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ABSTRACT

Maximum Likelihood Estimation and Lagrange Multiplier Tests for Panel Seemingly Unrelated Regressions with Spatial Lag and Spatial Errors: An Application to Hedonic Housing Prices in Paris*

This paper proposes maximum likelihood estimators for panel seemingly unrelated regressions with both spatial lag and spatial error components. We study the general case where spatial effects are incorporated via spatial errors terms and via a spatial lag dependent variable and where the heterogeneity in the panel is incorporated via an error component specification. We generalize the approach of Wang and Kockelman (2007) and propose joint and conditional Lagrange Multiplier tests for spatial autocorrelation and random effects for this spatial SUR panel model. The small sample performance of the proposed estimators and tests are examined using Monte Carlo experiments. An empirical application to hedonic housing prices in Paris illustrates these methods. The proposed specification uses a system of three SUR equations corresponding to three types of flats within 80 districts of Paris over the period 1990-2003. We test for spatial effects and heterogeneity and find reasonable estimates of the shadow prices for housing characteristics.

JEL Classification: C31, C33, R21

Keywords: hedonic housing prices, Lagrange multiplier tests, maximum likelihood, panel spatial dependence, spatial lag, spatial error, SUR

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1 Introduction

Zellner's (1962) pioneering paper considered the estimation and testing of seemingly unrelated regressions (SUR) with correlated error terms. SUR has been applied in many research areas in economics and other fields, see Srivastava and Giles (1987) and Fiebig (2001) for excellent surveys. It is by now clear that SUR achieves gains in efficiency by estimating a set of equations simultaneously rather than estimating each equation separately. Common factors affecting these equations allow such gains in efficiency and has been demonstrated in economics, for e.g., in studying demand systems and translog cost functions, to mention a few important applications.

Avery (1977) and Baltagi (1980) extended the SUR model to panel data models with error components. This extension allows one to take advantage of panel data which pools regions, counties, countries, neighborhoods over time. Besides the larger variation in the data across these regions, one is able to control for unobserved heterogeneity across these units of observation.

Anselin (1988) extended the SUR model to allow for spatial correlation in the data. This extension allows one to take advantage of spillover effects across regions. Here, we focus on combining the spatial and panel aspects of the data in a SUR context. In fact, Anselin (1988) and Elhorst (2003) among others provided maximum likelihood (ML) methods that combine panel data with spatial analysis, while Kapoor, Kelejian and Prucha (2007) provided a generalized moments estimators (GM) approach for estimating a spatial random effects panel model with SAR disturbances. Fingleton (2008a) extended the GM approach of Kapoor, Kelejian and Prucha to allow for spatial moving average disturbances, see Anselin, Le Gallo and Jayet (2008) for a recent survey.

This paper follows Wang and Kockelman (2007) who applied ML methods to a SUR model with spatial effects incorporated via autocorrelation in the spatial error terms and heterogeneity in the panel incorporated via random-effects. However, this paper extends the ML approach developed by Wang and Kockelman (2007) to the general case where spatial effects are incorporated via spatial error terms and via a spatial lag dependent variable and where the heterogeneity in the panel is incorporated via an error component specification.

We propose joint and conditional Lagrange Multiplier tests for spatial autocorrelation and random effects for this spatial SUR panel model. The small sample performance of the proposed estimators and tests are examined using Monte Carlo experiments. We show that ignoring these spatial effects and/or heterogeneity can lead to misleading inference.

An empirical application to hedonic housing prices in Paris illustrates these methods. The proposed specification uses a system of three SUR equations corresponding to three types of flats within 80 districts of Paris over the period 1990-2003.[1]. One of the main contributions of the paper is that it pays special attention to the heterogeneity and spatial variation in housing prices across districts and it tests for their existence.[2] We find significant spatial effects and heterogeneity across the Paris districts, and we show that ML methods that incorporate these effects lead to reasonable estimates of the shadow prices of housing attributes.

Section 2 sets up the panel SUR model with spatial lag and spatial error components. In section 3, we present the ML estimation under normality of the disturbances. Section 4 considers the problem of jointly testing for random effects as well as spatial correlation in the context of this spatial SUR panel model. This extends earlier work on testing in spatial panel models by Baltagi et al. (2007) from the single equation case to the SUR case. Section 5 performs Monte Carlo experiments which compare the small sample properties of the proposed ML estimators and LM tests. Section 6 provides an empirical application of these methods to the problem of estimating hedonic housing prices in Paris, while section 7 concludes. We recognize that there is a large literature on hedonic housing and that our application is only meant to illustrate our spatial panel ML methods and the the asscociated LM test statistics.

[1]Hedonic measures have a strong theoretical grounding and use regression techniques to control for compositional and quality change (see, for example, Arguea and Hsiao (1993), Can (1992), Dubin (1992), Dubin et al. (1999), Griliches (1971), Halvorsen and Pollakowski (1981) and Rosen (1974) to mention a few).

[2]For spatial effects in real estate (see Fingleton (2008b), Glaeser (2008), and Helpman (1998) to mention a few). For spatial econometric methods (see Anselin (1988), Anselin and Bera (1998), Anselin et al. (2008), Baltagi (2010), Baltagi et al. (2007), and Elhorst (2003, 2010) to mention a few).

2 The panel SUR with spatial lag and spatial error components

We consider a spatial system of equations model viewed as an extension of the single equation spatial model introduced by Cliff and Ord (1973, 1981). In particular, we specify a system of spatially interrelated panel equations corresponding to N cross sectional units over T time periods. The spatial SUR model for panel data is composed of M equations (each potentially having a different set of explanatory variables) for N regions which are observed over T time periods. Consider the set of M equations:

$$\begin{aligned} y_{jt} &= \gamma_j W y_{jt} + X_{jt}\beta_j + \varepsilon_{jt} \ , \ j = 1, ..., M, \ t = 1, ..., T \\ &= \gamma_j \overline{y}_{jt} + X_{jt}\beta_j + \varepsilon_{jt} \end{aligned} \tag{1}$$

where y_{jt} is a $(N \times 1)$ vector, W is an $(N \times N)$ spatial weights matrix[3], X_{jt} is a $(N \times k_j)$ matrix of exogenous variables, β_j is a $(k_j \times 1)$ vector of parameters and ε_{jt} is a $(N \times 1)$ vector of disturbances. The vector $\overline{y}_{jt}$ $(= W y_{jt})$ is typically referred to as the spatial lag of y_{jt}. In addition to allowing for general spatial lags in the endogenous variables, we also allow for spatial autocorrelation in the disturbances. In particular, we assume that the disturbances are generated either by a spatially autoregressive (SAR) process or a spatially moving average (SMA) process:

$$\varepsilon_{jt} = \begin{cases} \lambda_j W \varepsilon_{jt} + u_{jt} & \text{for SAR} \\ \lambda_j W u_{jt} + u_{jt} & \text{for SMA} \end{cases} \tag{2}$$

and u_{jt} is an error component:

$$u_{jt} = \mu_j + v_{jt} \tag{3}$$

When we pool the T time periods, we get:

$$y_j = \gamma_j \left(I_T \otimes W\right) y_j + X_j \beta_j + \varepsilon_j \ , \ \varepsilon_j = \begin{cases} \lambda_j \left(I_T \otimes W\right) \varepsilon_j + u_j & \text{for SAR} \\ \lambda_j \left(I_T \otimes W\right) u_j + u_j & \text{for SMA} \end{cases} \tag{4}$$

[3]For ease of presentation, we are assuming that the system involves only one weight matrix. This also seems to be the typical specification in applied work. Our results can be generalized in a straight forward way to the case in which the weight matrix varies across equations.

with

$$u_j = (\iota_T \otimes I_N)\,\mu_j + v_j \tag{5}$$

where $\mu_j = \left(\mu_{j1}, ..., \mu_{jN}\right)'$, $v_j = (v_{j11}, ..., v_{jN1}, ..., v_{j1T}, ..., v_{jNT})'$ and ι_T is a $(T \times 1)$ vector of ones, see Anselin, Le Gallo and Jayet (2008). So:

$$y = (\Gamma \otimes I_T \otimes W)\,y + X\beta + \varepsilon \ , \ \varepsilon = \begin{cases} (\Lambda \otimes I_T \otimes W)\,\varepsilon + u & \text{for SAR} \\ (\Lambda \otimes I_T \otimes W)\,u + u & \text{for SMA} \end{cases} \tag{6}$$

where $\Gamma = diag_{j=1}^{M}\left\{\gamma_j\right\}$ and $\Lambda = diag_{j=1}^{M}\left\{\lambda_j\right\}$. Then,

$$Ay = X\beta + \varepsilon \ , \ B\varepsilon = u \tag{7}$$

with

$$\begin{cases} A &= I_{NTM} - (\Gamma \otimes I_T \otimes W) \\ B &= \begin{cases} I_{NTM} - (\Lambda \otimes I_T \otimes W) & \text{for SAR} \\ \left[I_{NTM} + (\Lambda \otimes I_T \otimes W)\right]^{-1} & \text{for SMA} \end{cases} \end{cases} \tag{8}$$

or

$$A = \begin{pmatrix} I_T \otimes A_1 & & \\ & \ddots & \\ & & I_T \otimes A_M \end{pmatrix}, \ B = \begin{pmatrix} I_T \otimes B_1 & & \\ & \ddots & \\ & & I_T \otimes B_M \end{pmatrix} \tag{9}$$

with

$$A_j = I_N - \gamma_j W \ , \ B_j = \begin{cases} I_N - \lambda_j W = H_j & \text{for SAR} \\ (I_N + \lambda_j W)^{-1} = L_j^{-1} & \text{for SMA} \end{cases} \tag{10}$$

The variance-covariance matrix of ε is given by:

$$\Omega_\varepsilon = B^{-1}\Omega_u\,(B')^{-1} \tag{11}$$

where Ω_u is the variance-covariance matrix of the error component term, see Baltagi (1980):

$$\begin{aligned} \Omega_u &= [\Omega_{jl}] \text{ with } \Omega_{jl} = \sigma_{\mu_{jl}}\,(J_T \otimes I_N) + \sigma_{v_{jl}} I_{NT} \\ &= \Sigma_u \otimes I_N = \Omega_\mu \otimes J_T \otimes I_N + \Omega_v \otimes I_T \otimes I_N \\ &= \left(T\Omega_\mu + \Omega_v\right) \otimes \overline{J_T} \otimes I_N + \Omega_v \otimes E_T \otimes I_N \end{aligned} \tag{12}$$

with $\overline{J_T} = J_T/T$, $E_T = \left(I_T - \overline{J_T}\right)$ and J_T is a $(T \times T)$ matrix of ones.

$$\Omega_{\mu} = \begin{pmatrix} \sigma^2_{\mu_1} & \sigma_{\mu_{12}} & \cdots & \sigma_{\mu_{1M}} \\ \sigma_{\mu_{21}} & \sigma^2_{\mu_2} & \cdots & \sigma_{\mu_{2M}} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\mu_{M1}} & \sigma_{\mu_{M2}} & \cdots & \sigma^2_{\mu_M} \end{pmatrix} \text{ and } \Omega_{v} = \begin{pmatrix} \sigma^2_{v_1} & \sigma_{v_{12}} & \cdots & \sigma_{v_{1M}} \\ \sigma_{v_{21}} & \sigma^2_{v_2} & \cdots & \sigma_{v_{2M}} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{v_{M1}} & \sigma_{v_{M2}} & \cdots & \sigma^2_{v_M} \end{pmatrix}.$$

Based on a joint standard normal distribution for the error term $\nu = \Omega_u^{-1/2} B\,(Ay - X\beta)$, the log-likelihood function for the joint vector of observations y is proportional to:

$$\ell \propto -\frac{1}{2} \ln |\Omega_u| + \ln |B| + \ln |A| - \frac{1}{2} \nu' \nu \tag{13}$$

with

$$\begin{aligned} \nu'\nu &= (Ay - X\beta)'\, B' \Omega_u^{-1} B\,(Ay - X\beta) \\ &= (Ay - X\beta)'\, \Omega_\varepsilon^{-1}\,(Ay - X\beta) = \varepsilon' \Omega_\varepsilon^{-1} \varepsilon \end{aligned} \tag{14}$$

3 Maximum Likelihood Estimation

The log-likelihood function (13) can also be written as follows:

$$\ell \propto \begin{cases} -\frac{N}{2} \ln |\Sigma_u| + T \sum\limits_{j=1}^{M} \ln |B_j| + T \sum\limits_{j=1}^{M} \ln |A_j| \\ -\frac{1}{2}\,(Ay - X\beta)'\, B' \left(\Sigma_u^{-1} \otimes I_N\right) B\,(Ay - X\beta) \end{cases} \tag{15}$$

Using the results in Baltagi (1980) and Magnus (1982),

$$\begin{cases} |\Sigma_u| = \left|T\Omega_{\mu} + \Omega_{v}\right| \left|\Omega_{v}\right|^{T-1} \\ \Sigma_u^{-1} = \left(T\Omega_{\mu} + \Omega_{v}\right)^{-1} \otimes \overline{J_T} + \Omega_{v}^{-1} \otimes E_T \end{cases} \tag{16}$$

we can express the log-likelihood function as follows:

$$\begin{aligned} \ell \;\propto\; & -\frac{N}{2} \ln \left|T\Omega_{\mu} + \Omega_{v}\right| - \frac{N\,(T-1)}{2} \ln |\Omega_{v}| + T \sum_{j=1}^{M} \ln |B_j| + T \sum_{j=1}^{M} \ln |A_j| \\ & -\frac{1}{2}\,(BAy - BX\beta)' \left(\left(T\Omega_{\mu} + \Omega_{v}\right)^{-1} \otimes \overline{J_T} \otimes I_N\right) (BAy - BX\beta) \\ & -\frac{1}{2}\,(BAy - BX\beta)' \left(\Omega_{v}^{-1} \otimes E_T \otimes I_N\right) (BAy - BX\beta) \end{aligned} \tag{17}$$

Generalizing the Wang and Kockelman (2007) approach, the model can be estimated using a three-step method: First, β can be estimated using generalized least squares (GLS), conditional on Ω_{μ}, Ω_{v}, $\gamma = (\gamma_1, ..., \gamma_M)'$, and $\lambda = (\lambda_1, ..., \lambda_M)'$. Then Ω_{μ} and Ω_{v} can be estimated conditional on β, γ and λ. These first two steps are iterated until the optimal Ω_{μ}, Ω_{v}, and β are found (conditional on γ and λ). The third step is to substitute the estimated Ω_{μ}, Ω_{v}, and β and to maximize the concentrated log-likelihood function over γ and λ. The estimated γ and λ then re-enter the estimation of Ω_{μ}, Ω_{v}, and β. This procedure is iterated until convergence.
The estimation method proposed can be performed using the following steps:

3.1 Step 1: Estimate β conditional on Ω_{μ}, Ω_{v}, γ and λ

Note that $\overline{J_T} \otimes I_N$ denotes an average of the $(BAy - BX\beta)$ values over time for each equation, and $E_T \otimes I_N$ denotes each observation's deviation from these averages. If one lets $P'P = \left(T\Omega_{\mu} + \Omega_{v}\right)^{-1}$ and $Q'Q = \Omega_{v}^{-1}$, one can transform the data as follows:

$$\begin{cases} y^* & = & \left(Q \otimes I_{NT}\right) BAy - \left(\left(P - Q\right) \otimes I_{NT}\right) \overline{BAy} \\ X^* & = & \left(Q \otimes I_{NT}\right) BX - \left(\left(P - Q\right) \otimes I_{NT}\right) \overline{BX} \end{cases} \tag{18}$$

where bars indicate averages over time. In this way, the regression resembles a standard linear regression, with transformed data:

$$\widehat{\beta} = \left(X^{*\prime} X^*\right)^{-1} X^{*\prime} y^* \tag{19}$$

3.2 Step 2: Estimate Ω_{μ} and Ω_{v} conditional on β, γ and λ

Denote by $\widehat{e} = B\left(Ay - X\widehat{\beta}\right)$, the spatial-autocorrelated transformed residuals, then the last part in Eq.(17) (conditional on both β, γ and λ) is simply:

$$-\frac{1}{2}\widehat{e}' \left(\Omega_{v}^{-1} \otimes E_T \otimes I_N\right) \widehat{e} \tag{20}$$

This term is actually a scalar that equals its trace, so:

$$\begin{aligned} \widehat{e}' \left(\Omega_{v}^{-1} \otimes E_T \otimes I_N\right) \widehat{e} &= \operatorname{tr}\left(\widehat{e}' \left(\Omega_{v}^{-1} \otimes E_T \otimes I_N\right) \widehat{e}\right) \\ &= \operatorname{tr}\left(\widetilde{e}' \left(\Omega_{v}^{-1} \otimes I_{NT}\right) \widetilde{e}\right) = \operatorname{tr}\left(\left(\Omega_{v}^{-1} \otimes I_{NT}\right) \widetilde{e}\widetilde{e}'\right) \end{aligned} \tag{21}$$

with

$$\widetilde{e} = (I_M \otimes E_T \otimes I_N)\,\widehat{e} \tag{22}$$

Thus, $\widetilde{e}$ is simply the transformed residuals $\widehat{e}$ expressed in deviations from their time mean. Using $\widetilde{\Pi}$ (of dimension $NTM \times NTM$) to denote the matrix $\widetilde{e}\widetilde{e}'$, Eq.(21) can be further simplified as

$$\widehat{e}'\left(\Omega_{\mathrm{v}}^{-1} \otimes E_T \otimes I_N\right)\widehat{e} = \mathrm{tr}\left(\Omega_{\mathrm{v}}^{-1}\widetilde{\Theta}\right) \tag{23}$$

where $\widetilde{\Theta}$ is an $(M \times M)$ matrix in which each element is the trace of an $(NT \times NT)$ sub-block matrix of $\widetilde{\Pi}$:

$$\widetilde{\Theta}_{j,l} = \mathrm{tr}\begin{pmatrix} \widetilde{\Pi}_{(j-1)NT+1,(l-1)NT+1} & \widetilde{\Pi}_{(j-1)NT+1,(l-1)NT+2} & \cdots & \widetilde{\Pi}_{(j-1)NT+1,lNT} \\ \widetilde{\Pi}_{(j-1)NT+2,(l-1)NT+1} & \widetilde{\Pi}_{(j-1)NT+2,(l-1)NT+2} & \cdots & \widetilde{\Pi}_{(j-1)NT+2,lNT} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\Pi}_{jNT,(l-1)NT+1} & \widetilde{\Pi}_{jNT,(l-1)NT+2} & \cdots & \widetilde{\Pi}_{jNT,lNT} \end{pmatrix}, \forall j,l \tag{24}$$

Similarly, $\widehat{e}'\left(\left(T\Omega_{\mu} + \Omega_{\mathrm{v}}\right)^{-1} \otimes \overline{J_T} \otimes I_N\right)\widehat{e}$ can be simplified as $\mathrm{tr}\left(\left(T\Omega_{\mu} + \Omega_{\mathrm{v}}\right)^{-1}\overline{\Theta}\right)$, where $\overline{\Theta}$ also is an $(M \times M)$ matrix with each element being the trace of the corresponding sub-block matrix of $\overline{\Pi}$. This comes from the transformed residuals $\widehat{e}$ but now averaging them over time: $\overline{e} = \left(I_M \otimes \overline{J_T} \otimes I_N\right)\widehat{e}$. Thus Eq.(17) can be finally expressed as

$$\begin{aligned} \ell \ \propto\ & -\frac{N}{2}\ln\left|T\Omega_{\mu} + \Omega_{\mathrm{v}}\right| - \frac{N(T-1)}{2}\ln\left|\Omega_{\mathrm{v}}\right| + T\sum_{j=1}^{M}\ln\left|B_j\right| + T\sum_{j=1}^{M}\ln\left|A_j\right| \\ & -\frac{1}{2}\mathrm{tr}\left(\left(T\Omega_{\mu} + \Omega_{\mathrm{v}}\right)^{-1}\overline{\Theta}\right) - \frac{1}{2}\mathrm{tr}\left(\Omega_{v}^{-1}\widetilde{\Theta}\right) \end{aligned} \tag{25}$$

The first order conditions for ML estimation are obtained by setting the score vector equal to zero:

$$d = \left(\frac{\partial \ell}{\partial \theta}\right) = 0\ ,\ \theta = \left(\beta', \gamma_j, \lambda_j, \sigma_{\mu_{\mathrm{jl}}}, \sigma_{v_{\mathrm{jl}}}\right)'\ ,\ j = 1, ..., M \tag{26}$$

In particular,

$$
\begin{aligned}
\frac{\partial \ell}{\partial \Omega_\mu} &= -\frac{NT}{2}\left(T\Omega_\mu + \Omega_v\right)^{-1} + \frac{T}{2}\left(T\Omega_\mu + \Omega_v\right)^{-1}\overline{\Theta}\left(T\Omega_\mu + \Omega_v\right)^{-1} \\
\frac{\partial \ell}{\partial \Omega_v} &= -\frac{N}{2}\left(T\Omega_\mu + \Omega_v\right)^{-1} - \frac{N\left(T-1\right)}{2}\Omega_v^{-1} \\
&\quad + \frac{1}{2}\left(T\Omega_\mu + \Omega_v\right)^{-1}\overline{\Theta}\left(T\Omega_\mu + \Omega_v\right)^{-1} + \frac{1}{2}\Omega_v^{-1}\widetilde{\Theta}\Omega_v^{-1}
\end{aligned}
$$

which gives immediate solutions for Ω_μ and Ω_v:

$$
\begin{cases}
\Omega_v = \frac{1}{N(T-1)}\widetilde{\Theta} \\
\Omega_\mu = \frac{1}{NT}\overline{\Theta} - \frac{1}{N(T-1)}\widetilde{\Theta}
\end{cases} \tag{27}
$$

By iterating steps 1 and 2, the optimal values for Ω_μ, Ω_v and β can be obtained conditional on γ and λ.

3.3 Step 3: Estimate γ and λ conditional on Ω_μ, Ω_v and β

The optimized Ω_μ, Ω_v and β from the first two steps are substituted into the log-likelihood function, and the only parameters left are γ_j and λ_j, $j = 1, ..., M$. These can be estimated by iteratively maximizing Eq.(17) via $\ell(\gamma, \lambda|\beta, \Omega_\mu, \Omega_v)$ and $\ell(\beta, \Omega_\mu, \Omega_v|\gamma, \lambda)$ until convergence. The information matrix given by:

$$
\left[I\left(\theta\right)\right]^{-1} = -E\left[\frac{\partial^2 \ell}{\partial\theta\partial\theta'}\right]^{-1} \tag{28}
$$

is not block-diagonal between γ_j and λ_j (and γ_j and β). As a consequence, the expression for the inverse $\left[I\left(\theta\right)\right]^{-1}$ is not straightforward, but not analytically prohibitive due to the sparseness of the non-diagonal parts (see Anselin (1988)). The $I\left(\theta\right)$ elements are given in the Appendix. Derivations of the score vector and the information matrix are available upon request from the authors in the supplement material.

4 Joint and conditional LM tests

Testing for spatial dependence has been surveyed by Anselin (1988) and Anselin and Bera (1998). This has been extended to single equation spatial panels by Baltagi et al. (2007). Here we extend this to SUR spatial panels. Let us partition θ as follows: $\theta = [\theta_1', \theta_2']'$ where θ_1 pertains to the parameters included in the null hypothesis and θ_2 to the remainder parameters. The Lagrange Multiplier (LM) or score test statistic for testing, $H_0 : \theta_1 = 0$, may be written as:

$$LM_{\theta_1=0} = \widetilde{D}_{\theta_1}' \widetilde{J}_{\theta_1}^{-1} \widetilde{D}_{\theta_1} \tag{29}$$

where D_{θ_1} is the score of the log-likelihood with respect to θ_1. J_{θ_1} is the corresponding block of the information matrix pertaining to θ_1, and $\widetilde{D}$ denotes that D is evaluated under the null H_0. Under normality of the disturbances, this statistic is asymptotically distributed as $N \to \infty$, as a χ^2 with k_{θ_1} degrees of freedom, where k_{θ_1} denotes the number of parameters in the vector θ_1 (see Breusch and Pagan (1980)).
In the next sub-section, we consider a joint LM test for spatial dependence (in the form of an omitted spatially lagged variable $\left(\gamma_j = 0, \, \forall j\right)$ or spatial autocorrelation in the disturbance term $(\lambda_j = 0, \, \forall j)$) as well as heterogeneity (in the form of random effects $\left(\sigma_{\mu_{lm}} = 0, \, \forall l, m\right)$).

4.1 The joint LM test

For the general panel SUR with spatial lag and spatially correlated errors described by equations (4)-(5), testing for no spatial correlation and no random effect in this model amounts to jointly testing the three sources of misspecification:

$$H_0^a : \left[\gamma_j, \, \lambda_j, \, \sigma_{\mu_{lm}}\right]' = 0, \, \forall j, l, m = 1, .., M$$

In this case, model (4)-(5) reduces to the pooled homoskedastic SUR model:

$$y_j = X_j \beta_j + \varepsilon_j, \, \varepsilon_j = v_j, \, \forall j = 1, .., M$$

For the score vector D_{θ_1}, only $\left[\left(\frac{\partial \ell}{\partial \gamma_j}\right), \left(\frac{\partial \ell}{\partial \lambda_j}\right), \left(\frac{\partial \ell}{\partial \sigma_{\mu_{lm}}}\right)\right]'$ need to be considered since $\left(\frac{\partial \ell}{\partial \beta}\right)$ and $\left(\frac{\partial \ell}{\partial \sigma_{v_{lm}}}\right)$ are zero as a result of the conditions for maximum likelihood estimation. Under the null hypothesis, the corresponding LM statistic is given by:

$$LM_{H_0^a} = \widetilde{D}_{H_0^a}' \widetilde{J}_{H_0^a}^{-1} \widetilde{D}_{H_0^a}$$

where the score vector is:

$$\widetilde{D}_{H_0^a} = \begin{bmatrix} \varepsilon' \left(\Omega_v^{-1} F^{jj} \otimes I_T \otimes W\right) y \\ \varepsilon' \left(\Omega_v^{-1} F^{jj} \otimes I_T \otimes W\right) \varepsilon \\ -\frac{NT}{2} Tr\left[F^{jk}\Omega_v^{-1}\right] + \frac{T}{2}\varepsilon' \left[\Omega_v^{-1} F^{jk} \Omega_v^{-1} \otimes \overline{J_T} \otimes I_N\right] \varepsilon \end{bmatrix}$$

and

$$\widetilde{J}_{H_0^a} = \left(J_{11} - J_{12} J_{22}^{-1} J_{12}'\right)$$

with

$$J_{11} = \begin{pmatrix} \widetilde{I}_{\gamma\gamma} & \widetilde{I}_{\gamma\lambda} & 0 \\ & \widetilde{I}_{\lambda\lambda} & 0 \\ & & \widetilde{I}_{\sigma_\mu \sigma_\mu} \end{pmatrix}, \; J_{12} = \begin{pmatrix} 0 & \widetilde{I}_{\gamma\beta^\theta} \\ 0 & 0 \\ \widetilde{I}_{\sigma_\mu \sigma_v} & 0 \end{pmatrix}, \; J_{22} = \begin{pmatrix} \widetilde{I}_{\sigma_v \sigma_v} & 0 \\ 0 & \widetilde{I}_{\beta\beta^\theta} \end{pmatrix}$$

F^{jk} is an $(M \times M)$ matrix of zeroes except for its (j,k) and (k,j) elements, which are equal to one. Here j and k index equations 1 through M. $\widetilde{I}_{xy} = I_{xy}$ in which A and B reduce to I_{MNT} and $\Omega_\mu = 0$. Intermediate matrices[4], used in elements of the information matrix I_{xy} (see appendix) reduce to $D_j^A = D_j^B = S_j = R_j = U_j^B = W$. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^a, this statistic is expected to be asymptotically distributed as χ^2 with $\left(2M + \frac{M(M+1)}{2}\right)$ degrees of freedom. We do not formally establish the large sample distribution of the LM score tests derived in this paper, but we conjecture that they are likely to hold under similar sets of primitive assumptions developed in Kelejian and Prucha (2001) for the Moran I test and its close cousins the LM tests for spatial dependence. See also Pinkse (1998, 1999) who provided general conditions under which Moran I flavoured tests for spatial correlation have a limiting normal distribution in the presence of nuisance parameters in six frequently encountered spatial models.

4

$$D_j^A = WA_j^{-1}, \; D_j^B = \begin{cases} WH_j^{-1} & \text{for SAR} \\ L_j^{-1}W & \text{for SMA} \end{cases}, \; U_j^B = \begin{cases} H_j^{-1}W & \text{for SAR} \\ WL_j^{-1} & \text{for SMA} \end{cases}$$

$$S_j = \begin{cases} H_j W A_j^{-1} & \text{for SAR} \\ L_j^{-1} W A_j^{-1} & \text{for SMA} \end{cases} \text{ and } R_j = \begin{cases} H_j W A_j^{-1} H_j^{-1} & \text{for SAR} \\ L_j^{-1} W A_j^{-1} L_j & \text{for SMA} \end{cases}$$

4.2 Two-dimensional conditional LM tests

4.2.1 Conditional LM test for no spatial correlation and no spatial lag given random effects

Testing for no spatial correlation and no spatial lag given random effects amounts to jointly testing:

$$H_0^b : \left[\gamma_j, \lambda_j\right]' = 0, \forall j = 1, .., M; \text{allowing for random effects.}$$

In this case, model (4)-(5) reduces to the one-way error component SUR model:

$$y_j = X_j\beta_j + \varepsilon_j, \, \varepsilon_j = (\iota_T \otimes I_N)\, \mu_j + v_j, \, \forall j = 1, .., M$$

Under the null hypothesis, the corresponding LM statistic is given by:

$$LM_{H_0^b} = \widetilde{D}'_{H_0^b} \widetilde{J}^{-1}_{H_0^b} \widetilde{D}_{H_0^b}$$

where the score vector is:

$$\widetilde{D}_{H_0^b} = \left[\begin{array}{c} \varepsilon' \left\{ \left(T\Omega_\mu + \Omega_v\right)^{-1} F^{jj} \otimes \overline{J_T} \otimes W \right\} y + \varepsilon' \left\{ \Omega_v^{-1} F^{jj} \otimes E_T \otimes W \right\} y \\ \varepsilon' \left\{ \left(T\Omega_\mu + \Omega_v\right)^{-1} F^{jj} \otimes \overline{J_T} \otimes W \right\} \varepsilon + \varepsilon' \left\{ \Omega_v^{-1} F^{jj} \otimes E_T \otimes W \right\} \varepsilon \end{array} \right]$$

and

$$\widetilde{J}_{H_0^b} = \left(J_{11} - J_{12} J_{22}^{-1} J'_{12}\right)$$

with

$$J_{11} = \left(\begin{array}{cc} \widetilde{I}_{\gamma\gamma} & \widetilde{I}_{\gamma\lambda} \\ & \widetilde{I}_{\lambda\lambda} \end{array} \right), \, J_{12} = \left(\begin{array}{ccc} 0 & 0 & \widetilde{I}_{\gamma\beta^0} \\ 0 & 0 & 0 \end{array} \right), \, J_{22} = \left(\begin{array}{ccc} I_{\sigma_\mu\sigma_\mu} & I_{\sigma_\mu\sigma_v} & 0 \\ I'_{\sigma_\mu\sigma_v} & I_{\sigma_v\sigma_v} & 0 \\ 0 & 0 & \widetilde{I}_{\beta\beta^0} \end{array} \right)$$

where $\widetilde{I}_{xy}$ are elements of the information matrix (I_{xy}) in which matrices $A = I_{MNT}$, $B = I_{MNT}$, $D_j^A = D_j^B = S_j = R_j = U_j^B = W$. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^b, this statistic is expected to be asymptotically distributed as χ^2 with $(2M)$ degrees of freedom.

4.2.2 Conditional LM test for no spatial lag and no random effects given spatial error correlation

Testing for no spatial lag correlation and no random effect given spatial error correlation, amounts to jointly testing:

$H_0^c : \left[\gamma_j, \sigma_{\mu_{lm}}\right]' = 0, \forall j, l, m = 1, .., M;$ allowing for spatial error correlation.

In this case, model (4)-(5) reduces to the pooled SUR model with spatial errors:

$$y_j = X_j \beta_j + \varepsilon_j \ , \ \varepsilon_j = \left\{ \begin{array}{ll} \lambda_j \left(I_T \otimes W\right) \varepsilon_j + v_j & \text{for SAR} \\ \lambda_j \left(I_T \otimes W\right) v_j + v_j & \text{for SMA} \end{array} \right. , \forall j = 1, .., M$$

Under the null hypothesis, the corresponding LM statistic is given by:

$$LM_{H_0^c} = \widetilde{D}'_{H_0^c} \widetilde{J}^{-1}_{H_0^c} \widetilde{D}_{H_0^c}$$

where the score vector is

$$\widetilde{D}_{H_0^c} = \left[\begin{array}{c} \varepsilon' B' \left(\Omega_v^{-1} F^{jj} \otimes I_T \otimes B_j W\right) y \\ -\frac{NT}{2} Tr \left[F^{lm} \Omega_v^{-1}\right] + \frac{T}{2} \varepsilon' B' \left[\Omega_v^{-1} F^{lm} \Omega_v^{-1} \otimes \overline{J_T} \otimes I_N\right] B\varepsilon \end{array} \right]$$

and

$$\widetilde{J}_{H_0^c} = \left(J_{11} - J_{12} J_{22}^{-1} J'_{12}\right)$$

with

$$J_{11} = \left(\begin{array}{cc} \widetilde{I}_{\gamma\gamma} & \widetilde{I}_{\gamma\sigma_\mu} \\ & \widetilde{I}_{\sigma_\mu\sigma_\mu} \end{array} \right), \ J_{12} = \left(\begin{array}{ccc} \widetilde{I}_{\gamma\lambda} & \widetilde{I}_{\gamma\sigma_v} & \widetilde{I}_{\gamma\beta^0} \\ \widetilde{I}_{\sigma_\mu\lambda} & \widetilde{I}_{\sigma_\mu\sigma_v} & 0 \end{array} \right), \ J_{22} = \left(\begin{array}{ccc} \widetilde{I}_{\lambda\lambda} & \widetilde{I}_{\lambda\sigma_v} & 0 \\ & \widetilde{I}_{\sigma_v\sigma_v} & 0 \\ & & \widetilde{I}_{\beta\beta^0} \end{array} \right)$$

where $\widetilde{I}_{xy} = I_{xy}$ in which $A = I_{MNT}$, $\Omega_\mu = 0$, $D_j^A = W$, $S_j = \widetilde{S}_j = H_j W$ for SAR and $S_j = \widetilde{S}_j = L_j^{-1} W$ for SMA and $R_j = \widetilde{R}_j = H_j W H_j^{-1}$ for SAR and $L_j^{-1} W L_j$ and $R_j = \widetilde{R}_j = L_j^{-1} W L_j$ for SMA. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^c, this statistic is expected to be asymptotically distributed as χ^2 with $\left(M + \frac{M(M+1)}{2}\right)$ degrees of freedom.

4.2.3 Conditional LM test for no spatial error correlation and no random effects given a spatial lag

Testing for no spatial error correlation and no random effects given a spatial lag, amounts to jointly testing:

$$H_0^d : \left[\lambda_j,\, \sigma_{\mu_{lm}}\right]' = 0,\, \forall j, l, m = 1, .., M; \text{allowing for a spatial lag.}$$

In this case, model (4)-(5) reduces to the pooled SUR model with spatial lag:

$$y_j = \gamma_j \left(I_T \otimes W\right) y_j + X_j \beta_j + \varepsilon_j \ ,\ \varepsilon_j = v_j,\, \forall j = 1, .., M$$

Under the null hypothesis, the corresponding LM statistic is given by:

$$LM_{H_0^d} = \widetilde{D}'_{H_0^d} \widetilde{J}^{-1}_{H_0^d} \widetilde{D}_{H_0^d}$$

where the score vector is:

$$\widetilde{D}_{H_0^d} = \left[\begin{array}{l} \varepsilon' \left(\Omega_v^{-1} F^{jj} \otimes I_T \otimes W\right) \varepsilon \\ -\frac{NT}{2} Tr\left[F^{lm} \Omega_v^{-1}\right] + \frac{T}{2} \varepsilon' \left[\Omega_v^{-1} F^{lm} \Omega_v^{-1} \otimes \overline{J_T} \otimes I_N\right] \varepsilon \end{array} \right]$$

and

$$\widetilde{J}_{H_0^d} = \left(J_{11} - J_{12} J_{22}^{-1} J'_{12}\right)$$

with

$$J_{11} = \begin{pmatrix} \widetilde{I}_{\lambda\lambda} & 0 \\ & \widetilde{I}_{\sigma_\mu \sigma_\mu} \end{pmatrix},\ J_{12} = \begin{pmatrix} \widetilde{I}_{\lambda\gamma} & 0 & 0 \\ \widetilde{I}_{\sigma_\mu \gamma} & \widetilde{I}_{\sigma_\mu \sigma_v} & 0 \end{pmatrix},\ J_{22} = \begin{pmatrix} \widetilde{I}_{\gamma\gamma} & \widetilde{I}_{\gamma \sigma_v} & \widetilde{I}_{\gamma \beta^\theta} \\ & \widetilde{I}_{\sigma_v \sigma_v} & 0 \\ & & \widetilde{I}_{\beta \beta^\theta} \end{pmatrix}$$

where $\widetilde{I}_{xy} = I_{xy}$ in which $B = I_{MNT}$, $\Omega_\mu = 0$, $D_j^A = W A_j^{-1}$, $D_j^B = W$, $U_j^B = W$, $S_j = D_j^A$, $R_j = D_j^A$. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^d, this statistic is expected to be asymptotically distributed as χ^2 with $\left(M + \frac{M(M+1)}{2}\right)$ degrees of freedom.

4.3 One-dimensional conditional LM tests

4.3.1 Conditional LM test for no spatial lag correlation given spatial error correlation and random effects

Testing for no spatial lag correlation amounts to testing:

$H_0^e : \left[\gamma_j\right] = 0$, $\forall j = 1, .., M$; allowing for spatial error correlation and random effects.

In this case, model (4)-(5) reduces to the one-way error component SUR model with spatial errors:

$$\begin{aligned} y_j &= X_j\beta_j + \varepsilon_j \ , \ \varepsilon_j = \begin{cases} \lambda_j \left(I_T \otimes W\right) \varepsilon_j + u_j & \text{for SAR} \\ \lambda_j \left(I_T \otimes W\right) u_j + u_j & \text{for SMA} \end{cases} \\ \text{with } u_j &= \left(\iota_T \otimes I_N\right) \mu_j + v_j, \ \forall j = 1, .., M \end{aligned}$$

Under the null hypothesis, the corresponding LM statistic is given by:

$$LM_{H_0^e} = \widetilde{D}'_{H_0^e} \widetilde{J}^{-1}_{H_0^e} \widetilde{D}_{H_0^e}$$

where the score vector is:

$$\widetilde{D}_{H_0^e} = \varepsilon' B' \Omega_u^{-1} B \left(F^{jj} \otimes I_T \otimes W\right) y$$

and

$$\widetilde{J}_{H_0^e} = \left(J_{11} - J_{12} J_{22}^{-1} J_{12}'\right)$$

with

$$J_{11} = \left(\widetilde{I}_{\gamma\gamma}\right), \ J_{12} = \begin{pmatrix} \widetilde{I}_{\gamma\lambda} & \widetilde{I}_{\gamma\sigma_\mu} & \widetilde{I}_{\gamma\sigma_v} & \widetilde{I}_{\gamma\beta^0} \end{pmatrix}, \ J_{22} = \begin{pmatrix} I_{\lambda\lambda} & I_{\lambda\sigma_\mu} & I_{\lambda\sigma_v} & 0 \\ & I_{\sigma_\mu\sigma_\mu} & I_{\sigma_\mu\sigma_v} & 0 \\ & & I_{\sigma_v\sigma_v} & 0 \\ & & & I_{\beta\beta^0} \end{pmatrix}$$

where $\widetilde{I}_{xy} = I_{xy}$ in which $A = I_{MNT}$, $D_j^A = W$, $S_j = \widetilde{S}_j = H_j W$ for SAR and $S_j = \widetilde{S}_j = L_j^{-1} W$ for SMA and $R_j = \widetilde{R}_j = H_j W H_j^{-1}$ for SAR and $L_j^{-1} W L_j$ and $R_j = \widetilde{R}_j = L_j^{-1} W L_j$ for SMA. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^e, this statistic is expected to be asymptotically distributed as χ^2 with M degrees of freedom.

4.3.2 Conditional LM test for no spatial error correlation given a spatial lag and random effects

Testing for no spatial error correlation given a spatial lag and random effects, amounts to testing:

$$H_0^f : [\lambda_j] = 0\ ,\ \forall j = 1,..,M; \text{allowing for a spatial lag and random effects.}$$

In this case, model (4)-(5) reduces to the one-way error component SUR model with spatial lag:

$$y_j = \gamma_j \left(I_T \otimes W\right) y_j + X_j \beta_j + \varepsilon_j\ ,\ \varepsilon_j = \left(\iota_T \otimes I_N\right) \mu_j + v_j,\ \forall j = 1,..,M$$

Under the null hypothesis, the corresponding LM statistic is given by:

$$LM_{H_0^f} = \widetilde{D}'_{H_0^f} \widetilde{J}^{-1}_{H_0^f} \widetilde{D}_{H_0^f}$$

where the score vector is:

$$\widetilde{D}_{H_0^f} = \varepsilon' \Omega_u^{-1} \left(F^{jj} \otimes I_T \otimes W\right) y$$

and

$$\widetilde{J}_{H_0^f} = \left(J_{11} - J_{12} J_{22}^{-1} J'_{12}\right)$$

with

$$J_{11} = \left(\widetilde{I}_{\lambda\lambda}\right),\ J_{12} = \left(\begin{array}{cccc} \widetilde{I}_{\lambda\gamma} & 0 & 0 & 0 \end{array}\right),\ J_{22} = \left(\begin{array}{cccc} \widetilde{I}_{\gamma\gamma} & \widetilde{I}_{\gamma\sigma_\mu} & \widetilde{I}_{\gamma\sigma_v} & \widetilde{I}_{\gamma\beta^\theta} \\ & I_{\sigma_\mu\sigma_\mu} & I_{\sigma_\mu\sigma_v} & 0 \\ & & I_{\sigma_v\sigma_v} & 0 \\ & & & \widetilde{I}_{\beta\beta^\theta} \end{array}\right)$$

where $\widetilde{I}_{xy} = I_{xy}$ in which $B = I_{MNT}$, $D_j^A = WA_j^{-1}$, $D_j^B = W$, $U_j^B = W$, $S_j = D_j^A$, $R_j = D_j^A$. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^f, this statistic is expected to be asymptotically distributed as χ^2 with M degrees of freedom.

4.3.3 Conditional LM test for no random effects given a spatial lag and spatial error correlation

Testing for no random effects given a spatial lag and spatial error correlation, amounts to testing:

$H_0^g : [\sigma_{\mu_{lm}}] = 0$, $\forall l, m = 1, .., M$; allowing for a spatial lag and spatial error correlation.

In this case, model (4)-(5) reduces to the pooled homoskedastic SUR model with spatial lag and spatial errors:

$$y_j = \gamma_j (I_T \otimes W) y_j + X_j \beta_j + \varepsilon_j \ , \ \varepsilon_j = \begin{cases} \lambda_j (I_T \otimes W) \varepsilon_j + v_j & \text{for SAR} \\ \lambda_j (I_T \otimes W) v_j + v_j & \text{for SMA} \end{cases} , \forall j = 1, .., M$$

Under the null hypothesis, the corresponding LM statistic is given by (see Appendix 2):

$$LM_{H_0^g} = \widetilde{D}'_{H_0^g} \widetilde{J}^{-1}_{H_0^g} \widetilde{D}_{H_0^g}$$

where the score vector is:

$$\widetilde{D}_{H_0^g} = -\frac{NT}{2} Tr \left[F^{lm} \Omega_v^{-1} \right] + \frac{T}{2} \varepsilon' B' \left[\Omega_v^{-1} F^{lm} \Omega_v^{-1} \otimes \overline{J_T} \otimes I_N \right] B \varepsilon$$

and

$$\widetilde{J}_{H_0^g} = \left(J_{11} - J_{12} J_{22}^{-1} J'_{12} \right)$$

with

$$J_{11} = \left(\widetilde{I}_{\sigma_\mu \sigma_\mu} \right) , \ J_{12} = \begin{pmatrix} \widetilde{I}_{\sigma_\mu \gamma} & \widetilde{I}_{\sigma_\mu \lambda} & \widetilde{I}_{\sigma_\mu \sigma_v} & 0 \end{pmatrix} , \ J_{22} = \begin{pmatrix} \widetilde{I}_{\gamma\gamma} & \widetilde{I}_{\gamma\lambda} & \widetilde{I}_{\gamma\sigma_v} & \widetilde{I}_{\gamma\beta^\theta} \\ & \widetilde{I}_{\lambda\lambda} & \widetilde{I}_{\lambda\sigma_v} & 0 \\ & & \widetilde{I}_{\sigma_v\sigma_v} & 0 \\ & & & \widetilde{I}_{\beta\beta^\theta} \end{pmatrix}$$

where $\widetilde{I}_{xy} = I_{xy}$ in which $\Omega_\mu = 0$. Derivation of the corresponding LM statistic is available upon request from the authors in the supplement material. Under the null H_0^g, this statistic expected to be asymptotically distributed as χ^2 with $\left(\frac{M(M+1)}{2}\right)$ degrees of freedom.

5 Monte Carlo experiments for the ML estimates and the LM tests

5.1 The data generating process

Consider the spatial SUR panel data model composed of $M = 2$ equations for N individuals (cities, regions, countries, ...) and T time periods:

$$\begin{cases} y_j &= \gamma_j (I_T \otimes W) y_j + X_j \beta_j + \varepsilon_j \\ \varepsilon_j &= \begin{cases} \lambda_j (I_T \otimes W) \varepsilon_j + u_j & \text{for SAR} \\ \lambda_j (I_T \otimes W) u_j + u_j & \text{for SMA} \end{cases} \\ u_j &= (\iota_T \otimes I_N) \mu_j + v_j \text{ with } j = 1, 2 \end{cases}$$

Let $X_j = [X_{j1}, X_{j2}]$ and $\beta_j = [\beta_{j1}, \beta_{j2}]'$. We fix the spatial lag coefficients as $\gamma_1 = 0.8$, $\gamma_2 = 0.8$, the spatial error coefficients as $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, the β_j coefficients as $\beta_{11} = \beta_{12} = \beta_{21} = \beta_{21} = 1$. Following Nerlove (1971), we consider two explanatory variables $[X_{j1}, X_{j2}]$ generated by:

$$\begin{cases} X_{j,1,it} &= a_{1,1} t + a_{1,2} X_{j,1,it-1} + \omega_{j,1,it} \\ X_{j,2,it} &= a_{2,1} t + a_{2,2} X_{j,2,it-1} + \omega_{j,2,it} \end{cases}$$

where $\omega_{j,1,it}$ (resp. $\omega_{j,2,it}$) is a random variable uniformly distributed on the interval $[b_{1,1}, b_{1,2}]$ (resp. $[b_{2,1}, b_{2,2}]$) and where the value $X_{j,1,i0}$ (resp. $X_{j,2,i0}$) is chosen as $c_{1,1} + c_{1,2}\omega_{j,1,i0}$ (resp. $c_{2,1} + c_{2,2}\omega_{j,2,i0}$). We fix the parameters as:

$$\begin{cases} a_{1,1} = 0.1 \ , \ a_{1,2} = 0.5 \ , \ b_{1,1} = -0.5 \ , \ b_{1,2} = 0.5 \ , \ c_{1,1} = 5 \ , \ c_{1,2} = 10 \\ a_{1,1} = 0.2 \ , \ a_{1,2} = 0.3 \ , \ b_{1,1} = -0.6 \ , \ b_{1,2} = 0.6 \ , \ c_{1,1} = 10 \ , \ c_{1,2} = 5 \end{cases}$$

We use several weighting matrices W which essentially differ in their degree of sparseness. The first matrix is a "1 ahead and 1 behind" matrix such that it's i-th row $(1 < i < N)$ of the $N \times N$ matrix has non-zero elements in positions $i+1$ and $i-1$. So, that the i-th cross-sectional unit is related to the one immediately after it and the one immediately before it. This matrix is row normalized so that all its non-zero elements are equal[5] to $1/2$. The other weighting matrices are labelled as "l ahead and l behind" with the non-zero elements being $1/2l$, for $\forall l$. For each $X_{j,it}$, we generate $T + 10$ observations and we drop the first ten observations in order to reduce the dependency on

[5] The matrix is defined in a circular world so that the non-zero elements in rows 1 and N are, respectively, in positions $(1, N)$ and $(N, 1)$.

initial values and we keep the last T observations for estimation.
The $(2NT \times 1)$ vector of disturbances is $\varepsilon = B^{-1}[\mu + v]$ with

$$B = \begin{pmatrix} I_T \otimes B_1 & 0 \\ 0 & I_T \otimes B_2 \end{pmatrix}, \; B_j = \begin{cases} I_N - \lambda_j W = H_j & \text{for SAR} \\ (I_N + \lambda_j W)^{-1} = L_j^{-1} & \text{for SMA} \end{cases}, \; j = 1, 2$$

The inverse of the variance-covariance matrix is $\Omega_\varepsilon^{-1} = B' \Omega_u^{-1} B$ with $\Omega_u^{-1} = \Sigma_u^{-1} \otimes I_N$ where $(\Sigma_u \otimes I_N)$ is the variance-covariance of the error component term $(\mu + v)$ with:

$$\Sigma_u = \Omega_\mu \otimes J_T + \Omega_v \otimes I_T$$

and

$$\Omega_\mu = \begin{pmatrix} \sigma^2_{\mu_1} & \rho_\mu \sigma_{\mu_1}\sigma_{\mu_2} \\ \rho_\mu \sigma_{\mu_1}\sigma_{\mu_2} & \sigma^2_{\mu_2} \end{pmatrix}, \; \Omega_v = \begin{pmatrix} \sigma^2_{v_1} & \rho_v \sigma_{v_1}\sigma_{v_2} \\ \rho_v \sigma_{v_1}\sigma_{v_2} & \sigma^2_{v_2} \end{pmatrix}$$

where

$$\sigma^2_{\mu_1} = 1, \; \sigma^2_{\mu_2} = 0.5, \; \rho_\mu = 0.8, \; \sigma^2_{v_1} = 1, \; \sigma^2_{v_2} = 0.5, \; \rho_v = 0.6$$

In order to generate the vector of disturbances $(\mu + v)$, we use the Choleski decomposition[6]. For all estimators, 1000 replications are performed. We compute the bias and the RMSE[7] of the coefficients $\beta_{i,j}$ $(i, j = 1, 2)$, the spatial lag coefficients γ_j $(j = 1, 2)$, the spatial autoregressive or moving average coefficients λ_j $(j = 1, 2)$ and the variance components $(\sigma^2_{\mu_1}, \sigma^2_{\mu_2}, \sigma_{\mu_{12}}, \sigma^2_{v_1}, \sigma^2_{v_2}, \sigma_{v_{12}})$. We choose $N = (25, 50)$, $T = (5, 10)$, "1 ahead and 1 behind" and "5 ahead and 5 behind" weighting matrices.

[6] As $(\mu + v) \sim N(0, \Sigma_u \otimes I_N)$ and μ and v are uncorrelated, $\mu \sim N\left(0, \left(\Omega_\mu \otimes J_T\right) \otimes I_N\right)$ and $v \sim N(0, \Omega_v \otimes I_{NT})$, then,

$$v \simeq C_v \otimes I_N \begin{bmatrix} \widetilde{u}_1 \\ \widetilde{u}_2 \end{bmatrix} \text{ and } \mu \simeq \begin{pmatrix} \iota_T \otimes (C_\mu \otimes I_N)\widetilde{\mu}_1 \\ \iota_T \otimes (C_\mu \otimes I_N)\widetilde{\mu}_2 \end{pmatrix}$$

where $(\widetilde{u}_1, \widetilde{u}_2)$ and $(\widetilde{\mu}_1, \widetilde{\mu}_2)$ are standard normal. C_μ (resp. C_v) is the lower triangular matrix defined by the decomposition: $C_\mu C_\mu'$ (resp. $C_v C_v'$) namely

$$C_\mu = \begin{bmatrix} \sigma_{\mu_1} & 0 \\ \rho_\mu \sigma_{\mu_2} & \sigma_{\mu_2}\sqrt{1 - \rho^2_\mu} \end{bmatrix} \text{ and } C_v = \begin{bmatrix} \sigma_{v_1} & 0 \\ \rho_v \sigma_{v_2} & \sigma_{v_2}\sqrt{1 - \rho^2_v} \end{bmatrix}$$

(see Anderson (1984)).

[7] Following Kapoor, Kelejian and Prucha (2007), our measure of dispersion is closely related to the standard measure of the RMSE, but it is based on quantiles rather than moments because, unlike moments, quantiles are assured to exit. For ease of presentation,

5.2 The results for the ML estimates

Table 1 gives the results on the bias and RMSE of the ML estimators for the SUR parameters, the spatial lags and spatial errors coefficients for a SAR process. Results on the estimates of the variance components are deleted to save space, these are available upon request from the authors. We report the results for 8 cases with $N = 25, 50$, $T = 5, 10$ and for "1 ahead and 1 behind" and "5 ahead and 5 behind" weighting matrices. Table 1 suggests that the biases are small (less than 3%). These biases decrease as N increases from 25 to 50, $\forall T$. Increasing the number of neighbors from ($W = 1$ to $W = 5$) does not change the results significantly. The RMSE also improves as we double N from 25 to 50 holding T fixed. Also when we double T from 5 to 10 holding N fixed. Table 2 shows these results for the SMA specification. The results are similar but, the magnitude of these biases and RMSE are smaller in absolute value than those for the SAR process.

5.3 The results for the LM tests

5.3.1 Joint LM test for $H_0^a : \gamma_j = 0,\ \lambda_j = 0,\ \sigma_{\mu_{jk}} = 0,\ \forall j, k = 1, .., M$

We use the same experimental design for the Monte Carlo simulations as in subsection 5.1. Table 3 gives the frequency of rejections at the 5% level for the joint LM test for $H_0^a : \gamma_j = 0,\ \lambda_j = 0,\ \sigma_{\mu_{jk}} = 0,\ \forall j, k = 1, .., M = 2$. For 1000 replications, counts between 37 and 63 are not significantly different from 50 at the 0.05 level. The results are reported for $N = 25$, 50, $T = 5$, 10 and for "1 ahead and 1 behind" and "5 ahead and 5 behind" weighting matrices. Table 3 shows that at the 5% level, the size of the joint LM test is close to 0.05 and varies between 0.036 and 0.054 depending on N and T. The

we also refer to our measure as RMSE. It is defined by:

$$RMSE = \sqrt{\mathsf{bias}^2 + \left[\frac{IQ}{1.35}\right]^2}$$

where bias is the difference between the median and the true value and IQ is the interquantile range $Q_3 - Q_1$ where Q_3 is the 0.75 quantile and Q_1 is the 0.25 quantile. If the distribution is normal, the median is the mean and, aside from a slight rounding error, $IQ/1.35$ is the standard deviation.

power[8] of the joint LM test is reasonably high as long as γ_j or λ_j are larger than 0.2. In fact, if γ_j or $\lambda_j \geqslant 0.4$, this power is almost one in all cases. For a fixed γ_j or λ_j, this power dramatically improves as N and T increase. For instance, for $N = 25$, $T = 5$, $W = 1$, $\lambda_j = 0.2$, the power is around 69%. If we double T from 5 to 10, this power tends to 93%. Increasing the number of neighbors from one to five, (i.e., $W = 1$ to $W = 5$) does not change the results significantly but slightly reduces the speed of convergence of the power to one.

5.3.2 Two-dimensional conditional LM tests

Conditional LM test for no spatial correlation and no spatial lag given random effects $H_0^b : \gamma_j = 0$**,** $\lambda_j = 0$**,** $\forall j = 1, .., M$. Table 4 gives the frequency of rejections at the 5% level for the two-dimensional LM test for $H_0^b : \gamma_j = 0$, $\lambda_j = 0$, $\forall j = 1, 2$ (allowing $\sigma_{\mu_{jk}} \neq 0$). In particular, we use $\sigma^2_{\mu_1} = 1$, $\sigma^2_{\mu_2} = 0.5$ and $\rho_\mu = 0.8$. The size of this test is not significantly different from 0.05 for $N = 25$, $T = 5, 10$ and $W = 1$. However, it is undersized for $N = 50$, $T = 5, 10$ and $W = 5$. The power of this LM test is reasonably high as long as γ_j or λ_j are larger than 0.2. In fact, if $\lambda_j \geqslant 0.4$, this power is almost one in all cases. For a small γ_j or λ_j, this power strongly improves as N and T increase.

Conditional LM test for no spatial lag and no random effects given spatial error correlation $H_0^c : \gamma_j = 0$, $\sigma_{\mu_{jk}} = 0, \forall j, k = 1, .., M$. Table 5 gives the frequency of rejections at the 5% level for the two-dimensional LM test for $H_0^c : \gamma_j = 0$, $\sigma_{\mu_{jk}} = 0$, $\forall j, k = 1, 2$ (allowing $\lambda_j = 0.5$). For $N = 25$, $T = 5$, the test is over-sized (0.09) but if we double T from 5 to 10, or double N from 25 to 50, the size of this test becomes close to 0.05. The power of this LM test is reasonably high as long as γ_j is larger than 0.2. In fact, if $\gamma_j \geqslant 0.4$, this power is almost one in all cases. Increasing the number of neighbors ($W = 1$ to $W = 5$) does not change the results significantly but slightly reduces the speed of convergence of the power to one.

[8] We use the SAR specification:

$$\varepsilon_j = \lambda_j \left(I_T \otimes W \right) \varepsilon_j + u_j \text{ and } u_j = \left(\iota_T \otimes I_N \right) \mu_j + v_j, \forall j = 1, 2$$

Conditional LM test for no spatial error correlation and no random effects given a spatial lag $H_0^d : \lambda_j = 0, \sigma_{\mu_{jk}} = 0, \forall j = 1, .., M$. Table 6 gives the frequency of rejections at the 5% level for the two-dimensional LM test for H_0^d : $\lambda_j = 0$, $\sigma_{\mu_{jk}} = 0$, $\forall j, k = 1, 2$ (allowing $\gamma_j = 0.5$). The size of this test is not significantly different from 0.05 for $N = 25$, $T = 5$, but becomes slightly undersized as N, T and W increase. The power of this LM test is high as long as λ_j is larger than 0.2. In fact, if $\lambda_j \geqslant 0.4$, this power is always one. Increasing the number of neighbors ($W = 1$ to $W = 5$) slightly reduces the speed of convergence of the power to one.

5.3.3 One-dimensional conditional LM tests

Conditional LM test for no spatial lag correlation given spatial error correlation and random effects $H_0^e : \gamma_j = 0$**,** $\forall j = 1, .., M$. Table 7 gives the frequency of rejections at the 5% level for the one-dimensional LM test for H_0^e : $\gamma_j = 0$, $\forall j = 1, 2$ (allowing $\sigma_{\mu_{jk}} \neq 0$ and $\lambda_j = 0.5$). The size of this test is not significantly different from 0.05 for $N = 25$, $T = 5$, but becomes slightly undersized as N, T and W increase. The power is reasonably high as long as γ_j is larger than 0.2. If $\gamma_j \geqslant 0.4$, this power is almost one in all cases. For a fixed γ_j, this power improves as N and T increase. Increasing the number of neighbors ($W = 1$ to $W = 5$) slightly reduces the speed of convergence of the power to one.

Conditional LM test for no spatial error correlation given a spatial lag and random effects H_0^f : $\lambda_j = 0$, $\forall j = 1, .., M$. Table 8 gives the frequency of rejections at the 5% level for the one-dimensional LM test for $H_0^f : \lambda_j = 0$, $\forall j = 1, 2$ (allowing $\sigma_{\mu_{jk}} \neq 0$ and $\gamma_j = 0.5$). At the 5% level, the size of this LM test is not significantly different from 0.05 for all experiments involving $W = 1$. However, for $W = 5$, it becomes slightly undersized. The power is almost one as long as λ_j is larger than 0.2. For a fixed λ_j, this power improves as N and T increase.

Conditional LM test for no random effects given a spatial lag and spatial error correlation H_0^g : $\sigma_{\mu_{jk}} = 0$, $\forall j, k = 1, .., M$. Table 9 gives the frequency of rejections at the 5% level for the one-dimensional LM test for $H_0^g : \sigma_{\mu_{jk}} = 0$, $\forall j, k = 1, 2$ (allowing $\gamma_j = 0.5$ and $\lambda_j = 0.5$). At the 5% level, the size of this LM test is close to 0.05. The power is always one if

$\sigma_{\mu_{jk}} \neq 0$ ($\sigma^2_{\mu_1} = 1$, $\sigma^2_{\mu_2} = 0.5$, $\rho_{\mu} = 0.8$) whatever the size of N and T. This holds for both sets of W matrices considered.

6 An application to hedonic housing prices in Paris

We illustrate our spatial panel methods by estimating a three SUR equations for hedonic housing prices in Paris. As the capital of France, Paris represents one of the most important real estate markets. The city of Paris is divided into 20 arrondissements (administrative districts) which in turn are divided into 4 quartiers (quarters). Our units of observation are the 80 quartiers.

In France, the housing classification used for flats by real estate agencies and notaries is the following: the studio (or efficiency) which is the cheapest rents in a given area, and consist mainly of a large room which is the living, dining, and bedroom combined. The kitchen facilities is usually a part of this central room, but the bathroom is its own smaller separate room. The two rooms (F2) flats (or one-bedroom apartments in the US or Great Britain), in which one bedroom is separate from the rest of the apartment. The three rooms (F3) flats (or two-bedroom in the US or Great Britain), and the four rooms (F4) flats (or three-bedroom in the US or Great Britain), etc.

6.1 Data Description

The French institutional setting is characterized by a network of notaries who have a monopoly in registering real estate transactions. The data base "BIEN", managed by the Notary Chamber of Paris covers Ile-de-France, i.e. the city of Paris and the Paris region[9]. For each transaction, we have information on the price for which the property was sold, along with its detailed characteristics (size, number of rooms and bathrooms, floor level, whether it has a balcony, whether it has a garage, a maid's room, time of construction, etc.) and its precise localization (Lambert II grid coordinates) with a precision of the order of 5 meters.

The data base covers the period 1990-2003. The dependent variable is the

[9] The data on a particular sale is made on a voluntary basis. However, the rate of coverage in 2003 is estimated to be 83% in Ile-de-France. Moreover, the database is anonymous, to comply with the French law.

(log) mean price per square meter[10] in each quartier for each time period and the explanatory variables are the mean characteristics of properties in each quartier for each time period. Using this aggregated quartier data gives us a balanced panel data of $NT = 80 \times 14 = 1120$ observations per variable.[11]

Put Table 10 here

Table 10 gives some descriptive statistics for housing prices and housing characteristics by three types of flats sold in the 80 quartiers during the period $1990 - 2003$. We have dropped studios, and flats with more than 8 rooms. So, the statistics pertain to flats with two rooms, three rooms and four to seven rooms (hereafter F2, F3, F4m, respectively).

The mean price per square meter is about 3000 euros, this ranged from 932 to 1200 euros per square meter. The mean price of flats has followed a J-shape curve. We observe a decrease from 1990 to 1997 and a boom after. This downswing and then upswing are more pronounced for the larger flats (F4m) and lead to mean prices per square meter between 4000 and 4400 euros.

Note that 29% (resp. 26%, 16%) of the F2 flats (resp. F3 and F4m) are not equipped with a bathroom and 70% (resp. 70%, 63%) have one bathroom. The majority of properties are sold without a parking lot (90%, 85%, 75% respectively for F2, F3 and F4m) and without a maid's room (98%, 95%, 83% resp. for F2, F3 and F4m).

Less than 3% of the flats have a balcony. These properties are mainly located between the ground floor and the third floor (55%) and only 8.4% of buildings have more than 7 floors. The mean square footage of all the properties is around 60 m^2. About 80% of these buildings are located in

[10] Our SUR ML estimator with spatial lags and spatial errors is derived only for a balanced panel data set with three indexes (jit) where $j = 1, ..., M$ equations, $i = 1, ..., N$ individuals ("quartiers") and $t = 1, ..., T$ time periods. The initial data base "BIEN" covers more than $260,000$ transactions and is an unbalanced clustered panel data set with four indexes ($jlit$) where $l = 1, ..., L_i$ flats sold in "quartier" $i\,(= 1, ..., N)$. This is why we use mean price per square meter $\left(\sum_{l=1}^{L_i} p_{jlit}/L_i\right)$ instead of price of each flat (p_{jlit}) of type j in each "quartier" i at time t.

[11] Unfortunately, some variables of interest like property taxes, crime rates, etc., were not available in this data set at the quartier level. These unobservable characteristics of the Paris districts may account for the spatial correlation in the disturbances and may be the reason for their significance.

streets, followed by avenues ($7-10\%$) and boulevards ($5-10\%$). The mean distance between these flats and the barycenter of each quartier is around 360 m.

Put figure 1 here

Figure 1 summarizes the spatial localization of mean prices per square meter of properties in the Paris area. This graph reveals the spatial heterogeneous behavior of housing prices, with low prices ($<$ 2500 euros per sq.m) for some arrondissements as $XVIII^{th}$, XIX^{th} and XX^{th} which are the north side popular districts of Paris and high prices ($>$ 4000 euros per sq.m) for some arrondissements as V^{th}, VI^{th}, VII^{th}, $VIII^{th}$ and XVI^{th} which are the famous, young, trendy and fashionable districts of Paris.

Put figures 2 and 3 here

Figures 2 and 3 give the mean prices per square meter of the properties in Paris during the period 1990-2003. We observe a decrease from 1990 to 1997 and a boom after. These downswing and upswing are more pronounced for some arrondissements as V^{th}, VI^{th}, VII^{th} and XV^{th}. These graphs reveal the heterogeneity in house price movements across time and quartier. Figure 3 also gives the proportion of flats according to square footage, by arrondissement.

6.2 The model and estimation results

To our knowledge, there is no econometric study on hedonic housing prices for the Paris real estate market that uses both panel and spatial dimensions and also take into account both micro-markets and market segmentation between several kinds of flats.[12]

The hedonic price function describes the expected price (expressed in logs) as a function of the house characteristics described in the data section (see Rosen (1974)). However, here we generalize it by introducing both spatial lag and spatial errors:

$$\begin{aligned} \ln(Y_{jt}) &= \gamma_j W_{1j} \ln(Y_{jt}) + X_{jt}\beta_j + \varepsilon_{jt} \ , \ j=1,2,3, \ t=1,...,T \quad (30) \\ \text{with } \varepsilon_{jt} &= \lambda_j W_{2j}\varepsilon_{jt} + u_{jt} \text{ and } u_{jt} = \mu_j + v_{jt}. \end{aligned}$$

[12] Some of the hedonic housing studies for France include Gravel et al. (1997), David et al. (2002), Laferrère (2003), Meese and Wallace (2003), Le Blanc and Lagarenne (2004), Maurer et al. (2004), Nappi-Choulet and Maury (2009) and Fack and Grenet (2010).

Y_{jt} is the $(N \times 1)$ vector of mean price per square meter for time period $t = 1, ..., T$ and flat type $j = 1, 2, 3$. The vector of observations is over the $(N = 80)$ quartiers. X_{jt} is a $(N \times k_j)$ matrix of mean characteristics of properties in the quartiers for time period t and flat type j. β_j is a $(k_j \times 1)$ vector of parameters and ε_{jt} is an $(N \times 1)$ vector of disturbances. μ_j is an $(N \times 1)$ vector of unobserved quartiers effects and v_{jt} is an $(N \times 1)$ vector of remainder disturbances. In this standard SUR hedonic housing price specification, the coefficients β_j measure the shadow prices of average house attributes for flats of type j. W_{1j} and W_{2j} are $(N \times N)$ spatial weight matrices, usually containing functions of distance or contiguity relations. This is an extension of the single equation spatially autoregressive (SAR) process introduced by Cliff and Ord (1973, 1981) to the SUR case, see Anselin (1988). The vector $[W_{1j}y_{jt}]$ is typically referred to as the spatial lag of y_{jt}. In addition to allowing for general spatial lags in the endogenous variables, we also allow for spatial autocorrelation in the disturbances. In particular, we assume that the disturbances (ε_{jt}) are generated by a spatially autoregressive (SAR) process. γ_j is the coefficient of the spatially lagged dependent variable $W_{1j}y_{jt}$, while λ_j is the coefficient of the spatially correlated errors.

The Lambert II grid coordinates allow us to compute distances d_{pq} between flats of the same type j sold in the two quartiers p and q. As the relationship we are modelling varies over space, mean prices of transactions that are near should exhibit similar relationships and those that are more distant may exhibit dissimilar relationships. Each spatially lagged variable depends upon a weight matrix which may vary across equations: $W_j = \left\{ w_{pq}^{(j)} \right\}$ with $w_{pp}^{(j)} = 0$ and the weight $w_{pq}^{(j)}$ is defined by $w_{pq}^{(j)} = d_{pq}^{-1} / \left(\sum_{n=1}^{N} d_{pn}^{-1} \right)$ for $p \neq q$. This is row standardized, so that each row sums to 1. In this case, the spatial weight matrix is filled with $N(N-1) = 6320$ nonzero elements depending on d_{pq}.

Another possible source of locational information is contiguity, reflecting the relative position in space of one unit with respect to the other units. The spatial contiguity matrix is defined as $w_{pq}^{(j)} = 1$ for $p \neq q$, for entities that share a common edge; otherwise, this weight is equal to zero. We consider here the 16 nearest neighbors (i.e. quartiers) which roughly corresponding to the 4 nearest arrondissements. Regarding spatial dependence, neighboring quartiers should exhibit a higher degree of spatial dependence than quartiers located far apart. This contiguity matrix is also row-normalized. In this case, the spatial contiguity matrix is sparse and is filled with only $16N = 1280$

nonzero elements.

Table 11 gives the estimation results of our hedonic housing price SUR system with spatial lags and spatial errors where the weights matrices W_{1j} and W_{2j} are functions of distances[13]. The estimated values of the spatial dependence coefficients (γ_1, γ_2 and γ_3) are not significantly different from zero. In contrast, the estimated values of the spatial autocorrelation coefficients (λ_1, λ_2 and λ_3) are $(0.735, 0.756$ and $0.73)$ which are all statistically different from zero.

The estimated variance-covariance matrices of the disturbances shown in Table 11 report significant cross-correlations between the three types of flats. This is true for the unobserved quartiers effects as well as the remainder disturbances. These significant cross-correlations favor the use of a panel SUR model for hedonic housing prices in Paris.

Lagrange multiplier (LM) tests for spatial autocorrelation, spatial lags and random effects are also reported in Table 11. These LM tests do not reject zero spatial lag on the dependent variable but they do reject zero effects on the spatial autoregressive structure of the disturbances and also the zero variance-covariance effects from the random quartier effects.

In summary, with distance matrices for both spatial lags and spatial errors, our results seem to favor a hedonic housing price SUR system with spatial autoregressive disturbances and random quartier error components but without a spatial lag on the dependent variables.[14]

Put Table 11 here

Except for three specific dummies (upper, rich and golden districts) and two distances variables (distance from the center of the arrondissement and distance from the center of the quartier, which are expressed in meters), all the other explanatory variables are ratios. So, the shadow price for an attribute X_{kj} is computed at the average price per square meter of the flat of type j. For F2 flats, if the demand for these flats with one bathroom increases by 10%, shadow price is expected to be, on average, 361 euros per

[13]Time dummies have been removed to save space.

[14]As the estimated parameters $\widehat{\gamma}_{\mathrm{j}}$ were not statistically significant in Table 11, the model was re-estimated by dropping the spatial autoregressive lag in y_{j}, but not the spatial dependence in the disturbances, see (3). The results are practically the same and are not reported here to save space. They are available upon request from the authors. They are used here to compute the shadow prices for house attributes.

square meter (hereafter e.s.m) to get this property. If the demand for F2 flats with one maid's room increases by 10%, the shadow price is around 448 e.s.m. The impact of garage plot(s) is relevant for the largest F4m flats, and the shadow price is 123 e.s.m for one garage and 409 e.s.m for two garage plots, altough with wider confidence intervals. Shadow prices are expected to be higher for properties located at higher floor levels. For F2 flats, the shadow price of floor level (4 to 7) is 247 e.s.m. For F2 flats, the shadow price for larger square footage is 153 e.s.m as we go from $[20m^2 - 40m^2]$ to $[41m^2 - 60m^2]$, and 279 e.s.m as we go to $[61m^2 - 80m^2]$. The quality of flats is also linked to their date of construction. As compared to the reference period (1850-1913) which includes the 19^{th} century Hausmannian construction in Paris (1852-1870), old buildings built in the previous period are strongly demanded since their shadow prices are 608 e.s.m for F4m flats. The closest the flat is to the *quartier* (or *arrondissement*) barycenter, the higher is the shadow price. This price is expected to be between 388 and 806 e.s.m less on average if the distance to the center of the *quartier* is increased by 100 meters. Last, living in the rich districts of Paris strongly increase the average price per square meter of all kinds of flats (around 1700 e.s.m for the F3 flats). Fashionable districts have a premium, especially "upper-class areas" (XIV^{th} and XV^{th} *arrondissements*), "rich, famous, young and trendy areas" (V^{th} and VI^{th} *arrondissements*) and "golden adresses" (VII^{th}, $VIII^{th}$ and XVI^{th} *arrondissements*).

For robustness checks we also used the contiguity spatial weight matrix and we get similar results but with different magnitudes. The LM tests still reject the spatial lag but not the spatial autocorrelation. These results are reported in Table 12.

Put Table 12 here

7 Conclusion

This paper proposed ML estimators for a panel SUR with both spatial lag and spatial error components. It extends the MLE approach developed by Wang and Kockelman (2007) to the general case where spatial effects are incorporated *via* spatial error terms and *via* a spatial lag on the dependent variables and where the heterogeneity in the panel is incorporated *via* an error component specification. This panel SUR model can be estimated using an iterative three-step method.

We also considered the problem of testing for random effects as well as spatial correlation under normality of the disturbances, and proposed joint and conditional LM tests for several sources of misspecification. This extends earlier work by Baltagi, et al. (2007) on spatial panels from the single equation to the SUR case.

While we did not derive the asymptotic distribution of our test statistics, we conjectured that they are likely to hold under similar set of primitive assumptions described in Kelejian and Prucha (2001). We reported extensive Monte Carlo experiments on bias and RMSE relating to the ML estimators for the SUR parameters, the variance components, the spatial lags and spatial errors coefficients for SAR and SMA process.

We find that the biases are small (less than 3%) even when N is small. These biases decrease when we double N. The results are similar for the SMA specification but, on average, bias and RMSE are smaller than those of the SAR process.

The same experimental design for the Monte Carlo simulations was used to obtain the size and power for the joint LM test, the two-dimensional conditional LM tests and the one-dimensional conditional LM tests. At the 5% level, the size of these LM tests are close to 0.05 depending on N and T. The power of these tests is reasonably high as long as the spatial lag and the spatial error components are larger than 0.2.

The results in the paper should be tempered by the fact that in our Monte Carlo experiments, $N = 25$, 50 and $T = 5$, 10 and we consider only two equations. One could encounter more equations, and larger N in micropanels. Larger N will probably improve the performance of these tests whose critical values are based on their large sample distributions. However, it is well known that maximum likelihood and quasi-maximum likelihood estimation of the spatial autocorrelation coefficients can be computationally difficult, particularly when N is large.

The paper concludes with an empirical illustration involving hedonic housing prices in Paris. For the 80 *quartiers* data for the city of Paris observed over the period $1990-2003$, our results suggest that a reasonable specification is a hedonic housing price SUR system with spatial autoregressive disturbances and random *quartier* effects, but without a spatial lag on the dependent variables. Using this specification, we find statistically significant as well as reasonable estimates of the shadow prices for mean characteristics of three types of flats considered.

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8 Appendix: the information matrix

The information matrix given by:

$$[I(\theta)]^{-1} = -E\left[\frac{\partial^2 \ell}{\partial\theta\partial\theta'}\right]^{-1}$$

is not block-diagonal between γ_j and λ_j (and γ_j and β) and the $I(\theta)$ elements are:

$$\begin{aligned}
I_{\beta\beta^{\theta}} &= X'B'\Omega_u^{-1}BX \\
I_{\beta\gamma_j} &= X'B'\left(\left(T\Omega_\mu + \Omega_v\right)^{-1}F^{jj}\otimes\overline{J_T}\otimes S_j\right)X\beta \\
&\quad + X'B'\left(\Omega_v^{-1}F^{jj}\otimes E_T\otimes S_j\right)X\beta \\
I_{\beta\lambda_j} &= 0,\ I_{\beta\sigma_{\mu_{lm}}} = 0,\ I_{\beta\sigma_{v_{lm}}} = 0 \\
I_{\gamma_j\gamma_l} &= T\times Tr\left[F^{jj}F^{ll}\right]Tr\left[D_j^A D_l^A\right] \\
&\quad + Tr\left[\beta'X'\left(F^{ll}\left(T\Omega_\mu+\Omega_v\right)^{-1}F^{jj}\otimes\overline{J_T}\otimes S_l'S_j\right)X\beta\right] \\
&\quad + Tr\left[\beta'X'\left(F^{ll}\Omega_v^{-1}F^{jj}\otimes E_T\otimes S_l'S_j\right)X\beta\right] \\
&\quad + Tr\left[F^{ll}\left(T\Omega_\mu+\Omega_v\right)^{-1}F^{jj} + (T-1)\,F^{ll}\Omega_v^{-1}F^{jj}\right]Tr\left[R_l'R_j\right] \\
I_{\gamma_j\lambda_l} &= T\times Tr\left[F^{jj}F^{ll}\right]\times Tr\left[D_j^A U_l^B\right] \\
&\quad + Tr\left[F^{jj}\left(T\Omega_\mu+\Omega_v\right)^{-1}F^{ll}\left(T\Omega_\mu+\Omega_v\right)\right]Tr\left[R_j\left(D_l^B\right)'\right] \\
&\quad + (T-1)\times Tr\left[F^{jj}\Omega_v^{-1}F^{ll}\Omega_v\right]\times Tr\left[R_j\left(D_l^B\right)'\right] \\
I_{\gamma_j\sigma_{\mu_{lm}}} &= T\times Tr\left[F^{lm}\left(T\Omega_\mu+\Omega_v\right)^{-1}F^{jj}\right]Tr\left[R_j\right] \\
I_{\gamma_j\sigma_{v_{lm}}} &= Tr\left[F^{lm}\left(T\Omega_\mu+\Omega_v\right)^{-1}F^{jj} + (T-1)\,F^{lm}\Omega_v^{-1}F^{jj}\right]Tr\left[R_j\right]
\end{aligned}$$

$$
\begin{aligned}
I_{\lambda_j \lambda_l} &= T \times Tr\left[F^{jj}F^{ll}\right] Tr\left[D_l^B D_j^B\right] \\
&+ Tr\left[F^{ll}\left(T\Omega_\mu + \Omega_v\right)^{-1} F^{jj}\left(T\Omega_\mu + \Omega_v\right)\right] Tr\left[D_j^B\left(D_l^B\right)'\right] \\
&+ (T-1) \times Tr\left[F^{ll}\Omega_v^{-1}F^{jj}\Omega_v\right] Tr\left[D_j^B\left(D_l^B\right)'\right] \\
I_{\lambda_j \sigma_{\mu_{lm}}} &= T \times Tr\left[F^{lm}\left(T\Omega_\mu + \Omega_v\right)^{-1} F^{jj}\right] Tr\left[D_j^B\right] \\
I_{\lambda_j \sigma_{v_{lm}}} &= Tr\left[F^{lm}\left\{\left(T\Omega_\mu + \Omega_v\right)^{-1} + (T-1)\,\Omega_v^{-1}\right\} F^{jj}\right] Tr\left[D_j^B\right] \\
I_{\sigma_{\mu_{jk}} \sigma_{\mu_{lm}}} &= \frac{NT^2}{2} Tr\left[F^{jk}\left(T\Omega_\mu + \Omega_v\right)^{-1} F^{lm}\left(T\Omega_\mu + \Omega_v\right)^{-1}\right] \\
I_{\sigma_{\mu_{jk}} \sigma_{v_{lm}}} &= \frac{NT}{2} Tr\left[F^{jk}\left(T\Omega_\mu + \Omega_v\right)^{-1} F^{lm}\left(T\Omega_\mu + \Omega_v\right)^{-1}\right] \\
I_{\sigma_{v_{jk}} \sigma_{v_{lm}}} &= \frac{N}{2} Tr\left[F^{jk}\left(T\Omega_\mu + \Omega_v\right)^{-1} F^{lm}\left(T\Omega_\mu + \Omega_v\right)^{-1} + (T-1)\,F^{jk}\Omega_v^{-1}F^{lm}\Omega_v^{-1}\right]
\end{aligned}
$$

where

$$
D_j^A = WA_j^{-1},\ D_j^B = \begin{cases} WH_j^{-1} & \text{for SAR} \\ L_j^{-1}W & \text{for SMA} \end{cases},\ U_j^B = \begin{cases} H_j^{-1}W & \text{for SAR} \\ WL_j^{-1} & \text{for SMA} \end{cases}
$$

$$
S_j = \begin{cases} H_jWA_j^{-1} & \text{for SAR} \\ L_j^{-1}WA_j^{-1} & \text{for SMA} \end{cases} \text{ and } R_j = \begin{cases} H_jWA_j^{-1}H_j^{-1} & \text{for SAR} \\ L_j^{-1}WA_j^{-1}L_j & \text{for SMA} \end{cases}
$$

F^{jk} is an $(M \times M)$ matrix of zeroes except for its (j,k) and (k,j) elements, which are equal to one. Here j, k, l and m index equations 1 through M. Derivations of the score vector and the information matrix are available upon request from the authors in the supplement material.

Table 1 - Bias and RMSE of ML estimators and standard errors of estimators for panel SUR with spatial lag and spatial autoregressive errors (SAR)

		N=25, T=5, spatial lag and SAR errors								N=25, T=10, spatial lag and SAR errors							
		W=1				W=5				W=1				W=5			
		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.	
true value		bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse
β_{11}	1	0.01738	0.28376	-0.00557	0.01938	0.01407	0.29966	-0.00706	0.02209	0.02211	0.17986	-0.00090	0.00817	0.00237	0.18398	-0.00130	0.00975
β_{12}	1	0.02729	0.21200	-0.00371	0.01417	0.02357	0.21581	-0.00401	0.01991	0.02690	0.13011	-0.00026	0.00660	0.01162	0.13721	-0.00063	0.00986
λ_1	0.5	-0.00562	0.01990	0.00000	0.00336	-0.00243	0.01618	-0.00040	0.00620	-0.00539	0.01425	0.00018	0.00168	-0.00327	0.01255	0.00006	0.00334
γ_1	0.8	-0.00228	0.07582	-0.00016	0.00527	-0.04313	0.14258	0.00667	0.02194	0.00694	0.05353	-0.00040	0.00276	-0.02129	0.09120	0.00195	0.00964
$\sigma^2_{\mu 11}$	1	-0.07766	0.33669	-0.02377	0.09473	-0.07991	0.34847	-0.02549	0.09808	-0.05091	0.31141	-0.01547	0.08894	-0.05717	0.31981	-0.01605	0.08963
$\sigma^2_{\mu 12}$	0.56569	-0.03881	0.21092	-0.01492	0.06074	-0.04486	0.21885	-0.01521	0.06380	-0.02148	0.18879	-0.00727	0.05567	-0.02723	0.19788	-0.00893	0.05637
$\sigma^2_{\mu 22}$	0.5	-0.04632	0.17166	-0.01288	0.04858	-0.04935	0.17690	-0.01420	0.05037	-0.02410	0.15524	-0.00677	0.04442	-0.02505	0.16435	-0.00758	0.04709
β_{21}	1	0.01499	0.19823	-0.00415	0.01383	0.01969	0.22051	-0.00699	0.01613	0.01423	0.11973	-0.00150	0.00631	0.00441	0.14076	-0.00220	0.00762
β_{22}	1	0.02111	0.14887	-0.00315	0.01009	0.01571	0.14981	-0.00501	0.01284	0.01559	0.09000	-0.00102	0.00474	0.00310	0.10058	-0.00211	0.00624
λ_2	0.5	-0.00478	0.01325	-0.00032	0.00132	-0.00228	0.01173	-0.00071	0.00243	-0.00365	0.00954	-0.00013	0.00071	-0.00149	0.00854	-0.00033	0.00119
γ_2	0.8	0.00094	0.06660	-0.00063	0.00477	-0.03937	0.13532	0.00516	0.02088	0.00521	0.04774	-0.00050	0.00249	-0.02240	0.09475	0.00235	0.01100
σ^2_{v11}	1	-0.02411	0.15531	-0.00388	0.02318	-0.02769	0.14547	-0.00439	0.02082	-0.00901	0.09551	-0.00083	0.00961	-0.01304	0.09941	-0.00123	0.00945
σ^2_{v12}	0.42426	-0.00773	0.09045	-0.00171	0.01237	-0.00856	0.08616	-0.00207	0.01186	-0.00435	0.05480	-0.00067	0.00520	-0.00455	0.05502	-0.00067	0.00524
σ^2_{v22}	0.5	-0.00640	0.07125	-0.00131	0.01061	-0.01199	0.06739	-0.00167	0.00979	-0.00734	0.04795	-0.00078	0.00481	-0.00698	0.04820	-0.00072	0.00459

		N=50, T=5, spatial lag and SAR errors								N=50, T=10, spatial lag and SAR errors							
		W=1				W=5				W=1				W=5			
		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.	
true value		bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse
β_{11}	1	0.02383	0.18844	-0.00161	0.00813	0.02759	0.18416	-0.00131	0.01047	0.02344	0.12462	-0.00017	0.00439	0.01585	0.13400	0.00005	0.00460
β_{12}	1	0.03060	0.13593	-0.00062	0.00614	0.00472	0.13177	-0.00073	0.01061	0.03351	0.09400	0.00012	0.00322	0.02454	0.09750	0.00011	0.00558
λ_1	0.5	-0.00643	0.01461	0.00026	0.00161	-0.00382	0.01176	0.00011	0.00345	-0.00502	0.01103	0.00015	0.00087	-0.00360	0.00918	0.00014	0.00158
γ_1	0.8	0.00740	0.05262	-0.00052	0.00241	-0.01716	0.08896	0.00185	0.00966	0.00604	0.03674	-0.00025	0.00134	-0.00512	0.06173	0.00040	0.00487
$\sigma^2_{\mu 1}$	1	-0.03525	0.24036	-0.00795	0.04849	-0.03995	0.24037	-0.00764	0.04762	-0.03112	0.21536	-0.00588	0.04398	-0.03713	0.20341	-0.00753	0.04136
$\sigma_{\mu 12}$	0.56569	-0.01757	0.15150	-0.00477	0.02909	-0.02244	0.15397	-0.00642	0.02987	-0.01899	0.14176	-0.00430	0.02916	-0.02158	0.13704	-0.00461	0.02758
$\sigma^2_{\mu 2}$	0.5	-0.01179	0.11377	-0.00293	0.02284	-0.02248	0.11694	-0.00446	0.02325	-0.01920	0.11629	-0.00354	0.02341	-0.01680	0.10976	-0.00320	0.02187
β_{21}	1	0.01823	0.12709	-0.00168	0.00681	0.00812	0.13417	-0.00182	0.00705	0.01333	0.08695	-0.00046	0.00294	0.00591	0.09235	-0.00065	0.00327
β_{22}	1	0.02004	0.09145	-0.00131	0.00496	0.00164	0.09628	-0.00179	0.00589	0.01603	0.06625	-0.00037	0.00230	0.00696	0.06410	-0.00071	0.00304
λ_2	0.5	-0.00439	0.01022	-0.00012	0.00077	-0.00175	0.00843	-0.00030	0.00120	-0.00294	0.00751	-0.00002	0.00037	-0.00096	0.00532	-0.00014	0.00060
γ_2	0.8	0.00594	0.04664	-0.00055	0.00237	-0.01801	0.08541	0.00193	0.00943	0.00307	0.03430	-0.00025	0.00126	-0.00668	0.05829	0.00036	0.00464
σ^2_{v1}	1	-0.01641	0.09890	-0.00151	0.01041	-0.01492	0.10278	-0.00154	0.01036	-0.00182	0.07367	-0.00015	0.00515	-0.00327	0.06984	-0.00021	0.00473
σ_{v12}	0.42426	-0.00489	0.05559	-0.00068	0.00525	-0.00500	0.05903	-0.00072	0.00555	-0.00026	0.03756	-0.00013	0.00261	-0.00086	0.03930	-0.00011	0.00248
σ^2_{v2}	0.5	-0.00610	0.05379	-0.00061	0.00560	-0.00405	0.05089	-0.00041	0.00504	-0.00195	0.03248	-0.00012	0.00230	-0.00273	0.03393	-0.00018	0.00221

Table 2 - Bias and RMSE of ML estimators and standard errors of estimators for panel SUR with spatial lag and spatial moving average errors (SMA)

		N=25, T=5, spatial lag and SMA errors								N=25, T=10, spatial lag and SMA errors							
		W=1				W=5				W=1				W=5			
		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.	
true value		bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse
β_{11}	1	-0.00656	0.23468	-0.00692	0.01824	0.03623	0.32349	-0.00873	0.02289	-0.00277	0.13446	-0.00200	0.00785	0.01419	0.16933	-0.00232	0.00844
β_{12}	1	0.02333	0.16342	-0.00505	0.01294	-0.01120	0.21461	-0.00762	0.02136	0.00303	0.10445	-0.00168	0.00565	0.00160	0.12455	-0.00225	0.00842
λ_1	0.5	-0.00189	0.01374	-0.04525	0.04529	-0.00105	0.01206	-0.19801	0.19806	-0.00062	0.00940	-0.03196	0.03198	-0.00083	0.00835	-0.14080	0.14082
γ_1	0.8	0.00490	0.06361	0.04386	0.04412	-0.05999	0.27258	0.19232	0.19312	0.00261	0.04713	0.03126	0.03136	-0.01808	0.16857	0.13913	0.13933
$\sigma^2_{\mu 1}$	1	-0.08114	0.33823	-0.02539	0.09792	-0.08523	0.36689	-0.02565	0.10456	-0.05235	0.31129	-0.01609	0.08887	-0.06463	0.33346	-0.01875	0.09420
$\sigma_{\mu 12}$	0.56569	-0.03813	0.20971	-0.01310	0.05840	-0.03987	0.22547	-0.01625	0.06460	-0.03167	0.19716	-0.01086	0.05362	-0.03553	0.19476	-0.01224	0.05416
$\sigma^2_{\mu 2}$	0.5	-0.03305	0.17351	-0.00887	0.05000	-0.04172	0.16302	-0.01384	0.04654	-0.02429	0.14864	-0.00731	0.04278	-0.03677	0.15405	-0.01022	0.04354
β_{21}	1	0.02309	0.16222	-0.00451	0.01361	0.03784	0.23182	-0.00694	0.01646	0.02817	0.11127	-0.00109	0.00570	0.02215	0.12972	-0.00124	0.00683
β_{22}	1	0.05061	0.12700	-0.00281	0.01016	0.01582	0.15985	-0.00672	0.01397	0.03575	0.08453	-0.00074	0.00447	0.01893	0.09155	-0.00116	0.00571
λ_2	0.5	-0.00766	0.01331	-0.04546	0.04547	-0.00403	0.01051	-0.19854	0.19855	-0.00707	0.01009	-0.03198	0.03198	-0.00398	0.00782	-0.14094	0.14094
γ_2	0.8	0.01343	0.06843	0.04377	0.04408	-0.05951	0.25253	0.19279	0.19336	0.01123	0.04625	0.03121	0.03131	-0.01346	0.15725	0.13946	0.13962
σ^2_{v1}	1	-0.01860	0.15157	-0.00288	0.02392	-0.02993	0.14928	-0.00451	0.02167	-0.00528	0.09790	-0.00085	0.01063	-0.01856	0.08854	-0.00173	0.00858
σ_{v12}	0.42426	-0.00688	0.08478	-0.00058	0.01221	-0.01523	0.08441	-0.00281	0.01124	-0.00043	0.05869	-0.00025	0.00576	-0.00546	0.05729	-0.00081	0.00510
σ^2_{v2}	0.5	0.00021	0.07050	-0.00014	0.01121	-0.01616	0.06958	-0.00209	0.01015	-0.00034	0.04961	-0.00006	0.00532	-0.00574	0.05470	-0.00052	0.00522

		N=50, T=5, spatial lag and SAR errors								N=50, T=10, spatial lag and SAR errors							
		W=1				W=5				W=1				W=5			
		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.		coefficients		s.e of coeff.	
true value		bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse	bias	rmse
β_{11}	1	0.00057	0.16248	-0.00264	0.00915	-0.00340	0.19260	-0.00322	0.01025	0.00346	0.10259	-0.00104	0.00385	0.00570	0.11975	-0.00103	0.00413
β_{12}	1	-0.00142	0.11949	-0.00199	0.00610	0.00039	0.13677	-0.00242	0.00847	0.00440	0.08355	-0.00122	0.00278	-0.00418	0.08800	-0.00122	0.00420
λ_1	0.5	-0.00046	0.00911	-0.03094	0.03096	-0.00069	0.00862	-0.13868	0.13870	-0.00038	0.00634	-0.02250	0.02250	-0.00008	0.00625	-0.09952	0.09953
γ_1	0.8	-0.00062	0.04697	0.03031	0.03041	-0.02459	0.16665	0.13676	0.13696	0.00255	0.03189	0.02211	0.02214	-0.01257	0.11194	0.09866	0.09872
$\sigma^2_{\mu 1}$	1	-0.04023	0.24029	-0.00864	0.04807	-0.03979	0.23161	-0.00902	0.04735	-0.03131	0.20774	-0.00598	0.04150	-0.03213	0.21591	-0.00702	0.04290
$\sigma_{\mu 12}$	0.56569	-0.02140	0.15552	-0.00524	0.03070	-0.02089	0.15563	-0.00549	0.03067	-0.01501	0.13412	-0.00243	0.02609	-0.02105	0.14827	-0.00535	0.02886
$\sigma^2_{\mu 2}$	0.5	-0.01423	0.11806	-0.00278	0.02384	-0.02483	0.12434	-0.00475	0.02432	-0.00981	0.11142	-0.00182	0.02256	-0.01708	0.11496	-0.00359	0.02316
β_{21}	1	0.03053	0.12216	-0.00135	0.00715	0.01626	0.14206	-0.00236	0.00791	0.02431	0.07283	-0.00049	0.00306	0.02075	0.08611	-0.00065	0.00315
β_{22}	1	0.03927	0.09466	-0.00058	0.00489	0.01931	0.09951	-0.00161	0.00624	0.04464	0.07683	0.00025	0.00224	0.02018	0.06628	-0.00041	0.00259
λ_2	0.5	-0.00741	0.01025	-0.03086	0.03086	-0.00411	0.00766	-0.13891	0.13891	-0.00712	0.00883	-0.02245	0.02245	-0.00350	0.00611	-0.09946	0.09947
γ_2	0.8	0.01348	0.04764	0.03002	0.03013	-0.02255	0.16765	0.13689	0.13707	0.01112	0.03010	0.02209	0.02212	-0.00292	0.11714	0.09914	0.09920
σ^2_{v1}	1	-0.00617	0.10966	-0.00082	0.01236	-0.01733	0.10225	-0.00170	0.01053	-0.00580	0.06780	-0.00055	0.00510	-0.00857	0.06799	-0.00065	0.00466
σ_{v12}	0.42426	0.00367	0.06230	0.00013	0.00614	-0.00836	0.06234	-0.00100	0.00586	0.00034	0.03995	0.00019	0.00274	-0.00413	0.03954	-0.00038	0.00245
σ^2_{v2}	0.5	0.00470	0.05422	0.00047	0.00597	-0.00548	0.05202	-0.00056	0.00544	0.00825	0.03549	0.00066	0.00267	-0.00453	0.03400	-0.00034	0.00235

Table 3 - Joint LM test $H0_a$: $\gamma_j = 0$, $\lambda_j = 0$, $\sigma_{\mu jk} = 0$

N	T	γ_j	λ_j	W=1	W=5	N	T	γ_j	λ_j	W=1	W=5
25	5	0	0	0.054	0.061	50	5	0	0	0.036	0.037
25	5	0	0.2	0.690	0.505	50	5	0	0.2	0.986	0.290
25	5	0	0.4	1.000	0.854	50	5	0	0.4	1.000	0.800
25	5	0	0.8	1.000	1.000	50	5	0	0.8	1.000	1.000
25	5	0.2	0	0.386	0.173	50	5	0.2	0	0.755	0.280
25	5	0.2	0.2	0.856	0.238	50	5	0.2	0.2	1.000	0.381
25	5	0.2	0.4	1.000	0.663	50	5	0.2	0.4	1.000	0.916
25	5	0.2	0.8	1.000	1.000	50	5	0.2	0.8	1.000	1.000
25	10	0	0	0.040	0.039	50	10	0	0	0.023	0.024
25	10	0	0.2	0.928	0.427	50	10	0	0.2	1.000	0.362
25	10	0	0.4	0.999	0.770	50	10	0	0.4	1.000	0.973
25	10	0	0.8	1.000	1.000	50	10	0	0.8	1.000	1.000
25	10	0.2	0	0.806	0.327	50	10	0.2	0	0.986	0.486
25	10	0.2	0.2	0.996	0.560	50	10	0.2	0.2	1.000	0.688
25	10	0.2	0.4	1.000	0.842	50	10	0.2	0.4	1.000	1.000
25	10	0.2	0.8	1.000	1.000	50	10	0.2	0.8	1.000	1.000

Table 4 - Conditional LM test for no spatial correlation and no spatial lag given random effects $H0_b$: $\gamma_j = 0$, $\lambda_j = 0$

N	T	γ_j	λ_j	W=1	W=5	N	T	γ_j	λ_j	W=1	W=5
25	5	0	0	0.036	0.031	50	5	0	0	0.032	0.017
25	5	0	0.2	0.619	0.217	50	5	0	0.2	0.917	0.339
25	5	0	0.4	0.999	0.705	50	5	0	0.4	1.000	0.959
25	5	0	0.8	0.999	1.000	50	5	0	0.8	1.000	1.000
25	5	0.2	0	0.673	0.130	50	5	0.2	0	1.000	0.310
25	5	0.2	0.2	0.974	0.452	50	5	0.2	0.2	1.000	0.476
25	5	0.2	0.4	1.000	0.685	50	5	0.2	0.4	1.000	0.983
25	5	0.2	0.8	1.000	1.000	50	5	0.2	0.8	1.000	1.000
25	10	0	0	0.034	0.018	50	10	0	0	0.026	0.011
25	10	0	0.2	0.915	0.340	50	10	0	0.2	1.000	0.601
25	10	0	0.4	1.000	0.962	50	10	0	0.4	1.000	1.000
25	10	0	0.8	1.000	1.000	50	10	0	0.8	1.000	1.000
25	10	0.2	0	0.997	0.290	50	10	0.2	0	1.000	0.540
25	10	0.2	0.2	1.000	0.680	50	10	0.2	0.2	1.000	0.773
25	10	0.2	0.4	1.000	0.985	50	10	0.2	0.4	1.000	1.000
25	10	0.2	0.8	1.000	1.000	50	10	0.2	0.8	1.000	1.000

Table 5 - Conditional LM test for no spatial lag and no random effects given spatial error correlation $H0_c$: $\gamma_j = 0$, $\sigma_{\mu jk} = 0$

N	T	γ_j	W=1	W=5	N	T	γ_j	W=1	W=5
25	5	0	0.091	0.090	50	5	0	0.070	0.069
25	5	0.2	0.235	0.186	50	5	0.2	0.476	0.105
25	5	0.4	0.928	0.817	50	5	0.4	1.000	0.353
25	5	0.8	1.000	1.000	50	5	0.8	1.000	1.000
25	10	0	0.063	0.060	50	10	0	0.046	0.039
25	10	0.2	0.398	0.276	50	10	0.2	0.992	0.366
25	10	0.4	1.000	0.456	50	10	0.4	1.000	0.574
25	10	0.8	1.000	0.998	50	10	0.8	1.000	1.000

Table 6 - Conditional LM test for no spatial error correlation and no random effects given a spatial lag $H0_d$: $\lambda_j = 0$, $\sigma_{\mu jk} = 0$

N	T	λ_j	W=1	W=5	N	T	λ_j	W=1	W=5
25	5	0	0.043	0.027	50	5	0	0.032	0.022
25	5	0.2	0.707	0.117	50	5	0.2	0.978	0.246
25	5	0.4	1.000	0.641	50	5	0.4	1.000	0.963
25	5	0.8	1.000	1.000	50	5	0.8	1.000	1.000
25	10	0	0.032	0.021	50	10	0	0.025	0.017
25	10	0.2	0.965	0.180	50	10	0.2	1.000	0.460
25	10	0.4	1.000	0.935	50	10	0.4	1.000	0.963
25	10	0.8	1.000	1.000	50	10	0.8	1.000	1.000

Table 7 - Conditional LM test for no spatial lag correlation given spatial error correlation and random effects $H0_e$: $\gamma_j = 0$

N	T	γ_j	W=1	W=5	N	T	γ_j	W=1	W=5
25	5	0	0.040	0.038	50	5	0	0.037	0.036
25	5	0.2	0.264	0.235	50	5	0.2	0.554	0.427
25	5	0.4	0.964	0.783	50	5	0.4	0.998	0.819
25	5	0.8	0.990	0.994	50	5	0.8	1.000	1.000
25	10	0	0.030	0.028	50	10	0	0.030	0.027
25	10	0.2	0.418	0.327	50	10	0.2	0.932	0.521
25	10	0.4	0.999	0.814	50	10	0.4	1.000	0.921
25	10	0.8	0.998	0.997	50	10	0.8	1.000	1.000

Table 8 - Conditional LM test for no spatial error correlation given a spatial lag and random effects $H0_f$: $\lambda_j = 0$

N	T	λ_j	W=1	W=5	N	T	λ_j	W=1	W=5
25	5	0	0.050	0.030	50	5	0	0.051	0.031
25	5	0.2	0.929	0.414	50	5	0.2	0.999	0.519
25	5	0.4	0.999	0.857	50	5	0.4	1.000	0.993
25	5	0.8	1.000	1.000	50	5	0.8	1.000	1.000
25	10	0	0.050	0.014	50	10	0	0.059	0.021
25	10	0.2	0.998	0.433	50	10	0.2	1.000	0.796
25	10	0.4	1.000	0.992	50	10	0.4	1.000	1.000
25	10	0.8	1.000	0.999	50	10	0.8	1.000	1.000

Table 9 - Conditional LM test for no random effects given a spatial lag and spatial error correlation $H0_g$: $\sigma_{\mu jk} = 0$

		W=1		W=5	
N	T	$\sigma_{\mu jk} = 0$	$\sigma_{\mu jk} \neq 0$	$\sigma_{\mu jk} = 0$	$\sigma_{\mu jk} \neq 0$
25	5	0.042	1.000	0.047	1.000
25	10	0.038	1.000	0.040	1.000
50	5	0.039	1.000	0.041	1.000
50	10	0.036	1.000	0.037	1.000

Figure 1 – Spatial localization of mean prices per sq. meter of properties in Paris (1990-2003)

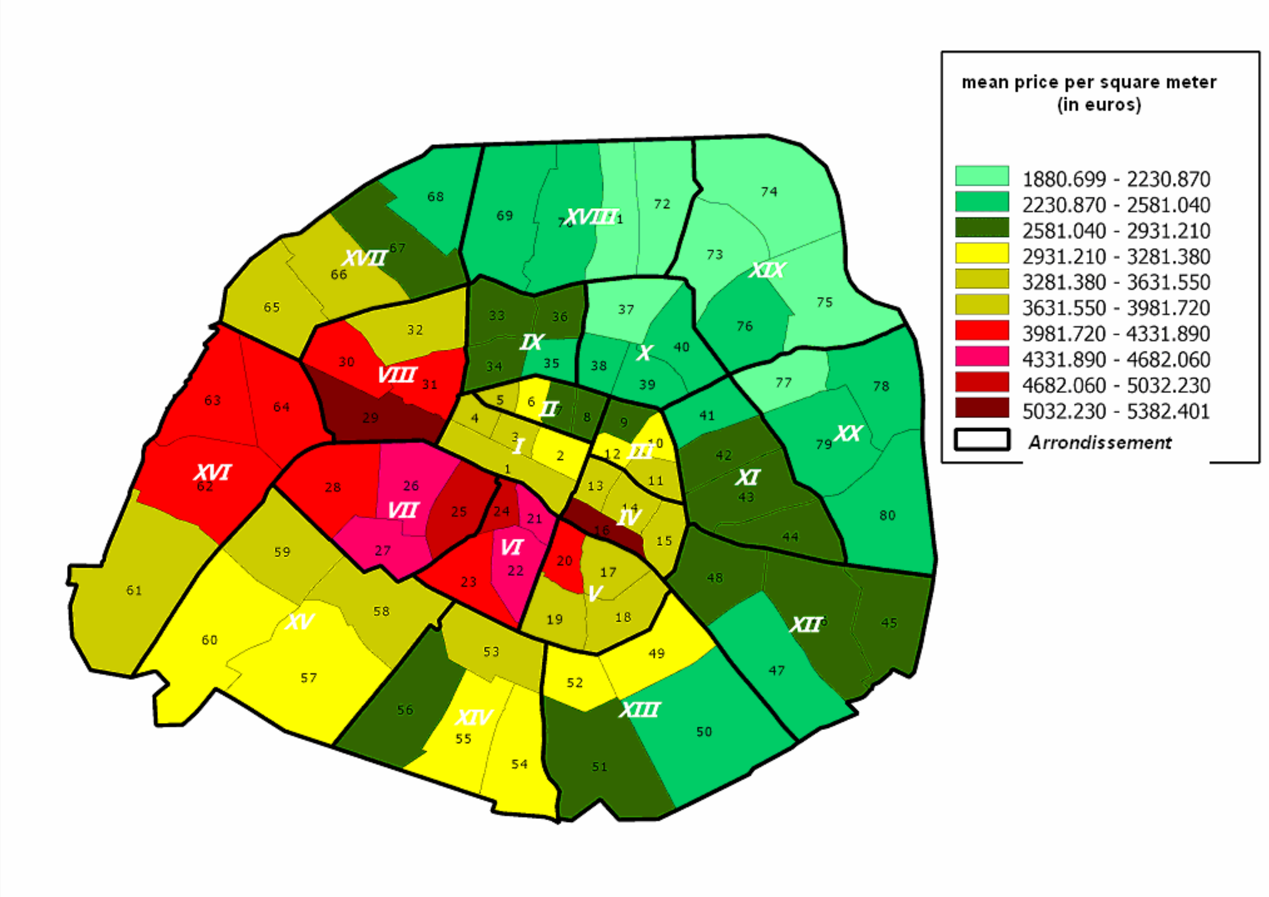

Figure 2 –Mean prices per sq. meter of properties in Paris' *quartiers* (areas) (1990 - 2003)

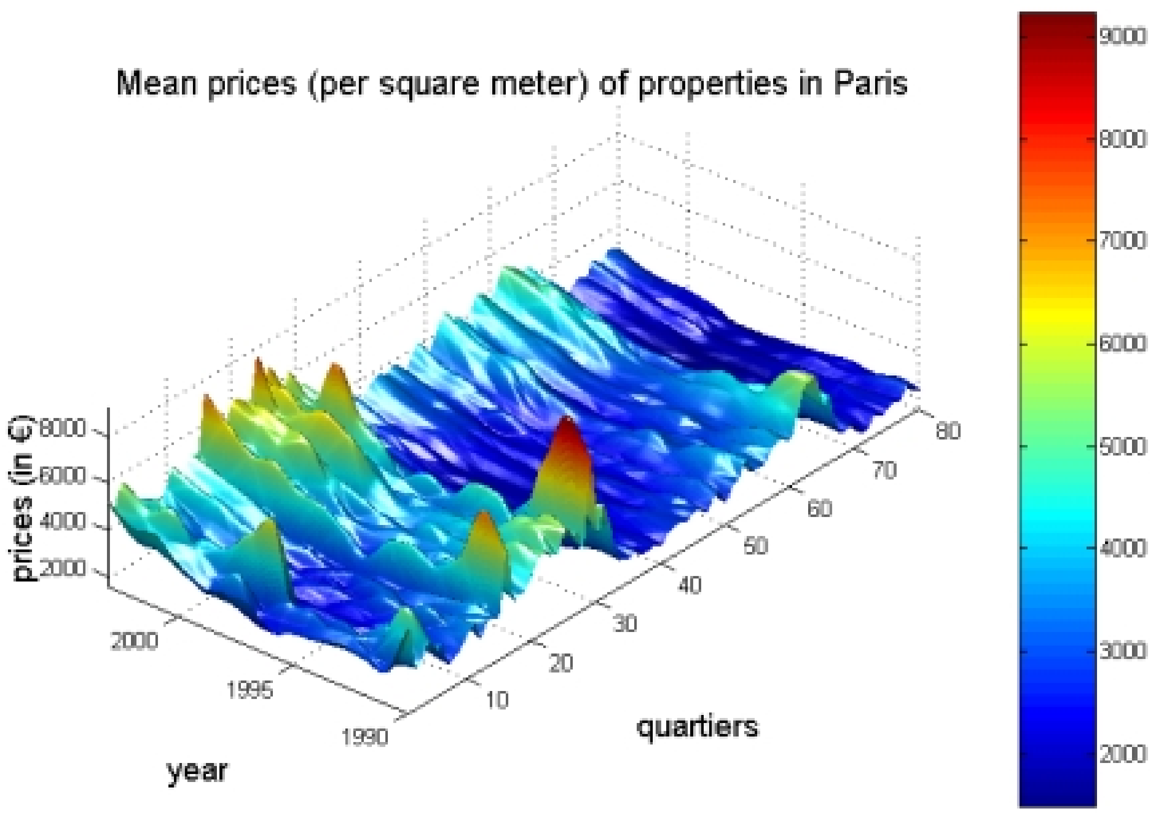

Table 10 - Descriptive statistics for hedonic housing prices in Paris (N=80 *quartiers*, 1990-2003)

		F2 flat Two rooms		F3 flat Three rooms		F4m flat More than three rooms	
	Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
	price per sq.meter (€) 1990	3067.193	932.153	3068.918	1063.746	3287.623	1203.218
Bathroom	no bathroom	0.287	0.146	0.258	0.168	0.166	0.160
	one bathroom	0.705	0.152	0.701	0.186	0.627	0.194
	two bathrooms	0.003	0.011	0.024	0.045	0.171	0.141
	three bathrooms and more	0.000	0.002	0.001	0.015	0.014	0.040
Maid's room	no maid's room	0.980	0.079	0.946	0.142	0.828	0.208
	one maid's room	0.014	0.042	0.034	0.071	0.119	0.137
	two maid's rooms and more	0.002	0.011	0.004	0.018	0.031	0.061
Garage lot	no garage plot	0.903	0.119	0.853	0.184	0.755	0.258
	one garage plot	0.091	0.101	0.125	0.145	0.195	0.210
	two garage plots	0.001	0.007	0.006	0.017	0.028	0.055
	three garage plots and more	0.000	0.001	0.000	0.002	0.001	0.006
Balcony	Balcony (yes or no)	0.015	0.028	0.020	0.037	0.032	0.055
Floor level	Floor level (0 to 3)	0.550	0.132	0.540	0.164	0.545	0.194
	Floor level (4 to 7)	0.421	0.129	0.413	0.154	0.384	0.166
	Floor level (8 to 11)	0.021	0.031	0.026	0.041	0.041	0.072
	Floor level (12 and more)	0.004	0.011	0.005	0.018	0.009	0.029
Square footage	Square footage (20 to 40 m2)	0.539	0.212	0.045	0.064	0.001	0.008
	Square footage (41 to 60 m2)	0.414	0.190	0.436	0.235	0.026	0.062
	Square footage (61 to 80 m2)	0.037	0.080	0.421	0.217	0.208	0.174
	Square footage (81 to 100 m2)	0.005	0.025	0.068	0.131	0.361	0.201
	Square footage (more than 100 m2)	0.001	0.006	0.014	0.037	0.382	0.264
Time of building	<1850	0.146	0.211	0.125	0.201	0.113	0.197
	1850-1913	0.487	0.191	0.502	0.224	0.469	0.264
	1914-1947	0.134	0.094	0.110	0.094	0.094	0.090
	1948-1969	0.104	0.096	0.121	0.127	0.127	0.137
	1970-1980	0.084	0.089	0.090	0.116	0.120	0.172
	1981-2003	0.020	0.057	0.014	0.040	0.018	0.049
Kind of streets	Avenue	0.069	0.101	0.083	0.120	0.101	0.137
	Boulevard	0.053	0.065	0.075	0.095	0.104	0.126
	Place	0.007	0.030	0.008	0.032	0.011	0.039
	Street	0.866	0.134	0.818	0.186	0.762	0.214
location	Distance to center of arrond.(m)	760.057	310.966	765.427	310.208	777.903	304.020
	Distance to center of quartier (m)	358.279	129.174	364.798	126.975	371.817	125.814
price per sq.m (€)	1990	3241.092	959.262	2907.220	1302.527	3546.180	1506.467
	1991	3399.057	964.035	3417.746	1185.554	3779.806	1332.065
	1992	2955.765	813.685	2925.903	980.454	3175.944	1184.537
	1993	2756.555	792.633	2839.746	914.806	3073.600	918.480
	1994	2817.402	599.574	2773.081	759.587	2956.089	999.147
	1995	2696.131	690.209	2694.468	692.725	2825.733	822.222
	1996	2503.367	485.931	2514.924	626.994	2635.913	738.734
	1997	2531.608	757.600	2494.286	743.385	2525.078	769.864
	1998	2716.929	806.392	2689.576	808.704	2724.691	735.732
	1999	2907.379	797.324	2880.633	827.330	3072.407	837.997
	2000	3204.227	891.160	3286.850	1000.489	3441.326	1053.854
	2001	3414.757	935.591	3486.653	1062.153	3820.322	1219.953
	2002	3706.720	886.354	3773.888	1056.617	4018.603	1276.669
	2003	4089.715	974.103	4279.882	1023.213	4431.024	1291.687

Table 11 - Hedonic housing price SUR Equations for Paris (N=80 *quartiers* , 1990-2003) (distances weight matrices W_{1j} and W_{2j})(*)

		SUR with spatial lags and spatial errors (SAR)					
		two rooms		three rooms		more than three rooms	
	ln(price per sq.meter)	Coeff.	T-stat	Coeff.	T-stat	Coeff.	T-stat
	Intercept	6.367	7.191	5.055	4.804	4.704	4.762
Bathroom	*no bathroom*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	one bathroom	1.178	10.401	1.636	12.434	1.876	14.255
	two bathrooms	1.169	1.037	1.527	3.291	2.004	10.160
	three bathrooms and more	1.319	0.257	3.639	3.127	2.332	4.655
Maid's roo	*no maid's room*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	one maid's room	1.463	4.803	1.276	4.707	1.204	6.803
	two maid's rooms and more	-0.690	-0.598	1.369	1.386	1.062	2.520
Garage plot	*no garage plot*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	one garage plot	0.265	1.098	-0.163	-0.633	0.375	1.794
	two garage plots	-2.005	-1.034	-1.359	-1.186	1.242	3.071
	three garage plots and more	0.490	0.039	-1.842	-0.162	-1.187	-0.358
Balcony	Balcony (yes or no)	-0.590	-1.241	0.274	0.539	0.352	0.927
Floor level	*Floor level (0 to 3)*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	Floor level (4 to 7)	0.807	7.573	1.236	10.341	0.700	5.733
	Floor level (8 to 11)	1.532	2.666	0.534	0.815	-0.120	-0.284
	Floor level (12 and more)	0.913	0.622	-0.149	-0.119	-1.112	-1.379
Square footage	*Square footage (20 to 40 m2)*	*ref.*	*ref.*	1.899	6.373	2.355	0.989
	Square footage (41 to 60 m2)	0.500	5.825	*ref.*	*ref.*	0.026	0.082
	Square footage (61 to 80 m2)	0.912	4.811	0.794	7.478	1.032	7.046
	Square footage (81 to 100 m2)	-0.348	-0.653	0.961	5.351	1.029	7.818
	Square footage (> 100 m2)	-1.338	-0.585	0.389	0.695	*ref.*	*ref.*
Time of building	<1850	0.736	6.822	1.107	8.386	1.848	13.907
	1850-1913	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	1914-1947	0.183	1.036	0.998	4.537	0.740	2.966
	1948-1969	0.128	0.678	0.068	0.311	0.355	1.671
	1970-1980	-0.658	-2.362	0.304	0.972	0.350	1.433
	1981-2003	0.805	3.035	0.876	1.713	0.023	0.049
Kind of street	Avenue	0.425	2.319	0.255	1.196	0.439	2.212
	Boulevard	0.836	3.687	1.412	6.300	0.930	5.283
	Place	0.630	1.425	1.928	3.262	0.978	1.899
	Street	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
location	Dist.center.arrond.(m)	-0.0003	-5.365	-0.0006	-6.473	-0.0007	-6.953
	Dist.center.quartier (m)	-0.0013	-8.325	-0.0022	-9.126	-0.0025	-10.009
Arrondisseme	Upper	0.340	3.492	0.554	3.566	0.340	2.241
	Rich	0.299	3.242	-0.026	-0.177	0.364	2.460
	Golden	0.282	3.750	0.526	4.341	0.654	5.397
	Others	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*

		SUR with spatial lags and spatial errors (SAR)					
		two rooms		three rooms		more than three rooms	
	ln(price per sq.meter)	Coeff.	T-stat	Coeff.	T-stat	Coeff.	T-stat
Spatial dep.	λ_j (j=1,2,3)	0.735	16.102	0.756	20.237	0.730	16.784
	γ_j (j=1,2,3)	-0.004	-0.051	0.007	0.083	0.008	0.103

		Coeff.	T-stat		Coeff.	T-stat
Error comp. Var-cov. Matrices	$\sigma^2_{\mu 1}$	0.271	6.033	σ^2_{v1}	0.182	22.724
	$\sigma^2_{\mu 2}$	0.394	5.829	σ^2_{v2}	0.466	22.739
	$\sigma^2_{\mu 3}$	0.480	5.920	σ^2_{v3}	0.457	22.735
	$\sigma_{\mu 12}$	0.331	6.099	σ_{v12}	0.089	9.381
	$\sigma_{\mu 13}$	0.355	6.076	σ_{v13}	0.019	2.130
	$\sigma_{\mu 23}$	0.435	5.990	σ_{v23}	0.202	12.920

log-likelihood	146.710	AIC	-35.428

	LM test	ddl	p-value
H_0^a : $[\gamma_j , \lambda_j , \sigma_{\mu lm}]' = 0$	43486.0	12	0
H_0^b : $[\gamma_j , \lambda_j]' = 0$	3208.6	6	0
H_0^c : $[\gamma_j , \sigma_{\mu lm}]' = 0$	48806.0	9	0
H_0^d : $[\lambda_j , \sigma_{\mu lm}]' = 0$	43402.0	9	0
H_0^e : $[\gamma_j] = 0$	0.0	3	1
H_0^f : $[\lambda_j] = 0$	4017.1	3	0
H_0^g : $[\sigma_{\mu lm}] = 0$	48782.0	6	0

(*): Regression includes time dummies

Table 12 - Hedonic housing price SUR Equations for Paris (N=80 *quartiers*, 1990-2003) (contiguity weight matrices W_{1j} and W_{2j})(*)

		SUR with spatial lags and spatial errors (SAR)					
		two rooms		three rooms		more than three rooms	
	ln(price per sq.meter)	Coeff.	T-stat	Coeff.	T-stat	Coeff.	T-stat
	Intercept	7.104	14.711	6.061	9.815	5.466	10.162
Bathroom	*no bathroom*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	one bathroom	1.149	10.111	1.601	12.106	2.007	15.240
	two bathrooms	1.166	1.027	1.639	3.546	2.075	10.411
	three bathrooms and more	2.487	0.485	4.015	3.459	2.503	4.959
Maid's roo	*no maid's room*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	one maid's room	1.507	4.947	1.322	4.855	1.170	6.582
	two maid's rooms and more	-0.738	-0.640	1.292	1.316	1.100	2.595
Garage plot	*no garage plot*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	one garage plot	0.392	1.610	-0.055	-0.213	0.381	1.804
	two garage plots	-2.175	-1.111	-1.127	-0.985	1.258	3.086
	three garage plots and more	0.018	0.001	-1.714	-0.149	-1.359	-0.405
Balcony	Balcony (yes or no)	-0.695	-1.460	0.340	0.670	0.371	0.963
Floor level	*Floor level (0 to 3)*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	Floor level (4 to 7)	0.790	7.375	1.248	10.397	0.645	5.234
	Floor level (8 to 11)	1.484	2.572	0.453	0.693	-0.161	-0.377
	Floor level (12 and more)	0.927	0.617	-0.230	-0.180	-0.942	-1.147
Square footage	*Square footage (20 to 40 m2)*	*ref.*	*ref.*	1.837	6.150	3.305	1.376
	Square footage (41 to 60 m2)	0.480	5.594	*ref.*	*ref.*	0.121	0.380
	Square footage (61 to 80 m2)	0.988	5.173	0.827	7.587	1.072	7.257
	Square footage (81 to 100 m2)	-0.787	-1.468	1.046	5.790	1.111	8.265
	Square footage (> 100 m2)	-1.304	-0.566	0.428	0.761	*ref.*	*ref.*
Time of building	<1850	0.717	6.688	1.129	8.483	1.774	13.095
	1850-1913	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
	1914-1947	0.225	1.282	0.976	4.376	0.842	3.353
	1948-1969	0.197	1.054	0.136	0.624	0.378	1.765
	1970-1980	-0.547	-1.954	0.383	1.211	0.362	1.457
	1981-2003	0.756	2.802	0.829	1.609	0.115	0.242
Kind of street	Avenue	0.507	2.700	0.274	1.267	0.492	2.441
	Boulevard	0.920	4.005	1.439	6.261	0.934	5.204
	Place	0.678	1.537	1.951	3.244	0.947	1.823
	Street	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
location	Dist.center.arrond.(m)	-0.0003	-4.881	-0.0006	-5.705	-0.0007	-6.632
	Dist.center.quartier (m)	-0.0013	-8.291	-0.0021	-8.582	-0.0024	-9.485
Arrondisseme	Upper	0.356	4.104	0.562	3.823	0.395	2.897
	Rich	0.354	4.185	-0.035	-0.245	0.428	3.117
	Golden	0.335	4.693	0.547	4.556	0.731	6.232
	Others	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*

		SUR with spatial lags and spatial errors (SAR)					
		two rooms		three rooms		more than three rooms	
	ln(price per sq.meter)	Coeff.	T-stat	Coeff.	T-stat	Coeff.	T-stat
Spatial dep.	λ_j (j=1,2,3)	0.575	11.930	0.634	16.213	0.571	12.448
	γ_j (j=1,2,3)	-0.045	-0.769	-0.065	-1.018	-0.043	-0.817

		Coeff.	T-stat		Coeff.	T-stat
Error comp. Var-cov. Matrices	$\sigma^2_{\mu 1}$	0.276	6.031	σ^2_{v1}	0.186	22.680
	$\sigma^2_{\mu 2}$	0.404	5.832	σ^2_{v2}	0.474	22.676
	$\sigma^2_{\mu 3}$	0.506	5.931	σ^2_{v3}	0.468	22.695
	$\sigma_{\mu 12}$	0.337	6.083	σ_{v12}	0.096	9.870
	$\sigma_{\mu 13}$	0.367	6.065	σ_{v13}	0.026	2.845
	$\sigma_{\mu 23}$	0.452	6.001	σ_{v23}	0.204	12.799

log-likelihood	103.730	AIC	50.534

	LM test	ddl	p-value
H_0^a : $[\gamma_j , \lambda_j , \sigma_{\mu lm}]' = 0$	2297300.0	12	0
H_0^b : $[\gamma_j , \lambda_j]' = 0$	1268.0	6	0
H_0^c : $[\gamma_j , \sigma_{\mu lm}]' = 0$	57351.0	9	0
H_0^d : $[\lambda_j , \sigma_{\mu lm}]' = 0$	51132.0	9	0
H_0^e : $[\gamma_j] = 0$	0.338	3	0.994
H_0^f : $[\lambda_j] = 0$	3255.6	3	0
H_0^g : $[\sigma_{\mu lm}] = 0$	47592.0	6	0

(*): Regression includes time dummies