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Christian Schwarz
Jens Suedekum

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Christian Schwarz
University of Duisburg-Essen

Jens Suedekum
University of Duisburg-Essen
and IZA

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IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

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ABSTRACT

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We develop a theory of a firm in an environment with incomplete contracts. The firm's headquarter decides on the complexity, the organization, and the global scale of its production process. Specifically, it decides: i) on the mass of symmetric intermediate inputs that are part of the value chain, ii) if the supplier of each component is an external contractor or an integrated affiliate, and iii) if the supplier is offshored to a foreign low-wage country. Afterwards we consider a related scenario where the headquarter contracts with a given number of two asymmetric suppliers. Our model is consistent with several stylized facts from the recent literature that existing theories of multinational firms cannot account for.

JEL Classification: F12, D23, L23

Keywords: multinational firms, outsourcing, intra-firm trade, offshoring, vertical FDI

Corresponding author:

Jens Suedekum
Mercator School of Management
University of Duisburg-Essen
Lotharstrasse 65
47057 Duisburg
Germany
Email: jens.suedekum@uni-due.de

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1 Introduction

The production of most final goods requires intermediate inputs. How thinly the value chain is "sliced", i.e., how many different inputs are combined in the production process for a particular final product, is a choice made by firms (Acemoglu et al., 2007): Some choose a setting with multiple highly specialized components and narrowly defined tasks, while other firms from the same industry rely on a substantially lower division of labor. We refer to the chosen mass of intermediate inputs as the degree of *complexity* of a firm's production process. For each component, a firm then needs to decide whether to manufacture that input inhouse or to outsource it to an external contractor. As is well known since Grossman and Hart (1986) and Hart and Moore (1990), these organizational decisions ("*make or buy*") matter in an environment with incomplete contracts, as they affect the suppliers' incentives to make relationship-specific investments. Finally, in a globalized world, firms also need to decide on the international scale of their value chain. Some firms only source domestically, while others collaborate with foreign suppliers either through arm's length transactions or through intra-firm trade (Grossman and Helpman, 2002).

An example that illustrates those different dimensions of a global value chain is the "Swedish" car *Volvo S40*, as discussed in Baldwin (2009). The production of this final good certainly is a complex process that consists of multiple intermediate inputs. A substantial share of those inputs is produced by independent suppliers, many of them from foreign countries: the navigation control is made by Japanese contractors, the side mirror and fuel tank by German, the headlights by American ones, and so on, while the airbag and the seats are outsourced domestically within Sweden. Yet other inputs are manufactured inhouse. Of those tasks, some are performed within the Swedish parent plants, while other components are manufactured by foreign subsidiaries which are directly owned and controlled by *Volvo*. Further examples of global sourcing strategies of multinational enterprises (MNEs) include *Nike*, which relies heavily on foreign outsourcing, or *Intel* which mainly engages in vertical foreign direct investment (FDI), see Antràs and Rossi-Hansberg (2009).

In this paper, we develop a theory of a firm which decides on the complexity, the organization, and the global scale of its production process. We build on the seminal approach by Antràs and Helpman (2004), who were the first to study global sourcing decisions under incomplete contracts. Their model is restricted to a setting with a headquarter and *one single* supplier, however. We extend that framework and consider multiple intermediate inputs. Our model is consistent with several stylized facts from the recent empirical literature that neither Antràs and Helpman (2004, 2008), nor other papers on the structure of MNEs can account for. It therefore further reconciles the theory and the empirics of multinational firms.[1]

Specifically, we first consider a model where the headquarter (the "producer") decides on the mass of (differentiated but symmetric) intermediate inputs that are part of the value chain, similar as in Ethier (1982) or Acemoglu et al. (2007). The larger this mass of components is, the more sliced is the value chain and the more specialized is the task that each single supplier performs. This specialization leads to efficiency gains, but it also generates endogenously larger fixed costs as it necessitates contracting with more input suppliers. The producer furthermore decides, separately for each component, if the respective supplier is an external contractor or an integrated affiliate, and if the supplier is offshored to a (low-wage, low-cost) foreign country. Our model firstly predicts that firms differ in the complexity of their production process, both within and across industries. Higher productivity and lower headquarter-intensity tend to increase the mass of suppliers that a firm chooses to contract with. Second, firms may outsource *some* of their inputs but vertically integrate others. This "hybrid sourcing" mode is prevalent in firms with medium-to-high productivity from sectors with low-to-medium headquarter-intensity. Third, firms may decide to offshore only *some* components, and this offshoring share tends to be higher in more productive firms and in less headquarter-intensive industries.

Afterwards, we turn to a related scenario where the producer contracts with a given and discrete number of *two* suppliers providing *asymmetric* components. These components can differ along two dimensions: i) the technological importance for the final product as measured by the input intensity, and ii) the bargaining power of the respective supplier. We show that firms from sectors with high (low) headquarter-intensity tend to integrate (outsource) both suppliers, particularly if the asymmetry across components is not too strong. With intermediate headquarter-intensity and for stronger asymmetries there is "hybrid sourcing", i.e., one integrated and one external supplier. The component with the higher input intensity is per se more likely to be outsourced, as this reduces the underinvestment problem for the supplier. Yet, that supplier is also likely to have higher bargaining power vis-a-vis the producer. If this latter effect is sufficiently strong, which may be the case for highly sophisticated and specific intermediate inputs, our model then

[1] Spencer (2005) provides a survey of the literature on international sourcing under incomplete contracts. In this literature, there has been no contribution that jointly analyzes the complexity, the organization, and the global scale of MNEs. A different model of multinational firms is Grossman and Rossi-Hansberg (2008). That model focuses particularly on the offshoring decision, but it is not based on incomplete contracts and it neglects the complexity and organizational choices of MNEs. Helpman (2006) presents a comprehensive overview of the recent literature on trade, FDI and firm organization.

predicts that the producer keeps the "more important" component, which generates more value added, within the boundaries of the firm.

The predictions of our model are then discussed in the light of the recent empirical literature on multinational firms.[2] That literature has started to carefully explore the internal structure of MNEs, and also to test particular aspects of the baseline model by Antràs and Helpman (2004) and the extension in Antràs and Helpman (2008). Several predictions of these models are supported by the empirical evidence.[3] Other features of the data are harder to understand with those baseline frameworks, however, while our model can account for these stylized facts.

For example, Kohler and Smolka (2009), Jabbour (2008) and Jabbour and Kneller (2010) show that most MNEs collaborate with *many* suppliers and often choose different sourcing modes for different inputs – as in the *Volvo*-example discussed above. In particular, Tomiura (2007) finds that firms which outsource *some* inputs while keeping others vertically integrated are more productive than firms which rely on a single sourcing mode in the global economy. Furthermore, Alfaro and Charlton (2009) show that firms tend to outsource low-skill inputs from the early stages, while high-skill inputs from the final stages of the production process are likely to be manufactured inhouse. Consistently, Corcos et al. (2009) find that inputs with a higher degree of specificity are less likely to be outsourced.

The rest of this paper is organized as follows. In Section 2 we present the basic structure of our model. Section 3 is devoted to the scenario with an endogenous mass of symmetric components, while Section 4 looks at the case with two asymmetric inputs. In Section 5 we conclude and contrast the predictions of our model with stylized facts on the structure of MNEs. In that section, we also point out some further testable predictions that have not yet been explored, in order to motivate future empirical research.

[2]The empirical literature has emphasized the significance of MNEs for world trade, which according to Corcos et al. (2009) are involved in about two thirds of all current international trade transactions. Feenstra (1995) and Feenstra and Hanson (1996) show that trade in intermediate inputs has increased much faster than trade in final goods over the last decades, which suggests a substantial increase in international outsourcing. The importance of intra-firm trade is stressed by Alfaro and Charlton (2009) and Badinger and Egger (2010), who consistently find that vertical FDI tends to dominate horizontal FDI.

[3]Consistent with Antràs and Helpman (2004), the study by Nunn and Trefler (2008) finds that intra-firm trade is most pervasive for highly productive firms in headquarter-intensive sectors, and Defever and Toubal (2007) find that highly productive firms tend to choose foreign outsourcing for components with high input intensity. Consistent with Antràs and Helpman (2008), who consider partial contractibility and cross-country differences in contracting institutions, the study by Corcos et al. (2009) finds that firms are more likely to offshore in countries with good contracting institutions, and Bernard et al. (2010) report that institutional improvements favor foreign outsourcing. The studies by Feenstra and Hanson (2005), Yeaple (2006), Marin (2006), and Federico (2010), among others, are also concerned with the internal structure of MNEs and obtain empirical findings broadly in line with those baseline models.

2 Model

2.1 Demand and technology

We consider a firm that produces a final good y for which it faces the following iso-elastic demand function:

$$y = Y \cdot p^{1/(\alpha-1)}. \quad (1)$$

The variable p denotes the price of this good, and $Y > 1$ is a demand shifter. The demand elasticity is given by $1/(1-\alpha)$ and is increasing in the parameter α (with $0 < \alpha < 1$). Production of this good requires headquarter services and manufacturing components, which are combined according to the following Cobb-Douglas production function:

$$y = \theta \cdot \left(\frac{h}{\eta^H}\right)^{\eta^H} \cdot \left(\frac{M}{1-\eta^H}\right)^{1-\eta^H}. \quad (2)$$

The parameter $\theta > 0$ is a productivity shifter; the larger θ is, the more productive is the firm. Headquarter services are denoted by h and are provided by the "producer". The parameter η^H (with $0 < \eta^H < 1$) is the exogenously given headquarter-intensity, and reflects the technology of the sector in which the firm operates. Consequently, $\eta^M = 1 - \eta^H$ is the overall component-intensity of production. There is a continuum of manufacturing components, with measure $N \in \mathbb{R}_+$. Each component is provided by a separate supplier. The supplier $i \in [0, N]$ delivers m_i units of its particular input, and the aggregate component input M is given by:

$$M = \exp\left\{\int_0^N \ln\left(\frac{m_i}{\eta_i}\right)^{\eta_i} \mathrm{d}i\right\}. \quad (3)$$

The parameter $\eta_i \in (0, 1)$ reflects the intensity of component i within the aggregate M, with $\int_0^N \eta_j \mathrm{d}j = 1$. The total input intensity of component i for final goods production is therefore given by $\eta^M \cdot \eta_i$.[4] Using equations (1), (2) and (3), total firm revenue can be written as follows:

$$R = \theta^\alpha \cdot Y^{(1-\alpha)} \cdot \left[\left(\frac{h}{\eta^H}\right)^{\eta^H} \cdot \left(\frac{M}{\eta^M}\right)^{\eta^M}\right]^\alpha, \quad (4)$$

which is increasing in the firm's productivity and demand level.

[4] If all components are symmetric, as will be assumed in Section 3, then each one has an individual input intensity equal to $(1-\eta^H)/N$.

2.2 Firm structure

The producer decides on the structure of the firm, and this choice involves three aspects: i) *complexity*, ii) *organization*, and iii) *global scale* of production. *Complexity* refers to the mass of components that are part of the production process. Recall that overall component-intensity η^M is exogenous and sector-specific. For example, intermediate inputs generally account for a larger share of total value added in the automobile than, say, in the software industry. Yet, within a sector, a producer can still decide on how thinly she wants to slice the value chain. If she chooses a "low" level of complexity, she relies on a setting with relatively few and broad components with a high average input intensity η_i. An increase in complexity lowers the average input intensity across the single components at constant overall component-intensity η^M. The inputs then become more specialized, and the respective suppliers have more narrowly defined tasks. For example, the carburetor system in car production may then no longer be provided by a single supplier, but different parts (like the choke and the throttle valve) are provided by different suppliers.

Secondly, turning to the organizational decision, the producer decides separately for each component if the respective supplier is integrated as a subsidiary within the boundaries of the firm, or if that component is outsourced to an external supplier. The crucial assumption is that the investments for all inputs are not contractible, as in Antràs and Helpman (2004). This may be due to the fact that the precise characteristics of the inputs are difficult to specify *ex ante* and also difficult to verify *ex post*. As a result of this contract incompleteness, the producer and the suppliers end up in a bargaining situation, at a time when their input investments are already sunk. Following the property rights approach of the firm, see Grossman and Hart (1986) or Hart and Moore (1990), we assume that bargaining also takes place within the boundaries of the firm in the case of vertical integration. This bargaining leads to a division of the total firm revenue as given in eq.(4) among the producer and the suppliers, where the bargaining power of the involved parties depends crucially on the firm structure, as will be explained below.

Finally, the producer decides on the global scale of production, i.e., on the location where each component is manufactured. The headquarter itself is located in a high-wage country 1, where final assembly of good y is carried out. Both under outsourcing and vertical integration, the respective input suppliers may either also come from country 1, or from a foreign low-wage country 2. In terms of the cross-country trade pattern, there is an arm's length transaction if the producer outsources a component to a foreign contractor, and intra-firm trade (vertical FDI) if a foreign supplier is vertically integrated.

2.3 Structure of the game

We consider a game that consists of seven stages. Our aim is to solve this game by backward induction for the subgame perfect Nash equilibrium. The timing of events is as follows:

1. The final goods producer enters and learns about the firm-specific productivity θ.

2. The producer decides whether to exit immediately, or to remain active in the market.

3. If the firm remains active, the producer simultaneously decides on: i) the complexity, ii) the organization, and iii) the global scale of the production process. In particular, i) she chooses the mass N of manufacturing components. ii) For each $i \in [0, N]$ the organizational choice is given by $\xi_i = \{O, V\}$. Here, $\xi_i = O$ denotes "outsourcing" and $\xi_i = V$ denotes "vertical integration" of supplier i. We order the mass N such that each supplier $j \in [0, N^O]$ is outsourced, and each supplier $k \in (N^O, N]$ is vertically integrated. Then, $\xi = N^O/N$ (with $0 \leq \xi \leq 1$) denotes the outsourcing share, and $(1 - \xi) = N^V/N$ is the share of vertically integrated suppliers/components. Finally, iii) for each $i \in [0, N]$ the producer decides on the country $r = \{1, 2\}$ where that component is manufactured. We order the mass of outsourced suppliers N^O such that each supplier $j \in [0, N_2^O]$ is offshored to the low-wage country 2, and each supplier $k \in (N_2^O, N^O]$ is located in the high-wage country 1. Then, $\ell^O = N_2^O/N^O$ denotes the offshoring share among all outsourced suppliers (with $0 \leq \ell^O \leq 1$). Similarly, $\ell^V = N_2^V/N^V$ (with $0 \leq \ell^V \leq 1$) is the offshoring share among all integrated suppliers, and the total offshoring share of the firm is given by $\ell = \xi \cdot \ell^O + (1 - \xi) \cdot \ell^V$.

4. Given the choice $\{N, \xi, \ell^O, \ell^V\}$, the producer offers a contract to potential input suppliers for every component $i \in [0, N]$. This contract includes an upfront payment τ_i (positive or negative) to be paid by the prospective supplier.

5. There exists a large pool of potential applicant suppliers for each manufacturing component in both countries. These suppliers have an outside opportunity (wage) equal to w_r^M in country $r = \{1, 2\}$. They are willing to accept the producer's contract if their payoff is at least equal to w_r^M. The payoff consists of the upfront payment τ_i and the revenue share β_i that supplier i anticipates to receive at the bargaining stage, minus the investment costs (which may differ across applicants). Potential suppliers apply for the contract, and the producer chooses one supplier (either from country 1 or from country 2) for each component $i \in [0, N]$.

6. The producer and the suppliers independently decide on their non-contractible input levels for the headquarter service (h) and the components (m_i), respectively.

7. Output is produced and revenue is realized according to (2), (3), and (4). The producer and the suppliers bargain over the division of the surplus value.

Starting with stage 7, following Grossman and Hart (1986) we assume that the producer and the suppliers cannot write down enforcable contracts that specify the division of revenue. The producer rather has to decide on the structure of the firm (complexity, organization, global scale) in order to affect the revenue distribution, since the firm structure pins down the bargaining power of the involved agents. We assume that the bargaining process follows a generalized simultaneous multi-party Nash bargaining.[5] The surplus value over which the $N+1$ agents bargain is the total revenue R as given in eq.(4), and the agents receive revenue shares that are reflective of their respective bargaining power. The revenue share of component supplier i is given by β_i, and $\beta^M = \int_0^N \beta_j \mathrm{d}j$ denotes the joint revenue share of all component suppliers. The revenue share realized by the producer is written as β^H, and we have $\beta^H = 1 - \beta^M$. The headquarter revenue share β^H reflects the *effective bargaining power* of the producer vis-a-vis the component suppliers. How the firm structure influences the bargaining power of the involved parties is analyzed later.

In stage 6, each component supplier i chooses m_i so as to maximize $\beta_i R - c^M_{i,r} m_i$ for each $i \in [0, N]$, where $c^M_{i,r}$ denotes the unit cost level of the supplier for component i that the producer has offered the contract. The producer chooses h in order to maximize $\beta^H R - c^H h$, where c^H denotes the unit cost of providing headquarter services. We show in Appendix A.1. that the agents choose the following levels of input provision:

$$m_i^* = \alpha \cdot \eta^M \cdot \eta_i \cdot \beta_i \cdot R^* \ / \ c^M_{i,r} \quad \text{and} \quad h^* = \alpha \cdot \eta^H \cdot \beta^H \cdot R^* \ / \ c^H, \tag{5}$$

with total revenue given by

$$R^* = (\alpha\theta)^{\alpha/(1-\alpha)} \cdot Y \cdot \left[\left(\frac{\beta^H}{c^H}\right)^{\eta^H} \cdot \left(\exp\left\{ \int_0^N \ln\left(\frac{\beta_j}{c^M_{j,r}}\right)^{\eta_j} \mathrm{d}j \right\} \right)^{\eta^M} \right]^{\frac{\alpha}{1-\alpha}}. \tag{6}$$

Everything else equal, the investment by supplier i relative to that of some other supplier j, (m_i^*/m_j^*), is increasing in supplier i's revenue share β_i and input intensity η_i. Similarly,

[5]We propose a Nash bargaining as in Antràs and Helpman (2004), since the mass of suppliers N is already determined at stage 7. We rule out the possibility of partial cooperation as in Acemoglu et al. (2007), where the Shapley value is used to account for potential coalition formation.

the producer invests relatively more the higher β^H and η^H are.

Next, in order to receive applications for each desired component input in stage 5, the producer must offer contracts in stage 4 that satisfy the suppliers' participation constraints. For supplier i this implies that the individual payoff from forming the relationship, given (5) and (6), must at least be equal to the attainable outside wage:

$$\beta_i R - c_{i,r}^M m_i + \tau_i \geq w_r^M. \tag{7}$$

In stage 3, the producer then chooses the structure of the firm so as to maximize her individual payoff, $\beta^H R - c^H h - \int_0^N \tau_j \mathrm{d}j$, subject to the revenue given in eq.(4), the incentive compatibility constraints (5), and the participation constraints (7). Since the producer can freely adjust the upfront payments τ_i, these participation constraints are satisfied with equality for all suppliers $i \in [0, N]$. Rearranging $\tau_i = w_r^M - \beta_i R + c_{i,r}^M m_i$, substituting this into the individual payoff of the producer, and recalling that $\beta^M = 1 - \beta^H$, it follows that the producer's problem is equivalent to maximizing the total payoff for all $N + 1$ involved parties, i.e.: $\pi = R^* - \int_0^N c_{j,r}^M m_j^* \mathrm{d}j - c_H h^* - f$, where f is the outside opportunity w_r^M aggregated across all (domestic and foreign) suppliers. Notice that the term f is increasing in N as long as $w_r^M > 0$, i.e., the participation constraints generate a "fixed cost" that is endogenously increasing in complexity, as this necessitates contracting with more suppliers.[6] We additionally allow for exogenous fixed costs $\bar{f}$ which arise independently of the participation constraints, e.g. for general overhead costs. With overall fixed costs given by $F = f + \bar{f}$, we can rewrite the total payoff as follows by using (4) and (5):

$$\pi = \Theta \cdot Y \cdot \Psi - F, \tag{8}$$

$$\Psi \equiv \left[1 - \alpha\left(\beta^H \eta^H + \eta^M \int_0^N \beta_j \eta_j \mathrm{d}j\right)\right]\left[\left(\frac{\beta^H}{c^H}\right)^{\eta^H}\left(\exp\left\{\int_0^N \ln\left(\frac{\beta_j}{c_{j,r}^M}\right)^{\eta_j} \mathrm{d}j\right\}\right)^{\eta^M}\right]^{\frac{\alpha}{1-\alpha}}, \tag{9}$$

where $\Theta = (\alpha\theta)^{\alpha/(1-\alpha)}$ is an alternative productivity measure.

Finally, similar as in Melitz (2003), a firm learns about its productivity level θ upon entry, which is drawn from some density function $g(\theta)$ with support $[\underline{\theta}, \infty]$, where $\underline{\theta} > 0$ denotes a lower bound. The firm only stays in the market (in stage 2) when the variable payoff $\Theta \cdot Y \cdot \Psi$ is sufficiently large to cover the fixed costs F.

[6] We assume that outside opportunities may differ across countries, but not across suppliers from the same country. This assumption could be relaxed without affecting our main results. Our main results only require that overall fixed costs for the firm are increasing in complexity N.

3 Symmetric components

In this section we consider the case of *symmetric* components. We assume that the individual input intensities of the single components are given by $\eta^M \cdot \eta_i = \left(1 - \eta^H\right)/N$ for all $i \in [0, N]$. We first abstract from the global scale dimension, and focus on the complexity and organization decision when all suppliers are located in country 1.

3.1 Closed economy

Notice that an increase in the complexity level N is associated with a uniform reduction of the individual input intensities of all suppliers, as each supplier now performs a more narrowly defined task. We assume that this specialization leads to efficiency gains, similar as in Acemoglu et al. (2007). Specifically, we assume that unit costs are the same for all suppliers, and are given by $c^M = c/N^s$, with $0 < s < 1$. The unit costs c^M are thus decreasing in N for all suppliers, and these cost savings are more substantial the larger s is. Without loss of generality, we normalize the parameter c to unity ($c = 1$).

With symmetric components, and using (5), (8) and (9), the producer's problem is to maximize the following total payoff:

$$\pi = \Theta \cdot Y \cdot \Psi - N \cdot w_1^M - \bar{f}, \tag{10}$$

$$\Psi \equiv \left[1 - \alpha\left(\beta^H \eta^H + \frac{\beta^M \eta^M}{N}\right)\right]\left[\left(\frac{\beta^H}{c^H}\right)^{\eta^H}\left(N^s \cdot \exp\left\{\frac{1}{N}\int_0^N \ln\left(\beta_j\right) dj\right\}\right)^{\eta^M}\right]^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

In subsection 3.1.1. we first study the case where enforcable contracts on the ex ante division of revenue are possible. In that case, the producer maximizes eqs.(10) and (11) simultaneously with respect to N and β^H. In subsection 3.1.2. we then study the incomplete contracts scenario where the producer cannot freely decide on the ex ante division of the surplus, but has to choose the complexity and the organization of the production process in order to affect the division of revenue that results in the bargaining stage.

3.1.1 Optimal mass of suppliers and revenue division

When the producer can freely choose the headquarter revenue share β^H, then each supplier receives a revenue share $\beta_i = (1 - \beta^H)/N$ due to symmetry. Using (10) and (11), the firm's variable payoff $\Theta \cdot Y \cdot \Psi\left(N, \beta^H\right)$ can then be simplified as follows:

$$\Theta \cdot Y \cdot \Psi = \Theta \cdot Y \cdot \left[1 - \alpha\left(\beta^H \eta^H + \frac{\left(1-\eta^H\right)\left(1-\beta^H\right)}{N}\right)\right]\left[\left(\frac{\beta^H}{c^H}\right)^{\eta^H}\left(\frac{1-\beta^H}{N^{1-s}}\right)^{1-\eta^H}\right]^{\frac{\alpha}{1-\alpha}}. \quad (12)$$

a) Zero outside opportunity. When setting the suppliers' outside opportunities to zero ($w_1^M = 0$), the producer's problem is equivalent to maximizing the variable payoff as given in (12). For this case, we can derive the following unique solution (see Appendix A.2.1.i):

$$N^*\left(w_1^M = 0\right) = \frac{\rho - s\left(1-\eta^H\right)\left(1+\alpha\eta^H\right)}{2\left(1-s\right)\eta^H} \equiv N_0^*, \quad (13)$$

$$\beta^{H*}\left(w_1^M = 0\right) = \frac{2\eta^H - \rho + s\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)}{2\eta^H} \equiv \beta_0^{H*} \quad (14)$$

with $\rho = \sqrt{s\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)\left(4\eta^H + s\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)\right)}$. Notice that $0 < \beta_0^{H*} < 1$ and $N_0^* > 0$ for all $0 < s < 1$, $0 < \eta_H < 1$, and $0 < \alpha < 1$. It directly follows from the solution in (13) and (14) that:

$$\frac{\partial \beta_0^{H*}}{\partial \eta^H} > 0, \qquad \frac{\partial N_0^*}{\partial \eta^H} < 0, \qquad \frac{\partial \beta_0^{H*}}{\partial s} < 0, \qquad \frac{\partial N_0^*}{\partial s} > 0.$$

Higher headquarter-intensity of final goods production leads to a larger optimal revenue share for the producer. The intuition for this result is similar as in Antràs and Helpman (2004, 2008): both the headquarter and the suppliers underinvest in the provision of their respective inputs, and this underinvestment problem is more severe for the headquarter (the mass of suppliers) the smaller (the larger) the revenue share β^H is. Ensuring ex ante efficiency requires that the producer should receive a larger share of the surplus in sectors where headquarter services are more intensively used in production.

The basic trade-off with respect to the complexity choice N is novel in our framework. It can be seen from (12) that the impact of N on the variable payoff is, *a priori*, ambiguous. Intuitively, higher complexity leads to stronger specialization (i.e., lower unit costs c^M), which tends to increase the firm's revenue and payoff. On the other hand, for a given share β^H, higher complexity also "dilutes" the investment incentives for every single supplier, because the individual input intensities decrease and the overall revenue share $\beta^M = 1 - \beta^H$ has to be split among more parties. This negatively impacts on the firm's payoff. The optimal complexity N_0^* balances the "cost saving" and the "dilution" effect. Higher headquarter-intensity η^H leads to a lower optimal complexity. The reason is the following:

The optimal joint revenue share for the suppliers (β^{M*}) is decreasing in η^H, which tends to jeopardize their investment incentives. To countervail this problem, the producer can concentrate on relatively few components with a high individual input intensity. Although the gains from specialization are smaller in that case, the resulting increases of β_i and η_i again raise the suppliers' incentives (see eq. (5)).[7]

The stronger the cost savings from specialization are (the larger s is), the more profitable is it to add components to the value chain, i.e., the higher is N_0^*. This increase in complexity is then accompanied by a decrease in the optimal revenue share β_0^{H*}, since the incentives for all component manufacturers must be maintained.[8] When s becomes very small, so does N_0^*. Intuitively, the "cost saving" effect disappears if s tends to zero. The "dilution effect" for the suppliers is still present, however, so that the optimal mass of components would then also become very small.[9] Notice that this is true even though contracting with more suppliers leads to no increase in fixed costs as long as $w_1^M = 0$.

Finally, notice that the payoff-maximizing choices (13) and (14) do not depend on Θ. Still, a firm needs to be sufficiently productive in order to remain in the market, since the variable payoff must be large enough to cover the fixed costs $\bar{f}$. Hence, only such firms survive whose productivity level is above some threshold $\hat{\Theta}_0$ given in Appendix A.2.1.v.

b) Positive outside opportunity. Turning to the case with $w_1^M > 0$, recall that a more complex production process leads to larger fixed costs $f = N \cdot w_1^M$, since the suppliers' participation constraints must be taken into account. With a positive outside opportunity there is thus an additional endogenous "complexity penalty" embedded in our model.

With $w_1^M > 0$, we cannot explicitly solve for N^* and β^{H*}. However, using the two first-order conditions for payoff maximization, it is possible to solve $\partial\pi/\partial\beta^H = 0$ for $\beta^H(N)$

[7] It is, thus, not clear if the optimal revenue share of a *single* supplier (β_{i0}^*) is increasing or decreasing in headquarter-intensity η^H; there is a larger joint revenue share β^M when η^H is low ("component-intensity effect"), but this share is then split among many suppliers ("complexity effect"). Using (13) and (14), it can be shown that $\beta_{i0}^* = (1 - \beta_0^{H*})/N_0^*$ is in fact hump-shaped over the range of η^H and achieves a maximum at some level η_{crit}^H (see Appendix A.2.1.ii). In other words, single suppliers receive the highest revenue shares in sectors with medium headquarter-intensity.

[8] We show in Appendix A.2.1.iii that $N_0^* = 1$ if s is equal to some s_{crit}. Suppose for the moment that the set of suppliers N is discrete, by assuming that the unit mass of inputs on the interval $[0, 1]$ is provided by a single supplier. In fact, if $s = s_{crit}$, choosing a unit mass of inputs is optimal for the producer. The corresponding optimal revenue share $\beta_0^{H*}(s = s_{crit})$ in that case is identical to eq. (10) in Antràs and Helpman (2004), where it is imposed exogenously that there is just one single component supplier. Their baseline model is thus included in our framework as a special case. When s is smaller (larger) than s_{crit}, it is optimal to have less (more) than a unit mass of inputs.

[9] See Appendix A.2.1.iv for an analytical decomposition that illustrates the trade-off between these effects more formally.

with $\partial\beta^H/\partial N < 0$, which does not depend on w_1^M (see eq.(23) in Appendix A.2.2.i). Substituting this into the other first-order condition, we can derive the following function:

$$\frac{\partial\pi}{\partial N} = \Theta \cdot Y \cdot \underbrace{\frac{\partial\Psi}{\partial N}\Big|_{\beta^H=\beta^H(N)}}_{\equiv\Psi'} - w_1^M = 0 \quad \Leftrightarrow \quad \Psi' = \frac{w_1^M}{\Theta \cdot Y}.$$

Ψ' depends only on N and represents the marginal change in the total payoff when raising complexity, taking into account that $\beta^H(N)$ is optimally adjusted. We know that $\Psi' = 0$ is solved by N_0^* as given in (14). With $w_1^M > 0$, the optimal mass of producers N^* is determined by setting Ψ' equal to $w_1^M/(\Theta \cdot Y) > 0$, and since $\partial\Psi'/\partial N < 0$ it follows that $0 < N^* < N_0^*$ and $0 < \beta_0^{H*} < \beta^{H*} < 1$, with:

$$\frac{\partial N^*}{\partial\Theta} > 0, \qquad \frac{\partial N^*}{\partial\eta^H} < 0, \qquad \frac{\partial N^*}{\partial w_1^M} < 0, \qquad \frac{\partial\beta^{H*}}{\partial\Theta} < 0, \qquad \frac{\partial\beta^{H*}}{\partial\eta^H} > 0, \qquad \frac{\partial\beta^{H*}}{\partial w_1^M} > 0.$$

The downward-sloping thick curve in Figure 1 illustrates the function Ψ'. The optimal mass of suppliers is where this curve cuts the horizontal line. An increase of w_1^M leads to an upward shift, and an increase of Θ to a downward shift of this horizontal line. For given values of w_1^M and η^H, more productive firms thus collaborate with more suppliers, since they can easier cope with the requirement to match their outside opportunities. Still, all firms choose a complexity level below N_0^*, i.e., the optimal complexity N^* is bounded. Furthermore, the Ψ'-curve shifts to the left as η^H increases. Hence, when comparing equally productive firms, those from headquarter-intensive industries have lower optimal complexity than those from component-intensive industries (see Appendix A.2.2.i).

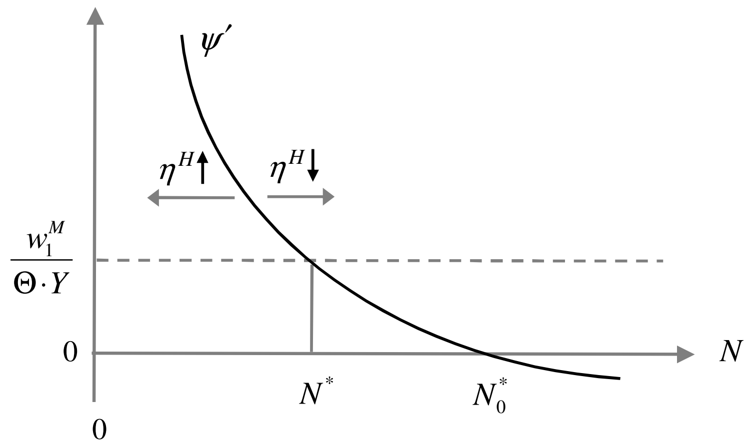

Figure 1: Optimal complexity with (N^*) and without (N_0^*) increasing fixed costs.

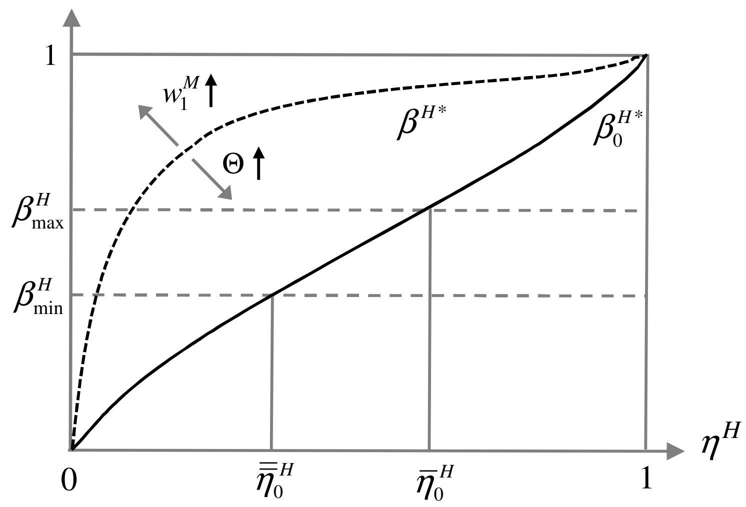

Figure 2: Distribution of revenue

In Figure 2 we illustrate the corresponding optimal headquarter revenue share. The figure firstly depicts the β_0^{H*}-curve for the benchmark case with $w_1^M = 0$. Since we know from the first-order conditions that $\partial\beta^H/\partial N < 0$ (see Appendix A.2.2.i), it is clear that the β^{H*}-curve stretches out to the left if $w_1^M > 0$, which implies a higher β^{H*} throughout the entire range of η^H. The reason is that an increase in w_1^M, by reducing the optimal complexity level, leads to a higher individual input intensity $\eta_i = \eta^M/N$ for each supplier. This raises the suppliers' incentives and thereby allows for a larger optimal revenue share β^{H*}. Yet, this share is lower in firms with higher productivity, i.e., the firm-specific β^{H*}-curve moves closer to the β_0^{H*}-curve. The intuition is that more productive firms operate more complex production processes, and to maintain the investment incentives, they need to leave a larger revenue share β^M for the suppliers. In the limit, β^{H*} converges to β_0^{H*}.

A stronger cost saving effect s naturally leads to more suppliers (a higher N^*) and, thus, to a lower β^{H*}.[10] Furthermore, higher productivity implies a higher total payoff π, despite the fact that more productive firms have more complex production processes and, thus, higher fixed costs. Higher productivity thus raises the variable payoff $\Theta \cdot Y \cdot \Psi$ stronger than the fixed costs $F = N^* \cdot w_1^M + \bar{f}$ (see Appendix A.2.2.ii). Ultimately, a firm only survives if it is sufficiently productive to cover these fixed costs, which are unambiguously larger than in the previous case with $w_1^M = 0$. It is thus clear that the threshold productivity $\hat{\Theta}$ is larger than the benchmark level $\hat{\Theta}_0$ given in Appendix A.2.1.v, even though we cannot solve for $\hat{\Theta}$ in closed form.

[10] Graphically, the Ψ'-curve in Figure 1 shifts to the right as s increases. In the corresponding Figure 2, both the β_0^{H*}- and the β^{H*}-curve stretch out to the right.

3.1.2 The make-or-buy decision under incomplete contracts

We now turn to the incomplete contracts scenario where the producer cannot "freely" decide on the ex ante division of the surplus, but has to choose the complexity and the organization of the firm in order to affect the division of revenue that results in the bargaining stage. Following Antràs and Helpman (2004), we assume that external suppliers are in a better bargaining position than integrated suppliers vis-a-vis the producer. This is due to the fact that the producer has no ownership of the assets of external suppliers, while she does have residual control rights over the assets of those suppliers that are integrated within the boundaries of the firm.

Specifically, we assume that if the producer has outsourced *all* suppliers ($\xi = 1$), she is able to realize an exogenously given revenue share β^H_{min}. Vice versa, if she has integrated *all* suppliers ($\xi = 0$), she is able to realize a larger revenue share, $\beta^H_{max} > \beta^H_{min}$, as a result of her asset ownership. For intermediate cases with $0 < \xi < 1$, her realized revenue share (her "effective bargaining power") can be written as:

$$\beta^H = \xi \cdot \beta^H_{min} + (1 - \xi) \cdot \beta^H_{max}. \tag{15}$$

The producer can thus affect her revenue share via the outsourcing share $\xi = N^O/N$, but she is contrained to the range between β^H_{min} and β^H_{max}.[11] The remaining share $\beta^M = 1 - \beta^H$ is left for the suppliers, and the individual revenue share of an outsourced and an integrated supplier is denoted by β^O_i and β^V_i, respectively. Since $\beta^M = N^O \cdot \beta^O_i + N^V \cdot \beta^V_i$ must hold, it follows that $N^O \cdot \beta^O_i = \xi \cdot (1 - \beta^H_{min})$ is the revenue share of the external contractors, and $N^V \cdot \beta^V_i = (1 - \xi) \cdot (1 - \beta^H_{max})$ the share of the integrated affiliates.[12]

a) Zero outside opportunity. As before we start with the case where the suppliers' outside opportunities are set to zero ($w^M_1 = 0$). In this case, the producer's problem is

[11] Notice that β^H_{min} and β^H_{max} are *independent* of N. The complexity of the production process, therefore, does not directly affect the bargaining power of the producer, which is plausible since the headquarter-intensity is also exogenous and independent of N. It is possible to analyze cases where complexity systematically affects the bargaining power (the realized revenue share) of the headquarter, but this complicates the analysis without adding many further insights.

[12] The joint revenue share of all suppliers (β^M) is thus unambiguously larger with complete outsourcing ($\xi = 1$) than with complete integration ($\xi = 0$). However, a single outsourced contractor in the first scenario does not necessarily obtain a larger revenue share than a single integrated affiliate in the second scenario. That is, β^O_i with $\xi = 1$ need not be larger than β^V_i with $\xi = 0$, because N^O and N^V need not be the same. Yet, in a constellation where outsourcing and integration coexist, it is clear that an external supplier receives a larger revenue share than an integrated supplier ($\beta^O_i > \beta^V_i$ with $0 < \xi < 1$).

equivalent to maximizing the variable payoff $\Theta \cdot Y \cdot \Psi\left(N, \beta^H(\xi)\right)$ with respect to N and ξ, subject to the constraint (15). The term Ψ is given by eq.(11).

As long as the constraint $\beta^H \in \left[\beta^H_{min}, \beta^H_{max}\right]$ is not binding, this maximization problem leads to an equivalent solution as described in subsection 3.1.1. In particular, if the producer is able to choose the outsourcing share ξ in such a way that β^H exactly matches β_0^{H*} as given in (14), she would target this payoff-maximizing revenue distribution with her organizational choice, and hence the corresponding complexity N_0^* given in (13). Since $\xi \cdot \beta^H_{min} + (1-\xi) \cdot \beta^H_{max} = \beta_0^{H*}$ in that case, this implies the following outsourcing share:

$$\xi_0^* = \left(\beta^H_{max} - \beta_0^{H*}\right) / \left(\beta^H_{max} - \beta^H_{min}\right) \qquad \text{for} \quad \beta^H_{min} \le \beta_0^{H*} \le \beta^H_{max}. \tag{16}$$

Notice, however, that this outsourcing share is feasible if and only if $\beta^H_{min} \le \beta_0^{H*} \le \beta^H_{max}$. Otherwise, if $\beta_0^{H*} < \beta^H_{min}$ or $\beta_0^{H*} > \beta^H_{max}$, she cannot achieve the unconstrained payoff-maximizing firm structure. She would then aim for an outsourcing share ξ that aligns the β^H given in eq. (15) as closely as possible with the optimal β_0^{H*}, and for the corresponding constrained optimal complexity level – also see Appendix A.3.

Figure 2 illustrates this problem. The figure depicts the payoff-maximizing β_0^{H*} that the producer aims for. If the firm operates in a headquarter-intensive sector, more precisely a sector with $\eta^H > \bar{\eta}_0^H$ where the threshold $\bar{\eta}_0^H$ is defined in Appendix A.3.1., we have $\beta_0^{H*} > \beta^H_{max}$ so that the producer cannot achieve β_0^{H*}. Firms from those sectors choose complete vertical integration, $\tilde{\xi}_0 = 0$, as this leads to the maximum possible revenue share β^H_{max} for the headquarter; the corresponding constrained optimal complexity level is analyzed soon. Vice versa, if the firm operates in a component-intensive sector, more precisely a sector with $\eta^H < \bar{\bar{\eta}}_0^H$ where the threshold $\bar{\bar{\eta}}_0^H$ is defined in Appendix A.3.1., the producer also cannot achieve β_0^{H*}, and she then aims for the highest possible revenue share for the suppliers by choosing complete outsourcing ($\tilde{\xi}_0 = 1$). In sectors with $\bar{\bar{\eta}}_0^H \le \eta^H \le \bar{\eta}_0^H$, the producer is not constrained by $\beta^H_{min} \le \beta_0^{H*} \le \beta^H_{max}$, and she therefore sets $\tilde{\xi}_0 = \xi_0^*$ as given in (16). In those sectors with medium headquarter-intensity we thus observe a coexistence of both organizational forms within the same firm (hybrid sourcing), with a higher outsourcing share in relatively more component-intensive industries within that range ($\partial \xi_0^* / \partial \eta^H < 0$ since $\partial \beta_0^{H*} / \partial \eta^H > 0$).[13]

[13] Du, Lu and Tao (2009) consider an extension of Antràs and Helpman (2004) where the *same* input can be provided by two suppliers. "Bi-sourcing" (one supplier integrated and the other outsourced) can arise in their model out of a strategic motive, because it systematically improves the headquarter's outside option and, thus, her effective bargaining power. Our model relies on an entirely different (non-strategic) mechanism why firms may choose different organizational modes for different inputs.

Turning to the corresponding complexity decision, let $\tilde{N}_0$ denote the complexity choice under incomplete contracts for the case with $w_1^M = 0$. To compute $\tilde{N}_0$, notice that in sectors with $\eta^H > \bar{\eta}_0^H$ and $\eta^H < \bar{\bar{\eta}}_0^H$, firms choose the same organizational form for *all* suppliers (complete vertical integration and, respectively, complete outsourcing). For these cases with a uniform organizational structure, we can simplify Ψ as given in eq.(11) by setting $\beta_j = (1 - \tilde{\beta}_0^H)/N$ where $\tilde{\beta}_0^H = \{\beta_{min}^H, \beta_{max}^H\}$. Solving $\Psi' = \partial\Psi/\partial N = 0$ then yields:

$$\tilde{N}_0 = \frac{\left(1 - \tilde{\beta}_0^H\right)\left(1 - s\alpha\left(1 - \eta^H\right) - \alpha\eta^H\right)}{(1 - s)\left(1 - \alpha\tilde{\beta}_0^H\eta^H\right)}. \tag{17}$$

It follows directly from (17) that $\tilde{N}_0^O \equiv \tilde{N}_0\left(\tilde{\beta}_0^H = \beta_{min}^H\right) > \tilde{N}_0\left(\tilde{\beta}_0^H = \beta_{max}^H\right) \equiv \tilde{N}_0^V$. That is, vertical integration is endogenously associated with less complexity than outsourcing, as the producer can reduce the underinvestment problem for the suppliers by choosing fewer intermediate inputs. Next, for the unconstrained firms in sectors with medium headquarter-intensity $\bar{\bar{\eta}}_0^H \leq \eta^H \leq \bar{\eta}_0^H$, where $\beta^H = \beta_0^{H*}$ and $0 \leq \tilde{\xi}_0 \leq \xi_0^* < 1$ holds, the mass of suppliers is given by (13), since it can be shown that $\tilde{N}_0\left(\tilde{\beta}_0^H = \beta_0^{H*}\right) = N_0^*$.[14]

Figure 3 summarizes the results. Active firms in sectors with low headquarter-intensity have a huge mass of suppliers ($\tilde{N}_0^O$), all of which are outsourced. Gradually moving to more headquarter-intensive sectors, we first see no change in the firms' organizational structures or the producer's revenue shares, since $\tilde{\xi}_0 = 1$ and $\beta^H = \beta_{min}^H$ as long as $\eta^H < \bar{\bar{\eta}}_0^H$. Yet, such a gradual increase of η^H leads to a decreasing mass of suppliers $\tilde{N}_0^O$, hence the most complex production processes prevail in the most component-intensive sectors.[15] Once we turn to sectors with a headquarter-intensity above $\bar{\bar{\eta}}_0^H$, there is a coexistence of both organizational forms within the same firm. The headquarter revenue share is gradually increasing, and the outsourcing share is gradually decreasing in η^H. Complexity $\tilde{N}_0$ continues to decrease in η^H and is equal to N_0^* in that range. Finally, once η^H goes beyond $\bar{\eta}_0^H$, further increasing the headquarter-intensity has again no impact on organizational structures or the producers' revenue shares, since $\tilde{\xi}_0 = 0$ and $\beta^H = \beta_{max}^H$ if $\eta^H > \bar{\eta}_0^H$. It still leads to a decreasing mass of suppliers, which is now given by $\tilde{N}_0^V$. Firms in the most headquarter-intensive sectors are thus the least complex ones, and fully vertically integrated.

[14] $\tilde{N}_0$ is continuous in η^H and $\tilde{\beta}_0^H$, so that it can be easily shown that $\tilde{N}_0^O > N_0^* > \tilde{N}_0^V$ holds.

[15] For a given ξ, higher headquarter-intensity is thus inversely related to complexity, similar as in subsection 3.1.1. where we have shown that the optimal mass of suppliers N_0^* also depends negatively on η^H. Formally, $\partial\tilde{N}_0^O/\partial\eta^H < 0$ only holds if $s < (1 - \tilde{\beta}_0^H)/(1 - \alpha\tilde{\beta}_0^H)$. To avoid undue case distinctions, we assume that the exogenous β_{max}^H is sufficiently small so that this restriction on s is satisfied.

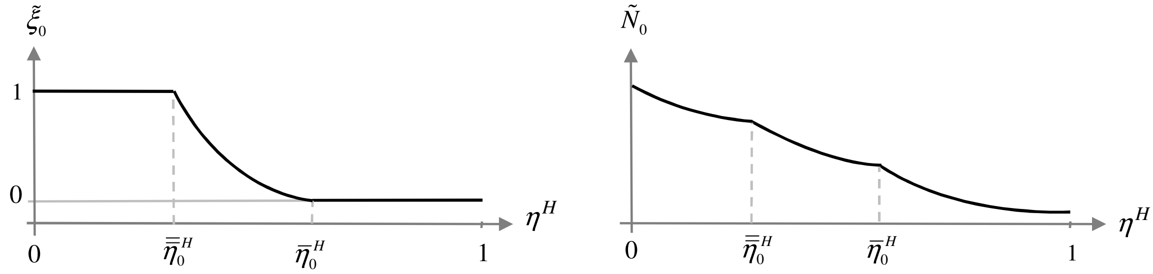

Figure 3: Organization and complexity decision for the case with $w_r^M = 0$.

The complexity and the organizational decision therefore have opposite incentive effects for the suppliers. Complete outsourcing (vertical integration) leaves a large (small) combined revenue share for the suppliers, but this share is then divided among many (few) of them. A stronger cost saving effect (a higher value of s) is associated with a larger mass of suppliers, other things equal.[16] Moreover, the β_0^{H*}-curve in Figure 2 stretches out to the right, and both $\bar{\eta}_0^H$ and $\bar{\bar{\eta}}_0^H$ go up when s increases (see Appendix A.3.1.i). Complete outsourcing is then chosen over a larger, and complete integration over a smaller domain of η^H the larger s is, as it becomes relatively more attractive to choose the organizational form that is endogenously associated with higher complexity, i.e., to choose outsourcing.

Finally, it is important to note that, as long as the suppliers' outside opportunities are set to zero ($w_1^M = 0$), there are no intra-sectoral differences in the complexity and the organization of firms. That is, for a given headquarter-intensity, all active firms in that industry (regardless of productivity) would choose the same mass of suppliers and the same outsourcing share. This is shown in the left panel of Figure 4. Here we depict the total payoff π as a function of Θ and η^H. A darker color indicates a higher complexity level. Within every sector (i.e., moving parallel to the Θ-axis), we see that higher productivity implies a higher total payoff, but it does not affect the firms' complexity or organization. Both differ only across sectors, such that a higher headquarter-intensity is associated with less suppliers and more vertical integration (as also shown in Figure 3). Figure 4a furthermore illustrates the decision whether to remain active in the market. For all firms in the hybrid range $\bar{\bar{\eta}}_0^H \leq \eta^H \leq \bar{\eta}_0^H$, the threshold producitivity for survival, $\tilde{\Theta}_0$, is identical to $\hat{\Theta}_0$ given in Appendix A.2.1.v, while $\tilde{\Theta}_0 > \hat{\Theta}_0$ must hold for all other firms, as they face the binding constraint $\beta^H \in \left[\beta_{min}^H, \beta_{max}^H\right]$ and cannot achieve the unconstrained payoff maximum. They hence need a higher productivity to break even.

[16]Formally, eq.(17) implies $\partial \tilde{N}_0 / \partial s > 0$ which applies for the ranges $\eta^H < \bar{\bar{\eta}}_0^H$ and $\eta^H > \bar{\eta}_0^H$, and eq.(13) implies $\partial N_0^* / \partial s > 0$ which applies for the range $\bar{\bar{\eta}}_0^H \leq \eta^H \leq \bar{\eta}_0^H$.

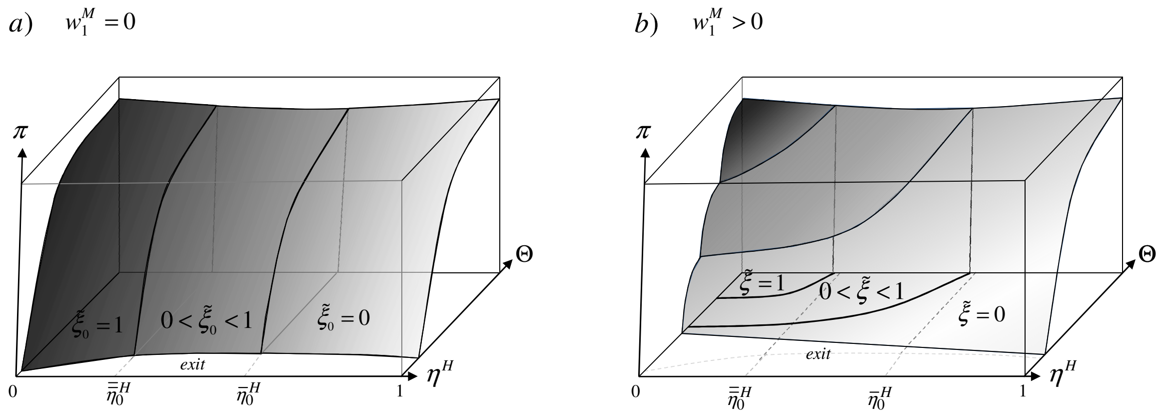

Figure 4: Total firm payoff, complexity and organization.

b) Positive outside opportunity. We now focus on the case with endogenous fixed costs ($w_1^M > 0$). We cannot explicitly solve for $\tilde{N}$ and $\tilde{\xi}$ in this case, but similar as in subsection 3.1.1. it is again possible to infer important comparative static results.

As in the previous case with $w_1^M = 0$, a single producer chooses the outsourcing share ξ so as to realign the revenue share β^H from eq. (15) as closely as possible with the payoff-maximizing revenue share β^{H*}, which then implies a corresponding complexity choice $\tilde{N}$. Comparing β^{H*} with the available range of revenue shares, $\beta^H \in \left[\beta^H_{min}, \beta^H_{max}\right]$, we can classify every firm into one of the following three groups:

1. firms with $\beta^{H*}\left(\eta^H, w_1^M, \Theta\right) > \beta^H_{max}$,
2. firms with $\beta^{H*}\left(\eta^H, w_1^M, \Theta\right) < \beta^H_{min}$,
3. firms with $\beta^H_{min} \leq \beta^{H*}\left(\eta^H, w_1^M, \Theta\right) \leq \beta^H_{max}$.

For the firms in group 3, the constraint $\beta^H \in \left[\beta^H_{min}, \beta^H_{max}\right]$ is not binding. These firms can choose an outsourcing share $\tilde{\xi} = \xi^* = \left(\beta^H_{max} - \beta^{H*}\right) / \left(\beta^H_{max} - \beta^H_{min}\right)$ so as to exactly match β^{H*}. For the other groups the constraint is binding, and all firms in group 1 choose complete vertical integration, while all firms in group 2 choose complete outsourcing.

The corresponding complexity choice can then be derived as follows: From eqs.(10) and (11) we know that $\tilde{N}$ is determined according to $\Psi' = w_1^M/(\Theta \cdot Y)$. For the unconstrained firms, which are able to achieve β^{H*} by setting $\tilde{\xi} = \xi^*$, their complexity choice $\tilde{N}$ is thus equivalent to the payoff-maximizing N^* described above. For the constrained firms, we can define the following functions: $\Psi^{O\prime} \equiv \Psi'\left(N, \beta^H = \beta^H_{min}, \beta_j = (1 - \beta^H_{min})/N\right)$ and

$\Psi^{V\prime} \equiv \Psi' \left(N, \beta^H = \beta^H_{max}, \beta_j = (1 - \beta^H_{max})/N\right)$, which depend negatively on N and depict the marginal change in the variable payoff for fixed values of β^H that correspond to the headquarter revenue share under complete outsourcing and integration, respectively. In Figure 5 we illustrate the curves $\Psi^{O\prime}$ and $\Psi^{V\prime}$, and it can be easily shown that the former curve always runs to the right of the latter (see Appendix A.3.2.).[17]

The complexity choice that corresponds to every possible organizational decision is determined by the intersection point of the respective downward-sloping Ψ'-curve with the horizontal line at $w^M_1/(\Theta \cdot Y)$. In Figure 5 we depict two firms from the same industry, one with "high" and one with "low" productivity. Suppose both firms have the same organizational structure. The highly productive firm then collaborates with more suppliers. More importantly, for given Θ and η^H, we have $\tilde{N^O} > \tilde{N}^{0<\xi<1} > \tilde{N^V} > 0$. Hence, vertical integration is endogenously associated with lower complexity. The intuition is similar as above: Since the suppliers receive a relatively small joint revenue share with vertical integration, decreasing complexity is a device to countervail their underinvestment problems.[18]

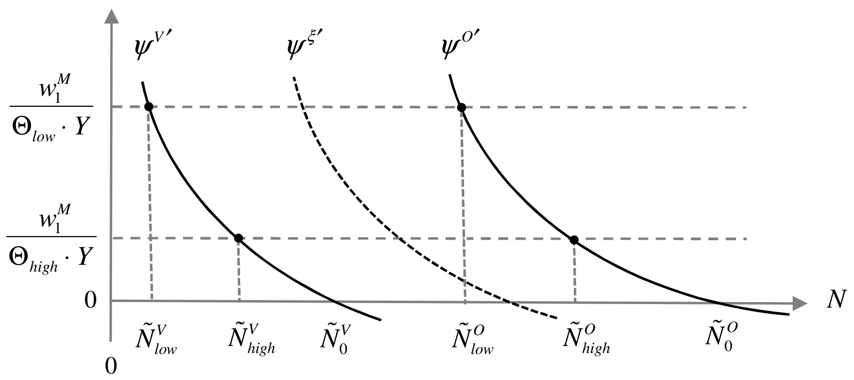

Figure 5: Payoff-maximizing mass of suppliers: The complexity decision.

To pin down the final complexity and organization decisions of firms in different industries, it is crucial to note that the three groups of firms defined above can no longer be delineated by the sectoral headquarter-intensity η^H alone. Recall from Figure 2 that the β^{H*}_0-curve is increasing in η^H, and that $w^M_1 > 0$ leads to an increase of β^{H*} that is larger for

[17]Since Ψ is continuous in β^H, it follows immediately that the Ψ'-curves for the intermediate cases with $0 < \tilde{\xi} < 1 \rightarrow \beta^H_{min} < \beta^H < \beta^H_{max}$ are located in between the $\Psi^{V\prime}$- and the $\Psi^{O\prime}$-curve.

[18]Notice that $\tilde{N}$ always remains below the respective $\tilde{N}_0$ for the same organizational structure, which is located at the intersection of the respective Ψ'-curve with the horizontal axis. Furthermore, it can be shown that an increase in the headquarter-intensity η^H shifts all Ψ'-curves to the left and, thus, leads to a smaller mass of suppliers for all possible productivities and organizational forms.

less productive firms. In other words, $\beta^{H*}\left(\eta^H, w_1^M, \Theta\right)$ is no longer the same for all firms from the same industry (with the same η^H), but it is now firm-specific as it depends on Θ. Hence, firms from the same industry no longer need to choose identical firm structures.

The final complexity and organization decisions are summarized above in the right panel of Figure 4. First, consider headquarter-intensive sectors with $\eta^H > \bar{\eta}_0^H$. All firms from those sectors belong to group 1, and thus choose complete vertical integration. This is for two reasons. This organization leads to the highest possible revenue share β^H_{max} for the producer. Now this choice is reinforced, since vertical integration is also associated with fewer suppliers and with lower fixed costs. There is, hence, no change in the organizational decision of firms in headquarter-intensive industries compared to the previous case with $w_1^M = 0$, which is depicted in Figure 4a. In other words, in sectors with $\eta^H > \bar{\eta}_0^H$, all firms (regardless of productivity) choose complete vertical integration. Figure 4b also shows that not only the total payoff π, but also the complexity level $\tilde{N}^V$ is now increasing in Θ. That is, within a given headquarter-intensive sector, more productive firms vertically integrate *more* suppliers. Furthermore, comparing two equally productive firms from two industries A and B with $\eta_A^H > \eta_B^H > \bar{\eta}_0^H$, it turns out that the firm in sector A chooses less complexity than the firm in the relatively more component-intensive sector B.

Now consider component-intensive sectors where $\eta^H < \bar{\bar{\eta}}_0^H$. Without the endogenous "complexity penalty", all firms in those sectors would belong to group 2 and choose complete outsourcing (see Figure 4a). With $w_1^M > 0$, we observe that some firms now switch to group 1, and this is more likely: i) the lower productivity is, since the increase of β^{H*} is then most substantial, and ii) the closer η^H is to the upper bound $\bar{\bar{\eta}}_0^H$, since the β^{H*} can then easier exceed β^H_{max}. Those firms now choose complete vertical integration, and this organizational form is chosen to keep the fixed costs f low. There are also firms whose β^{H*} increases by less, so that it now falls inside the range between β^H_{min} and β^H_{max}. These firms then belong to group 3, and can choose the unconstrained payoff-maximizing ξ^* (with $0 \leq \xi^* \leq 1$) and N^*. This is more likely to occur for firms with medium productivity, and in sectors with headquarter-intensity not too close to the upper bound $\bar{\bar{\eta}}_0^H$. For firms with high productivity, the increase of β^{H*} due to $w_1^M > 0$ is negligible, and they remain in group 2 and continue to choose complete outsourcing. Intuitively, the higher fixed cost under outsourcing play a minor role for these highly productive firms. Their main aim is to maximize the residual rights of the suppliers, whose inputs are intensively used in those sectors. Similarly, firms from highly component-intensive sectors are also more likely to remain in group 2, i.e., to choose complete outsourcing.

Summing up, the organization of firms in component-intensive industries now varies over the range of Θ, particularly if η^M is not too low. Low productive firms have few suppliers which are fully vertically integrated. With rising productivity, there is a gradual increase of complexity $\tilde{N}$ and the outsourcing share $\tilde{\xi}$, and the most productive firms collaborate with a huge mass of suppliers and choose complete outsourcing.[19]

Finally, the organizational decision of firms from sectors with medium headquarter-intensity, $\bar{\bar{\eta}}_0^H \leq \eta^H \leq \bar{\eta}_0^H$, is now also tilted towards more vertical integration. More precisely, all firms in those industries decrease their outsourcing share in response to an increase of w_1^M. Firms with low productivity see a larger increase in β^{H*}, so they are more likely to become constrained by β^H_{max} and thus choose $\tilde{\xi} = 0$. This switch from group 3 to group 1 is also more likely to happen in sectors where η^H is only slightly below $\bar{\eta}^H$, since the outsourcing share was already low there. Firms with high productivity and with headquarter-intensity relatively close to $\bar{\bar{\eta}}^H$ are, in contrast, more likely to continue to remain in the range between β^H_{min} and β^H_{max}. Those firms would then still belong to group 3 and choose hybrid sourcing. Yet, since β^{H*} has increased, this necessarily implies an outsourcing share $\xi^* = (\beta^H_{max} - \beta^{H*})/(\beta^H_{max} - \beta^H_{min}) < \xi_0^*$.[20] Overall, Figure 4b suggests that the coexistence of integration and outsourcing is most pervasive in firms with medium-to-high productivity in sectors with low-to-medium headquarter-intensity.

3.2 Open Economy

We now incorporate the global scale dimension into the producer's problem, who now also decides on the country $r \in \{1, 2\}$ where each component $i \in [0, N]$ is manufactured. We assume that unit costs of foreign suppliers are lower than for domestic suppliers, while the efficiency gains from specialization do not depend on the suppliers' country of origin. Specifically, domestic and foreign suppliers have unit cost equal to $c_1^M = 1/N^s$ and $c_2^M = \delta(\ell)/N^s$, respectively, with $0 < \delta(\ell) < 1$.

[19] Antràs and Helpman (2004) obtain the opposite result, namely that headquarter-intensive sectors are those where organizational structures are different across the productivity spectrum. That result is driven by the ad-hoc assumption that integration is associated with *exogenously* higher fixed costs than outsourcing. Grossman, Helpman and Szeidl (2005) consider the alternative ad-hoc assumption that outsourcing is associated with exogenously higher fixed costs. Our model is qualitatively more consistent with the latter paper, but in our model fixed cost differences between organizational modes emerge *endogenously* as they imply different optimal complexity levels. We could generate a similar sourcing pattern as Antràs and Helpman (2004) when assuming that $\bar{f}$ is sufficiently higher under integration than under outsourcing.

[20] If an increase of w_1^M overall leads to more or less hybrid sourcing is unclear, since there is exit from group 3 to group 1 but also entry from group 2 to group 3. To unambiguously sign the overall change would require more specific assumptions about the distribution of Θ and η^H across firms.

We assume the following specification for the "offshoring gain": $\delta(\ell) = \left(1 + \bar{\delta} \cdot \ell\right)^{-1/\ell}$, with $\bar{\delta} > 0$ (also see Appendix B.1.).[21] Using $\delta(\ell)$ and eq.(5), the producer's problem is to maximize the total payoff $\pi = \Theta \cdot Y \cdot \Psi - (1-\ell)N \cdot w_1^M - \ell N \cdot w_2^M - \bar{f}$, where Ψ is now given by:

$$\Psi \equiv \left[1 - \alpha\left(\beta^H \eta^H + \frac{\beta^M \eta^M}{N}\right)\right] \left[\left(\frac{\beta^H}{c^H}\right)^{\eta^H} \left((1+\bar{\delta}\ell)N^s \cdot \exp\left\{\frac{1}{N}\int_0^N \ln(\beta_j)\,\mathrm{d}j\right\}\right)^{\eta^M}\right]^{\frac{\alpha}{1-\alpha}} \tag{18}$$

3.2.1 Optimal mass of suppliers, revenue division, and offshoring share

Analogous to the closed economy case, we first analyze the scenario where the producer can freely assign the ex ante distribution of revenue. Taking into account that the optimal N^* and β^{H*} pin down $\beta_i^* = (1-\beta^{H*})/N^*$ due to symmetric input intensities, we can simplify the variable payoff $\Theta \cdot Y \cdot \Psi\left(N, \beta^H, \ell\right)$ from eq. (18) as follows:

$$\Theta{\cdot}Y{\cdot}\Psi = \Theta{\cdot}Y{\cdot}\left[1 - \alpha\left(\beta^H \eta^H + \frac{\left(1-\eta^H\right)\left(1-\beta^H\right)}{N}\right)\right] \left[\left(\frac{\beta^H}{c^H}\right)^{\eta^H} \left(\frac{\left(1-\beta^H\right)\cdot(1+\bar{\delta}\ell)}{N^{1-s}}\right)^{1-\eta^H}\right]^{\frac{\alpha}{1-\alpha}}.$$

Suppose the outside opportunity in both countries is equal to zero ($w_1^M = w_2^M = 0$). In that case, the producer's problem is equivalent to maximizing this variable payoff. We show in Appendix B.2. that the optimal complexity N_0^* and revenue share β_0^{H*} are identical to their closed economy counterparts given in eqs.(13) and (14). Furthermore, it directly follows that the variable payoff is unambigiously increasing in the offshoring share, i.e., $\partial\Psi/\partial\ell > 0$. Hence, in that case where endogenous fixed costs play no role, the optimal decision is to offshore *all* suppliers ($\ell_0^* = 1$) in order to take advantage of the lower unit costs in the foreign country. Now suppose that $w_1^M = w_2^M > 0$, i.e., fixed costs matter but there are no cross-country differences in the endogenous "complexity penalty". In that case we would also obtain analogous results for N^* and β^{H*} as in the closed economy case, and again have $\ell^* = 1$ since offshoring only generates advantages but no disadvantages.

However, as is widely known, offshoring in fact has disadvantages in terms of higher communication and transportation costs, more expensive managerial oversight, and so

[21] This particular functional form is chosen for analytical simplicity only. It implies that there are decreasing marginal returns from offshoring, i.e., the reduction of unit costs are most substantial for the first offshored component, and then become smaller as the offshoring share ℓ is increased. The strength of the offshoring gain is also stronger the larger the parameter $\bar{\delta}$ is. Our qualitative results would be similar for other specifications of the offshoring gain, though mathematically the model would become more difficult.

on. To take this into account, we assume that there is an extra fixed cost $f^X > 0$ per offshored component, capturing those higher transaction costs for the firm. Overall fixed cost are then given by $F = w_1^M \cdot (1-\ell)N + (w_2^M + f^X) \cdot \ell N + \bar{f}$, and we assume that $\Delta \equiv w_2^M + f^X - w_1^M > 0$, which allows us to rewrite fixed costs as $F = (w_1^M + \ell\Delta)N + \bar{f}$.[22] When it comes to the maximization of the total payoff $\pi = \Theta \cdot Y \cdot \Psi - F$ with respect to ℓ, there is thus a trade-off between the higher variable payoff ($\partial\Psi/\partial\ell > 0$) and the larger fixed costs ($\partial F/\partial\ell > 0$) under offshoring. The positive effect on the variable payoff is stronger the higher the productivity level is, while the fixed cost increase does not depend on Θ. This suggests that offshoring is relatively more attractive for highly productive firms. In fact, in Appendix B.2.2. we formally prove the following results:

$$\frac{\partial N^*}{\partial \Theta} > 0, \qquad \frac{\partial \beta^{H*}}{\partial \Theta} < 0, \qquad \frac{\partial \ell^*}{\partial \Theta} \geq 0, \qquad \frac{\partial N^*}{\partial \eta^H} < 0 \quad , \quad \frac{\partial \beta^{H*}}{\partial \eta^H} > 0, \qquad \frac{\partial \ell^*}{\partial \eta^H} \leq 0.$$

More productive firms thus have a higher optimal offshoring share ℓ^* (with $0 \leq \ell^* \leq 1$). Furthermore, as in the closed economy, they have a smaller optimal headquarter revenue share and more suppliers, hence larger fixed costs. Still, it can be shown that the total payoff is increasing in productivity, $\partial\pi/\partial\Theta > 0$. Second, firms from more headquarter-intensive industries have less suppliers and a larger optimal headquarter revenue share, as in the closed economy case. Other things equal, the optimal offshoring share is also lower in firms from more headquarter-intensive industries. Finally, it is also possible to show that $\partial\ell^*/\partial\Delta \leq 0$, $\partial N^*/\partial\Delta < 0$, and $\partial\beta^{H*}/\partial\Delta > 0$ (see Appendix B.2.2.). That is, lower offshoring costs Δ (holding domestic fixed costs w_1^M constant) not only lead to a higher optimal offshoring share, but they also boost complexity and thereby imply a lower optimal headquarter revenue share.

3.2.2 The make-or-buy decision under incomplete contracts

Turning now to the incomplete contracts environment, first suppose that fixed cost considerations play no role at all (i.e., $w_1^M = w_2^M = f^X = 0$). In that case, the producer would offshore *all* components ($\tilde{\ell}^O = \tilde{\ell}^V = 1$) while making the exact same complexity and organization decisions as shown in Figure 4a.[23] Put differently, all firms with $\eta^H < \bar{\bar{\eta}}_0^H$ would completely rely on arm's length transactions, those with $\eta^H > \bar{\bar{\eta}}_0^H$ on intra-firm trade, and

[22] Suppliers from country 1 probably have a better outside opportunity than those from the poor country 2. Assuming $\Delta > 0$ ensures that the offshoring cost f^X outweighs the difference in outside opportunities.

[23] This follows from the facts that: i) N_0^* and β_0^{H*} are the same as in the closed economy, and ii) that the available range $\beta^H \in [\beta^H_{min}, \beta^H_{max}]$ also does not change – see Appendix B.3.1. for more details.

those with $\bar{\bar{\eta}}_0^H \leq \eta^H \leq \bar{\eta}_0^H$ on a combination of the two global sourcing modes. Suppose now that fixed costs matter, $w_1^M > 0$, but there are no cross-country differences in overall fixed costs, $\Delta = 0$. In that case, the same pattern as in Figure 4b emerges, where more productive firms choose higher complexity and where the organizational decisions are tilted towards vertical integration in order to keep fixed costs low. Yet, all firms (regardless of productivity or headquarter-intensity) would only have foreign suppliers in that case.

The case with with $w_1^M > 0$ and $\Delta > 0$ is the most interesting one. We then have the aforementioned trade-off between higher fixed costs and higher variable payoffs under offshoring. The higher Θ is, the more important is the latter aspect, hence productivity and offshoring are positively related ($\partial\tilde{\ell}/\partial\Theta \geq 0$, see Appendix B.3.2.). Furthermore, since this trade-off does not depend on whether a supplier is external or internal, there are no differences in the organization-specific offshoring shares in our model with symmetric components, but $\tilde{\ell} = \tilde{\ell}^O = \tilde{\ell}^V$ holds. Summing up, the overall sourcing pattern in the open economy can be described as follows:

1. *Headquarter-intensive industries*: All firms choose complete vertical integration of all suppliers. The least productive among the surviving firms collaborate with few suppliers and only source domestically. As productivity rises, firms gradually increase the mass of suppliers and the offshoring share. The most productive firms collaborate with a huge mass of foreign suppliers that are integrated into the firm's boundaries.

2. *Component-intensive industries*: The least productive among the surviving firms have few suppliers, all of which are domestic and vertically integrated. As productivity increases, firms tend to increase the complexity $\tilde{N}$, the outsourcing share $\tilde{\xi}$, and the offshoring share $\tilde{\ell}$. The most productive firms collaborate with a huge mass of suppliers, all of which are outsourced and offshored.

3. *Industries with medium headquarter-intensity*: Low productive firms collaborate with few suppliers and tend to choose vertical integration and domestic sourcing. For given headquarter-intensity, increasing productivity is then associated with an increasing offshoring share and higher complexity. With respect to the organizational decision, firms in those sectors tend to choose hybrid sourcing, i.e., a coexistence of outsourcing and vertical integration within the same firm. Both the outsourcing and the offshoring share tend to be lower in relatively more headquarter-intensive industries within that range. The most productive firms have many suppliers and completely rely on foreign suppliers; they choose a combination of foreign outsourcing and intra-firm trade.

If this pattern with respect to $\tilde{N}$ and $\tilde{\xi}$ is similar as in the closed economy, it must still be noted that the possibility to engage in offshoring is positively correlated with complexity and outsourcing. To see this, consider a firm with given Θ and η^H, and compare the complexity and organization decision of that firm under autarky (with $w_1^M > 0$ and where $\ell = 0$ is imposed) and in the open economy (with the same $w_1^M > 0$, and given $\Delta > 0$). As shown in Appendix B.3.2., no firm would choose a lower mass of components or a lower outsourcing share after the economy has opened up, while some firms would choose a higher $\tilde{N}$ and $\tilde{\xi}$. In other words, opening up to trade in intermediate inputs boosts the slicing of the value chain and favors outsourcing. Notice that this "time series" correlation (identical firms tend to choose more outsourcing after the economy has opened up to trade) is consistent with a "cross-sectional" pattern across firms, where many choose vertical integration and domestic sourcing in order to keep fixed costs low.

4 Asymmetric components

In this last step of the analysis we consider a discrete setting with two asymmetric suppliers denoted by a and b.[24] These suppliers can differ along two dimensions in our model: i) with respect to their input intensities $\eta^M \cdot \eta_i$ for $i = a, b$ (with $\eta_a + \eta_b = 1$), and ii) with respect to their bargaining powers β_i^ξ, where $\xi = O, V$, which pin down the revenue shares that they ultimately receive. With our Cobb-Douglas production function, $\eta^M \cdot \eta_i$ is the partial output elasticity of component i and thus measures its technological importance for final goods production. If components differ in their input intensities, this is likely to be reflected in the bargaining power of the respective suppliers as well. Suppose one component is technologically more important than the other. The supplier of the more important input is then also likely to reap a larger revenue share from the producer than the supplier of the less important component.

To give a real world example, consider the production of perfume. Alcohol is the base material in this production process, and is needed in large quantities. But even though the quality of the alcohol (the binder) also matters, it still generates low value added as it is rather standardized. More value added is generated by the tiny amounts of the essential oils and aroma compounds (such as *ambra*) which are highly specific and characteristic

[24]It is straightforward to transform our model structure with a continuum of intermediate inputs into a discrete notation. Divide the intervall $[0, N]$ into X equally spaced subintervalls with all the intermediate inputs in each subintervall of length N/X performed by a single supplier. We restrict our attention to the case where complexity is exgenously given by $N = 2$, so that we neglect the cost saving effect s.

as they differentiate the fragrances. In terms of our model, if a and b stand for ambra and alcohol in perfume production, we thus have $\eta_a > \eta_b$ and $\beta_a^\xi > \beta_b^\xi$. That is, ambra is not only the technologically "more important" input, but its supplier also has higher bargaining power (and receives a larger revenue share) due to the indispensability of this particular component for the final product. Specifically, we assume that the exogenous revenue shares are such that $\beta_i^V > \beta_i^O$ for $i = a, b$, and $\beta_a^\xi > \beta_b^\xi$ for $\xi = O, V$. That is, outsourcing yields a larger revenue share than integration for each supplier, and the "more important" supplier a reaps a larger revenue share than b in either organizational form.[25]

It is useful to first analyze the impact of these two types of asymmetries separately, before considering them jointly. For brevity, we abstract from the global scale dimension in this last section and assume that both suppliers are located in country 1.[26] Given eqs. (5), (8) and (9) with $c^H = c_a = c_b = 1$, the producer's problem is to optimize the total firm payoff $\pi = \Theta \cdot Y \cdot \Psi - 2 \cdot w_1^M - \bar{f}$, where the term Ψ can now be written as follows:

$$\Psi \equiv \left[1 - \alpha \left(\beta^H \eta^H + \beta_a \eta^M \eta_a + \beta_b \eta^M \eta_b\right)\right] \left[\left(\beta^H\right)^{\eta^H} \left(\beta_a^{\eta_a} \cdot \beta_b^{\eta_b}\right)^{1-\eta^H}\right]^{\frac{\alpha}{1-\alpha}}. \qquad (19)$$

The producer has to choose among four possible organizational forms, which we denote as follows: $\{O, O\}$, $\{O, V\}$, $\{V, O\}$ and $\{V, V\}$, where the first (second) element depicts the organizational decision for input a (input b). This decision then pins down β_i^ξ and, residually, the producer's revenue share $\beta^H = 1 - \beta_a^\xi - \beta_b^\xi$.

First suppose that the input intensities η_a and η_b are the same, but that supplier a is ahead in terms of the exogenous bargaining power. We show in Appendix C.1. that β_i^* is identical for both suppliers since $\eta_a = \eta_b = 1/2$. Furthermore, $\beta_i^* = (1 - \beta^{H*})/2$ is increasing in the overall component-intensity $\eta^M = 1 - \eta^H$, as this raises the suppliers' total input intensity $\eta^M/2$. The producer's problem is equivalent to choosing the organization that aligns the β_i^ξ as closely as possible with the optimal revenue shares β_i^*. Figure 6 illustrates this problem. If component-intensity is sufficiently low, the producer vertically integrates both suppliers, $\{V, V\}$, as this leaves them with the lowest possible revenue shares and,

[25] Notice that this assumption is consistent both with $\beta_a^V > \beta_b^O$ and $\beta_a^V < \beta_b^O$. In Figures 6 and 7b below we depict the latter case, but all results would be similar with the alternative ranking $\beta_a^V > \beta_b^O$.

[26] It is possible to embed this model in an open economy context, where the producer may offshore both, one or none of the components to the low-wage country 2. One can again split the total payoff into two parts: the variable payoff and the fixed costs, which are both higher under offshoring. Yet, the former effect is magnified by firm productivity while the latter effect is not. This again implies that low productive firms source only domestically, while highly productive firms offshore both suppliers. Firms with medium productivity would offshore *one* component, and we can show that the producer would first tend to offshore the component with the higher input intensity.

in turn, maximizes $\beta^H = 1 - \beta_a^V - \beta_b^V \equiv \beta_{max}^H$. Conversely, if η^M is sufficiently high, she outsources both suppliers ($\{O, O\}$) as this leads to $\beta^H = 1 - \beta_a^O - \beta_b^O \equiv \beta_{min}^H$. For intermediate component-intensity the producer chooses hybrid sourcing, and she would then always outsource the "less important" input b while keeping the "more important" input a within the boundaries of the firm. That is, with $\beta_a^\xi > \beta_b^\xi$ and $\eta_a = \eta_b = 1/2$ there can be hybrid sourcing of the type $\{V, O\}$ but never of the type $\{O, V\}$. Asymmetry in bargaining powers thus favors integration of the "more important" input, as it increases the domain where the supplier can be properly incentiviced as an affiliated subsidiary.

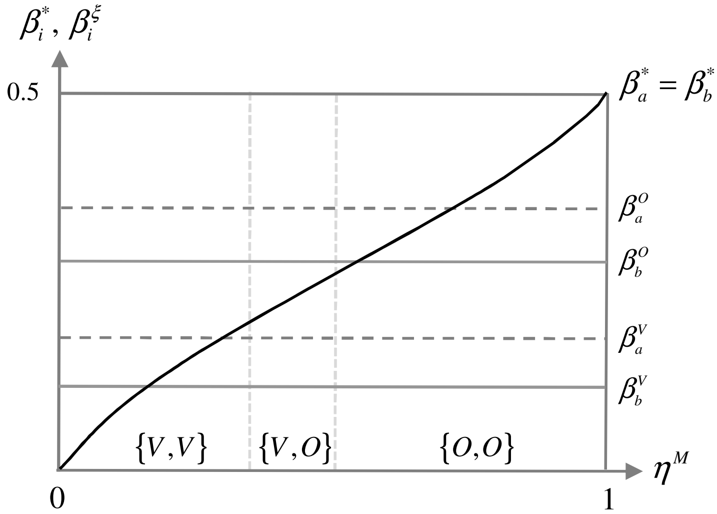

Figure 6: Revenue shares with two asymmetric components

Now consider the other case where the inputs a and b differ only in their input intensities, while the suppliers have identical bargaining powers $\beta_a^O = \beta_b^O > \beta_a^V = \beta_b^V$. In Appendix C.2. we provide an algorithm to derive closed form solutions for the optimal shares that the producer would choose if she could freely assign the ex ante revenue distribution (with $\beta_a^* + \beta_b^* = 1 - \beta^{H*}$). These solutions show that $\partial\beta^{H*}/\partial\eta^H > 0$, $\partial\beta_a^*/\partial\eta_a > 0$, and $\partial\beta_b^*/\partial\eta_b > 0$, which corroborates one key mechanism at work in this model: the higher the input intensity of a component, the higher is the optimal revenue share that should be assigned to its supplier. Clearly, with $\eta_a > \eta_b$ we have $\beta_a^* > \beta_b^*$. When the available revenue shares β^O and β^V are identical across suppliers, however, the producer would then easier outsource the "more important" component a in oder to reduce the underinvestment problem for the respective supplier.

This is illustrated in the left panel of Figure 7. On the horizontal axis we depict the headquarter-intensity of production, and on the vertical axis the technological asymmetry across inputs (where $\eta_a = 1/2$ is the symmetric benchmark case). The different colors indicate which organizational mode is payoff-maximizing. As before, the producer would vertically integrate (outsource) both suppliers for sufficiently high (low) values of η^H. Hybrid sourcing is chosen in sectors with intermediate headquarter-intensity, and within this range the producer tends to choose $\{O, V\}$ if $\eta_a > 1/2$.[27]

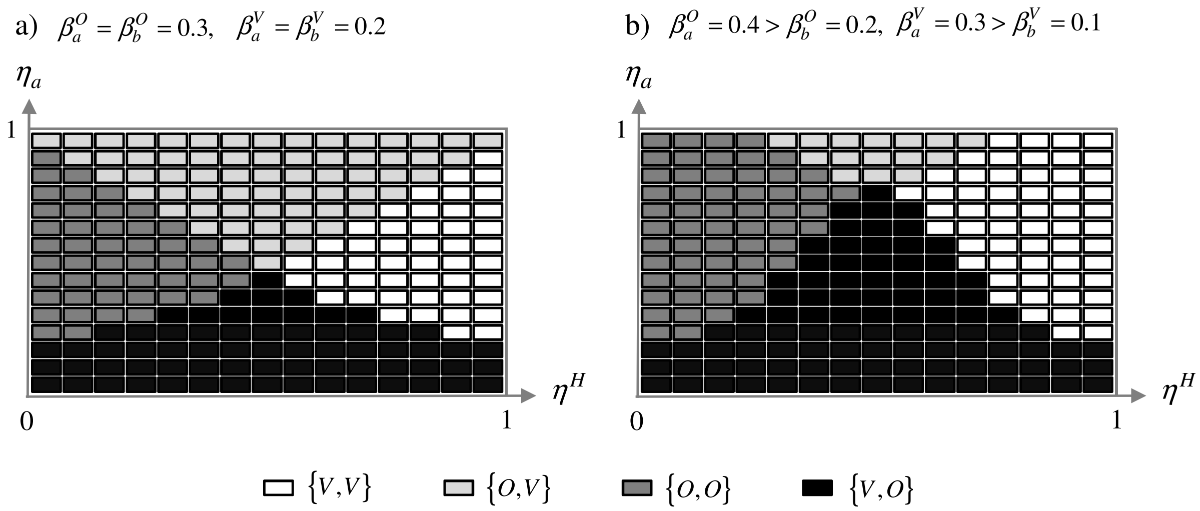

Figure 7: Organizational decision with two asymmetric components

The two different asymmetries thus have different implications for the firm structure in the hybrid range: While the asymmetry in bargaining powers favors vertical integration, the asymmetry in input intensities favors outsourcing of the "more important" component. As argued above, in practice both asymmetries are related and likely to emerge together. In the right panel of Figure 7 we consider such a case and illustrate the implications for the final organizational decision. In this example, we have $(\beta_a^\xi - \beta_b^\xi) = 0.2$ for $\xi = O, V$, and we focus on the range of intermediate headquarter intensity where hybrid sourcing can occur. As can be seen, for high values of η_a the producer would choose the mode $\{O, V\}$ and thus outsource input a, because the asymmetry in input intensities is relatively stronger than the asymmetry in the suppliers' bargaining powers. Yet, if the technological asymmetry is smaller (closer to 1/2), there is instead vertical integration of the "more important" input a and outsourcing of the "less important" input b, i.e., the mode $\{V, O\}$.

[27]Note that the producer's share is the same in both hybrid sourcing modes, $\beta^H = (\beta_{max}^H + \beta_{min}^H)/2$.

5 Conclusions

In this paper, we have developed a theory of a firm which decides on the complexity, the organization, and the global scale of its production process. The main results of our model can be summarized as follows:

i.) *Complexity*: Within a given industry, more productive firms choose to have more suppliers, i.e., more thinly sliced value chains or – in the terminology of our paper – more complex production processes. When comparing equally productive firms, we show that complexity is higher in more component-intensive industries, and higher in firms that choose outsourcing than in vertically integrated firms.

ii.) *Organization*: The organizational structure differs across firms, both within and across industries. As in Antràs and Helpman (2004), higher component-intensity tends to favor outsourcing. Yet, in contrast to that model, our framework predicts that firms may also choose a hybrid sourcing mode where *some* components are outsourced while others are vertically integrated within the firm's boundaries. This hybrid sourcing mode is most prevalent in firms with medium-to-high productivity from industries with low-to-medium headquarter-intensity.

iii.) *Global scale*: More productive firms tend to offshore more components, but only the most productive firms rely completely on foreign suppliers. Firm with medium productivity offshore *some* components but keep others domestic. For a given productivity, the offshoring share tends to be higher in more component-intensive industries. Furthermore, our model predicts that "globalization" boosts the slicing of the value chain and is positively correlated with outsourcing. More specifically, moving from an autarkic scenario to an open economy setting where trade in intermediate inputs is possible, we show that identical firms would choose more complexity and outsourcing in the open economy.

iv.) *Asymmetric components*: Finally, different asymmetries across components have different implications for the organizational structure of firms. A technological difference per se favors outsourcing of the "more important" input, as this reduces the underinvestment problem for the respective supplier. Yet, that supplier is also likely to have a higher bargaining power vis-a-vis the producer. Provided this latter effect is sufficiently strong, which may be the case for highly sophisticated and specific intermediate inputs, our model predicts that the producer keeps the "more important" component, which generates more value added, within the boundaries of the firm.

Several of those predictions are consistent with stylized facts from the recent empirical literature on multinational firms. For example, recent empirical work by Jabbour (2008) and Jabbour and Kneller (2010) shows that most MNEs are, in practice, characterized by a high degree of complexity (i.e., multiple suppliers) and by hybrid sourcing. Consistently, Kohler and Smolka (2009) emphasize that MNEs often choose different sourcing modes for different suppliers. In particular, Tomiura (2007) shows that firms which rely on hybdrid sourcing tend to be more productive than firms which rely on a single sourcing mode in the global economy. This finding is consistent with our framework for the case of intermediate headquarter-intensity, which is likely to encapsulate many industries in the data. Finally, Alfaro and Charlton (2009) show that firms tend to outsource low-skill inputs from the early stages, while high-skill inputs from the final stages of the production process – which generate a large share of total value added – are likely to be integrated. In line with this result, Corcos et al. (2009) find that inputs with a higher degree of specificity are less likely to be outsourced. Our theoretical framework is consistent with this finding if the technological importance of particular inputs is materialized in a high bargaining power of the respective suppliers. Our model may also motivate future empirical research, as it leads to several predictions that have – to the best of our knowledge – not been confronted with data yet. For example, it would be interesting to explore if trade integration has led to a stronger unbundling of the production chain, or if (conditional on productivity) firms from headquarter-intensive industries systematically have fewer suppliers than firms from component-intensive sectors.

The model in this paper is about single firms. It could potentially be embedded into a general equilibrium framework where firm interactions within and across industries are taken into account. Such a framework would be useful to explore more fully the repercussions of trade integration with cross-country differences in market conditions, factor prices and incomes, as well as their implications for global sourcing decisions. Furthermore, our model is based on a simple static Nash-bargaining. In practice, suppliers may care about long-term relationships, or may try to collude with other suppliers in order to induce pressure on the headquarter. Finally, we focus on horizontal "slicing" of the production chain in this paper, neglecting the fact that many components in reality consist themselves out of multiple intermediate inputs, as recently argued by Baldwin and Venables (2010). Exploring these and other extensions of our framework is left for future research.

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Appendix A: Closed Economy

Remark. To simplify notation, we denote the first-order partial derivative of a function f with respect to the argument x as f'_x in this Appendix. Analogously, the second-order partial derivative with respect to the argument y is denoted as f''_{xy}.

A.1. Input provision. Supplier $i \in [0, N]$ chooses the level of input provision m_i so as to maximize $\pi_i = \beta_i R - c_i^M m_i$. Using eqs.(3) and (4), the first-order-condition (FOC) for the maximization problem of supplier i can be written as follows:

$$\pi'_{m_i} = \beta_i \cdot R'_{m_i} - c_{i,r}^M = \beta_i \cdot \alpha \cdot \eta^M \cdot \eta_i \cdot R \,/\, m_i - c_{i,r}^M = 0. \tag{20}$$

It directly follows that $m_i^* = \alpha \cdot \eta^M \cdot \eta_i \cdot \beta_i \cdot R^* / \, c_{i,r}^M$ solves the FOCs, with R^* given by eq. (6). It remains to be shown that the second-order-conditions (SOC) are satisfied. Using the FOCs given by eq. (20), the SOCs simplify to

$$\pi''_{m_i m_i} = \beta_i \cdot \alpha \cdot \eta^M \cdot \eta_i \cdot \left(m_i \cdot R'_{m_i} - R\right) / m_i^2 = -\beta_i \cdot \alpha \cdot \eta^M \cdot \eta_i \cdot R \cdot \left(1 - \alpha \cdot \eta^M \cdot \eta_i\right) / \, m_i^2 < 0,$$

and are thus satisfied. Using a similar approach, it can be shown that $h^* = \alpha \cdot \eta^H \cdot \beta^H \cdot R^* / \, c^H$ maximizes the payoff for the producer, $\pi^H = \beta^H R - c^H h$.

A.2. Optimal mass of supplers and revenue division.

A.2.1. Zero outside opportunity

i.) Maximization problem: The first-order-conditions (FOCs) are given by $\pi'_N = \Theta \cdot Y \cdot \Psi'_N = 0$ and $\pi'_{\beta^H} = \Theta \cdot Y \cdot \Psi'_{\beta^H} = 0$. Using (12), the FOCs can simplified to:

$$\frac{\Psi'_N}{\Psi} = \frac{\alpha\eta^M \left(N\left(1-s\right)\left(1-\alpha\beta^H\eta^H\right) - \beta^M\left(1 - s\alpha\eta^M - \alpha\eta^H\right)\right)}{N\left(1-\alpha\right)\left(\alpha\beta^M\eta^M - N\left(1-\alpha\beta^H\eta^H\right)\right)} = 0, \tag{21}$$

$$\frac{\Psi'_{\beta^H}}{\Psi} = \frac{\alpha\left(\beta^H\left(N-\beta^M\right) - \left(N\left(1-\beta^H\left(\beta^M-\alpha\right)\right) - \beta^M\left(\alpha+\beta^H\right)\right)\eta^H - \alpha\left(\beta^M - N\beta^H\right)\eta^{H2}\right)}{\left(1-\alpha\right)\beta^H\beta^M\left(\alpha\beta^M\eta^M - N\left(1-\alpha\beta^H\eta^H\right)\right)} = 0. \tag{22}$$

With eqs. (21) and (22) it is straightforward to show that N_0^* and β_0^{H*} as given in (13) and (14) solve the FOCs. The matrix of SOCs can be expressed as follows:

$$\Gamma = \begin{bmatrix} \pi''_{NN} & , & \pi''_{N\beta^H} \\ \pi''_{\beta^H N} & , & \pi''_{\beta^H\beta^H} \end{bmatrix} = \Theta \cdot Y \cdot \begin{bmatrix} \Psi''_{NN} & , & \Psi''_{N\beta^H} \\ \Psi''_{\beta^H N} & , & \Psi''_{\beta^H\beta^H} \end{bmatrix}.$$

We now show that the matrix Γ is negative definite. We define $\Psi_0^* \equiv \Psi\left(N = N_0^*, \beta^H = \beta_0^{H*}\right)$ for notational convenience. The first diagonal element is given by $\Psi''_{NN} = -\Psi_0^* \cdot (T_1 \,/\, T_2)$, with

$$T_1 = 4\left(1-s\right)^3 \alpha\eta^M\eta^{H2}\left(1-\alpha\left(s\eta^M + \eta^H\right)\right) > 0, \quad T_2 = \left(1-\alpha\right)^2\left(\rho - s\eta^M\left(1+\alpha\eta^H\right)\right)^2 > 0.$$

Hence, Ψ''_{NN} is negative. The second diagonal element is given by $\Psi''_{\beta^H\beta^H} = -\Psi_0^* \cdot (T_3 \,/\, T_4)$, with

$$
\begin{aligned}
T_3 &= 16\eta^M\eta^{H2}\left(1-\alpha\eta^H\right)\left(s^3\eta^{M2}\left(1-\alpha\eta^H\right)^2-\eta^{H2}\rho-s^2\eta^M\left(1-\alpha\eta^H\right)\left(\eta^H\left(-5+2\eta^H\right)+\rho\right)\right)\\
&\quad +16\left(s\eta^H\left(\eta^M\eta^H\left(5-\alpha\eta^H\right)-3\rho+2\eta^H\rho\right)\right)<0\\
T_4 &= (1-\alpha)^2\left(s\eta^M\left(1-\alpha\eta^H\right)-\rho\right)^3\left(2\eta^H+s\eta^M\left(1-\alpha\eta^H\right)-\rho\right)^2<0.
\end{aligned}
$$

Hence, $\Psi''_{\beta^H\beta^H}$ is also negative. The determinant can be written as $|\Gamma|=\left(\Psi_0^*\right)^2\cdot(T_5\cdot T_6\,/\,T_7)$, with

$$
\begin{aligned}
T_5 &= -128\,(1-s)^3\,s^2\alpha^2\eta^{M3}\eta^{H4}\left(1-\alpha\eta^H\right)<0\\
T_6 &= -s^3\eta^{M3}\left(1-\alpha\eta^H\right)^3+s^2\eta^{M2}\left(1-\alpha\eta^H\right)^2\left(\eta^H\left(-7+\alpha\eta^H\right)+\rho\right)\\
&\quad +\eta^{H2}\left(5\rho+\eta^H\left(-4+4\alpha\eta^H-3\alpha\rho\right)\right)+s\eta^M\eta^H\left(1-\alpha\eta^H\right)\left(4\rho+\eta^H\left(-13+5\alpha\eta^H-\alpha\rho\right)\right)>0\\
T_7 &= -\left(1-\alpha\right)^3\left(2\eta^H+s\eta^M\left(1-\alpha\eta^H\right)-\rho\right)^2\left(-s\eta^M\left(1-\alpha\eta^H\right)+\rho\right)^4\\
&\quad \left(s\eta^M\left(1-\alpha\eta^H\right)+\rho\right)^2<0.
\end{aligned}
$$

It directly follows that $|\Gamma|>0$. Hence, Γ is negative definite, so that N_0^* and β_0^{H*} maximize (12).

ii.) Optimal revenue share of a single supplier. It is straightforward to show that β_{i0}^* is given by $\beta_{i0}^*=\left\{(1-s)\left(s\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)-\rho\right)\right\}/\left\{s\left(1-\eta^H\right)\left(1+\alpha\eta^H\right)-\rho\right\}$. It can be verified that $\beta^{*\,\prime}_{i0\eta^H}>0$ for $\eta^H<\eta^H_{crit}$ and $\beta^{*\,\prime}_{i0\eta^H}<0$ for $\eta^H>\eta^H_{crit}$, where

$$
\eta^H_{crit}=\left\{2-\sqrt{\left(4-4\sqrt{s}+s\left(1-\alpha\right)\right)\left(-1+\alpha\right)}-\sqrt{s}\left(1+\alpha\right)\right\}/\left\{2\left(1-\sqrt{s}\right)\alpha\right\}
$$

Hence, β_{i0}^* is hump-shaped over the range of η^H. Note that η^H_{crit} is unambigiously decreasing in s.

iii.) Antràs and Helpman (2004) is a special case of our framework: We claim in footnote 8 that there exists a s_{crit} such that $N_0^*=1$. This critical level is given by

$$
s_{crit}=\eta^H/\left\{\left(1-\alpha\left(1-\eta^H\right)\right)\eta^H+\sqrt{\left(1-\alpha\left(1-\eta^H\right)\right)\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)}\right\},
$$

and it can be verified that the optimal revenue share $\beta_0^{H*}\left(s=s_{crit}\right)$ is identical to eq. (10) in Antràs and Helpman (2004): $\beta_0^{H*}\left(s=s_{crit}\right)=\frac{\eta^H\left(\alpha\eta^H+1-\alpha\right)-\sqrt{\eta^H\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)\left(\alpha\eta^H+1-\alpha\right)}}{2\eta^H-1}$.

iv.) Cost saving versus dilution effect. In the following we restate operating profits as profit margin times the sold quantity. Profits are given by $\pi=\Theta\cdot Y\cdot\Psi=R-C$ where C denotes total variable costs. It follows from eq.(12) that optimal variable costs can be expressed as $C^*=\alpha\left[\beta^H\eta^H+\left(1-\beta^H\right)\left(1-\eta^H\right)/N\right]\cdot R^*$. We rewrite $\Psi=(1-C^*/R^*)\cdot R^*=(p^*-C^*/y^*)\cdot y^*$ where y^* and p^* denote the optimal quantity and price, respectively, and C^*/y^* are the average variable costs. The profit margin is given by $margin^*\equiv(p^*-C^*/y^*)$. Furthermore, R^* is given by

$$R^* = \left[\left(\frac{\beta^H}{c^H} \right)^{\eta^H} \left(\frac{1-\beta^H}{N^{1-s}} \right)^{1-\eta^H} \right]^{\frac{\alpha}{1-\alpha}},$$

so that we can restate p^* and y^* as: $y^* = (R^*)^{1/\alpha} Y^{(\alpha-1)/\alpha}$ and $p^* = (R^*)^{(\alpha-1)/\alpha} Y^{(-\alpha+1)/\alpha}$, and the average costs as $C^*/y^* = \alpha \left[\beta^H \eta^H + \left(1-\beta^H\right)\left(1-\eta^H\right)/N \right] \cdot p^*$.

To shed light on the two countervailing effects of raising complexity, we now we discuss comparative statics with respect to N for a given β^H. Since $R_N^{*\prime} < 0$ it directly follows $y_N^{*\prime} < 0$ and $p_N^{*\prime} > 0$. The profit margin can be written as

$$margin^* = \left[1 - \alpha \left(\beta^H \eta^H + \left(1-\beta^H\right)\left(1-\eta^H\right)/N \right) \right] \cdot p^*,$$

which is unambigiously increasing in N. Hence, higher complexity leads to a smaller quantity but a larger profit margin. For $s \to 1$, both $y_N^{*\prime} \to 0$ and $p_N^{*\prime} \to 0$. However, the profit margin is still strictly increasing in N. Hence, we have $N_0^* \to \infty$ for $s \to 1$. Although the dilution and the cost saving effect cannot be strictly decomposed analytically, we can still conclude that the cost saving effect dominates the dilution effect when we trace the impact of an increase in N on the profit margin, while the dilution effect dominates the cost saving effect when tracing the impact on the quantity.

v.) Cutoff productivity: The productivity threshold for survival is given by:

$$\hat{\Theta}_0 = \frac{\left(\frac{\bar{f}}{Y}\right) \cdot \left[\left(\frac{\beta_0^{H*}}{c^H} \right)^{\eta^H} \left(\frac{1-\beta_0^{H*}}{N_0^{*\,1-s}} \right)^{1-\eta^H} \right]^{-\alpha/(1-\alpha)}}{1 - \left(\alpha \beta_0^{H*} \eta^H + \left(1-\beta_0^{H*}\right)\left(1-\eta^H\right)/N_0^* \right)}$$

with N_0^* and β_0^{H*} given in (13) and (14). Note that $\hat{\Theta}_0$ is increasing in $\bar{f}$ and decreasing in Y.

A.2.2. Positive outside opportunity

i.) Maximization problem: The FOCs are given by $\pi_N' = \Theta \cdot Y \cdot \Psi_N' - w_1^M = 0$ and $\pi_{\beta}'^H = \Theta \cdot Y \cdot \Psi_{\beta^H}' = 0$. We can solve $\Psi_{\beta^H}' = 0$ for

$$\beta^H(N) = \frac{N - 1 + (1+N)(1-\alpha)\eta^H + (1+N)\alpha\left(\eta^H\right)^2 - \tilde{\rho}}{2\left(\eta^H(1+N) - 1\right)} \tag{23}$$

with $\tilde{\rho} = \sqrt{\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)\left((1-N)^2 - (1+N)(1+N(-3+\alpha)+\alpha)\eta^H + (1+N)^2\alpha\left(\eta^H\right)^2\right)}$.

Note that $\beta^H(N)$ as stated in eq. (23) does not depend on w_1^M. Furthermore, it directly follows that $\beta_N^{H'} < 0$. Using $\beta^H(N)$ in $\Psi_N' = 0$ allows us to derive the implicit condition $\Psi' = w_1^M / (\Theta \cdot Y)$, which we can solve for N^*. It then directly follows from Appendix A.2.1.i), and from the fact that $\Psi''_{NN} < 0$ in the relevant domain, that N^* solves the first- and second-order conditions and is, thus, the optimal complexity level. This N^* (as depicted in Figure 1) is then associated with an optimal headquarter revenue share $\beta^{H*} = \beta^H(N = N^*)$ from (23) (as depicted in Figure 2) that solves $\Psi'_{\beta^H} = 0$. From the condition $\Psi' = w_1^M / (\Theta \cdot Y)$ it also directly follows that $N^{*\prime}_{\Theta} > 0$ and $N^{*\prime}_{w_1^M} < 0$ with $N^* < N_0^*$, and hence (since $\beta_N^{H'} < 0$): $\beta_{\Theta}^{H*\prime} < 0$ and $\beta_{w_1^M}^{H*\prime} > 0$ with $\beta^{H*} > \beta_0^{H*}$. Finally, notice that $\Psi''_{N\eta^H} < 0$, hence $N^{*'}_{\eta^H} < 0$ and, thus, $\beta^{H*'}_{\eta^H} > 0$.

ii.) Total profits: We claim that more productive firms earn a higher total payoff π, despite that they have higher fixed costs. Total profits are given by $\pi = \Theta \cdot Y \cdot \Psi - w_1^M N$. The optimal mass of suppliers is implicitly given by $\pi'_N = \Theta \cdot Y \cdot \Psi'_N - w_1^M = 0$. It then directly follows that $\pi'_\theta = Y \cdot \Psi + N'_\Theta \underbrace{\left(\Theta \cdot Y \cdot \Psi'_N - w_1^M\right)}_{=0} = Y \cdot \Psi > 0$.

A.3. The make-or-buy decision under incomplete contracts.

Maximization problem: We know from Appendix A.2. that β^{H*} and the associated N^* (β_0^{H*} and the associated N_0^* for the case with $w_1^M = 0$) are payoff-maximizing if the producer is unconstrained in the choice of the revenue shares. Under incomplete contracts, since $\pi'_{\beta^H} > 0$ if $\beta^H < \beta^{H*}$ and $\pi'_{\beta^H} < 0$ if $\beta^H > \beta^{H*}$, it follows from continuity that the choice of ξ that aligns $\beta^H = \xi \cdot \beta^H_{min} + (1 - \xi) \cdot \beta^H_{max}$ as closely as possible with β^{H*} must be payoff-maximizing, given the constraint $\beta^H \in \left[\beta^H_{min}, \beta^H_{max}\right]$.

A.3.1. Zero outside opportunity

Definition of headquarter- and component-intensive industries: $\bar{\eta}_0^H$ and $\bar{\bar{\eta}}_0^H$ are given by

$$\bar{\eta}_0^H = \frac{1 + (s(1+\alpha) - 2)\beta^H_{max} + \left(\beta^H_{max}\right)^2 - \sqrt{(1 + \beta^H_{max}(s(1+\alpha) - 2 + \beta^H_{max}))^2 - 4s^2\alpha\left(\beta^H_{max}\right)^2}}{2s\alpha\beta^H_{max}}$$

$$\bar{\bar{\eta}}_0^H = \frac{1 + (s(1+\alpha) - 2)\beta^H_{min} + \left(\beta^H_{min}\right)^2 - \sqrt{\left(1 + \beta^H_{min}\left(s(1+\alpha) - 2 + \beta^H_{min}\right)\right)^2 - 4s^2\alpha\left(\beta^H_{min}\right)^2}}{2s\alpha\beta^H_{min}}$$

with $\beta^H_{min} < \beta^H_{max} \to \bar{\bar{\eta}}_0^H < \bar{\eta}_0^H$. Furthermore, $\bar{\eta}^{H'}_{0\beta^H_{max}} > 0$, $\bar{\bar{\eta}}^{H'}_{0\beta^H_{min}} > 0$, $\bar{\eta}^{H\prime}_{0s} > 0$, and $\bar{\bar{\eta}}^{H\prime}_{0s} > 0$.

A.3.2. Positive outside opportunity

Notice from eq.(11) that $\Psi''_{N\beta^H} = T_8/T_9$, with:

$$\begin{aligned}
T_8 &= -N(1-s)\alpha\beta^H + (1-s\alpha)\beta^M\beta^H + \alpha\left(\beta^{H2} - 1 + s\beta^M\left(\alpha + \beta^H\right)\right) + \\
&\quad \alpha\eta^H N(1-s)\left(\beta^M + \beta^H\left(\alpha + \beta^H\right)\right) + (1-s)\alpha^2\left(1 - \beta^H(1+N)\right)\eta^2 > 0, \\
T_9 &= (1-\alpha)\beta^M\beta^H\left(-\beta^M\left(1 - s\alpha\eta^M - \alpha\eta^H\right) + N(1-s)\left(1 - s\beta^H\eta^H\right)\right) < 0.
\end{aligned}$$

Hence, $\Psi''_{N\beta^H} < 0$. Since $\beta^H = \beta^H_{max}$ for $\Psi^{V'}$ and $\beta^H = \beta^H_{min}$ for $\Psi^{O'}$, the former curve must thus run to the left of the latter in Figure 5. Hence, we have $\tilde{N}^V < \tilde{N}^O$. The comparative static results for $\tilde{N}$ are similar as for N^* where the restriction $\beta^H \in \left[\beta^H_{min}, \beta^H_{max}\right]$ is not binding, see Appendix A.2.2.i). In particular, we have $\tilde{N}'_{\Theta} > 0$, $\tilde{N}'_{w_1^M} < 0$ and $\tilde{N}'_{\eta^H} < 0$ with $\tilde{N} < \tilde{N}_0$.

Appendix B: Open Economy

B.1. Cross-country cost difference. We assume the following specification for the "offshoring" gain: $\delta(\ell) = \left(1 + \bar{\delta} \cdot \ell\right)^{-1/\ell}$ with $\bar{\delta} > 0$. We have positive but decreasing marginal returns from offshoring since

$$\delta'_\ell = -\left(1 + \bar{\delta} \cdot \ell\right)^{-(1+\ell)/\ell} < 0 \quad \text{and} \quad \delta''_{\ell\ell} = (1+\ell)\left(1 + \bar{\delta} \cdot \ell\right)^{-(1+2\ell)/\ell} > 0.$$

It directly follows from $\delta(\ell = 0) = e^{-\bar{\delta}}$ and $\delta(\ell = 1) = 1/\left(1 + \bar{\delta}\right)$ that $0 < \delta(\ell) < 1$. Furthermore, the strength of the offshoring gain is stronger the larger the parameter $\bar{\delta}$ is.

B.2. Optimal mass of suppliers and revenue division.

B.2.1. Zero outside opportunity

Maximization problem: Using eq.(18) we have: $\pi'_N = \Theta \cdot Y \cdot \Psi'_N$, $\pi'_{\beta^H} = \Theta \cdot Y \cdot \Psi'_{\beta^H}$, and $\pi'_\ell = \Theta \cdot Y \cdot \Psi'_\ell$. Since $\Psi'_\ell = \Psi \cdot \left(\alpha\left(1 - \eta^H\right)\right) / \left((1-\alpha)(1+\ell)\right) > 0$, we hence have $\ell_0^* = 1$. It is then straightforward to see that the other two FOCs, $\pi'_N = 0$ and $\pi'_{\beta^H} = 0$, can be expressed as in eqs.(21) and (22) from Appendix A.2.1.i.), since the ℓ cancels out from those expressions. Hence, N_0^* and β_0^{H*} are the same as in the closed economy case. Furthermore, using a similar approach as in Appendix A.2.1.i, we can show that the SOCs are also satisfied.

B.2.2. Positive outside opportunity

i.) Maximization problem: Total profits are given by $\pi = \Theta \cdot Y \cdot \Psi - \left(w_1^M + \ell\Delta\right) N + \bar{f}$. The three FOCs are given by:

$$\pi'_N = \Theta \cdot Y \cdot \Psi'_N - \left(w_1^M + \ell\Delta\right) = 0, \quad \pi'_{\beta^H} = \Theta \cdot Y \cdot \Psi'_{\beta^H} = 0 \quad , \quad \pi'_\ell = \Theta \cdot Y \cdot \Psi'_\ell - \Delta N = 0.$$

As in the closed economy, it is possible to solve $\Psi'_{\beta^H} = 0$ for $\beta^H(N)$ with $\beta^H_N < 0$, which does not depend on w_1^M or Δ. Substituting $\beta^H(N)$ into the other two FOCs leads to:

$$\pi'_N = \Theta \cdot Y \cdot \underbrace{\Psi'_N \Big|_{\beta^H = \beta^H(N)}}_{\equiv \Psi'_N} - \left(w_1^M + \ell\Delta\right) = 0, \quad \pi'_\ell = \Theta \cdot Y \cdot \underbrace{\Psi'_\ell \Big|_{\beta^H = \beta^H(N)}}_{\equiv \Psi'_\ell} - \Delta N = 0. \qquad (24)$$

For sufficiently productive firms we have $\pi'_\ell > 0$ for all $\ell \in [0, 1]$, since $\Psi'_\ell > 0$ and N^* approaches

N_0^* and is bounded from above. Hence, the global maximum is given by $\ell^* = 1$. Vice versa, for firms with sufficiently low productivity, $\pi'_\ell < 0$ and hence $\ell^* = 0$.

We are now interested in the SOC for the case where $\ell^* \in (0,1)$. Assume that N^* and ℓ^* solve the system of FOCs given in (24). The SOCs are given by the following matrix K:

$$K = \begin{bmatrix} \pi''_{NN} & , & \pi''_{N\ell} \\ \pi''_{\ell N} & , & \pi''_{\ell\ell} \end{bmatrix} = \begin{bmatrix} \Theta \cdot Y \cdot \Psi''_{NN} & , & \Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta \\ \Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta & , & \Theta \cdot Y \cdot \Psi''_{\ell\ell} \end{bmatrix},$$

For negative definiteness of K we have to ensure that $\pi''_{\ell N} = \Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta$ is small, which can be achieved by setting the exogeneous parameter Δ sufficiently high. If this parameter restriction holds, Ψ''_{NN} and $\Psi''_{\ell\ell}$ are negative while the determinant $|K| = \Theta^2 \cdot Y^2 \cdot \Psi''_{NN}\Psi''_{\ell\ell} - (\Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta)^2 > 0$ is positive, so that the SOCs are unambiguously satisfied.

ii.) Comparative statics: We now use the implicit function theorem to derive the comparative statics $N_\Theta^{*\prime}$ and $\ell_\Theta^{*\prime}$:

$$N_\Theta^{*\prime} = \frac{\begin{vmatrix} -Y \cdot \Psi'_N & , & \Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta \\ -Y \cdot \Psi'_\ell & , & \Theta \cdot Y \cdot \Psi''_{\ell\ell} \end{vmatrix}}{|K|} = \frac{-\Theta \cdot Y^2 \cdot \Psi'_N \cdot \Psi''_{\ell\ell} + Y \cdot \Psi'_\ell \left(\Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta\right)}{|K|} > 0$$

$$\ell_\Theta^{*\prime} = \frac{\begin{vmatrix} \Theta \cdot Y \cdot \Psi''_{NN} & , & -Y \cdot \Psi'_N \\ \Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta & , & -Y \cdot \Psi'_\ell \end{vmatrix}}{|K|} = \frac{-\Theta \cdot Y^2 \cdot \Psi''_{NN} \cdot \Psi'_\ell + Y \cdot \Psi'_N \cdot (\Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta)}{|K|} \geq 0.$$

In words, more productive firms have more suppliers and a non-decreasing offshoring share (strictly increasing if $\ell^* \in (0,1)$). We can use these results to derive a relationship between the endogenous variables N^* and ℓ^*. With the help of the chain rule we can conclude that $N_{\ell^*}^{*\prime} = N_\Theta^{*\prime}/\ell_\Theta^{*\prime} > 0$ if $\ell^* \in (0,1)$ and zero otherwise. Furthermore, we know from solving the FOCs that $\beta_N^H < 0$. Hence, it directly follows that $\beta_{N^*}^{H^*\prime} < 0$, and since $N_{\ell^*}^{*\prime} > 0$, it also follows that $\beta_{\ell^*}^{H^*\prime} \leq 0$ if $\ell^* \in (0,1)$ and zero otherwise. For the comparative statics with respect to Θ it thus follows that $\beta_\Theta^{H^*\prime} < 0$, i.e., more productive firms have a lower headquarter revenue share. Next, we derive the comparative statics of N^* with respect to η^H:

$$\begin{aligned} N_{\eta^H}^{*\prime} &= \frac{\begin{vmatrix} -\Theta \cdot Y \cdot \Psi''_{N\eta^H} & , & \Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta \\ -\Theta \cdot Y \cdot \Psi''_{\ell\eta^H} & , & \Theta \cdot Y \cdot \Psi''_{\ell\ell} \end{vmatrix}}{|K|} \\ &= \frac{-\Theta^2 \cdot Y^2 \cdot \Psi''_{N\eta^H} \cdot \Psi''_{\ell\ell} + \Theta \cdot Y \cdot \Psi''_{\ell\eta^H} \cdot (\Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta)}{|K|} < 0, \end{aligned}$$

since $\Psi''_{N\eta^H} < 0$. The optimal complexity is thus lower in more headquarter-intensive industries. The comparative static results for β^{H*} and ℓ^* follow directly, since $\beta_{N^*}^{H^*\prime} < 0$ implies $\beta_{\eta^H}^{H*\prime} > 0$, and $N_{\ell^*}^{*\prime} > 0$ implies $\ell_{\eta^H}^{*\prime} < 0$ if $\ell^* \in (0,1)$ and zero otherwise. In words, the optimal offshoring share is smaller, while the optimal headquarter revenue share is larger in more headquarter-intensive industries. Finally, we derive

$$N_{\Delta}^{*\prime} = \frac{\begin{vmatrix} \ell & , & \Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta \\ N & , & \Theta \cdot Y \cdot \Psi''_{\ell\ell} \end{vmatrix}}{|K|} = \frac{\Theta \cdot Y \cdot \ell \cdot \Psi''_{\ell\ell} - N\left(\Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta\right)}{|K|} < 0.$$

The comparative static results for β^{H*} and ℓ^* follow again directly, since $\beta_{N^*}^{H^*\prime} < 0$ implies $\beta_{\Delta}^{H*\prime} > 0$, and $N_{\ell^*}^{*\prime} > 0$ implies $\ell_{\Delta}^{*\prime} < 0$ if $\ell^* \in (0,1)$ and zero otherwise.

iii.) Total profits: We claim that more productive firms earn a higher total payoff π, despite that they have higher fixed costs. The total payoff is given by $\pi = \Theta \cdot Y \cdot \Psi - \left(w_1^M + \ell\Delta\right) N + \bar{f}$. Recall that the FOCs are $\pi'_N = \Theta \cdot Y \cdot \Psi'_N - \left(w_1^M + \ell\Delta\right) = 0$ and $\pi'_\ell = \Theta \cdot Y \cdot \Psi'_\ell - \Delta N = 0$. It directly follows that $\pi'_\theta = Y \cdot \Psi + N'_\Theta \underbrace{\left(\Theta \cdot Y \cdot \Psi'_N - \left(w_1^M + \Delta\ell\right)\right)}_{=0} + \ell'_\Theta \underbrace{\left(\Theta \cdot Y \cdot \Psi'_\ell - \Delta N\right)}_{=0} = Y \cdot \Psi > 0$.

B.3. The make-or-buy decision under incomplete contracts.

B.3.1. Zero outside opportunity

Maximization problem: As shown in Appendix B.2.1., N_0^* and β_0^{H*} are identical to the closed economy case, see eq. (13) and eq. (14). The constraint $\beta^H \in \left[\beta_{min}^H, \beta_{max}^H\right]$ is also indentical as in the closed economy case. The constrained optimal complexity and organization choices are thus identical to the closed economy case, while the global scale choice is given by $\tilde{\ell}_0 = 1$. The thresholds $\bar{\eta}_0^H$ and $\bar{\bar{\eta}}_0^H$ as given in Appendix A.3.1. apply.

B.3.2. Positive outside opportunity

Maximization problem: In sectors with medium headquarter-intensity ($\bar{\bar{\eta}}_0^H < \eta^H < \bar{\eta}_0^H$) the producer can set the outsourcing share $\tilde{\xi} = (\beta_{max}^H - \beta^{H*})/(\beta_{max}^H - \beta_{min}^H)$ such that $\beta^H = \beta^{H*}$. This implies $\tilde{N} = N^*$ and $\tilde{\ell} = \ell^*$. The comparative static results are derived in Appendix B.2.2.ii. Since $\beta_{\eta^H}^{H^*\prime} > 0$ and $\tilde{\xi}'_{\beta^{H*}} < 0$ the outsourcing share is relatively lower the more headquarter-intensive the industry is. In headquarter-intensive ($\eta^H > \bar{\eta}_0^H$) and component-intensive industries ($\eta^H < \bar{\bar{\eta}}_0^H$), the outsourcing share is constant and given by $\tilde{\xi} = 0$ and $\tilde{\xi} = 1$, respectively. Conditional on $\tilde{\xi} = 0$ or $\tilde{\xi} = 1$ with $\beta^H = \beta_{min}^H$ and $\beta^H = \beta_{max}^H$, respectively, the optimal complexity level $\tilde{N}$ and offshoring share $\tilde{\ell}$ are determined according to

$$\pi'_N = \Theta \cdot Y \cdot \Psi'_N - \left(w_1^M + \ell\Delta\right) = 0 \quad \text{and} \quad \pi'_\ell = \Theta \cdot Y \cdot \Psi'_\ell - \Delta N = 0. \tag{25}$$

As in Appendix B.2.2., for sufficiently highly productive firms we have $\pi'_\ell > 0$ for all $\ell \in [0,1]$, since $\Psi'_\ell > 0$ and $\tilde{N}$ approaches $\tilde{N}_0$ and is bounded from above. Hence, the global maximum is given by $\tilde{\ell} = 1$. Vice versa, for firms with sufficiently low productivity: $\pi'_\ell < 0$ so that $\tilde{\ell} = 0$.

We are now interested in the SOC for the case where $\tilde{\ell} \in (0,1)$. Assume that $\tilde{N}$ and $\tilde{\ell}$ solve the system of FOCs given in (25). The SOCs are given by the following matrix $\tilde{K}$:

$$\tilde{K} = \begin{bmatrix} \pi''_{NN} & , & \pi''_{N\ell} \\ \pi''_{\ell N} & , & \pi''_{\ell\ell} \end{bmatrix} = \begin{bmatrix} \Theta \cdot Y \cdot \Psi''_{NN} & , & \Theta \cdot Y \cdot \Psi''_{N\ell} - \Delta \\ \Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta & , & \Theta \cdot Y \cdot \Psi''_{\ell\ell} \end{bmatrix},$$

For negative definiteness of $\tilde{K}$ we have to ensure that $\pi''_{\ell N} = \Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta$ is small, as in Appendix B.2.1.i, which can be achieved by setting Δ high enough. If this parameter restriction holds, the diagonal elements Ψ''_{NN} and $\Psi''_{\ell\ell}$ are negative while the determinant $\left|\tilde{K}\right| = \Theta^2 \cdot Y^2 \cdot \Psi''_{NN}\Psi''_{\ell\ell} - (\Theta \cdot Y \cdot \Psi''_{\ell N} - \Delta)^2 > 0$ is positive, so that the SOCs are unambiguously satisfied. Furthermore, if this parameter restriction holds, it is straightforward to prove the following comparative static results, which can be derived in a similar way as in Appendix B.2.2.ii: $\tilde{N}'_{\Theta} > 0$, $\tilde{\beta}^{H}_{\Theta}{}' < 0$, $\tilde{\ell}'_{\Theta} \geq 0$; $\tilde{N}'_{\eta^H} < 0$, $\tilde{\beta}^{H}_{\eta^H}{}' > 0$, $\tilde{\ell}'_{\eta^H} \leq 0$; $\tilde{N}'_{\Delta} < 0$, $\tilde{\beta}^{H}_{\Delta}{}' > 0$, $\tilde{\ell}'_{\Delta} \leq 0$.

Appendix C: Asymmetric components

C.1. Symmetric input intensities. The unique closed form solution for $\beta^*_a = \beta^*_b = \beta^*_i$ is given by:

$$\beta^*_i = \frac{3\left(1-\alpha\eta^H\right)\left(1-\eta^H\right) - \sqrt{\left(1-\eta^H\right)\left(1-\alpha\eta^H\right)\left(16-3\eta^H\left(5+3\alpha\left(1-\eta^H\right)\right)\right)}}{12\left(1-\eta^H\right)-8}. \tag{26}$$

It directly follows from (26) that $\beta^{*\prime}_{i\,\eta^H} < 0 \rightarrow \beta^{*\prime}_{i\,\eta^M} > 0$. Notice that $\beta^*_i = \left(1-\beta^{H*}\left(N=2, s=0\right)\right)/2$, with β^{H*} as given in eq. (14), leads to the same solution as (26).

C.2. Asymmetric input intensities. The FOCs reduce to $\Psi'_{\beta^H} = 0$ and $\Psi'_{\beta_a} = 0$. It is possible to solve $\Psi'_{\beta^H} = 0$ for $\beta^H(\beta_a)$. Using $\beta^H(\beta_a)$ in $\Psi'_{\beta_a} = 0$ leads to $\Psi'_{\beta_a}|_{\beta^H=\beta^H(\beta_a)} = 0$ and solely depends on β_a. To illustrate the alogrithm we assume in the following $\eta^H = \alpha = 1/2$. Then $\Psi'_{\beta_a}|_{\beta^H=\beta^H(\beta_a)} = 0$ is equivalent to finding a root β^*_a of the polynominal R given by

$$R = \beta_a^3 - \beta_a^2 \cdot \frac{-2+(\eta_a)^2(4+\eta_a)}{2(1-\eta_a)(1-2\eta_a)} + \beta_a \cdot \frac{9(\eta_a)^2(3+\eta_a)}{16(1-\eta_a)(1-2\eta_a)} - \frac{3(\eta_a)^2(3+\eta_a)}{16(1-\eta_a)(1-2\eta_a)}.$$

We propose the following change in variables that eliminates β_a^2 in R: $\beta_a^3 = Z - A/3$ with $A = \left[2-(\eta_a)^2(4+\eta_a)\right]/\left[2(1-\eta_a)(1-2\eta_a)\right]$. This leads to $R = Z^3 + Z \cdot P + Q$ where P and Q are given by:

$$P = \frac{(\eta_a)^2\left(145+\eta_a\left(\eta_a\left(17+22\eta_a-4(\eta_a)^2\right)-200\right)\right)-16}{48\left(1-3\eta_a+2(\eta_a)^2\right)^2},$$

$$Q = -\frac{(4-\eta_a)^2\left(\eta_a\left(-2+\eta_a\left(84+\eta_a\left(-205+\eta_a\left(148+\eta_a\left(1+2\eta_a\right)\left(4\eta_a-3\right)\right)\right)\right)\right)-4\right)}{864\left(1-3\eta_a+2\left(\eta_a\right)^2\right)^3},$$

respectively, which solely depend η_a. Cardano's formula leads to the solution Z^* that solves $R=0$. Since the discriminant $D = P^3/27 + Q^2/4$ is negative, the unique closed form solutions is piecewise defined from:

$$Z_1^* = \sqrt{-\frac{4P}{3}}\cos\left[\frac{1}{3}\arccos\left(-\frac{Q}{2}\sqrt{-\frac{27}{P^3}}\right)\right],$$

$$Z_2^* = -\sqrt{-\frac{4P}{3}}\cos\left[\frac{1}{3}\arccos\left(-\frac{Q}{2}\sqrt{-\frac{27}{P^3}}+\frac{\pi}{3}\right)\right],$$

$$Z_3^* = -\sqrt{-\frac{4P}{3}}\cos\left[\frac{1}{3}\arccos\left(-\frac{Q}{2}\sqrt{-\frac{27}{P^3}}-\frac{\pi}{3}\right)\right]$$

valid in corresponding η_a-domains which can be explicitly solved. Then, Z_1^*, Z_2^* and Z_3^* can be subsituted to yield $\beta_{a,1}^*$, $\beta_{a,2}^*$ and $\beta_{a,3}^*$. Taken together, these $\beta_{a,1}^*$-$\beta_{a,3}^*$ define the unique piecewise solution for β_a^*. The corresponding optimal share β^{H^*} can then be derived by using β_a^* in $\beta^H(\beta_a)$, derived from $\Psi'_{\beta^H} = 0$. The optimal share is then given by $\beta^{H^*} = \beta^H(\beta_a = \beta_a^*)$. The optimal share β_b^* for the other supplier is the residual share given by $\beta_b^* = 1 - \beta^{H*} - \beta_a^*$.

We have here illustrated the alogrithm for the example of $\eta^H = \alpha = 1/2$. Other parameter examples also reduce to a similar term as given by R (with polynomial degree of 3) and can be solved analogously with the help of Cardano's formula. Upon request we provide a *Mathematica* file with the algorithm behind Figure 7.