DISCUSSION PAPER SERIES

IZA DP No. 3298

Identification of New Keynesian Phillips Curves from a Global Perspective

Stephane Dees
M. Hashem Pesaran
L. Vanessa Smith
Ron P. Smith

January 2008

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

Identification of New Keynesian Phillips Curves from a Global Perspective

Stephane Dees
European Central Bank

M. Hashem Pesaran
CIMF, University of Cambridge, USC and IZA

L. Vanessa Smith
CFAP, University of Cambridge

Ron P. Smith
Birkbeck College, London

Discussion Paper No. 3298
January 2008

IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

IZA Discussion Paper No. 3298
January 2008

ABSTRACT

Identification of New Keynesian Phillips Curves from a Global Perspective*

New Keynesian Phillips Curves (NKPC) have been extensively used in the analysis of monetary policy, but yet there are a number of issues of concern about how they are estimated and then related to the underlying macroeconomic theory. The first is whether such equations are identified. To check identification requires specifying the process for the forcing variables (typically the output gap) and solving the model for inflation in terms of the observables. In practice, the equation is estimated by GMM, relying on statistical criteria to choose instruments. This may result in failure of identification or weak instruments. Secondly, the NKPC is usually derived as a part of a DSGE model, solved by log-linearising around a steady state and the variables are then measured in terms of deviations from the steady state. In practice the steady states, e.g. for output, are usually estimated by some statistical procedure such as the Hodrick-Prescott (HP) filter that might not be appropriate. Thirdly, there are arguments that other variables, e.g. interest rates, foreign inflation and foreign output gaps should enter the Phillips curve. This paper examines these three issues and argues that all three benefit from a global perspective. The global perspective provides additional instruments to alleviate the weak instrument problem, yields a theoretically consistent measure of the steady state and provides a natural route for foreign inflation or output gap to enter the NKPC.

JEL Classification: C32, E17, F37, F42

Keywords: Global VAR (GVAR), identification, New Keynesian Phillips Curve, trend-cycle decomposition

Corresponding author:

M. Hashem Pesaran
Faculty of Economics
University of Cambridge
Sidgwick Avenue
Cambridge, CB3 9DD
United Kingdom
E-mail: mhp1@econ.cam.ac.uk

* For Stephane Dees, the views expressed in this paper do not necessarily reflect the views of the European Central Bank or the Eurosystem. The other authors are grateful to the European Central Bank for financial support. We are grateful to Alex Al-Haschimi and Alex Chudik for comments.

1 Introduction

New Keynesian Phillips Curves (NKPC) have been widely used in the macroeconomic literature. Yet their empirical implementation raises a number of issues that continue to be of important concern. The first is whether such equations are identified. In order to determine whether the necessary and sufficient conditions for identification are satisfied one must specify the process determining the forcing variables and solve the full rational expectations model *jointly* in inflation and the forcing variables. In practice, it is common to estimate these equations by instrumental variables (IV) or the generalised methods of moments (GMM), and use statistical criteria to choose instruments from lagged observations. Since a valid instrument, besides being in the agent's information set, must also be sufficiently correlated with the target variables, an undiscriminating use of lagged variables might not help identification, or could do so only in a marginal sense, thus leading to the so called weak instrument problem. Secondly, the NKPC is usually derived from a dynamic stochastic general equilibrium (DSGE) model, which is solved by log-linearising around a steady state. Such a log-linearisation procedure is appropriate if the steady state exists and the deviations are taken around the correct steady state. In practice, the steady states are usually either assumed constant, e.g. for inflation, or estimated by some statistical procedure such as the Hodrick-Prescott (HP) filter, e.g. for output. How to identify and estimate the steady states is clearly an important consideration in the empirical analysis of NKPC. Inflation may not be stationary. For example, it could be a unit root process (at least in the case of some economies), or its mean might have been subject to structural breaks as some have argued in the case of the industrialised economies over the past two decades. It is also not clear that the HP filter is appropriate for the identification of the steady state of output across many different countries in the global economy. Thirdly, it could be argued that variables other than the output gap should enter into the Phillips curve. For instance, given the need to finance marginal costs, interest rates can enter through the cost channel, while domestic inflation may not be fully insulated from foreign inflation because of the nature of domestic monetary policy and because of incomplete pass through of exchange rate changes. Further, if inflation is I(1) or even approximately so, as it may be, and the output gap is I(0), as it certainly is, then the Phillips curve does not balance in the sense that it would imply explaining a highly persistent variable by a variable which exhibits a rather low level of persistence. Including foreign inflation, another potentially I(1) variable, restores the balance. This paper examines the identification of the NKPC, the construction of a theoretically consistent steady state and the role of variables other than the output gap in explaining inflation. The analysis of all three issues benefits from a global perspective. The global perspective provides additional instruments that can be interpreted in terms of a factor IV model, to reduce the weak instrument problem; it provides a theoretically consistent measure of the steady state and it establishes a natural route for foreign inflation or output to enter the NKPC.

Section 2 provides a short discussion of the evolution of the Phillips curve. Section 3 discusses the identification of the NKPC and shows how the use of global factors as instruments may reduce the weak instrument problem. Section 4 provides a brief overview of the cointegrating global vector autoregression, GVAR, of Dees, di Mauro, Pesaran and Smith, DdPS, (2007) which has the same form as the solution to a global DSGE model and therefore provides the global framework for our analysis.[1] Section 5 considers how the GVAR can be used to obtain theoretically consistent measures of the steady states. The natural definition of a steady state is the values of the variables to which the system would tend in the absence of further shocks, which is given by long horizon forecasts. The solution of the system thus involves both one period ahead rational expectations, which explain the deviations from the steady states, and the long horizon rational expectations which represent the steady states themselves.[2] If the variables in the system are I(1), this definition of the steady state corresponds to the permanent or trend component obtained from a multivariate Beveridge-Nelson (1981) decomposition (BN decomposition) and the deviation from steady state corresponds to the cyclical or transitory component.[3] We calculate such steady states from the GVAR which thus reflect all the open economy influences on the steady state and any long-run theoretical relations embodied in the cointegrating relations. We also use the BN decomposition to examine a range of questions about the system including the correlation between changes in trend and cycle, the smoothness of the permanent components and the importance of global cycles.

Section 6 reports estimates of the NKPC for the 26 countries under a variety of assumptions. The NKPC is estimated by instrumental variables, which allows consistent estimation of the coefficient of the output gap even if it is measured with errors, which it certainly is. We consider the choice of instruments. We compare the NKPC estimates obtained with the BN-GVAR measure of the output gap with those obtained using the HP measure. We also examine the effect of including foreign inflation, foreign output gaps and domestic interest rates. There is substantial heterogeneity across countries. This poses the danger that one may draw general conclusions from country specific results and the danger of data mining in trying to get models to fit on particular countries. To avoid these dangers we calculate mean group instrumental variable, MGIV, estimates over groups of countries. The non-parametric standard errors of the MGIV estimator also have the advantage that they are robust to serial correlation and heteroskedasticity in the equations for the individual countries. Section 7 provides some concluding comments.

2 The Evolution of the Phillips Curve

Our focus is on three issues associated with the New Keynesian Phillips Curve - identification, measurement of steady states and global influences - but we briefly comment on how these relate to a range of issues that are raised in the literature to motivate and interpret the Phillips curve. In Phillips (1958) the curve was a labour market relationship between wage inflation and unem-

[1] Data and code for the model are available on the Journal of Applied Econometrics data archive (http://qed.econ.queensu.ca/jae/).

[2] Lee and Nelson (2007) discuss expectations horizons from a different perspective.

[3] Although the permanent component may not look like a conventional trend and the cyclical component may not show periodic behaviour, we use the terms trend and cycle because they are so well established.

ployment; there was considerable emphasis on the non-linearity of the relationship; there was a distinction between the long-run pattern, that Phillips believed was measured by the curve, and the short run cyclical loops around the curve;[4] and there was some discussion of the impact of cost shocks coming from import prices. Expected inflation was treated as a given constant, which, empirically, seemed reasonable for the pre World War I gold standard data that he used. The curve was also treated as identifying a demand relationship: excess demand in the labour market pushing up wages. With the assumption that supply was relatively inelastic given by labour force, it was assumed that most of the variations in unemployment would reflect demand and that these would be uncorrelated with the exogenous cost shocks, e.g. from import prices. This allowed estimation by OLS, though Phillips himself used graphical methods.

The issue of identification surfaced shortly after Phelps (1967) and Friedman (1968) emphasised the importance of inflation expectations and a natural rate of unemployment, giving a vertical long-run Phillips curve. Unemployment or the output gap only influenced the difference between actual and expected inflation. Even in equations where the expectation of current inflation was a function of past data, the issue of testing whether the coefficient of expected inflation was unity raised identification issues. One can only test that the coefficient of expected inflation is unity in conjunction with particular identifying assumptions about how expectations are formed. Rational expectations may provide over-identifying restrictions and, if so, these can be tested. There are further issues when the expectation of future inflation appears, which are discussed below. Lucas (1972) changed the interpretation of the relationship to a supply curve, the amount of output produced depended on the difference between actual and expected prices (inflation). In econometric terms, the issue is whether inflation or output is regarded as the independent variable and the extent of the correlation of either or both with the error. There was a considerable literature explaining output by the money supply surprises that drove unexpected inflation. This literature terminated abruptly when it was appreciated that since output was a very persistent series, probably I(1), it could not be explained by surprises, which by construction were white noise. Pesaran and Smith (1995a) discuss this issue in detail. King and Watson (1994) discuss the effect of stochastic trends on estimation of the Phillips curve.

Recently, the NKPC has been derived from the solution of a DSGE model, which is obtained by log linearising around a steady state. Thus all variables are expressed as deviations from their steady states. The steady states are usually assumed either to be constants or, for trended variables like output, the steady state is measured by a statistical procedure such as the HP filter. The standard procedure thus does not use any economic information about the steady state and this is likely to produce misspecification of the estimated equations. Most statistical filters, like the HP filter, are two sided, using information about future values of the variables in calculation of the steady state values, rather than using the information available to agents at the time.[5] This not only raises

[4] Although the Phillips Curve is now usually treated as a short-run relation, there is still controversy about whether it exists as a long-run relation, e.g. Schreiber and Wolters (2007).

[5] Some like Beyer *et al.* (2007) recognise this problem and use one-sided HP filters.

problems for forecasting with models using HP filtered data, it also does not represent people's judgement about equilibrium output at the time.

Open economy influences on inflation have been an issue of continued concern. As noted above, Phillips raised the issue of import prices in his original article. Ihrig *et al.* (2007) provide a review of recent empirical work on external influences on inflation, with an emphasis on whether these have changed with the process of globalisation. There are a variety of results dependent on the choice of foreign variable (e.g. foreign inflation, foreign output gaps or real import prices), the specification used, e.g. including future variables or not, and the countries and time periods covered. Ihrig et al. also present a variety of estimates but they do not include expected future inflation explicitly so are not directly comparable with the NKPC results. Monacelli (2005) provides a theoretical discussion of the open economy NKPC. If there is full pass through of foreign prices in domestic currency to domestic prices, the open economy NKPC is isomorphic to the closed economy version: allowing for open-economy factors changes some slope coefficients but does not add variables. However, when there is incomplete pass through, extra terms are added to the Phillips curve that represent deviations from the law of one price and differences between the domestic and foreign consumption basket. Incomplete pass through breaks the usual proportionality relationship between real marginal cost and the output gap. There is substantial evidence for incomplete pass through, but the degree of pass depends on the volatility of the nominal exchange rate. The more likely that nominal exchange rate changes are perceived as transitory, the less likely that they will be passed through. This could differ from country to country. Since the pass through of exchange rates is likely to differ by country this suggests introducing exchange rate changes and foreign inflation separately in the NKPC.

3 Identification and Estimation of the Phillips Curve

We begin with a standard closed economy New Keynesian Phillips Curve (NKPC). For countries $i = 1, 2, ..., N$ and time periods $t = 1, 2, ..., T$, the NKPC links the deviations from steady state of inflation, $\widetilde{\pi}_{it}$ and a driving variable, $\tilde{y}_{it}$, by an equation of the form:

$$\widetilde{\pi}_{it} = \beta_{bi}\widetilde{\pi}_{i,t-1} + \beta_{fi}E(\widetilde{\pi}_{i,t+1} \mid \Im_{i,t-1}) + \gamma_i\widetilde{y}_{it} + \varepsilon_{it}, \tag{1}$$

where $E(\widetilde{\pi}_{i,t+1} \mid \Im_{i,t-1})$ denotes expectations formed conditional on information at time $t-1$. All variables are measured as deviations from their respective steady states. Denote the steady state or permanent value for a variable as x_{it}^P and the deviation from steady state as $\widetilde{x}_{it} = x_{it} - x_{it}^P$. The parameters are non-linear functions of underlying structural parameters such as the elasticity of substitution among differentiated goods, the elasticity of firms marginal costs to their own output, and the percentage of prices that are not reset optimally. There is no reason that these should be the same across countries with very different market institutions, so we make the parameters heterogeneous from the start.

Traditionally, the driving variable has been a measure of unemployment, or the output gap.

More recently measures of marginal cost and the share of labour have been used. This is partly motivated by the theoretical derivation and partly because it has been quite hard to get measures of the output gap significant in standard forward looking Phillips curves. The measures of marginal cost are more persistent, which may help with some of the econometric issues discussed below. We will use the output gap because it is the excess demand measure that is relevant to policy and the variable that appears in the standard three equation macro model. We will compare the performance of two measures of the output gap, the HP filter and the steady state measure obtained from the GVAR, but there are various issues of identification and estimation to be considered first.

It is common to assume that inflation is stationary, and that its steady state is a constant, say $\overline{\pi}_i$, then equation (1) becomes

$$\pi_{it} = (1 - \beta_{bi} - \beta_{fi})\overline{\pi}_i + \beta_{bi}\pi_{i,t-1} + \beta_{fi}E(\pi_{i,t+1} \mid \Im_{i,t-1}) + \gamma_i \widetilde{y}_{it} + \varepsilon_{it}. \tag{2}$$

The solution of the model depends on the process generating $\widetilde{y}_{it}$ and ε_{it}. It is typically assumed that ε_{it} is a martingale difference process, and $\widetilde{y}_{it}$ follows a stationary time series process. Consistent estimation of the NKPC critically depends on the nature of the $\widetilde{y}_{it}$ process. The empirical literature typically assumes suitable instruments (or moment conditions) exist and uses GMM to estimate the following version of the NKPC

$$\pi_{it} = (1 - \beta_{bi} - \beta_{fi})\overline{\pi}_i + \beta_{bi}\pi_{i,t-1} + \beta_{fi}\pi_{i,t+1} + \gamma_i \widetilde{y}_{it} + \xi_{i,t+1}, \tag{3}$$

or

$$\pi_{it} = \boldsymbol{\theta}_i' \mathbf{x}_{i,t+1} + \xi_{i,t+1},$$

where

$$\xi_{i,t+1} = \varepsilon_{it} - \beta_{fi}\upsilon_{i,t+1},$$

and $\upsilon_{i,t+1}$ is the expectations error of inflation, $\mathbf{x}_{i,t+1} = (\pi_{i,t-1}, \pi_{i,t+1}, \widetilde{y}_{it})'$, and $\boldsymbol{\theta}_i = (\beta_{bi}, \beta_{fi}, \gamma_i)'$.

The estimation of (3) requires at least three instruments that are

(a) not correlated with $\xi_{i,t+1}$, namely

$$E(\mathbf{z}_{i,t-1}\xi_{i,t+1} \,|\Im_{i,t-1}) = 0,$$

where $\mathbf{z}_{i,t-1}$ denotes the $s \times 1$ vector of instruments, and at the *same time* are

(b) sufficiently correlated with $\mathbf{x}_{i,t+1}$, such that

$$p \lim_{T\to\infty} \left(T^{-1} \sum_{t=1}^{T} \mathbf{x}_{i,t+1}\mathbf{z}_{i,t-1}' \right) = \text{Full Rank Matrix}.$$

Given the nature of the RE hypothesis there are no difficulties finding instruments that satisfy condition (a). Condition (b) is more problematic and whether it holds critically depends on the nature of the $\widetilde{y}_{it}$ process. To determine if the NKPC is identified requires solving the RE model.

3.1 Unique Stationary and Non-stationary Solutions

In the case where $\beta_{bi}, \beta_{fi} \geq 0$, $\beta_{fi}\beta_{bi} \leq 1/4$ and $\beta_{bi} + \beta_{fi} \leq 1$, the NKPC has the unique solution

$$\begin{aligned} \pi_{it} &= \frac{(1-\beta_{bi}-\beta_{fi})\overline{\pi}_i}{1-\beta_{fi}(1+\lambda_{bi})} + \lambda_{bi}\pi_{i,t-1} + \gamma_i \left[\widetilde{y}_{it} - E\left(\widetilde{y}_{it} \left| \mathfrak{I}_{i,t-1} \right. \right) \right] \\ & \left(\frac{\gamma_i}{1-\lambda_{bi}\beta_{fi}} \right) \sum_{j=0}^{\infty} \lambda_{fi}^{-j} E\left(\widetilde{y}_{i,t+j} \left| \mathfrak{I}_{i,t-1} \right. \right) + \varepsilon_{it}, \end{aligned} \tag{4}$$

where λ_{bi} and λ_{fi} are roots of

$$\beta_{fi}\lambda_i^2 - \lambda_i + \beta_{bi} = 0,$$

with $|\lambda_{bi}| \leq 1$ and $|\lambda_{fi}| > 1$. The condition $\beta_{bi} + \beta_{fi} < 1$ ensures that $|\lambda_{bi}| < 1$, and $|\lambda_{fi}| > 1$.

If $\beta_{bi} + \beta_{fi} = 1$, then $\lambda_{bi} = 1$ and $\lambda_{fi} = \beta_{fi}^{-1}(1-\beta_{fi}) > 1$ if $\beta_{fi} < 1/2$. In this case the solution is given by

$$\begin{aligned} \pi_{it} &= \pi_{i,t-1} + \gamma_i \left[\widetilde{y}_{it} - E\left(\widetilde{y}_{it} \left| \mathfrak{I}_{i,t-1} \right. \right) \right] \\ & \left(\frac{\gamma_i}{1-\beta_{fi}} \right) \sum_{j=0}^{\infty} \left(\frac{\beta_{fi}}{1-\beta_{fi}} \right)^j E\left(\widetilde{y}_{i,t+j} \left| \mathfrak{I}_{i,t-1} \right. \right) + \varepsilon_{it}. \end{aligned} \tag{5}$$

In the case where $\beta_{bi} + \beta_{fi} = 1$, the transversality condition needed for the existence of a unique solution will not be met if $\beta_{fi} > 1/2$. Since by construction $\widetilde{y}_{it}$ is a stationary process, then inflation will be $I(1)$ if $\beta_{bi} + \beta_{fi} = 1$. The above RE solutions are meaningful only if there are no feedbacks from past inflation to $\widetilde{y}_{it}$. More on this below.

3.2 Indeterminate Solutions

Indeterminate solutions arise if $\beta_{bi} + \beta_{fi} > 1$. In this case the above forward RE solutions are no longer applicable. To characterise the solutions in this case consider the inflation expectations errors

$$\mathfrak{m}_{i,t+1} = \widetilde{\pi}_{i,t+1} - E(\widetilde{\pi}_{i,t+1} \mid \mathfrak{I}_{i,t-1})$$

and note that under the RE hypothesis $\mathfrak{m}_{i,t+1}$ is an arbitrary martingale difference process, such that $E(\mathfrak{m}_{i,t+1} \mid \mathfrak{I}_{i,t-1}) = 0$. Using $\mathfrak{m}_{i,t+1}$, a general solution for the inflation process can be written as

$$\widetilde{\pi}_{it} = \beta_{fi}^{-1}\widetilde{\pi}_{i,t-1} - \beta_{fi}^{-1}\beta_{bi}\widetilde{\pi}_{i,t-2} - \beta_{fi}^{-1}\gamma_i\widetilde{y}_{i,t-1} + \mathfrak{m}_{it} - \beta_{fi}^{-1}\varepsilon_{i,t-1}. \tag{6}$$

When $\beta_{bi} + \beta_{fi} > 1$, (6) is a stable solution but it is not unique; there is a multiplicity of solutions indexed by $\mathfrak{m}_{it}$. Different stable solutions can be obtained for different choices of the martingale difference process, $\mathfrak{m}_{it}$. One possible choice for $\mathfrak{m}_{it}$ is the bubble free linear specification in terms of innovations to the forcing variable:

$$\mathfrak{m}_{it} = g_i(\widetilde{y}_{it} - E(\widetilde{y}_{it} \mid \mathfrak{I}_{i,t-1})),$$

where g_i is an arbitrary constant. This in itself gives a multiplicity of solutions, depending on the choice of g_i. In the context of the GVAR, the innovations to any of the variables in the global system may matter, in which case we could use the more general martingale difference process

$$\mathfrak{m}_{it} = \mathbf{g}_i'(\mathbf{x}_t - E(\mathbf{x}_t \mid \mathfrak{I}_{i,t-1})) + e_{it}$$

where e_{it} is any other martingale difference process. Similarly martingale difference processes involving non-linear terms such as $\tilde{y}_{jt}^2 - E(\tilde{y}_{jt}^2 \mid \mathfrak{I}_{i,t-1})$ for all i and j, could be used to allow volatilities to enter the system.

Consider now estimating (6). Since $\mathfrak{m}_{it}$ is a martingale difference, it is orthogonal to $\widetilde{\pi}_{i,t-1}$, $\widetilde{\pi}_{i,t-2}$, and $\widetilde{y}_{i,t-1}$. However the term $\varepsilon_{i,t-1}$ is correlated with $\widetilde{\pi}_{i,t-1}$ and $\widetilde{y}_{i,t-1}$ so they would need to be instrumented, but $\widetilde{\pi}_{i,t-3}$ and $\widetilde{y}_{i,t-2}$ are valid instruments for $\widetilde{\pi}_{i,t-1}$ and $\widetilde{y}_{i,t-1}$. The intractable problem is providing a proxy for $\mathfrak{m}_{it}$. Suppose we considered the innovations in x_{jt} as a candidate variable for estimating

$$\widetilde{\pi}_{it} = a_{0i}\widetilde{\pi}_{i,t-1} + a_{1i}\widetilde{\pi}_{i,t-2} + a_{3i}\widetilde{y}_{i,t-1} + \delta_i(x_{jt} - E(x_{jt} \mid \mathfrak{I}_{i,t-1})) + e_{it}.$$

This will not be feasible because $(x_{jt} - E(x_{jt} \mid \mathfrak{I}_{i,t-1}))$ is almost certainly correlated with e_{it} which will contain the innovations to the other variables and their powers. The theory is unlikely to put any restrictions on the nature of these correlations between expectational errors. In addition, since $(x_{jt} - E(x_{jt} \mid \mathfrak{I}_{i,t-1}))$ is a martingale difference, lagged values of any variables in the information set are not valid instruments.

3.3 Weak Instruments

To return to the determinate case, in the absence of feedbacks where $\widetilde{y}_{it}$ does not depend (directly or indirectly through a third variable) on past values of π_{it}, future inflation $\pi_{i,t+1}$ and $\widetilde{y}_{it}$ do not depend on $\pi_{i,t-1}$, $\pi_{i,t-2}$, or earlier. As a result apart from $\pi_{i,t-1}$ that enters (1), the use of inflation lagged two or more periods, namely, $\pi_{i,t-2}$, $\pi_{i,t-3}$, ..., cannot help identification and as a result do not contribute to meeting the full rank condition. Nevertheless, many papers in the literature routinely use second and higher order inflation lags as instruments. For example, Gali and Gertler (1999) use four lags of inflation, Batini, Jackson, and Nickell (2005, p. 1067) use five lags of inflation, and Gali, Gertler and Lopez-Salido (2005) use four lags of price inflation. Beyer *et al.* (2007) state that "as is usual" they use three lags of inflation, the output gap and the interest rates as instruments, but comment that it is questionable whether lags higher than one should be included.

As noted originally in Pesaran (1981, 1987, Ch. 7) identification of the structural parameters critically depends on the process generating $\widetilde{y}_{it}$. For example suppose that $\widetilde{y}_{it}$ follows the $AR(1)$ process

$$\widetilde{y}_{it} = \rho_i\widetilde{y}_{i,t-1} + v_{it}.$$

Then the RE solution is given by

$$\pi_{it} = a_{i0} + a_{i1}\pi_{i,t-1} + a_{i2}\widetilde{y}_{i,t-1} + u_{it}, \tag{7}$$

where

$$a_{i0} = \frac{(1-\beta_{bi}-\beta_{fi})\overline{\pi}_i}{1-\beta_{fi}(1+\lambda_{bi})},\ a_{i1} = \lambda_{bi} = \frac{1-\sqrt{1-4\beta_{fi}\beta_{bi}}}{2\beta_{fi}},$$

$$a_{i2} = \left(\frac{\gamma_i\rho_i}{1-\lambda_{bi}\beta_{fi}}\right)\left(\frac{1}{1-\rho_i\lambda_{fi}^{-1}}\right), \quad u_{it} = \varepsilon_{it} + \gamma_i v_{it}.$$

The reduced form for $(\pi_{it}, \widetilde{y}_{it})$ is a $VAR(1)$ that allows consistent estimation of the four parameters, a_{i0}, a_{i1}, a_{i2}, and ρ_i, whilst we have five unknown coefficients, $\bar{\pi}_i, \beta_{fi}, \beta_{bi}, \gamma_i$, and ρ_i. In this case the structural parameters β_{fi}, β_{bi} and γ_i are not identified. In other words although $\pi_{i,t-s}, \widetilde{y}_{i,t-s}$ for $s = 2, 3, ...$, are uncorrelated with $\xi_{i,t+1}$, their use as instruments will not help in identification. This is because once $\pi_{i,t-1}$ and $\widetilde{y}_{i,t-1}$ are included as instruments the additional lags do not contribute any further to the identification. Notice that the regression of the right hand side endogenous variables on the instruments may not be informative, (7) may fit very well even though the model is not identified.

More specifically, if $\mathbf{z}_{i,t-1} = (1, \pi_{i,t-1}, \widetilde{y}_{i,t-1})'$ does not ensure the rank condition because $\widetilde{y}_{it}$ follows an $AR(1)$ process, then adding $\pi_{i,t-s}, \widetilde{y}_{i,t-s}$ for $s = 2, 3, ...$, does not help, in the sense that the rank condition remains unfulfilled. The order of the $AR(p)$ process for the output gap must at least be equal to two. In general if the output gap, $\widetilde{y}_{it}$, is $AR(p)$, the form for the RE solution is $ARDL(1, p-1)$ in π_{it} and $\widetilde{y}_{it}$. Suppose that the model is an $AR(2)$

$$\widetilde{y}_{it} = \rho_{i1}\widetilde{y}_{i,t-1} + \rho_{i2}\widetilde{y}_{i,t-2} + v_{it}$$

then the extra instrument $\widetilde{y}_{i,t-2}$ exactly identifies the model. But the identification can be "weak" if ρ_2 is not statistically significant.

In fact, it can be readily shown that allowing for feedbacks from $\tilde{\pi}_{i,t-1}$ into $\tilde{y}_{it}$ will not resolve the weak instrument problem, unless it is assumed that the order of the lagged inflation term in the $\widetilde{y}_{it}$ equation is greater than the order of the lagged inflation term in the NKPC equation. For example, augmenting the $AR(1)$ equation of the output gap with lagged inflation, namely

$$\widetilde{y}_{it} = \rho_{iy}\widetilde{y}_{i,t-1} + \rho_{i\pi}\tilde{\pi}_{i,t-1} + v_{it},$$

does not alter the dynamic form of the inflation process and as before the lagged inflation terms, $\tilde{\pi}_{i,t-s}$, $s = 2, 3, ...$ will not be valid instruments for future inflation in the NKPC equation.

3.4 A Global Perspective

The argument put forth in Pesaran (1987) that to determine identification requires solving the rational expectations model, has been used more recently by a number of authors including Mavroeidis (2005) and Beyer *et al.* (2007) to argue that the NKPC may only be weakly identified which renders both GMM estimation and the usual tests for over-identifying restrictions unreliable. Adopting a global context, however does provide other instruments. Suppose that there are world cyclical influences represented by a vector of common factors $\mathbf{f}_t$

$$\widetilde{y}_{it} = a_i + \mathbf{b}_i'\mathbf{f}_t + \eta_{it} \quad (8)$$

and the idiosyncratic element is serially correlated

$$\eta_{it} = \rho_i \eta_{it-1} + v_{it}$$

giving

$$\widetilde{y}_{it} = a_i(1-\rho_i) + \rho_i \widetilde{y}_{i,t-1} + \mathbf{b}_i' \mathbf{f}_t + \rho_i \mathbf{b}_i' \mathbf{f}_{t-1} + v_{it},$$

which makes $\mathbf{f}_t$ and $\mathbf{f}_{t-1}$ relevant instruments. As long as the idiosyncratic components, η_{it}, are weakly dependent (i.e. no country is dominant), the global factor or factors can be estimated as principal components or cross-section averages of the $\widetilde{y}_{it}$. The cross-section average used to measure the global factor may be a country specific average. One possibility is to use trade weights such that a country specific estimate of $\mathbf{f}_t$ is estimated by $\widetilde{y}_{it}^* = \sum_{j=1}^N w_{ij} \widetilde{y}_{jt}$ with $w_{ii} = 0$. Notice that in constructing the cross-section averages we do not need all the $\widetilde{y}_{jt}$ to be uncorrelated with the Phillips curve error ε_{it} only that $\sum_{j=1}^N w_{ij} \widetilde{y}_{jt}$ is uncorrelated with the error. Suppose that the correlation between $\widetilde{y}_{jt}$ and ε_{it} is denoted δ_{ij} then we require the granularity condition that $\sum_{j=1}^N w_{ij} \delta_{ij} \to 0$ as $N \to \infty$. This weak exogeneity assumption can be tested and the results in DdPS indicate that it is accepted. As before to check identification, we need to find a solution for the rational expectations model and this requires providing a model for $\widetilde{y}_{it}^*$. This could also be an autoregression or the model could be provided by the GVAR which provides a consistent world estimate.

In (8) the global factors influence just the output gap, making them valid instruments. But it is also plausible that with an open economy NKPC the errors in the NKPC (1) will be subject to global factors, in which case current values of the factors should be included directly. This can be investigated by including measures of global inflation and the global output gap directly.

The global perspective both provides a theoretically consistent estimate of the steady state and a new set of instruments. However, given that the GVAR provides a very large number of potential instruments there is the danger that the IV estimator will just closely approximate the biased OLS estimator. Thus one needs to map the large number of potential instruments into a smaller number that satisfy the above two mentioned conditions. Kapetanios and Marcellino (2007) and Bai and Ng (2007) investigate estimating factor models and using the estimated factors as instruments. The GVAR provides an alternative mapping by measuring the factors, $\mathbf{f}_t$, as the trade weighted averages of the foreign variables corresponding to a particular domestic variable.

3.5 Unit Roots and Cointegration

The evidence presented in DdPS indicated that consumer prices might be $I(2)$, so that inflation could be $I(1)$. However, the output gap should certainly be $I(0)$, which would mean that the NKPC equation must be set out in changes in inflation and not in the level of inflation as is usually done, otherwise the residuals from estimated NKPC equations could be highly persistent.[6] The unit root test results on inflation are in line with the estimates of $\beta_b + \beta_f$ obtained in the literature where

[6] In addition for GMM to be valid, the variables must be stationary, e.g. Hall (2005). Li (2007) also discusses the influence of persistent data on inference in rational expectations models.

this sum is often estimated (taken) to be unity or close to unity (equal to the discount factor), see for instance the estimates in Gali, Gertler and Lopez-Salido (2005).

We have already seen that if $\beta_b + \beta_f = 1$ the RE solution of the NKPC does in fact imply a unit root in inflation. Failure to reject the unit root hypothesis, however, may occur for a variety of reasons, such as lack of power of the test used, or could be due to shifts in mean inflation, or other forms of non-linearities. In either case the steady state of inflation can no longer be assumed to be a fixed constant, and the evolution of the mean inflation needs to be modeled, possibly in terms of other factors.

An obvious way to correct this problem is to adopt an open economy NKPC and add the foreign inflation, π_{it}^*, and the foreign output gap, $\widetilde{y}_{it}^*$, to the NKPC. As discussed in the previous section, there are good theoretical and empirical reasons to expect foreign inflation to influence domestic CPI inflation (e.g. through cost shocks or exchange rates), but including it allows the two sides of the NKPC equation to have similar orders of integration, with the possibility of cointegration between domestic and foreign inflation. The foreign output gap, $\widetilde{y}_{it}^*$, does not solve this problem since this should certainly be $I(0)$. An open economy version of the NKPC, which includes foreign inflation and output gap is given by

$$\pi_{it} = a_{i\pi} + \beta_{ib}\pi_{i,t-1} + \beta_{if}E(\pi_{i,t+1} \mid \Im_{i,t-1}) + \beta_{*i}\pi_{it}^* + \gamma_i\widetilde{y}_{it} + \gamma_{*i}\widetilde{y}_{it}^* + \varepsilon_{it}.$$

There is an issue as to the transmission mechanism by which foreign inflation affects domestic inflation. As discussed above, if PPP held it would be cancelled out by exchange rate movements, but the evidence in favour of short-run PPP in low inflation environments is quite low or equivalently the exchange rate pass through is quite low. This might not be the case for high inflation economies, e.g. the Latin American ones in our sample. The extent to which domestic inflation is insulated from foreign inflation will, of course, depend on domestic monetary policy which will differ between countries so, like the other parameters, we would expect β_{*i} to differ between countries. Again we need to provide a model for foreign inflation. If this is approximated by an $AR(1)$ (the GVAR supplies a complete model), then th

e appropriate set of instruments would be $\pi_{i,t-1}$, π_{it}^*, $\pi_{i,t-1}^*$, $\widetilde{y}_{i,t-1}$, $\widetilde{y}_{it}^*$, $\widetilde{y}_{i,t-1}^*$. This is the initial set which is used in the empirical work below.

Interest rates can influence marginal costs, which determine inflation, through a cost channel since firms must finance their working capital such as labour costs. Thus various authors, e.g. Ravenna and Walsh (2006) and Chowdhury *et al.* (2006), have suggested that interest rates should appear in the Phillips curve. We investigate this issue and also estimate

$$\pi_{it} = a_{i\pi} + \beta_{ib}\pi_{i,t-1} + \beta_{if}E(\pi_{i,t+1} \mid \Im_{i,t-1}) + \beta_{*i}\pi_{it}^* + \gamma_i\widetilde{y}_{it} + \gamma_{*i}\widetilde{y}_{it}^* + \delta_i r_{it} + \varepsilon_{it} \qquad (9)$$

where r_{it} denotes domestic short-term interest rate.

Again a complete model that includes suitable specifications for the remaining variables $\pi_{it}^*, \widetilde{y}_{it}, \widetilde{y}_{it}^*$, and r_{it}, is needed to obtain the rational expectations solution and thus investigate the identification problem. In the case of a small open economy π_{it}^* and $\widetilde{y}_{it}^*$ can be treated as weakly exogenous and used as instruments. Contemporaneous and lagged values of the foreign interest rate as well as the lagged values of domestic interest rate are also potential instruments. But their effectiveness as instruments depends on the nature of the interlinkages between the economy under consideration and the rest of the world.

4 A Multicountry NKPC Model

To obtain the RE solution of the NKPC model in (9) we need a multicountry version of the familiar three equation macro model comprising a NKPC, an optimising IS curve and a Taylor rule, for example discussed in Pesaran and Smith (2006). For each country i we specify that

$$\begin{aligned}\mathbf{A}_{i0}\mathbf{x}_{it} &= \mathbf{a}_i + \mathbf{A}_{i1}\mathbf{x}_{i,t-1} + \mathbf{A}_{i2}E_{t-1}(\mathbf{x}_{i,t+1}) \\ &\quad + \mathbf{A}_{i3}\mathbf{x}^*_{it} + \mathbf{A}_{i4}\mathbf{x}^*_{i,t-1} + \mathbf{A}_{i5}E_{t-1}(\mathbf{x}^*_{i,t+1}) + \boldsymbol{\varepsilon}_{it},\end{aligned}$$

where $\mathbf{x}_{it} = (\pi_{it}, \tilde{y}_{it}, r_{it})'$, and $\mathbf{x}^*_{it} = (\pi^*_{it}, \tilde{y}^*_{it}, r^*_{it})'$ is the associated vector of foreign variables constructed as weighted cross section averages, defined as before by $\mathbf{x}^*_{it} = \sum_{j=1}^{N} w_{ij}\mathbf{x}_{jt}$ with $w_{ii} = 0$. Here expectations are taken with respect to a common global information formed as the union intersection of the individual country information sets, $\Im_{i,t-1}$. This formulation is sufficiently general for our purposes and represents an open economy version of the familiar three equation DSGE model composed of a NKPC, an output gap equation and an interest rule equation.[7] In the empirical applications we also examine the effect of exchange rate on inflation but will not include it here to simplify the exposition.

To obtain a solution to the above rational expectations model a statistical model for $(\mathbf{x}^*_{it}, \boldsymbol{\varepsilon}_{it})$ is clearly required. In the DSGE literature the foreign variables, $\mathbf{x}^*_{it}$, are typically assumed to be strictly exogenous, excluding any feedbacks from the lagged $\mathbf{x}_{it}$. However, due to the presence of common factors and dominant country effects $\mathbf{x}^*_{it}$ is unlikely to be strictly exogenous, and one needs to derive a globally consistent RE solution. This can be achieved by linking up the N country-specific DSGE models using the equations for $\mathbf{x}^*_{it}$. To see this let $\mathbf{z}_{it} = (\mathbf{x}'_{it}, \mathbf{x}^{*\prime}_{it})'$ and write the N country-specific DSGE models as

$$\mathbf{A}_{iz0}\mathbf{z}_{it} = \mathbf{a}_i + \mathbf{A}_{iz1}\mathbf{z}_{i,t-1} + \mathbf{A}_{iz2}E_{t-1}\left(\mathbf{z}_{i,t+1}\right) + \boldsymbol{\varepsilon}_{it}, \text{ for } i = 1, 2, ..., N. \tag{10}$$

But given that $\mathbf{x}^*_{it} = \sum_{j=1}^{N} w_{ij}\mathbf{x}_{jt}$, there must be a 'link' matrix $\mathbf{W}_i$ such that

$$\mathbf{z}_{it} = \mathbf{W}_i\mathbf{x}_t,$$

where $\mathbf{x}_t = (\mathbf{x}'_{1t}, \mathbf{x}'_{2t}, ..., \mathbf{x}'_{Nt})'$, and hence (10) can be written as

$$\mathbf{A}_{iz0}\mathbf{W}_i\mathbf{x}_t = \mathbf{a}_i + \mathbf{A}_{iz1}\mathbf{W}_i\mathbf{x}_{t-1} + \mathbf{A}_{iz2}\mathbf{W}_iE_{t-1}(\mathbf{x}_{t+1}) + \boldsymbol{\varepsilon}_{it}.$$

[7] As in DdPS, the three equation model can be readily extended to include exchange rates and other financial variables such as long term interest rate and real equity prices.

Stacking these models now yields

$$\mathbf{A}_0\mathbf{x}_t = \mathbf{a} + \mathbf{A}_1\mathbf{x}_{t-1} + \mathbf{A}_2 E_{t-1}(\mathbf{x}_{t+1}) + \boldsymbol{\varepsilon}_t, \tag{11}$$

where

$$\mathbf{A}_j = \begin{pmatrix} \mathbf{A}_{1zj}\mathbf{W}_1 \\ \mathbf{A}_{2zj}\mathbf{W}_2 \\ \vdots \\ \mathbf{A}_{Nzj}\mathbf{W}_N \end{pmatrix}, \ \mathbf{a} = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_N \end{pmatrix}, \ \boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Nt} \end{pmatrix}.$$

The solution properties of the RE model, (11), depends on the roots of the quadratic matrix equation[8]

$$\mathbf{A}_2\boldsymbol{\Phi}^2 - \mathbf{A}_0\boldsymbol{\Phi} + \mathbf{A}_1 = \mathbf{0}.$$

There will be a globally consistent RE solution if there exists a real matrix solution to the above equation such that all the eigenvalues of $\boldsymbol{\Phi}$ and $(\mathbf{I}_{3N} - \mathbf{A}_2\boldsymbol{\Phi})^{-1}\mathbf{A}_2$ lie inside or on the unit circle. In such a case the unique solution is given by

$$\mathbf{x}_t = \mathbf{b} + \boldsymbol{\Phi}\mathbf{x}_{t-1} + \mathbf{v}_t, \tag{12}$$

where

$$(\mathbf{A}_0 - \mathbf{A}_2\boldsymbol{\Phi} - \mathbf{A}_2)\,\mathbf{b} = \mathbf{a}$$

and

$$(\mathbf{A}_0 - \mathbf{A}_2\boldsymbol{\Phi})\,\mathbf{v}_t = \boldsymbol{\varepsilon}_t.$$

This solution shows that all first order lags of inflation rates, output gaps and interest rates can be used as instruments. But in the case where N is sufficiently large and there are only a few dominant economies and/or common factors, as shown in Chudik and Pesaran (2007), the reduced form model of the non-dominant (small) economies in (12) can be well approximated by the following VARX* model

$$\mathbf{x}_{it} = \mathbf{b}_i + \boldsymbol{\Phi}_{ii}\mathbf{x}_{i,t-1} + \boldsymbol{\Psi}_{i0}\mathbf{x}^*_{it} + \boldsymbol{\Psi}_{i1}\mathbf{x}^*_{i,t-1} + \mathbf{v}_{it},$$

where $\boldsymbol{\Phi}_{ii}$ is the 3×3 matrix on the i^{th} diagonal block of $\boldsymbol{\Phi}$, and $\mathbf{x}^*_{it}$ is a weighted cross section average with granular weights, such that for each i, $\Sigma_{j=1}^{N} w_{ij}^2 \to 0$, as $N \to \infty$.[9] Chudik and Pesaran show that although $\mathbf{x}^*_{it}$ and $\mathbf{v}_{it}$ are correlated for a fixed N, they become uncorrelated as $N \to \infty$. Therefore, for small open economies it is valid to use $\pi^*_{it}, \tilde{y}^*_{it}$, and r^*_{it} as instruments when estimating their NKPC equations.

The above results also establish that for estimation $\mathbf{x}^*_{it}$ can be treated as weakly exogenous with respect to the parameters of the conditional model, an assumption found acceptable when tested. The VARX* model is estimated separately for each country conditional on $\mathbf{x}^*_{it}$, taking into account the possibility of cointegration both within $\mathbf{x}_{it}$ and across $\mathbf{x}_{it}$ and $\mathbf{x}^*_{it}$. Although estimation is done on a country by country basis, the GVAR model needs to be solved for the world as a whole.

[8] See, for example, Binder and Pesaran (1995,1997).

[9] Variables of the dominant economy (if any) can also be added as additional regressors if needed.

5 The GVAR Model and the Estimation of the Permanent Components

Here we provide a brief account of the GVAR model developed by DdPS which we shall use for the estimation of the permanent components and the specification of the $\mathbf{x}_{it}^*$ variables. The model of DdPS comprises 33 countries, 8 grouped into a single euro area economy, covering 90% of world output. In total, there are 26 individual country models linked within a unified GVAR framework including Europe, the Anglo-Saxon world, Latin America, South East Asia, China, Korea, India, Saudi Arabia, Turkey and South Africa. For a detailed list of countries see DdPS. The model is estimated on quarterly data over the period 1979Q4-2003Q4. While some variables are not available for some countries, for most countries the variables included are given in Table 1 below, with the US treated differently given its importance in the world economy and the fact that US dollar is used as a reference currency.

Table 1. Domestic and Foreign Variables Included in the Individual Country Models

	All Countries Excluding US		US	
Variables	Endogenous	Foreign	Endogenous	Foreign
Real Output	y_{it}	y_{it}^*	$y_{us,t}$	$y_{us,t}^*$
Inflation	π_{it}	π_{it}^*	$\pi_{us,t}$	$\pi_{us,t}^*$
Real Exchange Rate	ep_{it}	-	-	$ep_{us,t}^*$
Real Equity Price	q_{it}	q_{it}^*	$q_{us,t}$	-
Short-Term Interest Rate	r_{it}^S	r_{it}^{*S}	$r_{us,t}^S$	-
Long-Term Interest Rate	r_{it}^L	r_{it}^{*L}	$r_{us,t}^L$	-
Oil Price	-	p_t^o	p_t^o	-

The GVAR model has 134 endogenous variables 71 stochastic trends and 63 cointegrating relations. All its roots either lie on or inside the unit circle. The long run forcing assumption, required for weak exogeneity is rejected only in 5 out of 153 cases. Evidence of structural instability is found primarily in the error variances (47% of the equations - clustered in the period 1985-1992). The model uses the exactly identified cointegrating vectors. Discussion of the effect of imposing over-identifying restrictions on the long run relations can be found in Dees, Holly, Pesaran and Smith (2007).

5.1 Estimation of the Steady States

In this section we discuss how we obtain the estimate of, say, the output gap as the deviation of output from its steady state, $\widetilde{y}_{it} = y_{it} - y_{it}^P$, from the decomposition of the variables in the GVAR into their permanent, $\mathbf{x}_t^P$, and transitory or cyclical components, $\mathbf{x}_t^C$ (or equivalently $\tilde{\mathbf{x}}_t$).

Denote the $k \times 1$ vector of endogenous variables in the global economy by $\mathbf{x}_t$, and consider the decomposition, $\mathbf{x}_t = \mathbf{x}_t^P + \mathbf{x}_t^C$. Suppose also that the permanent component, $\mathbf{x}_t^P$, is further subdivided into deterministic and stochastic components, $\mathbf{x}_t^P = \mathbf{x}_{dt}^P + \mathbf{x}_{st}^P$. The permanent-determinstic component, $\mathbf{x}_{dt}^P$, is defined by

$$\mathbf{x}_{dt}^P = \boldsymbol{\mu} + \mathbf{g}t,$$

where $\boldsymbol{\mu}$ and $\mathbf{g}$ are $k \times 1$ vectors of fixed constants, and t is a deterministic time trend. The permanent-stochastic component, $\mathbf{x}_{st}^P$, is then *uniquely* defined as the 'long-horizon forecast' (net of the permanent-deterministic component)[10]

$$\mathbf{x}_{st}^P = \lim_{h \to \infty} E_t \left(\mathbf{x}_{t+h} - \mathbf{x}_{d,t+h}^P \right) = \lim_{h \to \infty} E_t \left[\mathbf{x}_{t+h} - \boldsymbol{\mu} - \mathbf{g}(t+h) \right], \tag{13}$$

and $E_t(.)$ denotes the expectations operator conditional on the information available at time t, taken to include at least $\{\mathbf{x}_t, \mathbf{x}_{t-1}, ..., \mathbf{x}_0\}$.[11]

The above decomposition has a number of nice properties. The permanent stochastic component is identically equal to zero if the process generating $\mathbf{x}_t$ is trend stationary. On the other extreme $\mathbf{x}_{st}^P = \mathbf{x}_t$ if $\mathbf{x}_t$ is a pure unit root process and non-cointegrated. The GVAR provides a model of interest that lies somewhere in between these two extremes and allows derivation of permanent components that take account of unit roots and cointegration in the global economy. To illustrate some of these points and highlight the uniqueness of $\mathbf{x}_{st}^P$, as a simple example abstract from the deterministics and suppose that $\mathbf{x}_t$ follows a VAR of order 1 with the coefficient matrix $\boldsymbol{\Phi}$. It is then easily seen that $\mathbf{x}_{st}^P = \lim_{h \to \infty} E_t(\mathbf{x}_{t+h}) = \left(\lim_{h \to \infty} \boldsymbol{\Phi}^h \right) \mathbf{x}_t = \boldsymbol{\Phi}^{\infty} \mathbf{x}_t$. Hence, as indicated $\mathbf{x}_{st}^P = \mathbf{0}$, if the VAR(1) process is stationary and all eigenvalues of $\boldsymbol{\Phi}$ lie within the unit circle, $\mathbf{x}_{st}^P = \mathbf{x}_t$ if $\mathbf{x}_t$ is a unit root process with $\boldsymbol{\Phi} = \mathbf{I}_k$. But when $\mathbf{I}_k - \boldsymbol{\Phi}$ is rank deficient and some of the roots of $\boldsymbol{\Phi}$ lie exactly on the unit circle $\mathbf{x}_{st}^P$ will be determined by the linear combinations of $\mathbf{x}_t$ that are not cointegrated.

The GVAR is constructed from the underlying country-specific models and in its global error correction form is given by

$$\mathbf{G}\Delta\mathbf{x}_t = \mathbf{a} - \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}' \left[\mathbf{x}_{t-1} - \boldsymbol{\gamma}(t-1) \right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{x}_{t-i} + \mathbf{u}_t, \tag{14}$$

where $\mathbf{G}$ is a $k \times k$ matrix that reflects the contemporaneous interdependencies across countries, $\boldsymbol{\gamma}$ is a $k \times 1$ vector of fixed constants, $\tilde{\boldsymbol{\alpha}}$ is the $k \times r$ block-diagonal matrix of the global loading coefficients

$$\tilde{\boldsymbol{\alpha}} = \begin{pmatrix} \boldsymbol{\alpha}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\alpha}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{\alpha}_N \end{pmatrix}$$

[10] See also the discussion in Garratt et al. (2006).

[11] One could equally well have derived the long horizon forecast with respect to the information set at t-1. Here we have chosen to work with time t long-horizon expectations so that, as we shall see, the permanent-stochastic component coincides with that obtained in the Beveridge-Nelson decomposition. This should help the comparability of our results with those in the literature.

with $r = \sum_{i=1}^{N} r_i$ and r_i is the cointegrating rank for country i, and $\tilde{\boldsymbol{\beta}}$ is the global $k \times r$ cointegrating matrix[12]

$$\tilde{\boldsymbol{\beta}} = \begin{pmatrix} \mathbf{W}_1'\boldsymbol{\beta}_1, & \mathbf{W}_2'\boldsymbol{\beta}_2, & \ldots, & \mathbf{W}_N'\boldsymbol{\beta}_N \end{pmatrix}.$$

To derive the permanent components, we first write the global error correction model, (14), as the VAR(p) specification

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \sum_{i=1}^{p} \boldsymbol{\Phi}_i \mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t, \tag{15}$$

where

$$\mathbf{b}_0 = \mathbf{G}^{-1}(\mathbf{a} - \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'\boldsymbol{\gamma}), \;\; \mathbf{b}_1 = \mathbf{G}^{-1}\tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'\boldsymbol{\gamma}, \;\; \boldsymbol{\varepsilon}_t = \mathbf{G}^{-1}\mathbf{u}_t,$$

$$\boldsymbol{\Phi}_1 = \mathbf{G}^{-1}(\mathbf{G} + \boldsymbol{\Gamma}_1 - \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'), \; \boldsymbol{\Phi}_i = \mathbf{G}^{-1}(\boldsymbol{\Gamma}_i - \boldsymbol{\Gamma}_{i-1}), \; i = 2, ..., p-1, \; \boldsymbol{\Phi}_p = -\mathbf{G}^{-1}\boldsymbol{\Gamma}_{p-1}.$$

Using (15) we can now write down the solution of $\mathbf{x}_t$ as

$$\mathbf{x}_t = \boldsymbol{\mu} + \mathbf{g}t + \mathbf{C}(1)\,\mathbf{s}_{\varepsilon t} + \mathbf{C}^*(L)\,\boldsymbol{\varepsilon}_t, \tag{16}$$

where

$$\boldsymbol{\mu} = \mathbf{x}_0 - \mathbf{C}^*(L)\boldsymbol{\varepsilon}_0,$$

$$\mathbf{s}_{\varepsilon t} = \sum_{j=1}^{t} \boldsymbol{\varepsilon}_j, \;\; \mathbf{C}^*(L) = \sum_{j=0}^{\infty} \mathbf{C}_j^* L^j,$$

$$\mathbf{C}_j = \mathbf{C}_{j-1}\boldsymbol{\Phi}_1 + \mathbf{C}_{j-2}\boldsymbol{\Phi}_2 + \cdots + \mathbf{C}_{j-p}\boldsymbol{\Phi}_p, \text{ for } j = 1, 2, ...,$$

with $\mathbf{C}_0 = \mathbf{I}_k$, $\mathbf{C}_1 = -(\mathbf{I}_k - \boldsymbol{\Phi}_1)$, and $\mathbf{C}_j = \mathbf{0}$ for $j < 0$; $\mathbf{C}_j^* = \mathbf{C}_{j-1}^* + \mathbf{C}_j$, for $j = 1, 2, ...$, with $\mathbf{C}_0^* = \mathbf{C}_0 - \mathbf{C}(1)$, and $\mathbf{C}(1) = \sum_{j=0}^{\infty} \mathbf{C}_j$. Hence, it is easily seen that

$$\mathbf{x}_{st}^P = \lim_{h \to \infty} E_t \left[\mathbf{x}_{t+h} - \boldsymbol{\mu} - \mathbf{g}(t+h) \right] = \mathbf{C}(1) \sum_{j=1}^{t} \boldsymbol{\varepsilon}_j, \tag{17}$$

which is the multivariate version of the Beveridge-Nelson (BN) stochastic trend component. Note that $\mathbf{x}_{st}^P$ is uniquely determined from the time series observations on $\mathbf{x}_t$ and its lagged values. The identification problem with the BN decomposition discussed in the literature relates to separating the k shocks, $\boldsymbol{\varepsilon}_t$, into permanent (supply) or transitory (demand) shocks. A general discussion of this problem is provided by Pagan and Pesaran (2007).

The permanent-stochastic component can now be estimated directly from the parameters of the GVAR as $\hat{\mathbf{x}}_{st}^P = \hat{\mathbf{C}}(1) \sum_{i=1}^{t} \hat{\boldsymbol{\varepsilon}}_i$. The cyclical or the transitory component, $\widehat{\mathbf{x}}_t^C$, can then be estimated as

$$\hat{\mathbf{v}}_t = \mathbf{x}_t - \hat{\mathbf{x}}_{st}^P = \hat{\boldsymbol{\mu}} + \hat{\mathbf{g}}t + \widehat{\mathbf{x}}_t^C$$

[12] Note that for the deterministic trend properties of the variables to be the same in the global model as in the underlying country-specific models $\tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'\boldsymbol{\gamma} = \begin{pmatrix} (\boldsymbol{\alpha}_1\boldsymbol{\beta}_1'\mathbf{W}_1\boldsymbol{\gamma})', & (\boldsymbol{\alpha}_2\boldsymbol{\beta}_2'\mathbf{W}_2\boldsymbol{\gamma})', & \ldots, & (\boldsymbol{\alpha}_N\boldsymbol{\beta}_N'\mathbf{W}_N\boldsymbol{\gamma})' \end{pmatrix}'$ where $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ are the loading coefficients and the cointegrating matrix, respectively, of the individual country models.

with $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{g}}$ in turn estimated from the OLS regressions

$$\hat{\mathrm{v}}_{i,\ell t} = \mu_{i\ell} + g_{i\ell} t + \xi_{i,\ell t},\ i = 1, 2, ..., N;\ \ell = 1, ..., k_i$$

for variable ℓ in country i. In this way we are also able to impose a number of trend restrictions of interest. For example, we set $g_{i\pi} = g^S_{i,r} = g^L_{i,r} = 0$ in all countries, as it does not seem reasonable to allow for long-run trends in inflation and interest rates. The estimated cyclical component, $\hat{\mathbf{x}}^C_t$, is then the residual from the above regressions, that is $\widehat{\mathbf{x}}^C_t = (\hat{\boldsymbol{\xi}}'_1, \hat{\boldsymbol{\xi}}'_2 ..., \hat{\boldsymbol{\xi}}'_N)'$.

In the empirical applications we consider two measures of output gaps: one based on the GVAR and computed as above which we denote by $\tilde{y}_{gvar,it}$, and the familiar HP measure denoted by $\tilde{y}_{hp,it}$. Similarly, alternative measures of country-specific foreign output gaps are computed as $\tilde{y}^*_{gvar,it} = \sum_{j=1}^N w_{ij} \tilde{y}_{gvar,jt}$, and $\tilde{y}^*_{hp,it} = \sum_{j=1}^N w_{ij} \tilde{y}_{hp,jt}$.

Note that in contrast to $\tilde{y}_{hp,it}$, the output gap measures, $\tilde{y}_{gvar,it}$ will reflect the structure of the full GVAR model of the economy, including the variables chosen, the lag orders selected, the cointegrating relations imposed and the treatment of deterministic elements. Changing any of these will change the estimated decomposition. This seems a desirable feature as compared to statistical procedures like the HP filter where the estimate is invariant to the form of the economic model. However, where there is uncertainty about the form of the model and the appropriate sample to be used for estimation, in these circumstances one could use some form of model averaging to obtain a more robust decomposition. In the empirical exercise we shall use the published DdPS model for the decomposition, and leave the use of more robust approaches to future research.

5.2 Estimates of the Trends and Cycles

One reason that BN trends are not widely employed is that in the univariate BN decomposition much of the variation in output comes from variation in the trend and the cyclical component is small and noisy. This is in contrast to the smooth trends produced by the HP filter and unobserved-components, UC, models. However, this is a property of a univariate approach: if a single series is a pure random walk, the long-horizon forecast will always be the current value. This lack of smoothness need not carry over to a multivariate system, where the long horizon forecast for, say output, will reflect the information in the other variables and the cointegrating relations. This is the case here. Figure 1 shows actual and the permanent components of log US GDP. The permanent component is clearly quite smooth. This is generally the case for most of the countries, but is not universally so, e.g. the permanent components for the UK and Japan show a lot of variation.

Figure 1. US Log Output, Y, and BN Trend, Y_P

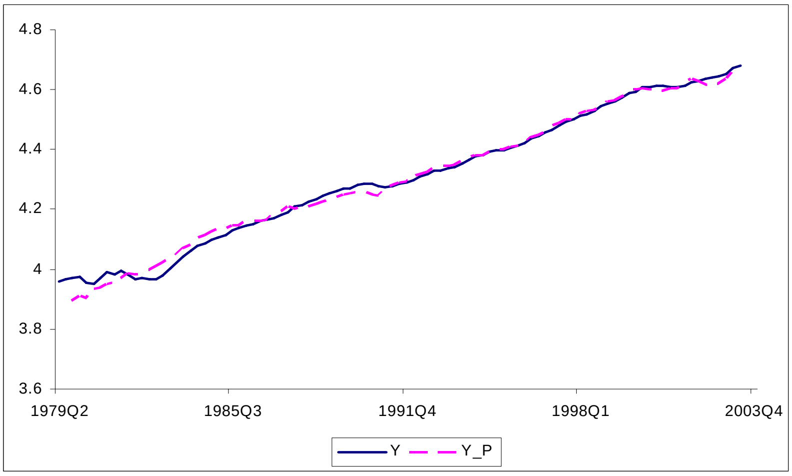
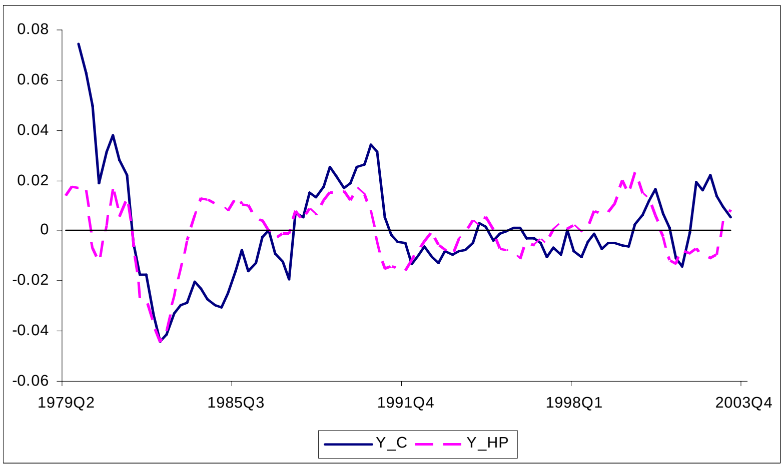


Figure 2 shows the estimates for the US of the BN cyclical component and the HP cyclical component (calculated letting the smoothing parameter equal 1600). While they show similarities, they also show clear differences. For instance, the HP estimate shows the recovery from the early 1980s recession happening much earlier than the BN estimate. This may be because the HP estimate, being a two sided filter can use information about the future recovery, which is not available to the BN estimate or economic agents.

Figure 2. BN (Y_C) and HP (Y_HP) Estimates of the US Output Cycle



We investigate the Phillips curve in more detail below but it is interesting to note that across the 26 countries the average correlation between inflation and the BN cyclical component of output is

0.17, which is significantly different from zero and the correlation is negative in only six countries: Argentina, Brazil, Chile, Peru, Singapore and India.

An issue that has attracted a lot of attention in the literature is the correlation between the innovations in the trend and cycle. For the univariate case, there is a perfect negative correlation between estimated BN trend and cycle innovations, whereas the unobserved-component, UC, model imposes zero correlation between the trend and cycle innovations, so the variance of output is the sum of the variance of the permanent and the cyclical components. This is a testable restriction, which is rejected by Morley, Nelson and Zivot (2003). They estimate the correlation to be -0.9 for a univariate model of US GDP and argue that the strong negative correlation strengthens the case for the importance of real shocks. For instance, a positive shock to productivity will immediately shift the long-run component of output upwards, leaving actual output below trend (a negative cyclical component) till it catches up. In the multivariate case, there are no explicit restrictions on the correlation of permanent and transitory components - they depend on the parameters of the model. The innovations to trend and cycle for a particular variable will be complicated functions of the innovations to all the variables, thus there is no simple representation, however we can look at the correlations between the changes.

The growth rate of actual output can be decomposed into the change in the permanent component and the change in the cyclical component, call the steady state y_{it}^P and the deviation from steady state $\widetilde{y}_{it}$ then the change in log output

$$\Delta y_{it} = \Delta y_{it}^P + \Delta \widetilde{y}_{it}.$$

Whereas actual output and the permanent component of output are non stationary, the changes in output and its permanent component and the cyclical component will be stationary. A number of simple correlations between Δy_{it}, Δy_{it}^P, $\Delta \widetilde{y}_{it}$ and $\widetilde{y}_{it}$ are given in Table 2 for 10 industrial countries.

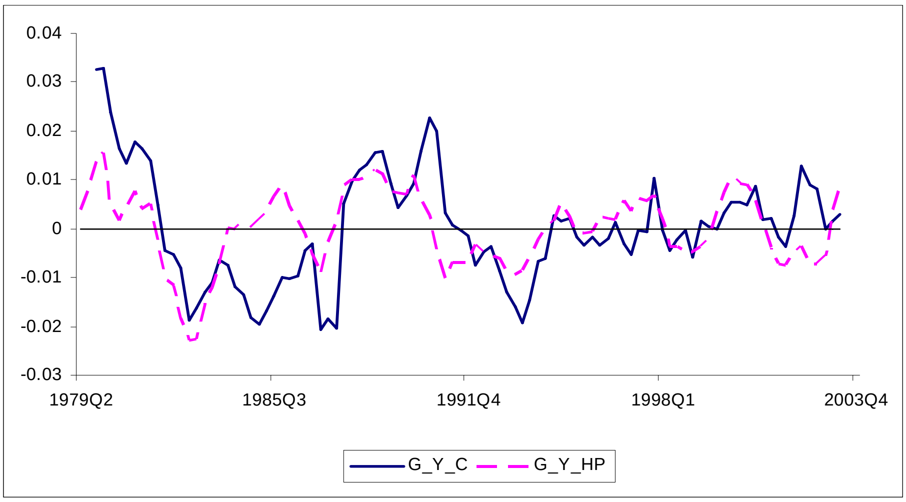
Table 2. Correlations for Output Decomposition, 10 Industrial Countries

Country	$Cor(\Delta y_{it}, \Delta y_{it}^P)$	$Cor(\Delta y_{it}, \Delta \widetilde{y}_{it})$	$Cor(\Delta \widetilde{y}_{it}, \Delta y_{it}^P)$	$Cor(\Delta y_{it}, \widetilde{y}_{it})$
US	0.39	0.51	-0.59	-0.26
Euro Area	0.19	0.46	-0.79	-0.13
Japan	-0.16	0.49	-0.94	-0.17
UK	-0.08	0.24	-0.99	-0.24
Canada	0.30	0.49	-0.67	-0.18
Australia	0.46	0.38	-0.65	-0.08
Sweden	0.10	0.87	-0.41	0.37
Switzerland	0.25	0.24	-0.88	-0.01
Norway	0.61	0.78	-0.01	0.40
New Zealand	0.62	-0.01	-0.80	-0.32

The correlations between the growth rate and the change in the permanent components are positive except in Japan and the UK, which did not show smooth trends and where the negative correlation between the changes in cyclical and permanent components is large. Among all the countries, Mexico and Peru also showed negative correlations. The correlation between growth and the change in the cyclical components is positive except in New Zealand. The correlations between the changes in the permanent and cyclical components is always negative in the whole sample of countries. The correlation between growth and the cyclical component is negative except in Norway and Sweden (and Peru, India and Turkey, which are not shown in Table 2). The actual adjustment processes reflect the complex dynamics of the whole system, which cannot be captured by simple correlations. This result looks sensible as large negative cyclical components should be removed by an increase in growth. Moreover, there is substantial evidence for a negative correlation between changes in the permanent and transitory component, but to a lesser extent than suggested by the univariate BN decomposition.

A natural measure of the world cycle is the weighted average of the cyclical components of GDP in each country, where the weights are based on PPP GDP. This is shown in Figure 3 for both the BN and HP measures of the cycles. As with the US case, there are some similarities and some differences between the two measures. The biggest difference is in the mid-1980s, when the HP shows a recovery not supported by the BN. The large reduction in the variance from the mid-1990s is marked. Although the sample contains a number of countries hit by the Asian crisis of 1997, their weight is quite small, though both measures show a drop around 1997. The corresponding permanent component of the BN measure is quite smooth.

Figure 3. BN (G_Y_C) and HP (G_Y_HP) Estimates of Global Cycle



To examine the possibility of common world cycles in more detail, we examined the correlation of the cyclical components of variables across countries. We call the correlation between country i and country j, r_{ij}, for $i, j = 1, 2, ..N$, then we calculate the average correlation between country i and the others $N^{-1} \sum_{j \neq i} r_{ij}$. We do this for various groups of countries. For actual GDP the average correlations are of course very high because of the trend. Similar estimates are obtained for cross correlations of the permanent components. The average correlation for the growth rates of actual output were much smaller varying from 0.16 for the US and 0.15 for the euro area, to 0.01 for India and the Philippines and 0.02 for China. The average correlation for the cyclical components was larger than the average correlation of the growth rates in 17 of the 26 countries, but was negative for China, Mexico and Peru. The results in DdPS also indicated that there have been breaks in the variances for many countries in the late 1980s or early 1990s, thus we examined the results for pre and post 1992.[13] The pre 1992 average correlation of the cyclical component was greater than that for the whole period in 17 of the 26 countries, the exceptions being New Zealand and some emerging markets. When attention is confined to 10 industrialised countries, the average correlations of the cyclical components of output were much higher than with all countries, ranging from 0.13 for New Zealand to 0.44 for Switzerland and 0.43 for US and euro area. The average correlation among the European countries was higher still.

For the other variables, we will focus on the ten industrial countries in our sample. Average correlations for actual inflation rates varied from 0.64 for the euro area (0.6 for US) to 0.45 for Japan and Switzerland. Average correlations for the permanent component varied from 0.46 for the US (0.39 for the euro area) to 0.00 for Australia. Average correlations for the cyclical component varied from 0.66 for the euro area (0.62 for US) to 0.44 for Japan. There is clearly a global component to inflation, rather more in the cyclical than the permanent components.

The average correlation for the permanent component of equities was high (>0.75) everywhere, except for Japan 0.25; the average correlation for the transitory component was larger than that for the change in equities (0.68 as compared to 0.36 in Switzerland), or in the few cases where it was not the US, Australia and New Zealand, the difference was very small <0.03. Thus if there is a common cycle the transitory component of equities seems to pick it up better than the returns themselves. For both short- and long-term interest rates, there seems to be a fairly high average correlation of the actual values and both the permanent and transitory components, with the correlations tending to be higher for the long rates. For instance, for the US the average correlation for long-term interest rates is 0.83, that of their permanent components is 0.60; that of their transitory components 0.91 and that of their changes 0.45 (all these average correlations are much smaller using just data up to 1992). This is consistent with the permanent components reflecting country effects more and thus being less correlated, while the transitory components reflect financial markets effects. The exchange rates are all against the US dollar so the average correlations for the levels of the exchange rate tend to be quite high, 19 of the 25 countries greater than 0.5, this all comes from the permanent component, the highest average correlation between transitory components is Chile at 0.17 and Turkey at -0.17.

Thus there does seem to be a world cycle as represented by the average correlations of the transitory components of output and a number of other interesting similarities. These global factors in output and inflation can thus play a role in identifying the NKPC.

[13] This is in line with the evidence on the "great moderation" found for the US.

6 Estimates of the NKPC

Following the discussion in Section 3, we begin with a standard NKPC equation,

$$\pi_{it} = \beta_{bi}\pi_{i,t-1} + \beta_{fi}E(\pi_{i,t+1} \mid \Im_{i,t-1}) + \gamma_i \widetilde{y}_{it} + \varepsilon_{it},$$

where we treat steady state inflation as a constant[14] and which we estimate with both measures of the output gap: either using the decomposition from the GVAR, $\widetilde{y}_{gvar,it}$ or the HP filter $\widetilde{y}_{hp,it}$. We then consider various extended models, discussed in Section 3, which include, country specific measures of foreign inflation and the output gap and domestic interest rates. The most general form is

$$\pi_{it} = \beta_{bi}\pi_{i,t-1} + \beta_{fi}E(\pi_{i,t+1} \mid \Im_{i,t-1}) + \beta_{*i}\pi^*_{it} + \gamma_i \widetilde{y}_{it} + \gamma_{*i}\widetilde{y}^*_{it} + \delta_i r_{it} + \varepsilon_{it}.$$

Although the data are seasonally adjusted, some residual seasonality seems to be present. All regressions are therefore run including a constant and three seasonal dummies in the NKPC equations.

Since identification of the NKPC depends on the nature of the process generating the output gap variable, we first estimated an $AR(2)$ model for the output gap, for the 26 countries of our sample using both the $\widetilde{y}_{hp,it}$ and the $\widetilde{y}_{gvar,it}$ measures. With the $\widetilde{y}_{gvar,it}$ measure only 6 of the 26 estimates of ρ_2 were significant, thus identification is clearly an issue. With the $\widetilde{y}_{hp,it}$ measure 18 of the 26 estimates were significant, though there is a danger, as Harvey and Jaeger (1993) point out, that the HP filter can induce spurious serial correlation. Thus identification is likely to be weak if we confine our empirical analysis to the standard NKPC. Recall that unless there are significant feedbacks from second or higher ordered lagged inflation to the output gap, the use of second or higher order lags of inflation as instruments cannot help identification. Also, even if such lagged inflation terms are included in the output gap equation, their use as instruments require their exclusion from the NKPC equation, which seems rather *ad hoc*. Therefore for identification, instruments other than the lagged values of output gap are needed. The discussion in Section 3 suggested that the appropriate instruments were $\pi_{i,t-1}$, π^*_{it}, $\pi^*_{i,t-1}$, $\widetilde{y}_{i,t-1}$, $\widetilde{y}^*_{it}$, $\widetilde{y}^*_{i,t-1}$, plus intercept and seasonals. In the case where the NKPC is extended to include the interest rate, we augment these instruments with $r_{i,t-1}$ and r^*_{it}.[15]

The NKPC equations are estimated for all 26 countries in the GVAR model over the period 1980Q2-2003Q4. Both the HP filter and the GVAR estimates of the trend may give poor estimates near the beginning of the sample because of sensitivity to initial values. However, we obtained similar results using shorter samples.

To deal with the residual serial correlation in the GMM estimation of the NKPC equations, all inference for individual countries is based on Newey West standard errors (using Bartlett weights with a window size of 8 quarters).

[14] We also investigated using deviations of inflation from its BN steady state, but for comparability with the literature we focus on the case where the steady state of inflation rate is treated as a constant. The regressions to be reported below all include an intercept but this is excluded to simplify the exposition.

[15] Although it might be optimal to use different sets of instruments for different models, for comparability we use the same instruments for the open and the closed economy versions of the NKPC.

Given the considerable degree of heterogeneity across countries, and to give an overall assessment of the results, we begin with the Mean Group IV (MGIV) estimates.[16] Also, since over the sample under consideration the Latin American economies have experienced very high rates of inflation and this could affect the average estimates, we report the MGIV estimates both for all 26 countries and for a sample of 21 countries that exclude the 5 Latin American economies.

6.1 MGIV Estimates

Consider first the MGIV estimates for the standard NKPC using the HP measure of the output gap, $\tilde{y}_{hp,it}$. The estimates are summarised in Table 3. The average estimate of the coefficient of $\tilde{y}_{hp,it}$ is negative and statistically insignificant for all the 26 countries and the sample excluding Latin America.[17] Most of the weight is on forward inflation and the estimate of $\beta_f + \beta_b$ is 1.02 in both samples, slightly larger than unity implying a multiplicity of solutions, though it is not that different from unity. When the foreign output gap is included the coefficient of the domestic gap remains negative and the foreign gap is insignificant. These estimates are in line with the ones obtained in the literature. Some authors, notably Gali, Gertler and Lopez-Salido (2005) have argued against the use of output gap measures, and instead have advocated that real marginal cost measures should be used. Here we stay with the output gap measure but instead consider the estimates obtained from the GVAR which are likely to be less ad hoc and globally more coherent.

Table 3. Mean Group IV Estimates of the NKPC for $\tilde{y}_{it} = \tilde{y}_{hp,it}$

Country Groups	$E(\pi_{i,t+1}\|\Im_{i,t-1})$	$\pi_{i,t-1}$	$\tilde{y}_{hp,it}$	$\tilde{y}^*_{hp,it}$
All 26 Countries	0.87	0.15	-0.07	
	(12.01)	(2.93)	(-0.95)	
Excluding 5 Latin American	0.89	0.13	-0.02	
	(11.21)	(2.60)	(-0.84)	
All 26 Countries	0.87	0.14	-0.05	0.05
	(11.26)	(2.69)	(-0.75)	(0.82)
Excluding 5 Latin American	0.89	0.12	0.00	0.01
	(10.41)	(2.25)	(-0.19)	(0.20)

Note: The MGIV estimates are based on individual country IV estimates of the NKPC regression estimated over the period 1980Q2-2003Q3 using the instrument set $\pi_{i,t-1}$, π^*_{it}, $\pi^*_{i,t-1}$, $\tilde{y}_{hp,i,t-1}$, $\tilde{y}^*_{hp,it}$, $\tilde{y}^*_{hp,i,t-1}$. An intercept and 3 seasonal dummies are included both in the regression and instrument set. The HP cyclical component was calculated setting the smoothing parameter equal to 1600.

[16] Mean Group estimates based on OLS regressions are introduced in Pesaran and Smith (1995b). The MGIV estimates and their standard errors are similarly computed.

[17] Note that the standard errors of the MGIV estimator are robust to residual serial correlation and/or error heteroskedasticity in the individual country equations.

Table 4 gives the estimates using the GVAR measure of the output gap. Unlike the HP measure the coefficient of the GVAR measure is positive in both samples and on the edge of significance for the sample of countries excluding Latin America. The estimate of $\beta_f + \beta_b$ is less than but not significantly different from unity. In terms of matching theoretical expectations the coefficient of the GVAR measure of the output gap performs better than the HP measure, having the correct sign, but as is common in the literature these coefficients are not well determined. The coefficients on future and past inflation and the output gap using the GVAR measure are very similar to those reported elsewhere in the literature, though these are usually estimates for the US rather than average measures over many countries. Adding the foreign output gap reduces the significance of the domestic output gap, and the foreign output gap is significant in the sample excluding Latin America. Thus we can reproduce some of the mixed evidence noted in the literature for the importance of the foreign output gap.

Table 4. Mean Group IV Estimates of the NKPC for $\tilde{y}_{it} = \tilde{y}_{gvar,it}$

Country Groups	$E(\pi_{i,t+1}\|\Im_{i,t-1})$	$\pi_{i,t-1}$	$\tilde{y}_{gvar,it}$	$\tilde{y}^*_{gvar,it}$
All 26 Countries	0.79	0.19	0.03	
	(14.54)	(4.75)	(0.74)	
Excluding 5 Latin American	0.81	0.17	0.05	
	(13.54)	(4.24)	(1.92)	
All 26 Countries	0.84	0.16	0.00	0.09
	(14.29)	(3.82)	(-0.02)	1.49
Excluding 5 Latin American	0.86	0.14	0.04	0.04
	(13.89)	(3.42)	(1.29)	(2.17)

Note: The MGIV estimates are based on individual country IV estimates of the NKPC regression estimated over the period 1980Q2-2003Q3 using the instrument set $\pi_{i,t-1}$, π^*_{it}, $\pi^*_{i,t-1}$, $\tilde{y}_{gvar,i,t-1}$, $\tilde{y}^*_{gvar,it}$, $\tilde{y}^*_{gvar,i,t-1}$. An intercept and 3 seasonal dummies are included both in the regression and instrument set.

Table 5 adds foreign inflation and the domestic interest rate to the individual country NKPC equations. When foreign inflation and the foreign output gap are added to the equation, foreign inflation is significant while the foreign output gap is not in both country groupings. The size of the coefficient of the domestic output gap increases substantially and it is on the edge of significance in the sample excluding Latin America. As one might expect, the coefficients on the domestic inflation terms get smaller, future domestic inflation becomes insignificant, and their sum is substantially less than unity. When domestic interest rates are included (and lagged domestic rates and current foreign rates are added to the instrument set) they are significant in both samples. The coefficients of the output gap do not change very much, but become significant in the full sample (which is now 25 countries, since there is no interest rate data for Saudi Arabia). The coefficient of foreign inflation is smaller and just on the edge of significance in the full sample. The foreign output gap remains statistically insignificant. We also run IV regressions with changes in the log of the effective exchange rate. When the exchange rate variable was added to the baseline NKPC, the MGIV estimates of its coefficient were significant in both samples. However, when it was included with foreign inflation it was not significant, consistent with the incomplete pass-through argument.[18]

Table 5. MGIV Estimates of the NKPC Including Foreign Inflation, Foreign Output and Domestic Interest Rates

Country Groups	$E(\pi_{i,t+1}\|\Im_{i,t-1})$	$\pi_{i,t-1}$	$\tilde{y}_{gvar,it}$	$\tilde{y}^*_{gvar,it}$	π^*_{it}	r^S_{it}
All 26 Countries	0.06	0.41	0.25	-0.01	0.41	
	(0.22)	(3.98)	(1.55)	(-0.14)	(2.95)	
Excluding 5 Latin American	0.03	0.37	0.13	0.03	0.35	
	(0.09)	(3.13)	(1.93)	(1.35)	(2.28)	
All 25 Countries	0.23	0.19	0.14	0.00	0.19	0.37
	(1.59)	(6.46)	(2.04)	(0.05)	(1.80)	(2.53)
Excluding 5 Latin American	0.37	0.18	0.09	0.04	0.35	0.13
	(2.47)	(5.45)	(1.74)	(1.59)	(4.31)	(1.79)

Note: The MGIV estimates are based on individual country IV estimates of the NKPC regression estimated over the period 1980Q2-2003Q3 using the instrument set $\pi_{i,t-1}$, π^*_{it}, $\pi^*_{i,t-1}$, $\tilde{y}_{gvar,i,t-1}$, $\tilde{y}^*_{gvar,it}$, $\tilde{y}^*_{gvar,i,t-1}$ for the cases without domestic interest rates in the equation, and that set plus $r_{i,t-1}$ and r^*_{it} for the equations with interest rates. An intercept and 3 seasonal dummies are included both in the regression and instrument sets.

These results indicate that: the foreign factors aid identification; the GVAR measure of $\tilde{y}_{it}$ performs better than the HP measure in terms of the sign and size of its coefficient; foreign inflation is significant and that when it is included the coefficients on domestic inflation are considerably reduced in size and that interest rates are also significant. However, when foreign inflation is included the foreign output gap is not significant.

6.2 US and Euro Area Estimates

To illustrate the performance of some country specific estimates Table 6 gives estimates for the US and Table 7 for the euro area, using the GVAR measure of the output gap.[19] In the NKPC the HP measure gave positive but insignificant estimates for both the US and euro area, unlike the MGIV estimates where the average coefficient was negative. For the individual countries, we calculated the Generalised R^2 for IV regressions of Pesaran and Smith (1994); a test for fourth order serial

[18] These results are available from the authors on request.

[19] A complete set of individual country results are available from the authors on request.

correlation, and the Sargan or Hansen's J test for overidentifying restrictions. In each case four specifications are presented. The standard NKPC is given first, then the foreign output gap, foreign inflation and the domestic interest rate are added sequentially. First consider the US estimates. The coefficient of the GVAR estimate of the domestic gap is always positive and is significant in the extended specifications which include foreign inflation. The coefficient of the foreign output gap is always insignificant and positive except in the specification also including foreign inflation and domestic interest rates, where it is negative and insignificant. The coefficient of foreign inflation is always positive and is significant when domestic interest rates are not included. Domestic interest rates are significant when included with the other variables. The coefficient of future inflation is positive in the first two specifications though not significant, but is negative though not significant in the final two specifications. This might be because foreign inflation and domestic interest rates are acting as better proxies for expected inflation than the predicted value of inflation one quarter ahead.

All the equations show significant serial correlation and the over-identifying restrictions are rejected in the first two equations which do not contain foreign inflation. Given the dominant position of the US in the world financial system, it might be argued that the foreign interest rate cannot be regarded as exogenous. The equation in the fourth row of Table 6 was re-estimated using lagged rather than current foreign rates as an instrument. The results were almost identical, with the coefficients on the output gap and domestic interest rates slightly smaller, but still both very significant, with the other variables remaining insignificant. However, unlike the version using the current value of the foreign interest rate, it just failed the Sargan test. Except for the negative (though insignificant) coefficient on future inflation the US estimates are broadly sensible and suggest, depending on specification, significant roles for the output gap and either foreign inflation or domestic interest rates.

Table 6. US IV Estimates

$E(\pi_{i,t+1} \| I_{i,t-1})$ β_{if}	$\pi_{i,t-1}$ β_{ib}	$\tilde{y}_{gvar,it}$ γ_i	$\tilde{y}^*_{gvar,it}$ γ_{*i}	π^*_{it} β_{*i}	r^S_{it} δ_i	GR^2	$\chi^2_{SC}(4)$	χ^2_{SM}
0.31	0.46	0.04				0.56	$37.78^\dagger$	$11.38^\dagger$
(1.78)	(6.90)	(1.70)						
0.32	0.46	0.04	0.02			0.56	$38.26^\dagger$	$11.06^\dagger$
(1.75)	(6.64)	(1.41)	(0.51)					
-0.54	0.61	0.13	0.02	0.15		0.61	$17.14^\dagger$	0.62
(-1.51)	(3.40)	(2.12)	(0.36)	(2.19)				
-0.54	0.28	0.17	-0.03	0.06	0.47	0.65	$26.42^\dagger$	2.01
(-1.84)	(1.56)	(3.07)	(-0.75)	(1.54)	(3.54)			

Note: The underlying Phillips Curve regressions are estimated by IV over the period 1980Q2-2003Q3 using the instrument set $\pi_{i,t-1}$, π^*_{it}, $\pi^*_{i,t-1}$, $\tilde{y}_{gvar,i,t-1}$, $\tilde{y}^*_{gvar,it}$, $\tilde{y}^*_{gvar,i,t-1}$ for the cases without domestic interest rates in the equation, and that set plus $r_{i,t-1}$ and r^*_{it} for the equations with interest rates. An intercept and 3 seasonal dummies are included both in the regression and instrument sets. GR^2 is the Pesaran and Smith (1994) Generalised R^2. $\chi^2_{SC}(4)$ is a test for fourth order serial correlation, χ^2_{SM} is Sargan's test for overidentifying restrictions, which has degrees of freedom 3, 2, 1 and 2 respectively. For the misspecification tests †indicates significant at the 5% level.

Now consider the euro area estimates given in Table 7. The coefficient of future inflation is positive and significant except in the specification that includes foreign inflation as well as the domestic and foreign output gaps, where it is large and negative though insignificant. The coefficient of the output gap is negative in the first two specifications, positive in the last two, but never significant. The foreign output gap has negative though insignificant coefficients in all the specifications where it is included. Domestic interest rates are not significant. The overidentifying restrictions are never rejected. The euro estimates show much less significant determinants of domestic inflation than the US estimates. To a certain extent this is not surprising, prior to the euro being established in 1999, the euro area data are constructed from aggregates of heterogeneous countries.

Table 7. Euro Area IV Estimates

$E(\pi_{i,t+1}\|I_{i,t-1})$ β_{if}	$\pi_{i,t-1}$ β_{ib}	$\tilde{y}_{gvar,it}$ γ_i	$\tilde{y}^*_{gvar,it}$ γ_{*i}	π^*_{it} β_{*i}	r^S_{it} δ_i	GR^2	$\chi^2_{SC}(4)$	χ^2_{SM}
0.66	0.34	-0.02				0.84	25.68†	6.05
(2.08)	(1.36)	(-0.68)						
0.61	0.37	-0.00	-0.01			0.84	24.58†	5.92
(2.32)	(1.76)	(-0.09)	(-0.90)					
-2.13	2.39	0.33	-0.12	0.16		0.85	2.76	0.01
(-0.55)	(0.80)	(0.92)	(-1.11)	(0.81)				
0.78	0.25	0.02	-0.03	0.05	-0.06	0.86	27.18†	4.30
(2.49)	(1.41)	(0.69)	(-1.51)	(0.92)	(-0.45)			

Note: The underlying Phillips Curve regressions are estimated by IV over the period 1980Q2-2003Q3 using the instrument set $\pi_{i,t-1}$, π^*_{it}, $\pi^*_{i,t-1}$, $\tilde{y}_{gvar,i,t-1}$, $\tilde{y}^*_{gvar,it}$, $\tilde{y}^*_{gvar,i,t-1}$ for the cases without domestic interest rates in the equation, and that set plus $r_{i,t-1}$ and r^*_{it} for the equations with interest rates. An intercept and 3 seasonal dummies are included both in the regression and instrument sets. GR^2 is the Pesaran and Smith (1994) Generalised R^2. $\chi^2_{SC}(4)$ is a test for fourth order serial correlation, χ^2_{SM} is Sargan's test for overidentifying restrictions, which has degrees of freedom 3, 2, 1 and 2 respectively. For the misspecification tests † indicates significant at the 5% level.

7 Conclusion

In this paper we have highlighted three issues that surround the NKPC and its estimation in a global context; namely identification, measurement of steady states and the role of global variables, particularly foreign inflation. We have argued that the measurement of the steady state and identification of NKPC need to be approached from an economic theory perspective and cannot be resolved in a satisfactory manner by resort to purely statistical reasoning. To determine instrument validity requires explicit solution of the rational expectations model and identification will depend on the form of the model for the driving processes. Similarly, to determine steady states requires an explicit long-run economic model. Unlike the HP filter, the BN estimates of permanent and cyclical components are model dependent. This seems to be a desirable feature as our estimate of steady state should reflect economic information.

The global perspective, using the GVAR as a framework, contributes to all three issues and this was illustrated using estimates of the NKPC from 26 countries. The GVAR provides global factors that are valid instruments and help alleviate the weak instrument problem. The global perspective provides a way to calculate theoretically consistent steady states from the BN decomposition that reflect global influences and any long-run theoretical relationships embodied in the cointegrating relations. Output gaps measured in this way produced results more in accord with the theory than those calculated using the HP filter. The global perspective provides a natural way for foreign inflation to enter the NKPC and we found foreign inflation, and domestic interest rates, to be significant, but not the foreign output gap once foreign inflation was included.

References

Bai, J., Ng, S., 2007. Instrumental variable estimation in a data rich environment. Manuscript, University of Michigan.

Batini, N., Jackson, B., Nickell, S., 2005. An open-economy new Keynesian Phillips curve for the U.K. Journal of Monetary Economics 52, 1061–1071.

Beveridge, S., Nelson, C.R., 1981. A new approach to the decomposition of economic time series into permanent and transitory components with particular attention to the measurement of the 'business cycle'. Journal of Monetary Economics 7, 151-174.

Beyer, A., Farmer, R.E.A., Henry, J., Marcellino, M., 2007. Factor analysis in a model with rational expectations. NBER Working Paper 13404.

Binder, M., Pesaran, M.H., 1995. Multivariate rational expectations models and macroeconomic modelling: a review and some new results. In : Pesaran, M.H., Wickens, W. (Eds.), Handbook of Applied Econometrics Vol. 1, 655-673.

Binder, M., Pesaran, M.H., 1997. Multivariate linear rational expectations models: characterization of the nature of the solutions and their fully recursive computation. Econometric Theory 13, 877-888.

Chowdhury, I., Hoffman, M., Schabert, A., 2006. Inflation dynamics and the cost channel of monetary transmission. European Economic Review 50, 995-1016.

Chudik, A., Pesaran, M.H., 2007. Infinite Dimension VARs and Factor Models. Manuscript, Faculty of Economics, University of Cambridge.

Dees, S., di Mauro, F., Pesaran, M.H., Smith, L.V., 2007. Exploring the international linkages of the euro area: a global VAR analysis. Journal of Applied Econometrics 22, 1-38.

Dees, S., Holly, S., Pesaran, M.H., and Smith, L.V., 2007. Long run macroeconometric relations in the global economy. *Economics* - The Open-Access, Open-Assessment E-Journal.

Friedman, M., 1968. The role of monetary policy. American Economic Review 58, 1-17.

Gali, J.,Gertler, M.,1999. Inflation dynamics: A structural econometric analysis. Journal of Monetary Economics 44, 195-222.

Gali, J., Gertler, M., Lopez-Salido, J.D., 2005. Robustness of the estimates of the hybrid New Keynesian Phillips curve. Journal of Monetary Economics 52, 1107-1118.

Garratt, A., Robertson, D., Wright, S., 2006. Permanent vs transitory components and economic fundamentals. Journal of Applied Econometrics 21, 521-542.

Hall, A.R., 2005. Generalized Method of Moments. Oxford University Press.

Harvey, A. C., Jaeger, A., 1993. Detrending, stylized facts and the business cycle. Journal of Applied Econometrics 8, 231-247.

Ihrig, J., Kamin, S.B., Lindner, D., Marquez, J., 2007. Some simple tests of the globalization and inflation hypothesis. Board of Governors of the Federal Reserve System, International Finance Discussion Papers.

Kapetanios, G., Marcellino, M., 2007. Factor-GMM estimation with large sets of possibly weak instruments. Manuscript, Queen Mary University of London.

King, R., Watson, N., 1994. The post-war US Phillips curve: a revisionist econometric history. Carnegie-Rochester Series on Public Policy 41, 157-219.

Lee, J., Nelson, C.R., 2007. Expectations horizon and the Phillips curve: the solution to an empirical puzzle. Journal of Applied Econometrics 22, 161-178.

Li, H., 2007. Small-sample inference in rational expectations models with persistent data. Economics Letters 95, 203-210.

Lucas, R. E., 1972. Expectations and the neutrality of money. Journal of Economic Theory 4,103-124.

Mavroeidis, S., 2005. Identification issues in forward looking models estimated by GMM with an application to the Phillips Curve. Journal of Money Credit and Banking 37, 421-448.

Monacelli, T., 2005. Monetary policy in a low pass through environment. Journal of Money Credit and Banking 37, 1047-1066.

Morley, J.C., Nelson, C.R., Zivot, E., 2003. Why are the Beveridge-Nelson and unobserved components decomposition of GDP so different? Review of Economics and Statistics 85, 235-243.

Pagan, A., Pesaran M.H., 2007. Econometric analysis of structural systems with permanent and transitory shocks. Revised version of the Cambridge Working Papers in Economics 0704, Faculty of Economics, University of Cambridge.

Phelps, E., 1967. Phillips curves, expectations of inflation and optimal unemployment over time. Economica 34, 254-281.

Phillips, W.A., 1958. The relationship between unemployment and the rate of change of money wages 1862-1957. Economica 34, 254-81.

Pesaran, M.H., 1981. Identification of rational expectations models. Journal of Econometrics 16, 375-98.

Pesaran, M.H., 1987. The Limits to Rational Expectations. Basil Blackwell, Oxford.

Pesaran, M.H., Smith, R.J., 1994. A generalised $\overline{R}^2$ criterion for regression models estimated by instrumental variable methods. Econometrica 62, 705-10.

Pesaran, M.H., Smith, R.P., 1995a. The natural rate hypothesis and its testable implications. In: Cross, R. (Eds.), The Natural Rate of Unemployment, 203-230. Cambridge University Press, Cambridge, UK.

Pesaran, M.H., Smith, R.P., 1995b. Estimation of long-run relationships from dynamic heterogeneous panels, Journal of Econometrics 68, 79-113.

Pesaran, M.H., Smith, R.P., 2006. Macroeconometric modelling with a global perspective. Manchester School, Supplement, 24-49.

Ravenna, F., Walsh, C.E., 2006. Optimal monetary policy with the cost channel. Journal of Monetary Economics 53, 199-216.

Schreiber, S., Wolters, J., 2007. The long-run Phillips curve revisited: is the NAIRU framework data-consistent? Journal of Macroeconomics 29, 355-367.