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Arnaud Chéron
Jean-Olivier Hairault
François Langot

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Institute for the Study
of Labor

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Arnaud Chéron
GAINS-TEPP, Université du Maine,
and EDHEC

Jean-Olivier Hairault
Paris School of Economics,
Université de Paris 1 Panthéon-Sorbonne
and IZA

François Langot
GAINS-TEPP, Université du Maine,
ERMES, Université de Paris 2,
CEPREMAP and IZA

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IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

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ABSTRACT

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This paper examines the age-related design of firing taxes by extending the theory of job creation and job destruction to account for a finite working life-time. We first argue that the potential employment gains related to employment protection are high for older workers, but higher firing taxes for these workers increase job destruction rates for the younger generations. On the other hand, age-decreasing firing taxes can lead to lower job destruction rates at all ages. Furthermore, from a normative standpoint, because firings of older (younger) workers exert a negative (positive) externality on the matching process, we find that the first best age-dynamic of firing taxes and hiring subsidies is typically hump-shaped. Taking into account distortions related to unemployment benefits and bargaining power shows the robustness of this result, in contradiction with the existing policies in most OECD countries.

JEL Classification: J22, J26, H55

Keywords: search, matching, endogenous destruction, older workers

Corresponding author:

Arnaud Chéron
Faculté de Sciences Economiques
Université du Maine
Avenue Olivier Messiaen
72000 Le Mans
France
E-mail: acheron@univ-lemans.fr

1 Introduction

Faced with low employment rates for older workers, most OECD countries have experimented with specific older worker employment protection in the form of taxes on firing and subsidies on hiring (see OECD [2006]). Additional penalties for firms that lay off older workers have been introduced, either in the form of a tax or higher social security contributions (*e.g.* Austria, Finland, France and Spain) or in the form of paying part or all of the costs of outplacement services to help workers find new jobs (*e.g.* Belgium and Korea). Older workers are also protected to a greater extent than younger workers by tenure-related provisions: workers with longer tenure (more likely to be older workers) are often required to be given longer notice periods in the case of dismissals and higher severance payments. In Sweden, the Last In-First Out rule implies that older workers are more protected in the event of lay-offs than younger workers since they usually have longer tenure. At the same time, various hiring and wage subsidy schemes encouraging employers to hire and to retain older workers have been introduced. The hirings of older workers lead to permanent reduction or exonerations in social security contributions in Austria, Belgium, Netherlands, Norway and Spain. Direct subsidies to employers who hire older workers also exist in Denmark, Germany, Japan and Sweden. Employment subsidy schemes for older workers (in-work benefits) are present in Austria, Germany, Japan, the United Kingdom and the United States. The main objective of these policies is to protect older workers from unemployment because these workers suffer from lower job finding rates. This paper aims at examining the positive and normative implications of these policies which increase the employment protection with the age of the workers.

The existence of higher taxes for firing and higher subsidies for hiring older workers in most developed countries calls indeed for a theory of age-dependent employment protection. Since the seminal work of Mortensen and Pissarides [1999], it is well-established that employment protection reduces job destruction but also reduces job creation. Combining firing taxes with hiring subsidies is then thought of as corresponding to a consistent policy set to boost the employment rate. This paper calls into question the higher employment protection for older workers by extending the theory of job creation and job destruction over a finite working life-time. What are the predicted effects of this policy on the older workers? If older workers benefit from the higher protection put in place in most developed countries, what are the consequences of this policy on younger workers? Can this policy be legitimized by welfare arguments? Surprisingly enough, the theoretical foundations and implications of the age-dependent employment protection have not been yet addressed. This contrasts with the extensive use of this policy in OECD countries (OECD [2006]). From this point of view, our paper fills a gap.

Our analysis of employment protection includes both positive and normative issues which are successively addressed. Unlike the large literature following MP, we consider a life cycle setting characterized by an exogenous age at which workers exit the labor market. The only heterogeneity across workers is the distance to retirement. We also assume that firms cannot ex-ante age-direct their search, that is vacancies cannot be targeted at a specific age group. However, once the contact with a worker is made, according to the observed productivity of the job-worker pair, the firm can make the choice of not recruiting the worker, and this productivity threshold decision obviously depends on the age of the worker. These assumptions allow us to be consistent with existing legislation prohibiting age-discrimination such as in the US and European countries, together with the observed discrimination against older workers (Neumark [2001]).

The shorter distance to retirement is then the key point to understanding the economics of older worker employment. We think that it is the only intrinsic characteristic of the older workers common to all countries. We then consider that this is a natural starting point, to which other potential sources of heterogeneity across workers of different ages could be added[1]. Because the horizon of older workers is shorter, the life-cycle labor market equilibrium shows that firms invest less in labor-hoarding activities at the end of the life cycle. This explains that the separation rate increases with the age of the worker and why there is concern to protect older workers.

We then propose a positive analysis of the older worker employment protection. We first argue that the impact of a firing tax is greater for older workers than for younger ones. Indeed, at the end of the working cycle, introducing a firing tax increases the present firing cost without any future consequences on the job value as the worker will be retired in the following periods. For younger workers, the present firing cost increases, but the job value also decreases, as the firm rationally expects the future cost of the firing tax. In some sense, retirement allows firms to avoid the firing tax, leading them to more labor hoarding of older workers.[2] Ultimately, this tax can be high enough so that it implies a decreasing age-dynamic of job destruction

[1]For instance, one could think that older workers have more job-specific skills and therefore suffer more from losing their job. The amount of idiosyncratic uncertainty could be weaker for older workers. The bargaining power of younger and older workers is not necessarily the same.

[2]In an infinite horizon economy such as MP, a firing tax has no impact on job destructions if the interest rate is zero, whereas in our model it still has strong impact on a firm's decision to fire older workers.

rates, the opposite of the laisser-faire equilibrium. Secondly, we emphasize the age-differentiated effects of higher employment protection for older workers. Even though higher employment protection for older workers decreases the job destruction rate for this age group, it increases the firings of younger ones.[3] On the other hand, we show that an age-decreasing path would allow to unambiguously decrease job destruction rates for all workers, because it gives firms incentives to keep a worker by expecting lower firing taxes in the future, whatever the age of the worker. This shape of the firing taxes allows the policy maker to reach his main objective: to sustain the older worker employment rate.

The second step of our analysis is related to normative considerations. In the context of matching frictions and wage bargaining, it is now well-established that the decentralized equilibrium is in general not optimal, except when the Hosios [1990] condition holds.[4] In our finite working cycle setting, we consider that each firm is engaged in a non age-directed search. In that context, the age distribution of the unemployed workers determines the return on vacancies. We show that older worker job destructions then exert a negative externality on the employment of the younger unemployed workers which is not internalized by firms in the decentralized equilibrium. This is why the Hosios condition is not enough to restore the social optimality of the labor market equilibrium: there are too many (not enough) older (younger) worker job destructions even though the optimal profile of job destructions is typically increasing with age as in the equilibrium outcome.

This result provides welfare foundations for age-dependent employment protection. In a first best perspective, it is optimal to implement a hump-shaped age-dynamics for the firing taxes and hiring subsidies which is at odds with the existing policies. We then further explore the optimal age-dependent employment protection in the context of distortions related to bargaining power and unemployment benefits. We show that high unemployment benefits (or high worker bargaining power) can even require strictly age-decreasing firing taxes and hiring subsidies. This result reflects the fact that distortions related to unemployment benefits are higher for younger workers.

Overall, the existing higher employment protection for older workers seems to be at odds with the policy recommendations which can be deduced from a life-cycle version of the MP model. Age-decreasing employment pro-

[3] These results are consistent with the recent empirical evidence of Behagel, Crépon and Sédillot [2008] which stresses perverse effects related to the French experience of higher employment protection for workers over the age of 50.

[4] This condition states that the elasticity of the matching friction with respect to vacancies should be equal to the worker's bargaining power (Hosios [1990]). This efficiency result could also be obtained in a competitive search equilibrium (Moen [1997]).

tection until retirement could be more efficient in terms of both employment and welfare. This result seems counter-intuitive if the objective is to reduce the job destructions of older workers, but it is the natural implication of their shorter distance to retirement. Our paper then makes the case for a tax policy determined by age. This statement echoes recent studies which also recommend making taxation dependent on age. For instance, Kremer [1999] advocates a lower income tax on younger workers. We also echo recent studies which try to legitimize individual-differentiated taxation (see for instance Alesina, Ichino and Karabarbounis [2007] for a gender-based approach).

The next section presents the benchmark model and the age-dynamic properties of the equilibrium. The third section addresses the impact of age-dependent employment protection on employment. The fourth section deals with the social efficiency of the equilibrium and presents the optimal age-dependent employment protection both in a first and second best environment. The final section concludes.

2 A Finite-Horizon Economy with Endogenous Job Creations and Job Destructions

The primary objective of this section is to show that extending the job creation - job destruction approach to take into account a finite life-time horizon of workers gives rise to an increasing (decreasing) age-dynamic of job destructions (creations). This provides some foundations for the observed low employment rate of older workers which has led some OECD countries to implement firing taxes and hiring subsidies targeted at these workers.

We consider an economy with labor market frictions *à la* Mortensen - Pissarides [1994] with endogenous job creation and job destruction decisions, extended to take into account for a finite life time horizon for workers. That is, instead of assuming infinite-lived agents, our setting is characterized by a deterministic age T at which workers exit the labor market. Workers only differ respectively in their age i, and so in their distance to retirement. The model is in discrete time and at each period the older worker generation retiring from the labor market is replaced by a younger worker generation of the same size (normalized to unity) so that there is no labor force growth in the economy. The economy is at steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labor market as unemployed.

We consider (un)employment policies: *(i)* a firing cost F_i which refers both to the implicit costs in employment protection legislation and to the experience-rated unemployment insurance taxes, *(ii)* a hiring subsidy H_i, that is a lump sum paid to the employer when a worker of age i is hired, *(iii)* unemployment benefits z.

2.1 Shocks and workers flows

Firms are small and each has one job. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. Then, for age $i \in (2, T-1)$, employed workers are faced with layoffs when their job becomes unprofitable. At the beginning of each age[5], a job productivity ϵ is drawn in the general distribution $G(\epsilon)$ with $\epsilon \in [0,1]$. The firms decide to close down any jobs whose productivity is below an (endogenous) productivity threshold (productivity reservation) denoted R_i.

Job creation takes place when a firm and a worker meet. The flow of newly created jobs result from a matching function, $M(v,u)$, the inputs of which are vacancies v and unemployed workers u. M is increasing and concave in both its arguments, and with constant returns-to-scale. We assume that firms cannot ex-ante age-direct their search and that the matching function embodies all unemployed workers. The flow of newly created jobs also depends on productivity thresholds R_i^0 because it is assumed that productivity values ϵ are known after firm and worker have met. R_i^0 may differ from R_i since firms are not liable for the firing cost at this stage.

Let $\theta = v/u$ denote the tightness of the labor market. It is then straightforward to define the probability for unemployed workers of age i to be employed at age $i+1$, as $jc_i \equiv p(\theta)[1 - G(R_{i+1}^0)]$ with $p(\theta) = \frac{M(u,v)}{u}$. Similarly, we define the job destruction rate for an employed worker of age i as $jd_i = G(R_i)$.

At the beginning of their age i, the realization of the productivity level on each job is revealed. Workers hired when they were $i-1$ years old (at the end of the period) are now productive. Workers whose productivity is below the reservation productivity R_i (R_i^0) are laid off (not hired, for those previously unemployed). For any age i, the flow from employment to unemployment is then equal to $G(R_i)(1-u_{i-1})$. The other workers who remain employed $(1-G(R_i))(1-u_{i-1})$ can renegotiate their wage. The age-dynamic of unemployment is then given by:

$$u_{i+1} = u_i \left[1 - p(\theta)(1 - G(R_{i+1}^0))\right] + G(R_{i+1})(1-u_i) \quad \forall i \in (1, T-1) \qquad (1)$$

[5]This assumption is made to allow for analytical results. The persistency of shocks is left for a quantitative empirical investigation of the model's performance.

for a given initial condition $u_1 = 1$. The overall level of unemployment is $u = \sum_{i=1}^{T-1} u_i$, so that the average unemployment rate is $u/[T-1]$.

2.2 Hiring and firing decisions

Any firm is free to open a job vacancy and engage in hiring. c denotes the flow cost of recruiting a worker and $\beta \in [0,1]$ the discount factor. Let V be the expected value of a vacant position and $J_i^0(\epsilon)$ the value of a filled job with productivity ϵ:

$$V = -c + \beta q(\theta) \sum_{i=1}^{T-2} \left[\frac{u_i}{u} \left(\int_{R_{i+1}^0}^1 \left[J_{i+1}^0(x) + H_{i+1} \right] dG(x) + G(R_{i+1}^0) V \right) \right] + \beta(1-q(\theta))V$$

where the hiring subsidy H_i is received by the firm when the job becomes productive. At this time, the age of the hired worker is perfectly observed.

Beyond the traditional matching externality, the heterogeneity across ages in filled job values and in productivity thresholds implies the existence of intergenerational externalities in the search process: the more older unemployed workers there are, the less is the expected return on a vacancy.

The zero-profit condition $V = 0$ allows us to determine the labor market tightness from the following condition:

$$\frac{c}{q(\theta)} = \sum_{i=1}^{T-2} \left[\frac{u_i}{u} \left(\int_{R_{i+1}^0}^1 \left[J_{i+1}^0(x) + H_{i+1} \right] dG(x) \right) \right] \tag{2}$$

We follow MP by considering that the wage structure that arises as a Nash bargaining solution has two tiers.[6] The first tier wage reflects the fact that the hiring subsidy is directly relevant to the decision to accept a match and that the possibility of incurring firing costs in the future affects the value the employer places on the match. In turn, the second tier wage applies when firing costs are directly relevant to a continuation decision. For a bargained outsider wage $w_i^0(\epsilon)$, the expected value $J_i^0(\epsilon)$ of a filled job by a worker of age i is defined, for $\forall i \in [1, T-1]$, by:

$$J_i^0(\epsilon) = \epsilon - w_i^0(\epsilon) + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left(V - F_{i+1} \right) \tag{3}$$

[6] Recently, this wage setting rule has been somewhat disputed. Shimer [2005] argue that, in the conventional matching model, the wage rate is close to being as cyclical as productivity, so that the model does not have enough power to generate the observed cyclical volatility of unemployment. Using microeconometric evidence, Pissarides [2008] however shows that the cyclical volatility of wages in the canonical matching model is about the same as the one estimated for new matches. Furthermore, the alternative "insider wage" rule is discussed in Appendix B and it shows that our main results are robust to the wage setting process.

whereas for a bargained insider wage $w_i(\epsilon)$, the expected value $J_i(\epsilon)$ of a filled job by a worker of age i is defined, for $\forall i \in [1, T-1]$, by:

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) + \beta G(R_{i+1}) (V - F_{i+1}) \quad (4)$$

Optimal decisions of the firm are then characterized by productivity thresholds $\{R_i, R_i^0\}$, which are the solution of:

$$J_i^0(R_i^0) = -H_i \;\; ; \;\; J_i(R_i) = -F_i$$

Adding the free entry condition, $V = 0$, it is straightforward to derive the following equations:

$$R_i = w(R_i) - F_i - \beta \left[\int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) - G(R_{i+1}) F_{i+1} \right] \quad (5)$$

$$R_i^0 = R_i + F_i - H_i + w(R_i^0) - w(R_i) \quad (6)$$

Equation (5) gives the lowest level of productivity (R_i) necessary to avoid a separation (firing decision) and the equation (6) determines the lowest productivity level (R_i^0) for a newly created job (hiring decision). The productivity threshold governing the hiring decision must be at least equal to the outsider wage ($w(R_i^0)$) net of the hiring subsidies (H_i). But firm also expects profits (continuation value). This continuation value is the same as for a job occupied by an insider. Because the labor hoarding has the same value for a newly created job as for an existing one, the link between R_i^0 and R_i is static (see equation (6)). The higher the wage, the higher the reservation productivity R_i (R_i^0), and hence the higher (lower) the job destruction (creation) flows. On the other hand, the higher the option value of filled jobs, the weaker the job destructions and the greater the job creations. Because the job value vanishes at the end of the working life, labor hoarding of older workers is less profitable. It is worth determining the terminal age conditions: $R_{T-1} = w_{T-1}(R_{T-1}) - F_{T-1}$ and $R_{T-1}^0 = R_{T-1} + F_{T-1} - H_{T-1} + w(R_{T-1}^0) - w(R_{T-1})$.

2.3 Nash bargaining

Values of insiders, outsiders (on a job of productivity ϵ) and unemployed workers of any age i, $\forall i < T$, are respectively given by[7]:

$$\mathcal{W}_i^0(\epsilon) = w_i^0(\epsilon) + \beta \left[\int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x) dG(x) + G(R_{i+1}) \mathcal{U}_{i+1} \right] \quad (7)$$

$$\mathcal{W}_i(\epsilon) = w_i(\epsilon) + \beta \left[\int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x) dG(x) + G(R_{i+1}) \mathcal{U}_{i+1} \right] \quad (8)$$

$$\mathcal{U}_i = b + z + \beta \left[p(\theta) \int_{R_{i+1}^0}^1 \mathcal{W}_{i+1}^0(x) dG(x) + p(\theta) G(R_{i+1}^0) \mathcal{U}_{i+1} + (1 - p(\theta)) \mathcal{U}_{i+1} \right] \quad (9)$$

with b and z denoting the domestic production and the unemployment benefit respectively.

For a given bargaining power of the workers, γ, considered as constant across ages, the global surplus generated by a job is divided according to the following two sharing rules which are the solution of the conventional Nash bargaining problems in the context of two-tier contracts[8]:

$$\mathcal{W}_i^0(\epsilon) - \mathcal{U}_i = \gamma \left[J_i^0(\epsilon) + H_i + \mathcal{W}_i^0(\epsilon) - \mathcal{U}_i \right] \quad (10)$$

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma \left[J_i(\epsilon) + F_i + \mathcal{W}_i(\epsilon) - \mathcal{U}_i \right] \quad (11)$$

so that the equations for the initial and subsequent wage bargaining are (see Appendix A for details on derivation):

$$w_i^0(\epsilon) = \gamma \left(\epsilon + c\theta\tau_i + H_i - \beta F_{i+1} \right) + (1 - \gamma)(b + z) \quad (12)$$

$$w_i(\epsilon) = \gamma \left(\epsilon + c\theta\tau_i + F_i - \beta F_{i+1} \right) + (1 - \gamma)(b + z) \quad (13)$$

where τ_i is defined by[9]

$$\tau_i \equiv \frac{\int_{R_{i+1}^0}^1 [J_{i+1}^0(x) + H_{i+1}] dG(x)}{\sum_{i=1}^{T-1} \left(\frac{u_i}{u} \int_{R_{i+1}^0}^1 [J_{i+1}^0(x) + H_{i+1}] dG(x) \right)} = \frac{\int_{R_{i+1}^0}^1 [1 - G(x)] dx}{\sum_{i=1}^{T-1} \left(\frac{u_i}{u} \int_{R_{i+1}^0}^1 [1 - G(x)] dx \right)}$$

τ_i gives the value of a worker hired at age i relative to the expected value of a job according to the age distribution of unemployed workers. τ_i decreases

[7] We assume that $\mathcal{W}_T = \mathcal{U}_T$ so that the social security provisions do not affect the wage bargaining and the labor market equilibrium.

[8] Again, see Appendix B for an examination of an alternative "insider wage" rule.

[9] To derive this expression, notice that ${J_{i+1}^0}'(\epsilon) = 1 - \gamma$ and $J_i^0(R_i^0) = -H_i$ implies that $J_i^0(\epsilon) = (1 - \gamma)\left(\epsilon - R_i^0\right) - H_i$. Furthermore, integrating by parts yields that $\int_{R_{i+1}^0}^1 \left(x - R_{i+1}^0\right) dG(x) = \int_{R_{i+1}^0}^1 [1 - G(x)] dx$.

with age[10]. Despite the assumption of an undirected search, implying a homogenous search cost for each match, wages are age-specific.

Contrary to Mortensen and Pissarides' framework, the equilibrium is no longer symmetrical. Due to the finite-lived assumption, the way turn-over costs interact with the wage bargaining process depends on the age of the workers through the variable τ_i. This means that ending up with a young worker is more worth while than hiring an older one for the firm. Hence, younger workers capture a larger fraction of the search costs than the older worker. A younger worker has then to be rewarded for more than the saving of the average search costs ($c\theta$).[11]

As in Mortensen and Pissarides [1999], the difference between the initial wage and subsequent renegotiation arises because hiring subsidies are sunk in the latter case but on-the-table in the former, and termination costs are not incurred if no match is formed initially but must be paid if an existing match is destroyed. From that point of view, the effects of firing taxes and hiring subsidies on these wage rules are conventional:

(i) The hiring subsidy H_i increases the initial wage $w_i^0(\epsilon)$ because this gain is conditional on agreement to form the match, whereas the discounted firing tax βF_{i+1} decreases this wage because it reduces the expected match surplus at the creation date. Workers then get a share γ of the expected net subsidy of the job, $H_i - \beta F_{i+1}$.

(ii) Once the job is created, the hiring subsidy no longer influence wages in continuing jobs $w_i(\epsilon)$, but in turn the employer is now liable for the firing tax F_i and this strengthens the workers's hand in the wage bargaining. Accordingly, wages for continuing workers are increased (decreased) by firing costs if the firm expects that keeping the worker accounts for an additional cost (gain), that is if $F_i < \beta F_{i+1}$ ($F_i > \beta F_{i+1}$).

[10] As shown hereafter, and at least in an economy without a labor market policy, $\tau_1 > 1$ for the youngest workers, $\tau_{T-1} = 0$ for the oldest ones, and $\tau_{i+1} \leq \tau_i \;\; \forall i$.

[11] For a given productivity level ϵ, the wage is lower for a worker of age $i+1$ than for a worker of age i, $w_{i+1}(\epsilon) \leq w_i(\epsilon)$. Age-decreasing dynamics of wages are obviously at odds with empirical findings. This shortcoming could easily be overcome by allowing the model to account for exogenous human capital accumulation. For instance, consider $h_{i+1} = (1+\mu)h_i$, where the productivity of the job is now given by $h_i\epsilon$, it can be the case that the growth rate $\mu \geq 0$ is high enough to imply $w_{i+1}(\epsilon) \geq w_i(\epsilon)$. Furthermore, even though a higher growth rate would account for higher labor market tightness, the shape of the age-dynamics of job creations and job destructions would be unaffected.

2.4 The labor market equilibrium

Proposition 1. *A labor market equilibrium with wage bargaining exists and it is characterized by:*

$$\frac{c}{q(\theta)} = \beta(1-\gamma)\sum_{i=1}^{T-2}\left(\frac{u_i}{u}\int_{R_{i+1}^0}^1 [1-G(x)]dx\right) \quad (14)$$

$$b+z+\frac{\gamma}{1-\gamma}c\theta\tau_i = R_i+\beta\int_{R_{i+1}}^1 [1-G(x)]dx + F_i - \beta F_{i+1} \quad (15)$$

$$R_i^0 = R_i + F_i - H_i \quad (16)$$

$$u_{i+1} = u_i\left[1-p(\theta)(1-G(R_{i+1}^0))\right] + G(R_{i+1})(1-u_i) \quad (17)$$

where $\tau_i = \frac{\int_{R_{i+1}^0}^1 [1-G(x)]dx}{\sum_{i=1}^{T-1}\left(\frac{u_i}{u}\int_{R_{i+1}^0}^1 [1-G(x)]dx\right)}$, *with terminal conditions* $R_{T-1} = b + z_{T-1} - F_{T-1}$, $R_{T-1}^0 = b + z_{T-1} - H_{T-1}$ *and a given initial condition* u_1.

Proof. Combining (2), (5), (6), (7) (13) and noticing that ${J_{i+1}^0}'(\epsilon) = 1-\gamma$ and $J_i^0(R_i^0) = -H_i$ implies that $J_i^0(\epsilon) = (1-\gamma)(\epsilon - R_i^0) - H_i$, as well as $J_{i+1}'(\epsilon) = 1-\gamma$ and $J_i(R_i) = -F_i$ implies $J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i$. Furthermore, integrating by parts yields that $\int_{R_{i+1}^0}^1 \left(x - R_{i+1}^0\right) dG(x) = \int_{R_{i+1}^0}^1 [1-G(x)]dx$ and $\int_{R_{i+1}}^1 (x - R_{i+1})\, dG(x) = \int_{R_{i+1}}^1 [1-G(x)]dx$. □

In particular, equation (15) shows that a job is destroyed when the expected profit from the marginal job -current product plus option value from expected productivity shocks- (right side of (15)) fails to cover the worker's reservation wage (left side). Without any policy distortions, the reservation productivity governing the hiring decision (R_i^0) is equal to the reservation productivity determining the firing decision because insider and outsider wages are equal and there are no termination and opening costs/gains. On the other hand, it is obvious that the age-dynamics of job creations and job destructions depend on the age design of employment protection.

2.5 The age-dynamics of job creations and job destructions in a *laissez-faire* economy

As a benchmark case, we first consider the equilibrium age-dynamics of job creations and job destructions without any labor market policies. This allows us to show that older workers face a higher (lower) probability of exit from employment (unemployment). Let us denote $\tilde{R}_i$ and $\tilde{\theta}$ the productivity thresholds and the labor market tightness respectively in the case of no labor market policy ($z = 0$ and $H_i = F_i = 0 \ \ \forall i$).

Proposition 2. *The equilibrium without any labor policies is characterized by* $\tilde{R}_{i+1} \geq \tilde{R}_i \quad \forall i \in [2, T-1]$.

Proof. The age-dynamics of job creations and job destructions is governed by the sequence $\{\tilde{R}_i\}_{i=2}^{T-1}$ which solves:

$$\tilde{R}_i \;\; = \;\; b - \beta[1 - \gamma p(\tilde{\theta})] \int_{\tilde{R}_{i+1}}^{1} [1 - G(x)]dx$$

with terminal conditions $\tilde{R}_{T-1} = b$, and where $\tilde{\theta}$ is defined by $\frac{c}{q(\tilde{\theta})} = \beta(1 - \gamma) \sum_{i=1}^{T-1} \left(\frac{u_i}{u} \int_{\tilde{R}_{i+1}}^{1} [1 - G(x)]dx \right)$ with $u_{i+1} = u_i \left[1 - p(\tilde{\theta})(1 - G(\tilde{R}_{i+1})) \right] + G(\tilde{R}_{i+1})(1 - u_i)$. Solving backward this equation (with $\beta[1 - \gamma p(\tilde{\theta})] < 1$) and starting with terminal condition $\tilde{R}_{T-1} = b$, we obtain $\tilde{R}_{i+1} \geq \tilde{R}_i$. □

Because the horizon of older workers is shorter, firms invest less in labor-hoarding activities at the end of the life cycle, and older workers are more vulnerable to idiosyncratic shocks. Otherwise stated, this reflects the fact that labor-hoarding decreases with the worker's age. It is then straightforward to see that this increasing (decreasing) age-dynamic of job destructions (creations) is, at least qualitatively, able to account for the observed low employment rate of older workers.[12]

3 The Impact of Firing Taxes Revisited

Faced with the low employment rate of older workers, most developed countries have experimented with higher employment protection combined with subsidies targeted at these workers. The main objective of this section is to question, from a positive point of view, the impact of such policies.

It is well-known following Mortensen and Pissarides [1999] that stricter employment legislation protects workers who already have a job, but at the expense of those without a job. The overall impact on employment is then theoretically ambiguous unless hiring subsidies offset the perverse effects of employment protection on job creation. It is straightforward to see that such a result also holds in our framework because the firing tax increases productivity thresholds at the time of job creation, whatever the worker's age (condition (16)). But the incidence of firing taxes on job creation and job destruction is still age-dependent.

[12] Quantitative assessment of the model is beyond the scope of this paper (see Chéron, Hairault and Langot [2008] for such an exercise).

3.1 On the age-differentiated effect of firing taxes

As a preliminary step, our objective is first to examine the age-differentiated effect of a constant firing tax ($F_i = F \;\; \forall i$) on the age-dynamics of job flows. Without loss of generality, we consider also that $H_i = F \;\; \forall i$ so that a firing tax unambiguously increases employment rates at all ages. However, the impact on the job destruction and job creation rates remains age-dependent. We argue indeed that, in a finite horizon setting, there is a specific intertemporal trade-off related to the introduction of a firing tax.

Proposition 3. *Consider $H_i = F_i = F > 0 \;\; \forall i$, the labor market equilibrium is characterized by:*

$$0 \;\; \geq \;\; \frac{dR_2}{dF} \;\; > \frac{dR_i}{dF} \; ... > \;\; \frac{dR_{T-1}}{dF} \qquad \forall i \in [2, T-1] \;\; and \;\; \frac{d\theta}{dF} > 0$$

Proof. Consider $z = 0$, $F_{i+1} = F_i \equiv F$ and $H_i = F$ so that $R_i^0 = R_i$ in Proposition 1, it yields that $\frac{dR_i}{dF} = -(1-\beta) + \frac{dR_{i+1}}{dF}\beta(1-\gamma p(\theta))\left[1 - G(R_{i+1})\right] + \frac{d\theta}{dF}\gamma p'(\theta)\beta\int_{R_{i+1}}^{1}[1 - G(x)]dx$ with $\frac{dR_{T-1}}{dF} = -1$ from Proposition 1. Then, it remains to iterate backward from $i = T - 1$ to $i = 2$, having in mind that $\beta \leq 1$, $p'(\theta) \geq 0$ and $\frac{d\theta}{dF} \geq 0$ (straightforward with $H_i = F$). □

A given firing tax is thus found to reduce the job destruction (creation) rate of older workers more (less) than younger ones. Otherwise stated, the potential employment gains related to F are greater for older workers.

To get further intuitions on this result, let us also assume $\beta \to 1$. It is straightforward to see that in an infinite horizon economy *à la* MP, we would no longer have the impact of the firing tax F on job destruction[13]: the value of avoiding punishment today is equal to the loss in job value induced by the expected firing taxes. In this extreme case, a constant firing tax is neutral. On the contrary, in our finite life time context, we still have $\frac{dR_{T-1}}{dF} = -1$ which implies $\frac{dR_i}{dF} < 0 \;\; \forall i$. At the end of the working cycle, introducing a firing tax increases the present firing cost without any future consequences on the job value as the worker will be retired in the next period. In other words, the value of avoiding punishment today is not canceled out by the expected future loss. Retirement allows firms to avoid the firing tax, leading them to more labor hoarding of older workers. This suggests that evaluating employment protection in an infinite-lived agent context understates the potential employment gains implied by this policy.

[13] To state this result, consider $R_i = R_{i+1} \equiv R$ in (15). For $\beta \to 1$ and $\gamma \to 0$, we then have $\frac{dR}{dF} = 0$.

We are even able to state that the impact of firing costs can be sizeable enough at the end of the life cycle to imply that older workers face a lower probability of job destruction than younger ones.

Proposition 4. *There exists* $\hat{F} > 0$ *such that if* $F \geq \hat{F}$*, then* $R_{T-1} \leq R_{i+1} \leq R_i \;\; \forall i \in [2, T-1]$.

Proof. Let us consider Proposition 1 with $z = 0$, $H_i = F \;\; \forall i$ and $F_{i+1} = F_i \equiv F$, and let us define $\Psi(y) \equiv (1 - \gamma p(\theta)) \int_y^1 [1 - G(x))\, dx$, so that $\Psi'(y) < 0$. By definition $R_i = b - (1-\beta)F - \Psi(R_{i+1})$, so that $R_i - R_{i+1} = \Psi(R_{i+2}) - \Psi(R_{i+1})$. Accordingly, $R_{T-2} \geq R_{T-1}$ is a sufficient condition to imply $R_i \geq R_{i+1} \;\; \forall i$. Then, $R_{T-2} \geq R_{T-1} \iff F \geq (1-\gamma p(\theta)) \int_{b-F}^1 [1-G(x)]dx$ which implies that $F \geq \int_{b-F}^1 [1-G(x)]dx$ is a sufficient condition for $R_{T-2} \geq R_{T-1}$, hence $R_i \geq R_{i+1}$. Otherwise stated, from $0 \leq G(x) \leq 1$, there exists a unique $\hat{F}$ solving $\hat{F} = \int_{b-\hat{F}}^1 [1-G(x)]dx$, such that $F \geq \hat{F}$ implies $R_i \geq R_{i+1} \;\; \forall i$. □

The intuition behind this result is the following. Without any policy instruments, older workers face a higher (lower) rate of job destructions (creations) because of their shorter horizon. In turn, the introduction of a firing tax has a greater impact on the probability of job destruction for older workers than for younger ones. Hence, allowing for sufficiently high firing taxes can reverse the age-dynamics of job destructions and job creations. More precisely, this is unambiguously the case, whenever the value of the labor hoarding for a worker of age $T-2$ is less than the firing cost F. Then, since the job destruction rate for a worker of age $T-2$ turns out to be higher than that of a worker of age $T-1$, this is the overall age-dynamic of job destructions which is reversed in the context of a constant firing tax[14].

To conclude, at this stage Properties 3 and 4 emphasize that the laying off of older workers is, in relative terms with respect to younger workers, very sensitive to the firing tax. Employment protection has an age-differentiated impact. It may make age-dependent employment protection unnecessary if the only objective is to stimulate more the employment of older workers.

3.2 The impact of age-increasing firing taxes

It is obvious that not only the level of firing taxes, but also their shape depending on age play a key role in the age-dynamic of job flows. A quick look at Proposition 1 shows that F_i tends to push down R_i by increasing the current cost of firing, while F_{i+1} increases R_i by reducing the value of

[14]Indeed, job destruction at age $T-3$ becomes higher than at age $T-2$ because $R_{T-2} \geq R_{T-1}$, and so on.

labor-hoarding. This suggests that the shape of the discounted firing costs is crucial for the separation decisions.

Proposition 5. *If* $F_{i+1} = (1+\Delta_F)F_i \;\; \forall i \in [2, T-2]$*, with* $\Delta_F \gtrless 0$*, and* $H_i = F_i \;\; \forall i$*, the impact of age-dependent employment protection is characterized by*[15]*:*

- $\frac{\partial R_{T-1}}{\partial F_{T-1}} < 0$
- *If* $\Delta_F < \frac{1}{\beta} - 1$*, then* $\frac{\partial R_i}{\partial F_i} \leq 0 \;\; \forall i \in [2, T-1]$
- *If* $\Delta_F > \frac{1}{\beta} - 1$*, then there exists a threshold age* $\tilde{i}$ *such that* $\frac{\partial R_i}{\partial F_i} \geq 0 \;\; \forall i \in [2, \tilde{i}]$ *and* $\frac{\partial R_i}{\partial F_i} \leq 0 \;\; \forall i \in [\tilde{i}, T-1]$

Proof. See Appendix C.1. □

Proposition 5 first stresses that introducing a firing tax unambiguously decreases firings of the oldest workers. This is because, by definition, retirement avoids this tax: this gives firms incentives to wait for one period instead of firing the worker at age $T-1$ and being liable for the firing cost F_{T-1}. For all the other workers, the impact of firing taxes on job destruction rates depends on the age-dynamics of taxes. If firing taxes decrease with age or do not rise too fast ($\Delta_F < \frac{1}{\beta} - 1$), they decrease job destruction rates for all ages. It is in firms' interest to reduce firings because the value of avoiding punishment is higher than the expected loss induced by future taxes. Ultimately, firms avoid these costs by waiting for workers' retirement. In some sense, it generalizes to all ages the idea that labor hoarding allows firms to avoid (at least partially) firing costs.

On the other hand, the growth rate of firing taxes with age can be so high that it increases job destruction rates for younger workers: the value of avoiding punishment today is lower than the expected increase in future taxes, leading firm to fire early. For older workers, the proximity to retirement compensates for the impact of the expected growth in the firing tax.

Overall, Proposition 5 highlights that age-increasing firing taxes have some perverse effects by increasing the job destruction rates for younger workers. It is worth emphasizing that these results can give theoretical support to the empirical findings of Behaghel, Crépon and Sédillot [2008] who use microeconometric estimates to assess the French experiment (since 1987) with a higher firing tax for workers of 55 years old or more. The estimates mainly show that firings of people under 55 have increased.

[15] A similar statement can be derived when assuming $H_i = 0 \;\; \forall i$, $\gamma \rightarrow 0$ and $\forall F_{T-1} > 0$ (available upon request).

4 Optimal age-dependent employment protection

At this stage, we have mainly argued that the potential employment gain for older workers of higher firing taxes and hiring subsidies targeted at them could be largely due to the short distance to retirement. However, such a policy may have some perverse effects on younger generations. On the other hand, age-decreasing firing taxes lead to decreased job destructions for all workers.

It is so important to go beyond this positive approach by proposing a welfare analysis of age-dependent employment protection. Is an age-increasing dynamic consistent with optimal age-dependent employment protection?

4.1 An intergenerational externality

Traditionally, the equilibrium unemployment framework is known to generate congestion effects which take the decentralized equilibrium away from the efficient allocation. However, when the elasticity relative to vacancies in the matching function is equal to the bargaining power of firms (Hosios condition), social optimality can be reached. As demonstrated hereafter, this result no longer holds here, because there is a specific intergenerational externality.

We derive the optimal allocation by maximizing the steady-state output with respect to labor market tightness $\theta^\star$ and reservation productivity for each age, $R_i^\star$. The problem of the planner is stated as follows:

$$\max_{\left\{R_i^\star \geq 0\right\}_{i=1}^{T-1}, \theta^\star \geq 0} \sum_{i=1}^{T-1} \left[y_i + bu_i - \frac{c\theta^\star u^\star}{T-1} \right]$$

where $u^\star = \sum_{i=1}^{T-1} u_i$ and subject to the unemployment dynamic and the output equation, respectively:

$$u_{i+1} = G(R_{i+1}^\star)(1-u_i) + u_i \left(1 - p(\theta^\star)[1 - G(R_{i+1}^\star)]\right) \quad (18)$$

$$y_{i+1} = u_i p(\theta^\star) \int_{R_{i+1}^\star}^1 x dG(x) + (1-u_i) \int_{R_{i+1}^\star}^1 x dG(x) \quad (19)$$

Proposition 6. *Let* $\eta = 1 - \frac{\theta^\star p'(\theta^\star)}{p(\theta^\star)}$, *the maximum value of steady-state* *output is reached when:*

$$\frac{c}{q(\theta^\star)} = (1-\eta)\sum_{i=1}^{T-1}\frac{u_i}{u}\left(\int_{R^\star_{i+1}}^{1}[1-G(x)]dx\right) \quad (20)$$

$$R^\star_i + \int_{R^\star_{i+1}}^{1}[1-G(x)]dx = b + \frac{\eta}{1-\eta}c\theta^\star\tau^\star_i + c\theta^\star(\tau^\star_i - 1) \quad \forall i \in [2, T-2] \quad (21)$$

$$R^\star_{T-1} = b - c\theta^\star \quad (22)$$

where $\tau^\star_i \equiv \frac{\int_{R^\star_{i+1}}^{1}[1-G(x)]dx}{\sum_{i=1}^{T-1}\left(\frac{u_i}{u}\int_{R^\star_{i+1}}^{1}[1-G(x)]dx\right)}$.

Proof. See Appendix C.2. □

Equation (20) is similar to equation (14) obtained in the decentralized equilibrium, on condition that the worker share of employment surplus (γ) is now replaced by the elasticity relative to unemployment in the matching function ($\eta(\theta^\star)$). Equation (21) shows the optimal allocation of the age i labor force: the expected profit from the marginal employed worker (the current product plus the option value for expected productivity shocks) must be equal to the social return of the search activity which corresponds to the allocation as an unemployed worker. At the equilibrium, the return on unemployment is simply given by the reservation wage. For the social planner, the return on an additional age i unemployed worker is reduced by the cost of each vacancy per age i unemployed worker, which is equal to $c\theta^\star$. The social value of the search activity is not symmetrical: because a young (old) worker increases (decreases) the average search value in the economy, the social value of the young unemployed worker is larger than that of the old unemployed worker. At the end of the life-cycle ($i = T-1$), the social return on a worker occupied in the search process is at its lowest value: the oldest workers can be contacted by a firm whereas the surplus associated with this match is nil. Indeed, the relative surplus of an age $T-1$ worker (the oldest workers) to the average of the employment surplus is equal to zero ($\tau^\star_{T-1} = 0$): the return of the search is zero ($\frac{\eta}{1-\eta}c\theta^\star\tau^\star_{T-1} = 0$) and the size of the intergenerational externality takes its maximum value $c\theta^\star$. Then, the social value of unemployment for workers of age $T-1$ turns out to be $b - c\theta^\star$ (equation (22)).

Proposition 7. *Efficient allocation is characterized by* $R^\star_{i+1} \geq R^\star_i$*, so that* $jd^\star_{i+1} \geq jd^\star_i$ *and* $jc^\star_{i+1} \leq jc^\star_i$.

Proof. First note that $R_i^\star$ can be re-stated as follows:

$$R_i^\star = b-[1-p(\theta^\star)]\int_{R_{i+1}^\star}^1 [1-G(x)]dx-(1-\eta)p(\theta^\star)\sum_{i=1}^{T-1}\frac{u_i}{u}\left(\int_{R_{i+1}^\star}^1[1-G(x)]dx\right)$$

If $b > c\theta^\star$, the proof is straightforward by solving this equation backward, and by noticing that $0 < [1-p(\theta^\star)] < 1$ and $(1-\eta)p(\theta^\star)\sum_{i=1}^{T-1}\frac{u_i}{u}\left(\int_{R_{i+1}^\star}^1[1-G(x)]dx\right)$ is not age-dependent. If the condition $b > c\theta^\star$ does not hold, then $R_{T-1}^\star$ is bounded by zero, and it unambiguously appears that[16] $R_i^\star = R_{i+1}^\star = 0 \quad \forall i$. □

Proposition 7 first emphasizes that higher (lower) job destruction (creation) rates for older workers are typically an efficient age-pattern of labor market flows, as in the decentralized equilibrium. Because of their shorter horizon, older workers must be fired more and hired less. Despite the fact that the shape is qualitatively the same, this does not mean that the equilibrium job destruction and job creation rates are consistent with their efficient counterparts.

Importantly, when the job destruction rate is strictly positive, the difference between the private and the social value of unemployment emphasizes the existence of inefficiencies. Unlike the firms, the planner takes into account the impact of a particular unemployed worker of age i on the search process. Compared to Pissarides [2000], our life-cycle framework introduces another externality, namely an intergenerational externality. Firms neither take into account the facts that firings of older workers reduce the average value of a vacancy nor that firings of younger workers increase this average value.

Proposition 8. *The Hosios condition, $\eta = \gamma$, does not achieve efficiency.*

Proof. Straightforward by comparing the expression of $R_i^\star$ in Proposition 6 and R_i in Proposition 1 and considering $\gamma = \eta$. □

This proposition states clearly that the Hosios condition $\gamma = \eta$ allows the private agents to internalize traditional search externalities in the decentralized equilibrium, but not the intergenerational externalities in the matching process. This last result suggest that age-specific labor market policies are needed to improve the outcome of the decentralized equilibrium. This could

[16]This result is straightforward by noticing that the efficient productivity threshold can be restated as $R_i^\star = b - c\theta^\star[1-p(\theta^\star)] - \int_{R_{i+1}^\star}^1[1-G(x)]dx$.

give some theoretical foundation to the age-specific firing taxes and hiring subsidies in place in a large set of OECD countries[17].

4.2 The optimal age-dynamics of firing costs and hiring subsidies

Labor market policies designed by age may allow firms and workers to internalize the intergenerational externality. To focus on the impact of intergenerational externalities on the age design of labor market policies, we assume throughout this section that $\eta = \gamma$.

Proposition 9. *Assuming $\beta \to 1$, $z = 0$ and $\eta = \gamma$, an optimal age-sequence for firing taxes and hiring subsidies $\{F_i^\star, H_i^\star\}_{i=1}^{T-1}$ solves:*

$$\begin{array}{rcl} F_i^\star - F_{i+1}^\star & = & c\theta^\star (1 - \tau_i^\star) \quad \forall i \in [2, T-2] \quad \text{and} \quad F_{T-1} \; = \; c\theta^\star \\ H_i^\star & = & F_i^\star \quad \forall i \in [2, T-1] \end{array}$$

where $\{R_i^\star\}_{i=2}^{T-1}$ and $\theta^\star$ are defined in Proposition 6.

Proof. Straightforward by comparing Propositions 1 and 6 when assuming $\eta = \gamma$ and $z = 0$. □

Proposition 10. *There exists an age $\tilde{i}$ defined by $\tau_{\tilde{i}} = 1$ such that $F_{i+1}^\star \geq F_i^\star \;\; \forall i \leq \tilde{i}$ and $F_{i+1}^\star \leq F_i^\star \;\; \forall i \geq \tilde{i}$.*

Proof. Straightforward from Proposition 9 by recalling that $\tau_1^\star > 1$ and $\tau_{T-1}^\star = 0$. □

Firings of the workers aged more than $\tilde{i}$ account for a negative externality by increasing the average search cost. Otherwise stated, there are too many firings of that type of worker in equilibrium. So, it is optimal to implement an age-decreasing path of firing taxes for older workers: as emphasized earlier, the expectation of lower taxes in the future gives the right incentives for firms to keep workers (to postpone job destruction). On the other hand, for workers at age $i \leq \tilde{i}$, who reduce the average search cost in the case of firings, an age-increasing dynamic of firing taxes turns out to be the optimal age-dependent employment protection.

The first best age-dynamic of firing taxes and hiring subsidies is typically hump-shaped over all the life cycle. The optimal age-design employment protection would require implementing an age-decreasing firing tax for the older workers.

[17] Let us note that these policies would become pointless if an equilibrium with directed search were sustainable.

4.3 Second best analysis

Turning to a second best perspective, we examine how the optimal age-dynamic of firing taxes and hiring subsidies is affected by the introduction of unemployment benefits and the reconsideration of the Hosios condition.

Proposition 11. *Assuming $\beta \to 1$, $z > 0$ and $\eta = \gamma$, an optimal age-sequence for firing taxes and hiring subsidies $\{F_i^\star, H_i^\star\}_{i=1}^{T-1}$ solves:*

$$\begin{aligned} F_i^\star - F_{i+1}^\star &= c\theta^\star (1-\tau_i^\star) + z \quad \forall i \in [2, T-2] \quad \textit{and} \quad F_{T-1}^\star = z + c\theta^\star \\ H_i^\star &= F_i^\star \quad \forall i \in [2, T-1] \end{aligned}$$

where $\{R_i^\star\}_{i=2}^{T-1}$ and $\theta^\star$ are defined in Proposition 6.

Proof. Straightforward by comparing Propositions 1 and 6 when assuming $\eta = \gamma$ and $z > 0$. □

It is straightforward to see that the existence of unemployment benefits accounts for a lower threshold age $\hat{i} < \tilde{i}$ above which the firing tax is decreasing with age. Indeed, since unemployment benefits implicitly generate a constant tax on labor according to the flow z, this increases wages and productivity thresholds, and therefore requires the taxation of firings and the subsidization of hirings. This tax would also incorporate the distortion effects of expected taxes in the future. In a symmetrical equilibrium (if $\tau_i = 1 \ \forall i$), this would imply that $F_{T-1}^\star = z$ and $F_i^\star = \sum_{j=i}^{T-1} \beta^{T-j-1} z$. In other words, in order to avoid inefficient firings, the punishment costs associated with a layoff should be equal to the discounted sum of unemployment benefits. This gives an additional foundation for age-decreasing firing taxes.

Taking into account intergenerational externalities ($\tau_i \neq 1$), the higher the unemployment benefits, the sooner the age-decreasing path of firing taxes and hiring subsidies. Accordingly, jobs destructions turns out to be too high for some workers below $\tilde{i}$, despite the firings of these workers account for positive externalities. Hence, to reduce latter's firings, it is required to introduce age-decreasing employment protection from a critical age $\hat{i}$ below age $\tilde{i}$. Ultimately, z could be large enough to imply a monotonously age-decreasing dynamics of optimal firing taxes and hiring subsidies.

Proposition 12. *Assuming $\beta \to 1$, $z = 0$ and $\eta < \gamma$, an optimal age-sequence for firing taxes and hiring subsidies $\{F_i^\star, H_i^\star\}_{i=1}^{T-1}$ solves:*

$$\begin{aligned} F_i^\star - F_{i+1}^\star &= c\theta^\star (1-\tau_i^\star) + c\theta^\star \tau_i^\star \left(\frac{\gamma - \eta}{(1-\eta)(1-\gamma)} \right) \quad \forall i \in [2, T-2] \\ \textit{and} \quad F_{T-1}^\star &= c\theta^\star \end{aligned}$$

where $\{R_i^\star\}_{i=2}^{T-1}$ and $\theta^\star$ are defined in Proposition 6.

High worker bargaining power also leads to increased wages and productivity thresholds so that it also makes it desirable to implement an age-decreasing firing tax. However, in contradiction with the constant distortion z related to unemployment benefits, the distortion induced by the high worker's bargaining power ($\gamma > \eta$) now depends on the age of the worker. This distortion decreases with the worker's age because τ_i decreases with i. This is because of the wage bargaining process, which generates lower distortions in the allocation of older workers. Indeed, the older workers have a relative low labor market value and then capture a lower fraction of the search costs than the younger workers. This compensates for the high bargaining power of the workers at the end of the life-cycle. This lower distortion at the end of the life-cycle gives further foundation to the implementation of lower firing taxes for older workers.

Proposition 12 shows the optimal age-design of firing taxes in that context. Ultimately, $F^{\star}_{T-1} = c\theta^{\star}$, since the value of a worker of age $T-1$ in the future turns out to be zero, so that the optimal tax collapses to the first best policy. On the other hand, unemployment benefits increase the firing tax even in this last stage of the life cycle ($F^{\star}_{T-1} = z + c\theta^{\star}$ in Proposition 11).

Overall, this second-best analysis highlights the robustness of our results, which call into question the current OECD practice of higher employment protection for older workers.

5 Conclusion

Generally, our theory of age-dependent employment protection implies of implementing age-decreasing firing taxes for the older workers. This is at odds with the current practise in most OECD countries, which have implemented an age-increasing employment protection. This discrepancy between theory and practise is even greater in a second-best perspective when the distortions created by the existence of unemployment benefits are taken into account. This last point is not anecdotal as the European countries which have implemented the highest employment protection for older workers also provide the most generous unemployment benefits. We then conclude that the existing policies present strong perverse effects, both in terms of overall employment and of social welfare. It is important to note that this reconsideration is not due to a conflict in terms of objectives: our framework is consistent with the idea that older workers' jobs must be protected. But reaching this objective implies adopting age-decreasing firing taxes for older workers' jobs. Finally, we show that at least it would be more efficient to return to age-constant employment protection, which has the advantage of better preserving older workers' jobs without distorting the job creations and destructions of younger workers.

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A Wage equations under a two-tier structure

The sharing rules can be written as:

$$-\gamma H_i - (1-\gamma)\mathcal{U}_i = \gamma\left[J_i^0(\epsilon) + \mathcal{W}_i^0(\epsilon)\right] - \mathcal{W}_i^0(\epsilon) \quad (23)$$

$$-\gamma F_i - (1-\gamma)\mathcal{U}_i = \gamma\left[J_i(\epsilon) + \mathcal{W}_i(\epsilon)\right] - \mathcal{W}_i(\epsilon) \quad (24)$$

From value functions, it turns out that:

$$\begin{aligned}\gamma\left[J_i(\epsilon) + \mathcal{W}_i(\epsilon)\right] - \mathcal{W}_i(\epsilon) &= \gamma\epsilon - w_i(\epsilon) + \gamma\beta\int_{R_{i+1}}^1 \left[J_{i+1}(x) + \mathcal{W}_{i+1}(x)\right]dG(x)\\ &\quad -\beta\int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x)dG(x)\\ &\quad -(1-\gamma)\beta G(R_{i+1})U_{i+1} - \gamma\beta G(R_{i+1})F_{i+1}\\ &= \gamma\epsilon - w_i(\epsilon) - (1-\gamma)\beta U_{i+1} - \gamma\beta F_{i+1} \qquad (25)\end{aligned}$$

Combining this with (24) yields:

$$w_i(\epsilon) = \gamma\left(\epsilon + F_i - \beta F_{i+1}\right) + (1-\gamma)\left[\mathcal{U}_i - \beta\mathcal{U}_{i+1}\right] \quad (26)$$

Similarly,

$$\gamma\left[J_i^0(\epsilon) + \mathcal{W}_i^0(\epsilon)\right] - \mathcal{W}_i^0(\epsilon) = \gamma\epsilon - w_i^0(\epsilon) - (1-\gamma)\beta\mathcal{U}_{i+1} - \gamma\beta F_{i+1} \quad (27)$$

implies by combining with (23):

$$w_i^0(\epsilon) = \gamma\left(\epsilon + H_i - \beta F_{i+1}\right) + (1-\gamma)\left(\mathcal{U}_i - \beta\mathcal{U}_{i+1}\right)$$

Then, let us notice that the unemployed value solves in equilibrium:

$$\begin{aligned}\mathcal{U}_i &= b + \beta\left[p(\theta)\int_{R_{i+1}^0}^1 \left(\mathcal{W}_{i+1}^0(x) - \mathcal{U}_{i+1}\right)dG(x) + \mathcal{U}_{i+1}\right]\\ &= b + \beta\left[p(\theta)\frac{\gamma}{1-\gamma}\int_{R_{i+1}^0}^1 \left(J_{i+1}^0(x) + H_{i+1}\right)dG(x) + \mathcal{U}_{i+1}\right]\\ &= b + \frac{\gamma}{1-\gamma}c\theta\frac{\int_{R_{i+1}^0}^1 \left(J_{i+1}^0(x) + H_{i+1}\right)dG(x)}{\sum_{i=1}^{T-1}\left(\frac{u_i}{u}\int_{R_{i+1}^0}^1 \left(J_{i+1}^0(x) + H_{i+1}\right)dG(x)\right)} + \beta\mathcal{U}_{i+1}\\ &= b + \frac{\gamma}{1-\gamma}c\theta\frac{\int_{R_{i+1}^0}^1 [1-G(x)]dx}{\sum_{i=1}^{T-1}\left(\frac{u_i}{u}\int_{R_{i+1}^0}^1 [1-G(x)]dx\right)} + \beta\mathcal{U}_{i+1}\end{aligned}$$

where using $p(\theta)/q(\theta) = \theta$ and $\int_{R^0_{i+1}}^1 \left(J^0_{i+1}(x) + H_{i+1}\right) dG(x) = (1-\gamma) \int_{R^0_{i+1}}^1 [1-G(x)]dx$ by integrating by parts. Substituting for $\mathcal{U}_i - \beta\mathcal{U}_{i+1}$ from this expression into $w_i(\epsilon)$ and $w_i^0(\epsilon)$ one gets (13) and (12).

B Insider wage equilibrium

Outsiders, once hired, have an *ex-post* incentive to renege on the two-tier structure by demanding the insider wage. A two-tier wage structure might not be feasible. At least, if $F_i \geq H_i$, from (12) and (13), it appears that $w_i^0(\epsilon) > w_i(\epsilon)$. In this section, as for instance Pissarides [2008], we look at the implications of a pure insider wage equilibrium, that is when the second tier wage sharing rule applies initially as well as to subsequent renegotiations.

Otherwise stated, we now consider that $w_i^0(\epsilon) = w_i(\epsilon)$ (hence $R_i^0 = R_i$), and the only relevant sharing rule is:

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma \left[J_i(\epsilon) + F_i + \mathcal{W}_i(\epsilon) - \mathcal{U}_i\right]$$

In such circumstances, the unemployed value now solves:

$$\begin{aligned}
\mathcal{U}_i &= b + \beta \left[p(\theta) \int_{R_{i+1}}^1 \left(\mathcal{W}_{i+1}(x) - \mathcal{U}_{i+1}\right) dG(x) + \mathcal{U}_{i+1}\right] \\
&= b + \beta \left[p(\theta) \frac{\gamma}{1-\gamma} \int_{R_{i+1}}^1 \left(J_{i+1}(x) + F_{i+1}\right) dG(x) + \mathcal{U}_{i+1}\right] \\
&= b + \beta \left[p(\theta) \frac{\gamma}{1-\gamma} \int_{R_{i+1}}^1 \left(J_{i+1}(x) + H_{i+1}\right) dG(x) + \mathcal{U}_{i+1}\right] \\
&\quad + \beta p(\theta) \frac{\gamma}{1-\gamma} \int_{R_{i+1}}^1 \left(F_{i+1} - H_{i+1}\right) dG(x) \\
&= b + \frac{\gamma}{1-\gamma} c\theta\tau_i + \beta\mathcal{U}_{i+1} + \beta p(\theta) \frac{\gamma}{1-\gamma} [1 - G(R_{i+1})] \left(F_{i+1} - H_{i+1}\right)
\end{aligned}$$

Substituting for $\mathcal{U}_i - \beta\mathcal{U}_{i+1}$ from this expression into $w_i(\epsilon)$, one gets:

$$w_i(\epsilon) = \gamma \left[\epsilon + c\theta\tau_i + F_i - \beta F_{i+1} + \beta p(\theta)[1 - G(R_{i+1})] \left(F_{i+1} - H_{i+1}\right)\right] + (1-\gamma)(b + z)$$

So, the productivity threshold turns out to be defined by:

$$\begin{aligned}
& b + z_i + \frac{\gamma}{1-\gamma} \left[c\theta\tau_i + \beta p(\theta)[1 - G(R_{i+1})] \left(F_{i+1} - H_{i+1}\right)\right] = \\
& R_i + \beta \int_{R_{i+1}}^1 [1 - G(x)]dx + F_i - \beta F_{i+1}
\end{aligned}$$

Then, if we assume $\gamma = \eta$, it appears that

$$F_i^\star - F_{i+1}^\star = z + c\theta^\star (1-\tau_i^\star) + p(\theta^\star)\frac{\eta}{1-\eta}[1-G(R_{i+1}^\star)]\left(F_{i+1}^\star - H_{i+1}^\star\right)$$

It is thus obvious that if hiring subsidies only aim at countering the negative impact of firing costs on recruitment policy ($H_i^\star = F_i^\star$), the age design of firing taxes is unaffected by the wage bargaining process, either the two-tier wage structure or the insider wage assumption. This is true for $\gamma = \eta$, which implies that an optimal age-sequence for firing taxes and hiring subsidies $\{F_i^\star, H_i^\star\}_{i=1}^{T-1}$ solves proposition 9.

In other words, in the particular case where the labor market policy aims at internalizing intergenerational inefficiencies, since the optimal policy solves $H_i^\star = F_i^\star$, the optimal two-tier wage structure collapses to the insider wage solution, *i.e.* $w_i^0(\epsilon) = w_i(\epsilon)$.

C Proofs of propositions

C.1 The impact of Δ_F on R_i

Let us consider $H_i = F_i$, which implies $R_i^0 = R_i$, and

$$R_i = b + z - F_i\left[1-\beta(1+\Delta_F)\right] - \beta(1-\gamma p(\theta))\int_{R_{i+1}}^1 [1-G(x)]dx$$

Accordingly, it appears that

$$dR_i = \beta(1-\gamma p(\theta))[1-G(R_{i+1})]dR_{i+1} - \left[1-\beta(1+\Delta_F)\right]dF_i \quad \forall i \leq T-2$$

Reasoning backward, starting with $dR_{T-1} = -dF_{T-1}$, it is straightforward to see that $\Delta_F < \frac{1}{\beta} - 1$ leads to $1-\beta(1+\Delta_F) > 0$ hence $\frac{\partial R_i}{\partial F_i} \leq 0 \ \ \forall i \leq T-2$.

In turn, for $\Delta_F > \frac{1}{\beta} - 1$, $1-\beta(1+\Delta_F) < 0$, let us note that

$$\begin{aligned} dR_{T-2} &= -\beta(1-\gamma p(\theta))[1-G(R_{T-1})]dF_{T-1} + \left[\beta(1+\Delta_F)-1\right]dF_{T-2} \\ dR_{T-3} &= \beta(1-\gamma p(\theta))[1-G(R_{T-2})]\{-\beta(1-\gamma p(\theta))[1-G(R_{T-1})]dF_{T-1} + \\ & \quad \left[\beta(1+\Delta_F)-1\right]dF_{T-2}\} + \left[\beta(1+\Delta_F)-1\right]dF_{T-3} \\ & \ldots \end{aligned}$$

Hence, since $\beta(1-\gamma p(\theta))[1-G(R_i)] < 1 \ \ \forall i$, there exists a threshold age $\tilde{i}$ such that $\frac{\partial R_i}{\partial F_i} > 0 \ \ \forall i \leq \tilde{i}$.

C.2 The efficient allocation

Let us denote λ_i and μ_i the Lagrange multiplier associated with constraints (18) and (19); optimal decision rules with respect to R_{i+1}, θ and u_i, y_i are respectively given by:

$$\begin{aligned}
\lambda_i &= \mu_i R_{i+1} \\
\sum_{i=1}^{T-1} c\left(\frac{\sum_{i=1}^{T-1} u_i}{T-1}\right) &= p'(\theta)\sum_{i=1}^{T-1} u_i\left(\mu_i \int_{R_{i+1}}^{1} xdG(x) - \lambda_i[1-G(R_{i+1})]\right) \\
\lambda_{i-1} &= b - \frac{\sum_{i=1}^{T-1} c\theta}{T-1} + \lambda_i\left[1-p(\theta)[1-G(R_{i+1})] - G(R_{i+1})\right] \\
&\quad + \mu_i\left[p(\theta)\int_{R_{i+1}}^{1} xdG(x) - \int_{R_{i+1}}^{1} xdG(x)\right] \\
\mu_i &= 1
\end{aligned}$$

Substituting for $\mu_i = 1$, hence $\lambda_i = R_{i+1}$, the remainder of the proof is straightforward with the definition $p'(\theta) = [1-\eta(\theta)]q(\theta)$.