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Supply Theory sans Profit-Maximization

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ABSTRACT

Supply Theory *sans* Profit-Maximization*

We utilize the analytical construct of a stochastic supply function to provide an aggregate representation of a finite collection of standard deterministic supply functions. We introduce a consistency postulate for a stochastic supply function that may be satisfied even if no underlying deterministic supply function is rationalizable in terms of profit maximization. Our consistency postulate is nonetheless equivalent to a stochastic expansion of supply inequality, which summarizes the predictive content of the traditional theory of competitive supply. A number of key results in the deterministic theory follow as special cases from this equivalence. In particular, it yields a probabilistic version of the law of supply, which implies the traditional specification. Our analysis thus provides a necessary and sufficient axiomatic foundation for a de-coupling of the predictive content of the classical theory of competitive firm behavior from its *a priori* roots in profit maximization, while subsuming the traditional theory as a special case.

JEL Classification: D21

Keywords: supply aggregation, stochastic supply function, stochastic consistency, weak axiom of profit maximization, stochastic supply inequality, law of supply

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I. Introduction

Suppose an industry contains 100 price-taking firms, which face identical input-output price configurations. Suppose further 50 of these firms choose the net output vector x under some price vector p, while the remaining 50 choose x' (say, because their technology is different). Then a natural way to model aggregate industry output is in terms of a *stochastic supply function* (SSF), which stipulates a choice of x and x', each with probability $\frac{1}{2}$, under p. Analogously, when a competitive firm faces technology shocks governed by some exogenous stochastic process, its supply behavior can be modeled in terms of an SSF. Thus, for example, suppose the technology set available to some firm producing wheat is τ if it snows at the beginning of the crop cycle, and τ' otherwise. Given some price vector p, the firm chooses the net output vector x if its realized (available) technology is τ; it chooses x' otherwise. The probability of snowfall is $\frac{1}{2}$. Then the supply of this firm can be modeled in terms of the SSF outlined earlier. Both situations turn up routinely in applications of the theory of competitive firm behavior.[1]

What kind of minimal, intuitive *a priori* restriction would ensure that an SSF satisfies analogues of the empirical content of the traditional theory of competitive firm behavior? The classical theory of firm behavior posits that the output-input choices of a competitive firm, summarized by a *deterministic supply function* (DSF), satisfy the Weak Axiom of Profit Maximization (WAPM). WAPM implies Supply Inequality (SI), which in turn yields the Law of Supply (LS). For a firm's choices to be rationalizable in terms of profit-maximization with respect to some collection of feasible input-output combinations, it is necessary and sufficient that its DSF satisfy WAPM.[2] Thus, predictions regarding competitive supply behavior, whether at the firm level or at the industry level, are derived from WAPM. Once firms are permitted to violate WAPM, the classical theory fails to generate any empirical content whatsoever.

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[1] The first situation, for example, is commonly considered in applied studies of competitive industry behavior and efficiency measurement, stemming from the seminal theoretical work of Debreu (1951) and Farrell (1957). These contexts have given rise to a large microeconometric literature on stochastic production frontier estimation (see, for example, Kumbhakar and Lovell (2000)). In macroeconomics, the literature on real business cycles flowing from Kydland and Prescott (1982) routinely models firm technology as subject to stochastic shocks.

[2] For formal definitions, see Section 2. Samuelson (1947) introduced WAPM and SI, showed that the former implies the latter (and thereby, LS), and noted that WAPM is necessary for profit maximization. See also Debreu (1959, p. 47). Hanoch and Rothschild (1972) showed that WAPM is sufficient for a firm's choices to be rationalizable in terms of profit-maximization with reference to some technology set. The name, WAPM, is due to Varian (1984).

Violations of WAPM appear to be frequently encountered in empirical studies. It has been argued that large departures from the profit maximization hypothesis, and, indeed, from maximizing behavior per se, are in fact routine in reality.[3]

In a well-known response, Becker (1962) argued that, even if individual firms do not maximize profits, LS may hold in the aggregate as a market-wide phenomenon. An extension of his claim suggests that, in contexts with stochastic firm technology, LS may also hold in the aggregate as a *firm-level* phenomenon even if the firm does not maximize profits with respect to its realized technology. Becker however based his case on examples. He did not provide general non-profit-maximizing behavioral foundations that would prove both necessary and sufficient for the empirical content of the traditional theory (as summarized by SI), to hold as an aggregate phenomenon. Recently, Dasgupta (2005) has posited a consistency restriction for a DSF which is weaker than WAPM, yet implies SI. However, Dasgupta left unexamined the connection between this restriction and aggregate supply behavior.

Regardless of whether an SSF provides an aggregate representation of the supply behavior of multiple price-taking firms with deterministic (but possibly different) technologies, or of a single competitive firm endowed with a stochastic technology, what kind of *minimal*, intuitively plausible, *a priori* consistency restriction can one posit for SSFs, that would imply aggregate supply behavior broadly in consonance with the classical (deterministic) theory? Would such a *necessary and sufficient* consistency restriction on the aggregate SSF representation be compatible with violation of WAPM by *every* underlying constituent DSF? Furthermore, would this restriction be compatible even with violation of the condition in Dasgupta (2005) by at least some underlying DSFs? If so, the empirical content of the classical theory (i.e. SI), suitably reinterpreted, would prove robust both: (i) as a market-wide phenomenon even when *no* individual firm exhibits choices that can be rationalized in terms of profit-maximization, and (ii) as a firm-level phenomenon even when the firm's choices on the basis of its realized technology set *invariably* violates profit-maximization. Such an analysis would provide a general axiomatic foundation for a complete de-linkage between the empirical

[3] See, for example, Leibenstein (1976, 1979, 1983) and Simon (1979). Varian (1985) provides a formal treatment of the notion of a "large" violation in this context. Public sector firms are often actively prevented from maximizing profits, in pursuance of other public objectives, even though they are price-takers in some competitive markets and face politically determined prices in others. 'Firms' in our usage also includes foundations and other private non-profit organizations which are small enough to be reasonably thought of as price-takers.

content of the classical theory of competitive firm behavior and the assumption of profit maximization, while subsuming both the analysis in Dasgupta (2005) and the traditional, WAPM-based theory, as special cases. Thus, it would identify the exact sense in which "… anti-marginalists can believe that firms are irrational, marginalists that market responses are rational, and both can be talking about the same economic world" (Becker, 1962: p.12).

This paper offers such an analysis. Furthermore, we show that the empirical content of the classical theory (i.e. a general formulation of SI) can be derived from an *a priori* behavioral restriction with independent intuitive appeal that does not presuppose even *cost minimization*.

Section 2 introduces the basic notation and definitions. In particular, we introduce our notion of a stochastic supply function and discuss how it can be used as an aggregate analytical representation of a finite class of deterministic supply functions. Section 3 introduces our *a priori* consistency postulate for SSFs, which we term *stochastic consistency* (SC), and discusses its properties. We show that the aggregate SSF representation of a finite class of DSFs may satisfy SC even if *no* underlying DSF can be rationalized in terms of profit maximization (nor satisfy cost minimization). Our consistency postulate may hold even if some of the underlying DSFs violate the consistency restriction proposed by Dasgupta (2005). Thus, SC completely delinks firm behavior from profit-maximization. Section 4 introduces a general probabilistic expansion of the classical Supply Inequality, and establishes its equivalence with our SC. Existing results in the deterministic framework, such as the relationship between WAPM and SI and the equivalence between Dasgupta's (2005) condition and SI, fall out as special cases of this general equivalence. Our general equivalence condition also yields a probabilistic version of the law of supply, from which the traditional version falls out as a special case. Section 5 concludes. Detailed proofs are presented in the Appendix.

2. Notation and preliminaries

Let n be the number of commodities and let $N = \{1,2,...n\}$. Let $\Re$ and $\Re_{++}$ denote, respectively, the set of real numbers and that of positive real numbers. A competitive firm faces n-dimensional vectors of commodity prices and produces n-tuples of net outputs. We shall denote price vectors by p, p' etc. and net output vectors by x, x' etc. The set of all possible price vectors is $\Re^n_{++}$. Given a net output vector x, x_i will denote the amount of the $i-th$ commodity contained in x. Given a price vector p, p_i will denote the price of the $i-th$ commodity. We shall denote the power set (i.e., the set of all possible subsets) of $\Re^n$ by $\pi(\Re^n)$. Given any finite set Ω, $|\Omega|$ will denote the number of elements in Ω.

Definition 2.1. *A stochastic supply function* (SSF) is a rule *s*, which specifies, for every $p \in \Re^n_{++}$, exactly one finitely additive probability measure *t* on ($\Re^n$, $\pi(\Re^n)$) ($\Re^n$ being the set of outcomes and $\pi(\Re^n)$ being the relevant algebra in $\Re^n$).

Given an SSF, *s*, let $t = s(p)$, and let *A* be a subset of $\Re^n$. Then $t(A)$ represents the probability a net output vector will be chosen from the set *A*, under the price vector *p*.

Definition 2.2. A *deterministic supply function* (DSF) is a rule *S*, which specifies, for every $p \in \Re^n_{++}$, exactly one element of R^n.

Given a DSF, *S*, and given any price vector p, $S(p)$ is the net output vector (uniquely) chosen. A DSF can evidently be identified with a *degenerate* SSF.[4]

An SSF may be used as an analytical construct to aggregate deterministic supply behavior by *n* competitive firms, represented by *n* (possibly different) DSFs, all facing the same price vector, *p*. Group (industry) supply can be modeled via a 'representative' competitive firm facing *p* and choosing according to an SSF such that, for any subset *A* of $\Re^n$, the probability of choosing a net output vector from *A* is simply the proportion of firms who do so.

Definition 2.3. For all $j \in M = \{1,...,m\}$, let S_j be a given DSF ($S_1,...,S_m$ need not all be distinct). We say that that an SSF, *s*, *aggregates* $\langle S_1,...,S_m \rangle$ iff, for every $p \in \Re^n_{++}$, and every $A \subseteq \Re^n$, [$t(A) = \frac{|\{i \in M \mid S_i(p) \in A\}|}{m}$], where $t = s(p)$.

An SSF may also represent supply behavior of a single competitive firm subject to technology shocks. The intuitive interpretation here is that there exists a given *m*-tuple of technology sets (collections of feasible net output vectors), say $\langle G_1,...,G_m \rangle$. If G_j happened to be the technology set actually facing the firm, the firm would choose according to some DSF S_j. Given a price vector p, some exogenous process ('nature') randomly determines which

[4] An SSF, *s*, is degenerate iff, for every $p \in \Re^n_{++}$, there exists $x \in R^n$ such that $t(\{x\}) = 1$, where $t = s(p)$.

technology set is *realized*: the firm subsequently chooses according to the corresponding DSF. The probability that any technology set G_j will be realized is $1/m$, but since $\langle G_1,...,G_m \rangle$ need not all be distinct, two distinct DSFs may have different probabilities of realization.

We now summarize, for later reference, the classical theory of competitive firm behavior.

Definition 2.4. A DSF, *S*, satisfies the *Weak Axiom of Profit Maximization* (WAPM) iff, for every ordered pair of price vectors $\langle p, p' \rangle$, $[p(x - x') \geq 0]$, where $x = S(p), x' = S(p')$.

WAPM requires that, if a competitive firm happens to choose some net output vector x when faced with the price vector *p*, then it cannot choose any net output vector under another price situation which would give it a higher profit under *p*. The firm's supply function, *S*, can be rationalized, i.e. interpreted, as being driven by the goal of profit-maximization, if one can construct a set of net output vectors, say Γ, such that, if the firm's technology set was indeed Γ, and it wished to maximize its profit, then it would be able to do so by choosing according to *S*.[5] A *closed and convex* set $\Gamma \subseteq \Re^n$ exists which rationalizes *S* in terms of profit maximization if, and only if, *S* satisfies WAPM.[6] The primary empirical implication of the *a priori* behavioral restriction imposed by WAPM is the so-called Supply Inequality.

Definition 2.5. A DSF, *S*, satisfies *supply inequality* (SI) iff,

for every pair of price vectors $\langle p, p' \rangle$: $(p - p')(x - x') \geq 0$, (2.1)

where $x = S(p), x' = S(p')$.

SI yields the law of supply (the supply of any output by a competitive firm must be non-decreasing in its own price, and the use of any input non-increasing), along with positive semi-definiteness of the substitution matrix. The predictive content of the classical choice-based theory of firm behavior is entirely specified by SI, which in turn is derived as the implication of WAPM. Thus, WAPM provides the *a priori* behavioral foundation for the classical theory.

[5] A DSF, *S*, is *rationalizable in terms of profit maximization with respect to a technology set* iff there exists some $\Gamma \subseteq \Re^n$ such that, for all $p \in \Re^n_{++}$, (i) $S(p) \in \Gamma$, and (ii) $[[pS(p) \geq pv'] \;\; \textit{for all } v' \in \Gamma]$. The set Γ is said to *rationalize S* in terms of profit maximization.

[6] See Varian (1984).

3. Stochastic consistency

We now introduce our consistency restriction for an SSF. Let $x^* \in \Re^n$ be an arbitrary net output vector, and suppose the price vector changes from some initial situation p to p'. Consider the collection of all net output vectors whose *attractiveness relative to the reference vector* x^* *does not decline* due to the price change. This is the set of all net output vectors which continue to yield at least as much profit (or as low a loss) *relative to* x^*, despite the price change. Thus, this collection consists of all $x \in \Re^n$ which satisfy [$p'(x-x^*) \geq p(x-x^*)$]. It seems reasonable to require that a net output vector be chosen from this collection at least as frequently as earlier: the price shift has not reduced the attractiveness of its members. Thus, the probability that a net output vector is chosen from this set should not decline. An analogous condition should hold for the set of net output vectors which are made *strictly more attractive* by the price shift.[7]

Definition 3.1. An SSF, *s*, satisfies *stochastic consistency* (SC) iff, for every pair of price vectors $\langle p, p' \rangle$, and for every $x^* \in \Re^n$:

$$t'\left(\left\{x \in \Re^n \mid p'\left(x-x^*\right) \geq p\left(x-x^*\right)\right\}\right) \geq t\left(\left\{x \in \Re^n \mid p'\left(x-x^*\right) \geq p\left(x-x^*\right)\right\}\right), \tag{3.1}$$

and

$$t'\left(\left\{x \in \Re^n \mid p'\left(x-x^*\right) > p\left(x-x^*\right)\right\}\right) \geq t\left(\left\{x \in \Re^n \mid p'\left(x-x^*\right) > p\left(x-x^*\right)\right\}\right). \tag{3.2}$$

where $t = s(p)$ and $t' = s(p')$.

Stochastic consistency is the probabilistic analogue of a restriction on an individual firm's deterministic supply behavior, introduced by Dasgupta (2005), which we term 'consistency' here. Suppose the firm chooses, respectively, the net output vectors $\tilde{x}, \tilde{x}'$, under p, p'. Suppose further that, by choosing $\tilde{x}$ instead of the feasible alternative $\tilde{x}'$ under p, the firm loses some amount, say \$10. Consistency requires that the loss entailed by the choice of $\tilde{x}$ instead of $\tilde{x}'$ under p' must be at least \$10 (since otherwise, intuitively, a consistent firm should have persisted with $\tilde{x}$ under p', instead of switching to $\tilde{x}'$).

Definition 3.2. A DSF, *S*, satisfies *consistency* (C) iff, for every pair of price vectors $\langle p, p' \rangle$:

$$p'\left(\tilde{x}' - \tilde{x}\right) \geq p\left(\tilde{x}' - \tilde{x}\right); \tag{3.3}$$

7 Intuitively, our condition thus treats profit as *a* consideration in choice, but not necessarily the only consideration.

where $\tilde{x}' = S(p'), \tilde{x} = S(p)$.

Observation 3.3. A degenerate SSF, *s*, satisfies SC iff the DSF corresponding to *s* satisfies C.

Proof: See the Appendix.

Dasgupta (2005) has shown that, for a DSF, C is weaker than WAPM (recall Definitions 3.2 and 2.4 and note Example 3.7 below). By Observation 3.3, the following relationship thus holds between WAPM and SC for a degenerate SSF.

Observation 3.4.

(i) A degenerate SSF, *s*, may satisfy SC even if the DSF corresponding to *s* violates WAPM.

(ii) If a DSF, *S*, satisfies WAPM, then the degenerate SSF corresponding to *S* must satisfy SC.

When an SSF aggregates a finite class of DSFs (recall Definition 2.3), SC turns out to be a weaker restriction than the condition that the constituent DSFs all individually satisfy C.

Lemma 3.5. Let $M = \{1,...,m\}$ be a given finite set, $|M| \geq 1$.

(i) For every *m*-tuple of DSFs $\langle S_1,..,S_m \rangle$ such that [for all $j \in M$, S_j satisfies C], the SSF aggregating $\langle S_1,..,S_m \rangle$ satisfies SC.

(ii) For all $m \geq 2$, there exists an *m*-tuple of DSFs $\langle S_1,..,S_m \rangle$ such that: [for some $j \in M$, S_j violates C], but the SSF aggregating $\langle S_1,..,S_m \rangle$ satisfies SC.

Proof: See the Appendix.

Recall now that, for a DSF, WAPM necessarily implies C; but a DSF may satisfy C, yet violate WAPM (see Example 3.7 below). In light of this, Lemma 3.5 immediately yields the following.

Proposition 3.6. Let $M = \{1,...,m\}$ be a given finite set, $|M| \geq 1$.

(i) For every *m*-tuple of DSFs $\langle S_1,..,S_m \rangle$ such that [for all $j \in M$, S_j satisfies WAPM], the SSF aggregating $\langle S_1,..,S_m \rangle$ satisfies SC.

(ii) There exists an *m*-tuple of DSFs, $\langle S_1,..,S_m \rangle$, such that: [for every $j \in M$, S_j violates WAPM], but the SSF aggregating $\langle S_1,..,S_m \rangle$ satisfies SC.

Proposition 3.6 summarizes the central finding of this section. Proposition 3.6(ii) brings into focus the disconnect between profit maximization and SC: the aggregate SSF representation may satisfy SC even if *not a single* underlying DSF can be rationalized via profit-maximization.[8] Thus, given any finite collection of DSFs, SC for their aggregate SSF representation is weaker than the requirement that all constituent DSFs be rationalizable in terms of profit maximization.

Our framework takes, as its theoretical prior, a *given* finite collection of DSFs, and derives an SSF as its aggregate representation. This seems the natural modeling approach in the case of aggregation over a collection of competitive firms. In the case of an SSF that models supply behavior of an individual firm subject to stochastic technology shocks, in some empirical contexts, the underlying collection of alternative DSFs over which 'nature' randomizes may not be directly observable. Proposition 3.6(ii) implies that one can have a *given* prior collection of DSFs, all of whom violate WAPM, yet whose aggregate SSF representation satisfies SC. But, given an SSF, *s*, satisfying SC, does there necessarily exist *some m*-tuple of DSFs, at least one of whom satisfies WAPM, and whose aggregate representation is s?

The following example shows that even this is not the case. An SSF, s, may satisfy SC, yet *every* possible finite collection of DSFs that yields s as its aggregate representation may exhibit violation of WAPM by *all* its constituents. Thus, an SSF may satisfy SC even when it is completely impervious to *ex post* rationalization in terms of profit-maximization.

Example 3.7. Let s be a degenerate SSF specified as follows: for all $p \in \Re^n_{++}$, $t\left(\left\{\left(1,-\frac{1}{p_2},0,...,0\right)\right\}\right)=1$; where $t = s(p)$. It can be checked that s satisfies SC. Evidently, any

[8] The disconnect between C and SC, while present (recall Lemma 3.5(ii)), is not so absolute. It can be checked that an aggregate SSF representation must violate SC if all underlying DSFs violate C.

m-tuple of DSFs $\langle S_1,...,S_m \rangle$ that *s* can be said to aggregate must exhibit, for all $j \in \{1,2,...,m\}$, $S_j = S$; where $S(p) = \left(1, -\frac{1}{p_2}, 0,...,0\right)$. S violates WAPM, while satisfying C. ◊

Remark 3.8. The example above also shows that an SSF may satisfy SC, yet its constituent DSFs may all violate *cost-minimization*. Furthermore, an SSF may satisfy SC even when it is inconsistent with any collection of DSFs satisfying cost-minimization.[9]

4. Stochastic supply inequality and the law of supply

As discussed in Section 3, stochastic consistency for an aggregate SSF representation is a weaker restriction than the requirement that all underlying DSFs be rationalizable in terms of profit maximization. It is even weaker than the restriction that the underlying DSFs all satisfy deterministic consistency. Can such a weak behavioral restriction suffice to generate a probabilistic analogue of deterministic Supply Inequality, which summarizes the main empirical content of the classical theory of competitive firm behavior? We now address this question.

Definition 4.1. An SSF, *s*, satisfies *stochastic supply inequality* (SSI) iff, for every pair of price vectors $\langle p, p' \rangle$, and for every $z \in \Re$:

$$t'\left(\left\{x \in \Re^n \mid (p'-p)x \geq z\right\}\right) \geq t\left(\left\{x \in \Re^n \mid (p'-p)x \geq z\right\}\right), \tag{4.1}$$

and

$$t'\left(\left\{x \in \Re^n \mid (p'-p)x > z\right\}\right) \geq t\left(\left\{x \in \Re^n \mid (p'-p)x > z\right\}\right); \tag{4.2}$$

where $t = s(p)$ and $t' = s(p')$.

Let z be any arbitrary real number, and suppose the price vector changes from some initial situation p to p'. Consider the collection of all net output vectors whose profitability increases by at least z due to the price change. SSI requires that the probability of choosing from this collection should not fall under p'. An analogous requirement should also hold with regard to the collection of all net output vectors whose profitability increases by more than z.

SSI provides the natural stochastic expansion of deterministic Supply Inequality (recall Definition 2.5). More formally, it is easy to check that the following must hold.

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[9] The DSF in Example 3.7 captures something of the intuitive flavor of an X-efficiency model, where the firm improves its technical efficiency, by economizing on its input use, in response to a rise in input price.

Observation 4.2. A degenerate SSF, *s*, satisfies SSI iff the DSF corresponding to *s* satisfies SI.

Proposition 4.3. An SSF satisfies SSI iff it satisfies SC.

Proof of Proposition 4.3: See the Appendix.

Proposition 4.3 subsumes a number of results in the standard deterministic theory of competitive firm supply, while also encompassing probabilistic and not-necessarily profit-maximizing behavior. Notice first that, in light of Observations 3.3 and 4.2, Proposition 4.3 yields the following central result in the deterministic theory, noted by Dasgupta (2005).

Corollary 4.4. (Dasgupta 2005) A DSF satisfies C iff it satisfies SI.

Since WAPM is a stronger restriction than C for DSFs (recall Example 3.7), Proposition 4.3 also yields, via Corollary 4.4, the following result, familiar in the traditional deterministic theory.

Corollary 4.5. A DSF satisfies SI if it satisfies WAPM; there exist DSFs which satisfy SI but violate WAPM.

Lastly, Proposition 4.3 yields a probabilistic expansion of the law of supply, which subsumes the traditional, deterministic, version. Suppose the price of a commodity rises, all other prices remaining invariant. Our (stochastic dominance) formulation of LS requires that, for every real number β, neither the probability of producing at least β amount of the commodity, nor the probability of producing more than β amount of the commodity, should decrease.

Notation 4.6. For every $i \in N$, let K_i denote the set of all ordered pairs of price vectors $\langle p, p' \rangle$ such that (i) [$p_j = p'_j$ for all $j \in N \setminus \{i\}$] and (ii) $p_i' > p_i$.

Definition 4.7. An SSF, *s*, satisfies the *law of supply* (LS) iff, for all $\beta \in R$, for all $i \in N$, and for all ordered pairs $\langle p, p' \rangle \in K_i$, we have:

$$t'\left(\left\{x \in \Re^n \mid x_i \geq \beta\right\}\right) \geq t\left(\left\{x \in \Re^n \mid x_i \geq \beta\right\}\right), \tag{4.3}$$

and

$$t'\left(\left\{x \in \Re^n \mid x_i > \beta\right\}\right) \geq t\left(\left\{x \in \Re^n \mid x_i > \beta\right\}\right); \quad (4.4)$$

where $t = s(p)$ and $t' = s(p')$.

Corollary 4.8. If an SSF satisfies SC, then it must satisfy LS.

Proof of Corollary 4.8: See the Appendix.

In light of Corollary 4.4 (or, alternatively, in light of Corollary 4.8 and Observation 3.3) the law of supply in its standard, deterministic, version also follows from Proposition 4.3.

Corollary 4.9. If a DSF, *S*, satisfies C, then, for all $i \in N$, and for all ordered pairs $\langle p, p' \rangle \in K_i$, [$x_i' \geq x_i$], where $x = S(p), x' = S(p')$.

5. Concluding remarks

This paper extends and completes a line of investigation initiated by Becker (1962) and continued by Dasgupta (2005), which argued that the primary empirical/predictive content of the traditional theory of competitive firm behavior can be delinked from its behavioral presumption of profit-maximization. Our analysis provides a necessary and sufficient axiomatic foundation for such de-linkage, while subsuming both the contribution of Dasgupta (2005) and the traditional, WAPM-based theory as special cases. In so doing, we have also provided a supply theoretic parallel to the revealed preference treatment of stochastic demand theory recently developed by Bandyopadhyay *et al.* (2004, 2002, 1999) and Dasgupta and Pattanaik (2007).[10]

We have utilized the analytical construct of a stochastic supply function to provide an aggregate representation of a finite class of standard deterministic supply functions. We have introduced an intuitively plausible consistency postulate for a stochastic supply function that may be satisfied even if no underlying deterministic supply function is open to rationalization in terms of profit maximization (nor, indeed, satisfies cost-minimization). In this sense, our consistency postulate provides a complete conceptual departure from the traditional presumption of profit-maximization. Despite this departure, our consistency postulate turns out to be equivalent to a stochastic analogue of the deterministic condition of supply inequality, which summarizes the

[10] See also McFadden (2005) for a recent discussion of the literature on probabilistic revealed preference.

predictive content of the traditional theory of competitive firm behavior. This finding provides the central equivalence in the theory of competitive firm behavior, in that a number of results in the deterministic theory follow as special cases. In particular, our equivalence result yields a probabilistic version of the law of supply, which implies the traditional specification. It is difficult to see how a theory of supply with any applicability can afford to dispense with the law of supply, at least in its probabilistic version. In this sense, our analysis also appears to set the conceptual limits beyond which the behavioral presumptions of the traditional theory cannot be substantively relaxed without seriously undermining its predictive import.

If not profit, exactly what is the objective function (if any) that a competitive firm need maximize to satisfy our consistency condition? In other words, can one characterize some objective function, maximization of which over probabilistic convex technology sets would provide a necessary and sufficient rationalization of firm choice behavior that satisfies our stochastic consistency? Would such an objective function be open to intuitive interpretation? These questions, which relate to a preference-based counterpart of the choice-based supply theory developed in this paper, suggest themselves as useful candidates for future investigations.

Appendix

Proof of Observation 3.3:

First consider a degenerate SSF, *s*, satisfying SC. Since *s* is degenerate, there must exist some $\tilde{x}', \tilde{x} \in \Re^n$ such that $t(\{\tilde{x}\}) = 1 = t'(\{\tilde{x}'\})$. Since, given SC, (3.1) holds for all $x^* \in \Re^n$, it must also hold for $x^* \equiv \tilde{x}$. Thus, (3.1) implies: [$t'(\{x \in \Re^n \mid p'(x - \tilde{x}) \geq p(x - \tilde{x})\}) = 1$]; i.e., (3.3).

Now consider a DSF, *S*, satisfying C. Given any $x^* \in \Re^n$, (3.3) is equivalent to:

$$(p' - p)(\tilde{x}' - x^*) \geq (p' - p)(\tilde{x} - x^*). \tag{X1}$$

The degenerate SSF corresponding to *S* must therefore satisfy, for all $x^* \in \Re^n$,

$$t'(\{x \in \Re^n \mid (p' - p)(x - x^*) \geq 0\}) \geq t(\{x \in \Re^n \mid (p' - p)(x - x^*) \geq 0\}); \tag{X2}$$

$$t'(\{x \in \Re^n \mid (p' - p)(x - x^*) > 0\}) \geq t(\{x \in \Re^n \mid (p' - p)(x - x^*) > 0\}). \tag{X3}$$

Respectively, (X2) and (X3) imply (3.1) and (3.2). ◊

Proof of Lemma 3.5.

(i) Suppose, for all $j \in M$, S_j satisfies C. Consider any $p, p' \in \Re^n$ and any $x^* \in \Re^n$. Let $S_j(p) = \tilde{x}^j$, $S_j(p') = \tilde{x}^{j'}$. Then, from (3.3), we get:

$$\text{for all } j \in M, \left[(p'-p)\left(\tilde{x}^{j'} - x^*\right) \geq (p'-p)\left(\tilde{x}^j - x^*\right)\right]. \qquad \text{(X4)}$$

Notice now that the aggregate SSF representation, *s*, of $\langle S_1,...,S_m \rangle$ must satisfy:

$$t\left(\left\{x \in \Re^n \mid (p'-p)(x - x^*) \geq 0\right\}\right) = \frac{\left|\left\{j \in M \mid (p'-p)\left(S_j(p) - x^*\right) \geq 0\right\}\right|}{m};$$

$$t'\left(\left\{x \in \Re^n \mid (p'-p)(x - x^*) \geq 0\right\}\right) = \frac{\left|\left\{j \in M \mid (p'-p)\left(S_j(p') - x^*\right) \geq 0\right\}\right|}{m}.$$

By (X4), $\left\{j \in M \mid (p'-p)\left(S_j(p) - x^*\right) \geq 0\right\} \subseteq \left\{j \in M \mid (p'-p)\left(S_j(p') - x^*\right) \geq 0\right\}$. Condition (3.1) follows. An analogous argument establishes (3.2).

(ii) Let p_i be the price of commodity $i \in N$, and let the net output vector supplied according to the DSF S_j be denoted by x^j, $j \in M$. Let the individual supply functions be: $x^1 = (1,-3,0,...,0)$ if $p_2 \geq 1$; $x^1 = (1,-1,0,...,0)$ otherwise ; $x^2 = (1,-1,0,...,0)$ if $p_2 \geq 1$, $x^2 = (1,-3,0,...,0)$ otherwise; $x^j = (0,...,0)$ for all $j \in \{3,...,m\}$. S_1 violates C. Consider the SSF aggregation, *s*, of $\langle S_1,...,S_m \rangle$: regardless of the price vector, the net output vectors $(1,-3,0,...,0)$ and $(1,-1,0,...,0)$ must each be chosen with probability $\frac{1}{m}$, while the net output vector $(0,...,0)$ must be chosen with the remaining probability $\frac{m-2}{m}$. Clearly, *s* satisfies SC. ◊

Proof of Proposition 4.3.

First let *s* be an SSF satisfying SC, and consider $x^* \in \Re^n$. Denoting $z^* \equiv (p'-p)x^*$, by (3.1):

$$t'\left(\left\{x \in \Re^n \mid (p'-p)x \geq z^*\right\}\right) \geq t\left(\left\{x \in \Re^n \mid (p'-p)x \geq z^*\right\}\right). \qquad \text{(X5)}$$

Since, given any pair of price vectors $\langle p, p' \rangle$, (3.1) must hold for every $x^* \in \Re^n$, and noting that, given any $z \in \Re$ and any pair of price vectors $\langle p, p' \rangle$, there exists $x \in \Re^n$ such that

$(p'-p)x = z$, it follows from (X5) that SC implies (4.1) must hold for every $z \in \Re$. Noting (3.2), an analogous argument establishes (4.2) for every $z \in \Re$. Hence, SC implies SSI.

Now let s be an SSF satisfying SSI, and consider any $z \in \Re$. Then, for any $\hat{x} \in \Re^n$ which satisfies [$(p'-p)\hat{x} = z$], by (4.1):

$$t'\left(\left\{x \in \Re^n \mid p'(x-\hat{x}) \geq p(x-\hat{x})\right\}\right) \geq t\left(\left\{x \in \Re^n \mid p'(x-\hat{x}) \geq p(x-\hat{x})\right\}\right). \quad \text{(X6)}$$

Since, given any pair of price vectors $\langle p, p' \rangle$, (4.1) holds for every $z \in \Re$, and since, given any $x^* \in \Re^n$ and any $\langle p, p' \rangle$, there exists $z \in \Re$ such that $(p'-p)x^* = z$, (X6) implies (3.1) must hold for all $x^* \in \Re^n$. Noting (4.2), an analogous argument establishes (3.2) for all $x^* \in \Re^n$. ◊

Proof of Corollary 4.8:

Noting (4.1), if an SSF satisfies SSI, for all $i \in N$, for all $\langle p, p' \rangle \in K_i$, and for every $z \in \Re$:

$$t'\left(\left\{x \in \Re^n \mid x_i \geq \frac{z}{\left(p_i' - p_i\right)}\right\}\right) \geq t\left(\left\{x \in \Re^n \mid x_i \geq \frac{z}{\left(p_i' - p_i\right)}\right\}\right). \quad \text{(X7)}$$

Hence, given any $\beta \in \Re$, (X7) holds for $z = \beta\left(p_i' - p_i\right)$. Condition (4.3) follows. Noting (4.2), an analogous argument establishes (4.4). Corollary 4.8 follows from Proposition 4.3. ◊

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