let us study the possibility of an equilibrium with an interior value of ω. For this to exist, it must be that

$$f(\omega) = a\omega^2 - b\omega + c = 0, \tag{27}$$

where $a = \frac{mk}{c_O - c_N}$, $b = k(1+\mu)$, $c = c_O k\left(\mu + 1/2\right)/m - l$.

Assume $c < 0$. Then (27) has a unique positive root $\bar{\omega}$. f is negative for $0 < \omega < \bar{\omega}$ and positive for $\omega > \bar{\omega}$. Therefore, $\bar{\omega} < \frac{c_O - c_N}{m}$ iff $f(\frac{c_O - c_N}{m}) > 0$.

We have that

$$f(\frac{c_O - c_N}{m}) = k\frac{c_O - c_N}{m} - k(1+\mu)\frac{c_O - c_N}{m} + c < 0.$$

Therefore, if $c < 0$ there is no type I high equilibrium. The condition $c < 0$ is equivalent to

$$ml > k\psi_3.$$

Consequently, in this zone, there is a unique type II equilibrium.

Now consider the case such that $c > 0$.Then, (27) has a solution iff

$$k^2(1+\mu)^2 + \frac{4mkl}{c_O - c_N} > 4\frac{k^2}{c_O - c_N}c_0\left(\mu + 1/2\right) \tag{28}$$

Both roots are positive but the lowest one is unstable. The candidate equilibrium is the largest root, which must be smaller than $\frac{c_O - c_N}{m}$. This largest root is equal to

$$\omega^+ = \frac{k(1+\mu) + \sqrt{k^2(1+\mu)^2 + \frac{4mkl}{c_O - c_N} - 4\frac{k^2}{c_O - c_N}c_0\left(\mu + 1/2\right)}}{2mk}(c_O - c_N).$$

The condition $\omega^+ < \frac{c_O - c_N}{m}$ is equivalent to

$$\sqrt{k^2(1+\mu)^2 + \frac{4mkl}{c_O - c_N} - 4\frac{k^2}{c_O - c_N}c_0\left(\mu + 1/2\right)} < k(1-\mu).$$

Since both sides are positive, we can compare their square. Rearranging, this inequality is equivalent to

$$ml < kc_0 (\mu + 1/2) - k\mu(c_0 - c_N) = k\psi_2.$$

By continuity, then, $\omega^+ = \frac{c_O - c_N}{m}$ at $k = \frac{ml}{\psi_2}$.

This proves that if $k\psi_2 < ml < k\psi_3$, there are two equilibria: a type II equilibrium and a low equilibrium.

If $ml < k\psi_2$, then the type I high equilibrium exists if and only if (28) holds. Rearranging, this is equivalent to

$$\begin{aligned} ml &> k\left[-\frac{1}{4}(1+\mu)^2(c_O - c_N) + c_O(\mu + 1/2)\right] \\ &= \psi_1 k. \end{aligned}$$

We check that $\psi_1 < \psi_2$:

$$\begin{aligned} \psi_2 - \psi_1 &= c_O/2 + \mu c_N - \frac{c_O + c_N}{4} - \mu\left(\frac{c_N + c_O}{2}\right) + \frac{\mu^2}{4}(c_O - c_N) \\ &= \frac{c_O - c_N}{4}(1-\mu)^2 > 0. \end{aligned}$$

This proves that a type I high equilibrium exists iff $\psi_1 k < ml < \psi_2 k$, which completes the proof.

QED

9.10 Proof of Proposition 9

Let us construct a steady state of type II such that $\omega = \frac{c_O - c_N}{m}$. The corresponding value of k must solve for $k = \frac{\theta}{1+\theta}\left(\frac{\beta}{c_O}\omega k + \alpha\right)$. Therefore, necessarily

$$k = \frac{\theta \alpha ml}{(1+\theta)ml - \theta\alpha\mu(c_O - c_N)} = k_{II} \tag{29}$$

Assume the expression on the RHS is positive, which is a necessary condition. For this value of k, an equilibrium such that $\omega = \frac{c_O - c_N}{m}$ has to exist. From Proposition 8, this is equivalent to $k_{II} > ml/\psi_2$, that is

$$\begin{aligned} \frac{\theta\alpha ml}{(1+\theta)ml - \theta\alpha\mu(c_O - c_N)} &< \frac{ml}{c_O/2 + \mu c_N} \\ &\iff \theta\alpha(c_O/2 + \mu c_N) < (1+\theta)ml - \theta\alpha\mu(c_O - c_N) \\ &\iff m > m_0. \end{aligned}$$

Conversely, if $m > m_0$, the denominator of (29) is clearly positive. If the economy is such that $k = k_{II}$, by construction there exists a type II equilibrium such that $\omega = \frac{c_O - c_N}{m}$ and $k' = k = k_{II}$. This proves the existence of the steady state, and therefore claim (i).

Let us construct a low steady state such that $\omega = 0$. Clearly, it must then be that

$$k = \frac{\theta}{1+\theta}\alpha = k_L.$$

From Proposition 8, an equilibrium such that $\omega = 0$ exists provided $k_L > \frac{ml}{\psi_3}$, i.e.

$$\frac{\theta}{1+\theta}\alpha > \frac{ml}{c_O(\mu + 1/2)},$$

which is trivially equivalent to $m < m_0$. Conversely if $m < m_0$ and $k = k_L$, then by construction there exists a low equilibrium and $k' = k_L = k$ in this low equilibrium. This proves claim (ii).

Turning now to claim (iii), let us construct a type I high equilibrium. Such an equilibrium must satisfy a number of conditions. First, the equilibrium ω must solve for (27). Second, the resulting value of ω must be such that $0 < \omega < \frac{c_O - c_N}{m}$. Third, k must be stationary, i.e. such that

$$k = \frac{\theta\alpha l}{(1+\theta)l - \theta\alpha\mu\omega}.$$

Substituting this expression into the values of a, b and c in (27) and rearranging, we get that ω must be a solution to

$$f^*(\omega) = a^*\omega^2 - \alpha\omega + c^* = 0, \tag{30}$$

where $a^* = \frac{\alpha m}{c_O - c_N}$ and $c^* = \frac{\alpha c_O}{m}(\mu + 1/2) - \frac{1+\theta}{\theta} l$.

For (30) to have a solution, it is necessary that

$$\Delta = \alpha^2 - 4a^*c^* > 0,$$

or equivalently, rearranging,

$$ml > \frac{\theta\alpha}{1+\theta}\left(c_O\mu + \frac{c_O + c_N}{4}\right) = lm_1 - \frac{\theta\alpha}{1+\theta}(c_O - c_N)\mu^2. \tag{31}$$

The largest root is then

$$\omega^+ = \frac{\alpha + \sqrt{\Delta}}{2a^*}$$

This expression does not exceed $\frac{c_O - c_N}{m}$ provided $\Delta < \alpha^2$, which is equivalent to $c^* > 0$, i.e.

$$m < m_0.$$

We note that the smallest root ω^- is such that $\omega^- < \frac{\alpha}{2a^*} = \frac{c_O - c_N}{2m} < (1 + \mu)\frac{c_O - c_N}{2m}$. Since the roots of (27), for any k, lie on both sides of the quantity $(1+\mu)\frac{c_O - c_N}{2m}$, ω^- can never be the largest root of $f()$ and therefore does not qualify as a stable equilibrium.

Conversely, assume that

$$m_1 < m < m_0.$$

Then, clearly, (31) holds, and the highest root of (30) ω^+ is lower than $\frac{c_O - c_N}{m}$.

Assume that

$$k = k_I = \frac{\theta\alpha l}{(1+\theta)l - \theta\alpha\mu\omega^+}.$$

It is easy to check that $k_I > 0$.[9] Then by construction ω^+ is a solution to (27) with $k = k_I$, since that is equivalent to being solution to (30). This implies, in particular, that those solutions exist. Since $\omega^+ < \frac{c_O - c_N}{m}$, from the proof of Proposition 8 we deduct that if $k = k_I$ both roots of (27) are positive and lower than $\frac{c_O - c_N}{m}$. To complete the proof, we just need to show that ω^+ is the highest root of (27), implying the equilibrium is stable. This is equivalent to $f'(\omega^+) > 0$, or equivalently

$$\omega^+ > \frac{(c_O - c_N)(1+\mu)}{2m}.$$

Substituting the expression for ω^+, and rearranging, we get that this is equivalent to

$$\Delta > \alpha^2\mu^2.$$

Rearranging, this is equivalent to

$$m > m_1,$$

which is true by assumption.

Hence ω^+ defines a stable equilibrium, and by construction $k' = k_I$, which proves that we have constructed a steady state in the required regime.

[9] Proof. This is true iff

$$\begin{aligned} \omega^+ &< \frac{1+\theta}{\theta}\frac{l}{\alpha\mu} \\ \iff & \frac{\alpha+\sqrt{\Delta}}{2} < \frac{1+\theta}{\theta}\frac{l}{\mu}\frac{m}{c_O - c_N}. \end{aligned}$$

Since $m > m_1$, a sufficient condition for this to hold is

$$\begin{aligned} \frac{\alpha+\sqrt{\Delta}}{2} &< \frac{1+\theta}{\theta}\frac{l}{\mu}\frac{m_1}{c_O - c_N} \\ &= \frac{\alpha}{\mu}\frac{c_O\mu + \frac{c_O + c_N}{4}}{c_O - c_N}, \\ \iff & \sqrt{\Delta} < \frac{3\alpha}{2}\frac{c_O + c_N}{c_O - c_N}, \end{aligned}$$

which is trivially true since $\Delta < \alpha$.

QED

9.11 Proof of Proposition 10

For a surge to be possible, an equilibrium with $\omega > 0$ must exist for $k = k_L$. From Proposition 8, this is true if and only if

$$k_L = \frac{\theta}{1+\theta}\alpha < \frac{ml}{\psi_1}.$$

Using the formula for ψ_1, this is equivalent to $m > m_2$. Comparing the expression for m_2 and m_1, it is clear that $m_2 < m_1$.

For a collapse scenario, one must have $k > \frac{ml}{\psi_3}$. For $m < m_0$, k_L satisfies this inequality, since a low steady state exists. Furthermore, in a type I steady state, $k_I > k_L$ and $m < m_0$.Therefore type I steady states can always collapse. A type II steady state can collapse if and only if

$$k_{II} = \frac{\theta\alpha ml}{(1+\theta)ml - \theta\alpha\mu(c_O - c_N)} > \frac{ml}{\psi_3}.$$

Rearranging and using the formula for ψ_3, this is equivalent to $m < m_3$. Again, it is trivial to show that $m_3 > m_0$.

QED

9.12 Proof of Proposition 11

We show that, if the economy selects an equilibrium with $\omega > 0$, implying that $k < \frac{ml}{\psi_1}$, then $k' < \frac{ml}{\psi_1}$.

Assume first that $k < \frac{ml}{\psi_2}$, implying that $\omega = \frac{c_O - c_N}{m}$. Then

$$k' = \frac{\theta\alpha}{1+\theta}(1 + \frac{\mu k}{l}\frac{c_O - c_N}{m}).$$

This expression is increasing in k and therefore in this zone it reaches its maximum for $k = \frac{ml}{\psi_2}$. The corresponding k' is lower than $\frac{ml}{\psi_1}$ iff

$$m > \frac{\theta}{1+\theta}\frac{\alpha}{l}\psi_1\left(1+\mu\frac{c_O-c_N}{\psi_2}\right) = \tilde{m}_4,$$

which is true by assumption. Note that

$$\tilde{m}_4 < m_0 \Longleftrightarrow \psi_1 < \frac{c_0}{2} + \mu c_N = \psi_2,$$

which has already been shown to be true.

Also

$$\begin{aligned}
\tilde{m}_4 \quad &> \quad m_1 \Longleftrightarrow \psi_1\frac{c_O(\mu+1/2)}{c_O/2+\mu c_N} > c_O(\mu+1/2) - \frac{1-\mu^2}{4}(c_O-c_N) \\
&\Longleftrightarrow \quad c_O(\mu+1/2)(\psi_1 - c_O/2 - \mu c_N) > -\frac{1-\mu^2}{4}(c_O-c_N)(c_O/2+\mu c_N) \\
&\Longleftrightarrow \quad -(1-\mu)^2\frac{c_O-c_N}{4}c_O(\mu+1/2) > -\frac{1-\mu^2}{4}(c_O-c_N)(c_O/2+\mu c_N) \\
&\Longleftrightarrow \quad (1-\mu)c_O(\mu+1/2) < (1+\mu)(c_O/2+\mu c_N).
\end{aligned}$$

Since $(1-\mu)(\mu+1/2) < \frac{1+\mu}{2}$ and the RHS is increasing in c_N, this inequality is always true.

Now consider the dynamics for $\frac{ml}{\psi_2} < k < \frac{ml}{\psi_1}$. We have that

$$\frac{dk'}{dk} \propto \omega + k\frac{d\omega}{dk}.$$

From (27),

$$(2a\omega - b)\frac{d\omega}{dk} + \left(\frac{m}{c_O-c_N}\omega^2 - (1+\mu)\omega + \frac{c_O}{m}(\mu+1/2)\right) = 0.$$

Using (27) again,

$$\frac{d\omega}{dk} = -\frac{l}{k(2a\omega-b)} < 0.$$

The sign comes from the fact that ω is the largest root, i.e. $\omega > b/(2a)$. Hence

$$\begin{aligned}
\frac{dk'}{dk} \quad &\propto \quad 2a\omega^2 - b\omega - l \\
&= \quad k\omega(1+\mu) + l - \frac{c_O k}{m}(2\mu+1).
\end{aligned}$$

This expression is < 0 iff

$$(1+\mu)\omega + \frac{l}{k} < \frac{c_O}{m}(2\mu+1). \tag{32}$$

Since $\frac{d\omega}{dk} < 0$, the LHS falls with k. Therefore, it cannot exceed its value at $k = \frac{ml}{\psi_2}$for which $\omega = \frac{c_O - c_N}{m}$. This quantity is itself lower than the RHS iff

$$\begin{aligned}(1+\mu)(c_O - c_N) + \psi_2 \quad &< \quad c_O(2\mu+1) \\ &\iff \quad \sigma + \mu > \frac{1}{2},\end{aligned}$$

where $\sigma = c_N/c_O$.

Hence, if this inequality holds and if $m > m_4$, the property that $k' < \frac{ml}{\psi_1}$ holds.

Now assume that $\sigma + \mu < \frac{1}{2}$. Then $\frac{dk'}{dk} > 0$ at $k = \frac{ml}{\psi_2}$. Assume $k = \frac{ml}{\psi_1}$. Then $\omega = \frac{b}{2a} = \frac{(c_O - c_N)(1+\mu)}{2m}$. At this point (32) holds if and only if

$$\begin{aligned}(1+\mu)^2(c_O - c_N) + \psi_1 \quad &< \quad c_O(2\mu+1) \\ &\iff \quad \frac{c_O + c_N}{4} + \mu\left(\frac{c_N + c_O}{2}\right) - \frac{\mu^2}{4}(c_O - c_N) \\ &< \quad c_O(2\mu+1) - (1+\mu)^2(c_O - c_N) \\ &\iff \quad 0 < c_O(1 + 3\mu - \mu^2) + c_N(1+\mu)^2,\end{aligned}$$

which always holds since $\mu < 1$.

It follows that there is a unique $k_m \in (\frac{ml}{\psi_2}, \frac{ml}{\psi_1})$ such that $dk'/dk = 0$ at $k = k_m$, and this is the point where k' reaches its global maximum for $k < \frac{ml}{\psi_1}$. Solving for the joint system defined by (27) and equality in (32) we get that

$$k_m = \frac{ml}{c_O(2\mu+1) - (1+\mu)z},$$

where

$$z = \sqrt{c_O(c_O - c_N)\left(\mu + \frac{1}{2}\right)}$$

and the corresponding wage is

$$\omega_m = \frac{z}{m}.$$

The corresponding value of k' is

$$\begin{aligned} k'_m &= \frac{\theta\alpha}{1+\theta}(1 + \frac{\mu k_m}{l}\omega_m) \\ &= \frac{\theta\alpha}{1+\theta}(1 + \frac{\mu z}{c_O(2\mu+1) - (1+\mu)z}). \end{aligned}$$

The condition $k'_m < \frac{ml}{\psi_1}$ is equivalent to

$$m > \tilde{m}_5 = \frac{\theta}{1+\theta}\frac{\alpha}{l}\psi_1(1 + \frac{\mu z}{c_O(2\mu+1) - (1+\mu)z}).$$

We prove that $\tilde{m}_5 < m_0$, or equivalently

$$c_O(\mu + \frac{1}{2}) > \psi_1(1 + \frac{\mu z}{c_O(2\mu+1) - (1+\mu)z}.$$

This is equivalent to

$$\psi_1\left(c_O(2\mu+1) - z\right) < c_O(\mu + \frac{1}{2})\left[c_O(2\mu+1) - (1+\mu)z\right].$$

Using the expression for ψ_1 and rearranging, and letting $z' = \sqrt{(1-\sigma)(\mu+\frac{1}{2})}$, this is equivalent to

$$z'\left(\frac{1-\sigma}{4} + \mu(1-\sigma/2) + \frac{5-\sigma}{4}\mu^2\right) < \frac{1-\sigma}{4} + \mu(1-\sigma) + \frac{5}{4}\mu^2(1-\sigma) + \mu^3\frac{1-\sigma}{2} \tag{33}$$

First note that $z' < 1$ since $z' < 1-\sigma$ as by assuption $\sigma + \mu < 1/2$. Therefore, $z'\frac{1-\sigma}{4} < \frac{1-\sigma}{4}$

Second, for the same reason, $z'(1-\sigma/2) < z' < 1-\sigma$. Therefore $z'\mu(1-\sigma/2) < \mu(1-\sigma)$.

Third, let us show that

$$z'\frac{5-\sigma}{4} < \frac{5}{4}(1-\sigma) + \mu\frac{1-\sigma}{2}.$$

This is equivalent to

$$z'(5-\sigma) < (1-\sigma)(5+2\mu),$$

which is true since $z' < 1-\sigma$ and $5-\sigma < 5+2\mu$.

Putting all this together, it follows that (33) always holds. The required property then holds for $m > \max(\tilde{m}_4, \tilde{m}_5) \equiv m_4$.

QED

9.13 Proof of Proposition 12

To prove (i), just note that since $\omega = 0$ at the time of collapse, $k' = k_L$. Since a low steady state exists, the economy may remain there forever, making the collapse permanent.

To prove (ii), recall that if $m > m_0$, then $k_L < \frac{ml}{\psi_3}$. Consequently, from Proposition 8, the only equilibrium at the following date is such that $\omega = \frac{c_O - c_N}{m}$. The dynamics as long as a type II equilibrium exists are given by

$$k' = \frac{\theta\alpha}{1+\theta}(1 + \mu\frac{c_O - c_N}{ml}k).$$

Since $m > m_0 > \frac{\theta\alpha}{1+\theta}\mu\frac{c_O - c_N}{l}$, this mapping is a contraction and the sequence converges monotonically from below to k_{II}. As $k_{II} < \frac{ml}{\psi_2}$, the type II equilibrium exists for all those values of k, proving that this sequence defines a valid equilibrium trajectory for the economy. Since, from the proof of Proposition 10, $k_{II} > \frac{ml}{\psi_3}$, by continuity there exists a date T such that $k > \frac{ml}{\psi_3}$ for $t > T$. Therefore, a low equilibrium exists for $t > T$,implying the economy may collapse again.

QED

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Figure 1: multiple equilibria

Equilibrium *i** as a function of ω according to each equilibrium condition