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Self-Employment, Wage Employment and Informality in a Developing Economy

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ABSTRACT

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We construct a simple model incorporating various urban labour market phenomena obtaining in developing economies. Our initial formulation assumes an integrated labour market and allows for entrepreneurship, self-employment and wage employment. We then introduce labour market segmentation. In equilibrium voluntary and involuntary self-employment, formal and informal wage employment, and formal and informal entrepreneurship may all coexist. We illustrate the model by an example calibrated on Latin American data, examining individual labour market transitions and implications of education/training and labour market policies. To diminish informality, cutting the costs of formality is more effective than raising those of informality.

JEL Classification: O17, J23

Keywords: self-employment, wage employment, informality

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1 Introduction

Urban labour markets in developing economies exhibit considerable diversity, typically including significant segments of both voluntary and involuntary self-employment, as well as formal and informal wage employment. In Latin America and the Caribbean, for example, Perry et al. (2007) find that, on average, 24% of urban employment is informal self-employment and 30% is informal wage employment, with each of these segments having significant voluntary and involuntary elements.[1] However, theoretical literature generally focuses on a specific form of employment relationship, e.g., modeling either self-employment or informal wage employment, but not both. In the present paper we attempt to develop this literature by constructing a simple model that incorporates all of the labour market states referred to above, as well as different types of entrepreneurship.

We assume that each agent can allocate his or her labour to one of three activities: self-employment, wage employment, or entrepreneurship (running a firm and providing wage employment to others). An agent is characterized in terms of two skills, Y and Z, where, loosely speaking, Y is the ability to produce and sell an output, and Z is managerial ability. Success as a self-employed worker would depend on the amount y of skill Y possessed; but, following the Lazear (2005) 'jack-of-all-trades' formulation, success as an entrepreneur would depend on applying both skills together, specifically, on the value of $\min(y, z)$, where z is the amount of Z the agent possesses.[2]

[1] See Fields (2009) for a general discussion of the complexity of labour markets in developing economies and of approaches to modeling them.

[2] Since we are concerned with relatively small firms, the productive and sales skills of the entrepreneur will generally matter for a firm's success. De Mel, McKenzie and Woodruff

In wage employment, however, everyone is assumed to be equally able.

We develop two versions of the model. First, in our benchmark case, we assume that the market for wage employment clears, so that all labour states are 'voluntary.' This is useful for expositional purposes, providing the groundwork for the second version of the model, in which we assume labour market segmentation as a result of a minimum wage rate; but it is also of interest in its own right because empirical evidence suggests that in some countries labour markets may largely be integrated (see, e.g., El Badaoui, Strobl and Walsh (2008) on South Africa and for general discussion of the empirical literature).

For our benchmark model, we begin by characterizing the supply function of an agent to the three activities. Two cases are developed (depending on parameter values) which may be interpreted as corresponding to different underlying macroeconomic conditions. We find, for example, that the pattern of labour market transitions that may occur as education or training improves an agent's skills differs between the two cases. We specify when such transitions may be non-monotonic and when direct transitions from wage employment to entrepreneurship may occur. The model also generates a demand for wage labour for those agents who choose entrepreneurship. Given the joint distribution of y and z across individuals, and a flexible wage rate, we characterize the labour market equilibrium and examine its comparative

(2008) suggest that the case for a jack-of-all-trades characterization is stronger if the market for business services is thin, as typically obtains in developing economies. Bloom et al. (2011) report on recent field experiments in developing countries that show that some forms of basic business training and advice can have significant effects on performance in small enterprises. See Nichter and Goldmark (2009) on the characteristics of successful entrepreneurs in developing economies.

statics.

In the second, segmented-labour market, version of the model we assume that a minimum wage rate is fixed above the market-clearing level, but that this applies only for firms above a certain size, with such firms being regarded as formal, and smaller firms as informal.[3] We show that informal and formal wage employment can coexist with voluntary and involuntary self-employment. If an agent who is rationed out of a formal wage job chooses self-employment, this is involuntary in the sense that it is not the agent's first choice (though it is voluntary in the sense that it is chosen freely from the remaining options). Involuntary entrepreneurship may obtain, i.e., agents who are rationed out of formal wage employment may choose, according to their second preference, to run a firm and employ others. We do not specify the rationing scheme that allocates to formal jobs a subset of the agents who would like them; but we note that a potential inefficiency exists (in addition to the distortion caused by the minimum wage rate) in that agents who would be relatively productive in self-employment may gain wage jobs in which their skills are not valuable. In equilibrium this has an adverse effect on output by both informal and formal firms.

To illustrate the model we explore an example calibrated so that it generates values that correspond broadly to Latin American data. We focus on the effects of policy changes aimed at diminishing informality, finding that a reduction in the cost of formality may be preferred to an increase in the

[3]In practice, informality is generally associated with smaller size (Perry et al., 2007). Formal regulations may only apply to larger firms, and, insofar as they apply to all firms, informal firms may eschew larger size to avoid detection (see Ahsan and Pages (2007) on India, and Almeida and Carneiro (2009) on Brazil).

cost of informality, and that it is more effective to provide education and training that improves the ability to produce and sell, rather than managerial skills. We also find that policy changes that might have been expected to favour entrepreneurship may reduce the total number of entrepreneurs, while increasing (formal) employment and output by the relatively able ones.

Following Rauch (1991), many papers in the informality literature develop models based on Lucas (1978), where agents differ with respect to a single ability parameter.[4] These include Fortin, Marceau and Savard (1997), Amaral and Quintin (2006), Galiani and Weinschelbaum (2007) and De Paula and Scheinkman (2008). Also, Antunes and Cavalcanti (2007) add 'bequests' as a second dimension by which agents are characterized. However, these contributions do not allow for self-employment, treating informal wage labour as the only type of informal work. Gollin (2008) is an exception: he allows for only one ability parameter, but incorporates self-employment into the Lucas framework by assuming that an agent's time may be split between running a firm (with no employees) and working as a wage employee in another firm. However, his focus is different to ours, solving numerically a dynamic equilibrium model of capital accumulation to examine the relationship between aggregate productivity, firm size and the extent of self-employment.

Informal self-employment is modeled by Loayza and Rigolini (2006) and Fiess, Fugazza and Maloney (2010) primarily to examine its cyclical behaviour. Each assume that a Lucas-type ability parameter affects an agent's output in self-employment. Loayza and Rigolini treat agents and firms in-

[4] Jovanovic (1994) generalizes the Lucas model in a different direction, including both heterogeneity of both labour and management skills, but he is not concerned with informality.

terchangeably, each firm constituting self-employment. An agent can choose between formality, which has a fixed compliance cost, and informality, with the effect of greater ability on output being greater for formality. Fiess et al. formulate a two-sector dynamic macro-model, with an informal self-employed nontradables sector, with output depending on ability, and a formal tradables sector with wage employment.

These models of informality are generally based on the assumption of homogeneity, in the sense that either, because of segmentation, all informal work is involuntary (e.g., Rauch, 1991; Fortin et al., 1997) or, because the labour market is unsegmented, all informal work is voluntary (e.g., Amaral and Quintin, 2006; Loayza and Rigolini, 2006).[5] However, Fields (1990) classifies informality into upper and lower tiers, one segmented and one not, and the evidence cited by Perry et al. (2007) and others is consistent with this view. Although our model stresses heterogeneity of employment status, it does not correspond neatly to this two-tier distinction, as it includes informal wage employment, involuntary self-employment (which is usually regarded as informal) and voluntary self-employment (which is sometimes regarded as informal).

Perry et al. (2007) also note the attachment of significant value to the non-pecuniary benefits of independent work, including the desire for flexibility, and so we include a non-pecuniary benefit for both the self-employed and

[5] A separate branch of the literature develops search-and-matching models of informality. In the formulation by Albrecht, Navarro and Vroman (2009), e.g., informality is equated with unregulated self-employment, and ability is assumed only to affect an agent's productivity in a formal sector job. Informal sector employment is assumed to preclude search for a formal jobs. It is examined how workers respond to informal or formal job offers according to their ability and, using simulations, the effects of different tax policies are analyzed.

entrepreneurs in the model. An implication is that in equilibrium, depending on ability, some agents could earn more in informal wage employment than from self-employment, while for others the reverse is true. This is consistent with the mixed empirical evidence on which of these types of earnings is the higher (see, e.g., Agénor, 2005).

Section 2 formulates the benchmark version of the model, with a market-clearing wage rate. Section 3 introduces labour market segmentation, and Section 4 applies the model in the Latin American context, examining the policy conclusions. Section 5 concludes. Proofs are given in the Appendix.

2 The Benchmark Model

Consider a large population P of agents, each of which is characterized in terms of two skills, Y and Z. Y may be thought of as the ability to produce and sell, and Z as managerial skill. An agent's levels of $Y \geq 0$ and $Z \geq 0$ are written $y \in (\underline{y}, \bar{y}]$ and $z \in (\underline{z}, \bar{z}]$, respectively. Skills are distributed across P according to $f(y, z)$. Throughout, for simplicity, we assume that $f(.)$ is continuous and positive for all y and z.

Any agent may have one of three possible occupations: wage employment, self-employment or entrepreneurship. We assume a self-employed person does not employ others - rather, any employment of others qualifies the person to be categorized as an entrepreneur.[6] Regardless of an agent's (y, z)-characteristics, he or she has the same ability to do wage work as any other person. However, for self-employment and entrepreneurship, ability matters.

[6] Our assumption is consistent with the definition that Lazear (2005) gives of an entrepreneur as being conceptually distinct from a self-employed person.

If an agent with characteristics (y, z) is self-employed, he or she produces the quantity y; that is, for self-employment 'the ability to produce and sell' matters, but 'managerial skills' do not. If, alternatively, he or she is an entrepreneur, the relevant measure of skill is $\min(y, z) \equiv A$; that is, a balance of both types of skill matters. Such a person runs a firm for which the production function is

$$x = Al^{\alpha}, \quad \alpha \in (0, 1), \tag{1}$$

where x is output and l is the number of people the firm employs.

Let q and p be the prices for the output of the self-employed and entrepreneurial firms, respectively, and let w be the money wage rate. An entrepreneur's profit is therefore $px - wl$, which, given (1), is maximized at $l = \hat{l}(A)$, where

$$\hat{l}(A) = \left(\frac{Ap\alpha}{w}\right)^{\frac{1}{1-\alpha}}. \tag{2}$$

We assume that both self-employment and entrepreneurship give an agent a non-pecuniary benefit, v, which may be thought of as a benefit from independence. Thus, letting UW, US and UE denote the utility from working, self-employment and entrepreneurship, respectively, we have

$$UW = w; \qquad US = qy + v; \qquad UE = pA\hat{l}^{\alpha} - w\hat{l} + v. \tag{3}$$

We shall only consider cases in which $w > v$, which is necessary for wage employment to exist in equilibrium.

If the two production sectors make the same good or service, we may expect self-employment, much of which might be classified as informal, to produce a good of lower quality than the entrepreneurial sector (see Banerji and Jain, 2007). If we regard prices q and p as quality-adjusted, we would

then have that $q < p$. Our analysis nonetheless also covers what happens if $q \geq p$.[7] A relatively high p/q might also be interpreted as reflecting strong aggregate demand, being tilted towards the largely higher-quality goods and services produced by entrepreneurial firms.

We partition P into three sets, W, S and E, according to whether an agent's first preference is for wage employment, self employment or entrepreneurship, respectively.[8] Thus, the sets are defined by

$$\begin{aligned} W &: UW > \max(UE, US); \\ S &: US > \max(UW, UE); \\ E &: UE > \max(UW, US). \end{aligned} \tag{4}$$

Using (2) and (3), we can determine the borderline values of parameters underlying (4):

$$UW \gtrless UE \text{ as } A \lessgtr B(w); \tag{5}$$

$$UW \gtrless US \text{ as } y \lessgtr C(w); \tag{6}$$

$$UE \gtrless US \text{ as } A \gtrless [D(w)]^{\alpha} y^{1-\alpha} \equiv \tilde{z}(y). \tag{7}$$

where

$$\begin{aligned} B(w) &\equiv \frac{1}{p}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{w-v}{1-\alpha}\right)^{1-\alpha}; \\ C(w) &\equiv \frac{1}{q}(w-v); \\ D(w) &\equiv \frac{w}{p\alpha}\left(\frac{q}{p(1-\alpha)}\right)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

[7] In practice self-employment covers a wide range of activity. Self-employed production with low y may be, e.g., construction work or street vending, while that with high y may be, e.g., professional work. A similar comment applies to entrepreneurial output.

[8] Throughout, we simplify the exposition by only specifying strong preference.

Note that $B(w) - C(w) \gtreqless 0$ as $q/p \gtreqless Q(w)$, where

$$Q(w) = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \left(\frac{w-v}{w} \right)^{\alpha}. \tag{8}$$

Since $\alpha^{\alpha} (1-\alpha)^{1-\alpha} \in (1/2, 1)$, $Q(w) \in (0,1)$. Thus, if $q < p$, we may have either $q/p > Q(w)$ or $q/p < Q(w)$; but if $q \geq p$, $q/p > Q(w)$.

Using (5)-(8), Proposition 1 characterizes the allocation of agents to the three sets, W, S and E and Figures 1-2 give an intuitive illustration.

Proposition 1 *Consider agent i_{yz} with characteristics (y,z). (i) for $q/p > Q(w)$, $i_{yz} \in W$ if $y < C(w)$; $i_{yz} \in S$ if either $y \in (C(w), D(w))$ or both $y > D(w)$ and $z < \tilde{z}(y)$; and $i_{yz} \in E$ otherwise. (ii) for $q/p < Q(w)$, $i_{yz} \in W$ if either $y < B$, or both $y \in (B(w), C(w))$ and $z < B(w)$; $i_{yz} \in S$ if $y > C(w)$ and $z < \tilde{z}(y)$; and $i_{yz} \in E$ otherwise.*

In Figure 1 $q/p > Q(w)$ and in Figure 2 $q/p < Q(w)$. For simplicity, it is assumed in these figures that $\bar{y} = \bar{z}$ and $\underline{y} = \underline{z} = 0$.[9] Consider Figure 1(a), in which $q/p > Q(w)$. For individuals with $y < C(w)$ self-employment and entrepreneurship both offer relatively low rewards and so wage employment is preferred. For $y > C(w)$, however, either self-employment or entrepreneurship is preferred. In this range of y, we might expect that, since Y and Z are perfect complements in production, set E would be represented by a square at the north-east corner of the figure for $y > D(w)$, with set S occupying an L-shaped area around it to the south-west, and at least one of Y and Z

[9] Without these assumptions it is simply necessary to trim Figures 1(a) and 2(a) accordingly. If, e.g., $\bar{y}$ were cut by δ we would delete the area in each panel between $\bar{y} - \delta$ and $\bar{y}$. When, below, we interpret the figures as representing labour market equilibrium, the trimming would affect the equilibrium values of w, B, C and D, but the appropriately redrawn figures would still have the same general characteristics as Figures 1(a) and 2(a).

taking lower values than in E. This is not quite what obtains, however, for when $y > D(w)$, along the borderline between S and E, which is defined by $z = \tilde{z}\,(y)$, we find that $dz/dy > 0$.[10]

[Figure 1 about here]

The figure may be interpreted in terms of an agent's transition between labour market states as skills are acquired.

Remark 1 *For some agents, even for changes in y alone, the transition between labour market states may be non-monotonic.*

Consider, for example, an agent with skill $z = z_1$ in Figure 1(a). Starting from a low level, the acquisition of greater skill y enables a transition from W to S, and then from S to E; but the acquisition of sufficiently high skill y enables a transition back to S.[11] This is also illustrated in panel (b), which plots the agent's utility for $z = z_1$. The non-monotonicity suggests a potential complication for empirical specifications of labour market transitions.

Figure 2 illustrates the corresponding results for $q/p < Q(w)$. In this case self-employment is relatively unattractive, compared to entrepreneurship, for any given w. Panel (a) contains a significant difference to Figure 1(a) in that, as y rises, an agent may switch directly from W to E, without an intermediate stage of S. An implication is that, for increases in y (or z) alone, monotonicity obtains, though, as in Figure 1, some agents belong to

[10] When $y = D$, $UE = US$ on the 45°- line, and, from this corner point of set E, $z = \tilde{z}(y)$ slopes up. Intuitively, a shift to a (y, z)-combination due east of the corner point raises US, but, since z is unchanged, it leaves UE unaffected; i.e., points due east of the corner point are interior to the set S. $\tilde{z}''(y) < 0$ because entrepreneurship exhibits increasing returns to A, whereas self-employment income exhibits constant returns to y.

[11] If there is incremental acquisition of both skills y and z, and this occurs sequentially over time, then repeated switches between E and S are also possible.

S at the highest values of y even though at lower y they would belong to E. Panel (b) plots the levels of utility for $z = z_2$.

[Figure 2 about here]

The difference between Figures 1(a) and 2(a) with regard to horizontal transitions suggests the following:

Remark 2 *The mobility implications of education and training that affect individuals' ability Y can depend on macroeconomic factors (p and q) as well as individual-specific ones (Z here).*

In Figure 1(a), where p/q is high, which may reflect strong aggregate demand, transitions may occur straight from W to E, however small the increase in Y. But in Figure 2(a), where p/q is low, perhaps reflecting weak aggregate demand, only a relatively large addition to skill Y would enable direct transition from W to E; in the absence of such large additions to skill, self-employment may play an important transitional role. Aggregating over P, we obtain the supplies of labour to the three activities. We denote the total supplies to wage employment, self-employment and entrepreneurship by L^s, SE^s and E^s, respectively. For each entrepreneur the demand for labour is given by $\hat{l}\,(A)$ in (2) and thus we obtain the total demand for labour, L^d.

Lemma 1 *The comparative statics of the supply and demand for wage labour are as follows:*

$$L_p^d > 0;\ L_q^d < 0;\ L_w^d < 0;\ L_q^s < 0;\ L_w^s > 0;\ L_v^s < 0;$$
$$L_v^d \begin{cases} = 0 & \text{for } q/p > Q(w) \\ > 0 & \text{for } q/p < Q(w) \end{cases};\ L_p^s \begin{cases} = 0 & \text{for } q/p > Q(w) \\ < 0 & \text{for } q/p < Q(w) \end{cases}.$$

The demand for wage labour is increasing in the price of the firms' output and decreasing in the money wage. It is decreasing in the price paid for the output of the self-employed because a higher price for this output makes entrepreneurship relatively less attractive. If the benefit from independence, v, is greater, then entrepreneurship (as well as self employment) is more attractive relative to wage employment. But it is only if there are agents on the margin of choice between entrepreneurship and wage employment (if $q/p < Q(w)$) that this is associated with more agents choosing to be entrepreneurs.

The supply of wage labour is increasing in the money wage rate, and decreasing in the price of self-employed output and benefit from independence. If the output price p is higher then, again, provided there are agents on the margin of choice between entrepreneurship and wage employment (if $q/p < Q(w)$), wage employment becomes less attractive relative to entrepreneurship for these agents, and so the supply of wage labour is lower.

We can now specify sufficient conditions for equilibrium in the labour market, including the coexistence of wage employment and self employment. We denote the lowest and highest levels of A in P by $\underline{A}$ and $\overline{A}$, respectively, and we define $\underline{w}$ and $\overline{w}$ as

$$\begin{aligned} UE\left(\underline{A}, \underline{w}\right) &= UW\left(\underline{w}\right); \\ UE\left(\overline{A}, \overline{w}\right) &= UW\left(\overline{w}\right). \end{aligned} \tag{9}$$

Thus, $\underline{w}$ is the level of the wage w at which an agent with $A = \underline{A}$ would be indifferent between being an entrepreneur and a worker, and $\overline{w}$ is defined similarly for $A = \overline{A}$.

Proposition 2 *If* $UE\left(\underline{A},\underline{w}\right) < US\left(\underline{y}\right) < UW\left(\overline{w}\right) < US\left(\bar{y}\right) < UE\left(\overline{A},\underline{w}\right)$ *then there exists a wage* $w^* \in \left(\underline{w},\overline{w}\right)$ *such that* $L^d\left(w^*\right) = L^s\left(w^*\right)$ *and the sets* E, S, W *are non-empty.*

Depending on whether the market-clearing wage rate w^* is such that $q/p > Q(w^*)$ or $q/p < Q(w^*)$, Figure 1(a) or 2(a), respectively, can be interpreted as representing this equilibrium. With $w = w^*$, the set of entrepreneurs E in the figure generates an aggregate demand for wage labour that equals the supply of wage labour, set W.

The comparative statics of this equilibrium, with w adjusting endogenously, are as follows:

Lemma 2 *In equilibrium* $(w = w^*)$, $dw/dp > 0$, $dw/dq \gtrless 0$ *and* $dw/dv > 0$; *and total wage employment* L *satisfies*

$$\begin{aligned} \frac{dL}{dp} &> 0 \begin{cases} \text{if } q/p > Q(w^*), \\ \text{if } q/p < Q(w^*) \text{ and } L_p^d L_w^s - L_w^d L_p^s > 0; \end{cases} \\ dL/dq &< 0; \\ \frac{dL}{dv} &= \begin{cases} < 0 & \text{if } q/p > Q(w^*), \\ \gtrless 0 & \text{if } q/p < Q(w^*). \end{cases} \end{aligned}$$

If the price p of the entrepreneurial output is higher then L^d is greater, as is w^*. Set W is therefore larger, subject, when $q/p < Q(w^*)$, to a stability condition. If the output price q for the self-employed is higher, the greater attractiveness of self-employment is associated with both a lower supply of and a lower demand for wage labour, the latter effect arising because the supply of entrepreneurship is smaller. Thus, W is smaller, but the net effect on w^* may be of either sign. A greater desire for independence v stimulates both self-employment (reducing the supply of wage labour) and entrepreneurship

(increasing the demand for wage labour). The latter effect implies a greater demand for wage labour, but as the supply of wage labour is smaller we can only sign the effect on W when $q/p > Q(w^*)$.

3 Labour Market Segmentation

We now examine the equilibrium that obtains when the wage rate w is fixed, e.g. by law, at w_f, above the market-clearing level w^*. As first specified by Rauch (1991), we assume that only firms above a certain threshold employment level $l = l_0$ pay the minimum wage w_f, whereas firms with $l \leq l_0$ pay the market-clearing wage $w = w_i$. The former firms are denoted 'formal' and the latter 'informal'. In Rauch's model (in which skill is one-dimensional) there is a critical entrepreneurial skill level above which formality is chosen, with informality being chosen otherwise. In our model there is a critical level of A, $A = \tilde{A}$, that plays a similar role. This is the level of A at which the entrepreneur achieves the same utility from operating informally at the maximum employment level l_0 as from operating formally at the higher, profit-maximizing employment level $\hat{l}(A)$; i.e.,

$$UE(\tilde{A}, w_i, l_0) = UE(\tilde{A}, w_f, \hat{l}(\tilde{A})). \tag{10}$$

Of agents choosing entrepreneurship, those with $A > \tilde{A}$ choose formality. As in Rauch's model there is a gap in the size-distribution of firms at $A = \tilde{A}$.

With this revised model, we must respecify the choices facing agents. The utility from self-employment is the same as in (3), but we now distinguish the respective utilities, U_f and U_i from formal and informal wage work:

$$US = qy + v; \qquad UW_f = w_f; \qquad UW_i = w_i. \tag{11}$$

The utilities from formal and informal entrepreneurship are denoted by UE_f and UE_i, where

$$UE_j = pAl_j^{\alpha} - w_j\hat{l}_j + v, \quad j = f, i. \tag{12}$$

For an informal entrepreneur ($A \leq \tilde{A}$), if there were no constraint on informal employment we would have $l_i = \hat{l}_i(A) = (Ap\alpha/w_i)^{1/(1-\alpha)}$. So the constraint $l \leq l_0$ binds exactly if $(Ap\alpha/w_i)^{1/(1-\alpha)} = l_0$; i.e., if $A = w_i l_0^{1-\alpha}/p\alpha \equiv A_0$.Thus, for firms operating informally,

$$l_i = \begin{cases} \hat{l}_i(A) & \text{if } A_0 > A; \\ l_0 & \text{if } \tilde{A} \geq A \geq A_0; \end{cases}$$

and for firms operating formally

$$l_f = \hat{l}_f = \left(\frac{Ap\alpha}{w_f}\right)^{\frac{1}{1-\alpha}}.$$

The population P can be partitioned into four sets according to their first preferences in the labour market.[12] In the appendix we specify the equalities parallel to (5)-(8) that underlie these first preferences (as well as those underlying second preferences). Our notation will be to write in parentheses f for formal and i for informal, and then to add a subscript V for voluntary and I for involuntary if a further distinction is necessary. Thus, all agents belong to one of the following sets.

1. Formal entrepreneurship, denoted $E(f)$; defined by $UE_f > \max(UE_i, US, UW_f)$.

2. Voluntary informal entrepreneurship, denoted $E_V(i)$; defined by $UE_i > \max(UE_f, US, UW_f)$.

[12]For each agent, the first preference is 'voluntary', but we only use this term in naming a set if the distinction will be necessary below where we specify 'involuntary' sets (for which a similar comment applies).

3. Voluntary self-employment, denoted S_V; defined by $US > \max(UE_f, UE_i, UW_f)$.

4. Formal employment, denoted $W(f)$; defined by $UW_f > \max(UE_f, UE_i, US)$.

Set $W(f)$ can be partitioned into two subsets - those agents who obtain a formal job (set $W(f)^+$) and those who do not (set $W(f)^-$). Members of set $W(f)^-$ attain their second preferences; i.e., they allocate their labour 'involuntarily'. Each belongs to one of the following sets.[13]

1. Involuntary informal entrepreneurs, denoted $E_I(i)$; defined by $UE_i > \max(US, UW_i)$.

2. Involuntary self-employed, denoted S_I; defined by $US > \max(UE_i, UW_i)$.

3. Informal employees, denoted $W(i)$; defined by $UW_i > \max(UE_i, US)$.

With this more complex model, we have the following.

Proposition 3 *Suppose firms may be formal, with $l > l_0$ and paying wage w_f, where $w_f > w^*$, or informal, with $l \leq l_0$ and paying the market clearing wage w_i. Then the sets $E(f)$, $E_V(i)$, S_V, $W(f)^+$, $E_I(i)$, S_I and $W(i)$ may, simultaneously, all be non-empty in equilibrium.*

An analytical proof of this proposition would involve numerous interacting side conditions, but it can be proved by example. We delay giving an example

[13] No (y, z)-combinations exist for which both (i) formal employment is first preference and (ii) formal entrepreneurship second preference; i.e., involuntary formal entrepreneurship is not feasible. Agents who choose entrepreneurship are in the highest A-range in P, and, amongst these, those with $A > (\leq)\ \tilde{A}$ choose formality (informality). Someone may be on the margin of choice between formal employment and informal entrepreneurship, slightly preferring the former, but because of formal employment rationing, engaging in the latter; they cannot be on the margin of choice between formal employment and formal entrepreneurship.

until Section 5, where we relate it to Latin American data, because we wish to consider this example in its own right. It should be emphasized that the sets listed in the proposition are not necessarily non-empty, and degenerate equilibria may easily be formulated (e.g., if q/p were sufficiently high all agents would belong to set S_V). But we shall focus on cases in which all the sets (except possibly $E_I(i)$) are non-empty because these correspond to the labour markets observed in practice.

Before considering the numerical example, we illustrate the proposition in Figure 3, which is a development of Figures 1(a) and 2(a), and can be interpreted as representing the equilibrium with endogenous adjustment of w_i. As previously, the cases shown correspond to different ranges of q/p relative to Q, but whereas in Figures 1(a) and 2(a) Q was a function of the single wage rate w^*, now there are two wage rates, w_f and w_i in the model. The relevant formulation is $Q(w_i, w_j)$ with $j = f, i$, where the first argument is the unit cost of labour to the entrepreneur and the second is the wage earned in activity j.[14] This is derived in the appendix, along with the borderline parameter values $B(.,.)$, $C(.)$ and $D(.)$ shown in Figure 3. Panel (i) illustrates the case in which $q/p > Q(w_i, w_f) > Q(w_i, w_i)$, which corresponds to the case shown in Figure 1(a); in panel (ii) $Q(w_i, w_f) > q/p > Q(w_i, w_i)$, which is essentially a hybrid of the Figure 1(a)- and Figure 2(a)-cases; and in panel (iii) $Q(w_i, w_f) > Q(w_i, w_i) > q/p$, which corresponds to

[14] In Figures 1(a) and 2(a) $Q(w^*)$ is the critical value of q/p determining whether the borderline value of y (and of z for entrepreneurship) at which $UW = UE$ is greater or less than that at which $UW = US$. Now two different values of Q come into play, depending on whether an employed agent earns w_f or w_i. In each case in Figure 3, the relevant labour cost to the entrepreneur is w_i because this is the wage rate that applies to marginal decisions between entrepreneurship and self-employment. It can be seen that marginal choices between self employment and formal entrepreneurship do not come into play.

Figure 2(a). Each of the panels can be explained in three steps.[15]

[Figure 3 about here]

First, using equations (1)-(7) with $w = w_f$ and (10), we determine the (y, z)-characteristics of the members of the 'first-preference' sets $E(f)$, $E_V(i)$, S_V and $W(f)$. The first three of these sets are shown unshaded, while set $W(f)$ is shown by the entire shaded area in each panel.

Second, because the rationing scheme has not been specified, we note that membership of set $W(f)^+$ may come from anywhere in the shaded area (set $W(f)$) in each panel.

Third, disregarding temporarily the allocation of agents to set $W(f)^+$, we treat the shaded area in the same way as we did the whole of (y, z)-space in Figures 1(a) and 2(a). Thus, for the agents concerned, we show the preference among the three options of entrepreneurship, self-employment and informal wage employment, given that all three options are involuntary in the sense that these agents would have preferred to have formal wage employment. Thus we determine the 'second-preference' sets $E_I(i)$, S_I and $W(i)$, with the proviso that a selection of agents with (y, z)-characteristics consonant with these sets, belong instead to set $W(f)^+$.

For a given (y, z)-distribution, we assume that w_i adjusts endogenously such that the supply of informal wage labour (from set $W(i)$) equals the demand for informal wage labour (from set $E_V(i) \cup E_I(i)$). The other allocations are determined simultaneously. The figures can then be interpreted as representing the equilibrium for three different cases. It can be seen that

[15] If w_f is not significantly above w^* the horizontal boundary of the set $E(f)$ will meet the upward-sloping boundary of set $E_V(i)$ and terminate there.

relatively highly-skilled agents with a balanced skill set become formal entrepreneurs, while those not quite so highly skilled and/or with not quite so balanced skill sets become voluntary informal entrepreneurs. Agents with a high y, but sufficiently low z, become voluntarily self-employed.

In panel (i), the return to self-employment is relatively high ($q/p > Q(w_i, w_f) > Q(w_i, w_i)$). As a result, there is no involuntary informal entrepreneurship, involuntary self-employment being preferred instead. However, the return to self-employment is not so high in panels (ii) and (iii) and so some involuntary entrepreneurship obtains, with the agents concerned having lower values of $A = \min(y, z)$ than voluntary entrepreneurs. Roughly speaking, involuntary informal entrepreneurs have high values of z, but intermediate values of y, although a member of set S_I may have more of both skills than a member of set $E_I(i)$.

Remark 3 *The 'rationing scheme' for formal wage employment may create an (additional) inefficiency, with output being forgone from self-employment, and both informal and formal entrepreneurial firms.*

Unless the formal wage employees are those with the smallest y endowments in the shaded area in each panel of Figure 3, some output by the involuntarily self-employed is forgone. Also, in panels (ii) and (iii), insofar as some agents from the shaded area associated with $E_I(i)$ gain formal employment, there is a negative effect on the demand for informal wage labour and the supply of informal output. This impacts negatively on the informal wage rate w_i, causing some substitution out of formal wage employment and output.

As in the benchmark model, individual labour market transitions may be non-monotonic. We end this section by considering more generally whether the transitions predicted by the model are consistent with those observed in practice. The empirical literature on Latin America indicates that young people tend to get informal jobs when they leave school, and these jobs are used as a stepping-stone to acquire skills. Formal employment may later be obtained, but for many the ultimate destination is voluntary self-employment (Perry et al., 2007; Bosch and Maloney, 2010; Cunningham and Salvagno, 2011).

In Figure 3(i) an agent with low skills will - unless they manage to obtain formal wage employment - begin work-life in set $W(i)$. As they acquire skills, they will move north-east in the figure, perhaps shifting into set S_I. Depending on the rationing scheme, as time goes by they may eventually get a formal wage job. Nonetheless, if their skills develop sufficiently, they will join set S_V, and possibly $E_V(i)$ or $E(f)$. The case shown in Figure 3(ii) is similar, except that, here, with q/p not as great as in Figure 3(i), if a formal wage job is not obtained, an agent with sufficiently high z/y will move from S_I to $E_I(i)$, without first being in S_V. Finally, in Figure 3(iii), where q/p is lower still, an agent with sufficiently high z/y will move straight from $W(i)$ to $E_I(i)$, without an intervening stage of self-employment. Our model is thus broadly consistent with the empirical evidence on Latin American transitions, with the case shown in Figure 3(i) perhaps the closest to this experience.

4 An Application to Latin American Data

To illustrate the model Table 1 shows a numerical example, calibrated so that the values it generates correspond broadly to Latin American data. We assume that the rationing scheme for allocating individuals in set $W(f)$ to set $W(f)^+$ is random. The baseline parameter values are shown under the table and the resulting values derived from the model are shown in the first column. We also vary parameter values to derive the comparative statics for the baseline case. The last three columns represent a positive incremental shift in the distributions shown.[16] The rest of the table shows the signs for small increases in the value of each of the parameters listed.

In the baseline case we assume that the price of entrepreneurial output p is almost 2/3 higher than the price of self-employed output q. The benefit from independence v is assumed to be nearly 10% of the minimum wage w_f. In their survey of informality, Oviedo, Thomas and Karakuram-Özdemir (2009) note that informal firms 'mostly' have five or fewer employees, and so we set $l_0 = 5$. We assume a joint log-normal distribution of skills Y and Z.[17]

The results generated for the baseline case can be compared to those in Table 2.1 of Perry et al. (2007), which gives data for recent years for various

[16] We do not set out the general comparative statics for the model with segmentation because it is complex, involving the interplay of numerous inequalities, and not greatly informative.

[17] The bivariate log-normal function is

$$f(y,z) = \left(\frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}yz}\right)\exp^{-\frac{k}{2(1-\rho^2)}},$$

where $k = \left[(\log y - \delta_y)^2 + (\log z - \delta_z)^2 - 2\rho(\log y - \delta_y)(\log z - \delta_z)\right]/\sigma^2$ and ρ, σ are constants.

Table 1: An example for Latin America

	Parameter Change							
Baseline (% of P)	q	p	v	w_f	l_0	Y	Z	Y, Z
$\lvert E(f)\rvert = 0.63$	0	+	+	−	−	+	+	+
$\lvert E_V(i)\rvert = 6.86$	−	−	−	+	−	−	+	−
$\lvert E_I(i)\rvert = 0.09$	−	−	−	+	−	−	−	−
$\lvert E(f) \cup E_V(i) \cup E_I(i)\rvert$	−	+	−	+	−	−	+	−
$\lvert S_V\rvert = 23.35$	+	−	+	−	+	+	−	+
$\lvert S_I\rvert = 7.77$	+	−	+	+	−	−	−	−
$\lvert S_V \cup S_I\rvert$	+	−	+	+	−	+	−	+
$\lvert W(f)^+\rvert = 35.40$	0	+	+	−	−	+	+	+
$\lvert W(i)\rvert = 25.99$	−	−	−	+	+	−	+	−
$\lvert W(f)^+ \cup W(i)\rvert$	−	+	−	−	+	−	+	−
$w_i/w_f = 0.5807$	−	+	+	−	+	+	+	+

$q = 0.55, p = 0.9, \alpha = 0.5, v = 0.1, w_f = 1.1, l_0 = 5, \rho = 0, \sigma = 1.63.$

countries in Latin America and the Caribbean.[18] For their three categories of formal waged, informal waged and self-employed, our results deviate by less than 2% from their data.[19] Although Perry et al. find wide variations for individual countries, the results that we have generated seem appropriate as a broad representation of the relative size of the different components of the

[18] We can also consider the equilibrium for the parameter values used in Table 1, but with the assumption that there is no minimum wage. We are then in the world of our benchmark model. In the notation we used earlier, we find that $w^* = 1.02$, $E = 4.87\%$, $S = 32.71\%$ and $W = 62.42\%$. Starting at this equilibrium, with the imposition of the minimum wage the most able entrepreneurs adopt formality, but reduce their demand for labour. As these are the biggest employers, this puts a substantial number of workers out of a job, with a correspondingly large negative effect on the market (informal) wage rate. This makes informal entrepreneurship more attractive, and, as a result, it is found that there are more entrepreneurs in the equilibrium with the minimum wage than without it. Because the formal wage is high, self-employment diminishes as a first preference, but the rationing of formal wage jobs results in many choosing self-employment as a second preference. The net effect on total self employment is relatively small.

[19] Using the social protection/legal definition of informality, Perry et al. find the proportions of paid private nondomestic employment in urban areas to be: formal waged 37%; informal waged 28%; and self-employed 34%. Their data do not appear to include entrepreneurs. If, for comparison, we recalculate the figures in our baseline example excluding entrepreneurs, they come to within 1% of the Perry et al. data.

labour force in the region. There is also wide variation across countries and types of worker in the gap between formal and formal wage rates (Perry et al., Table 3.1). Our baseline case generates an informal wage that is 58% of the formal wage, which, for example, is only 1% less than the corresponding figure for an average-earnings job in Argentine.

The comparative statics can be understood intuitively for any parameter by first considering the effect on first preferences and then on second preferences. Consider, for example, an increase in the self-employed price q. Because there are no agents on the borderline of choice between formal entrepreneurship and self-employment, this has no effect on set $E(f)$, and therefore none on $W(f)^+$. However, it causes a switch in first preferences away from formal employment and informal entrepreneurship towards self-employment, and so set $E_V(i)$ becomes smaller and S_V larger. The higher value of q also causes a shift towards self-employment as a second preference, and so set S_I becomes larger, but $W(i)$ and $E_I(i)$ smaller. Although the supply of informal labour falls, the decrease in the demand dominates and so w_i falls. Overall, as we would expect, there are more self-employed and fewer entrepreneurs and wage workers. Similar explanations can be given for other parameter changes, but, for brevity, we focus on some potential policy tools.

First, consider changes in parameters w_f and l_0. l_0 can be regarded as a policy tool even if it is not fixed by government regulation; changes in the probability of detection of informality or in the penalties when caught would affect the informal employment level that entrepreneurs are willing to set. A lower w_f is a reduced cost of formality, while a lower l_0, limiting informal firm

size further, can be interpreted as an increased cost of informality. We might expect that each of these changes would result in less informality. Indeed, reducing w_f does cause $E(f)$ and $W(f)^+$ to become larger, while $E_V(i)$ and $W(i)$ become smaller. However, a while a reduction in l_0 also causes $E(f)$ and $W(f)^+$ to expand and $W(i)$ to contract, the effect on informal activity is not entirely clear-cut, for it expands $E_V(i)$ and $E_I(i)$. Thus, in this example, if the aim is to reduce informality, a reduction in w_f might be preferred.

Second, the results are suggestive of the effects of different types of education/training. In Table 1 a general increase in skill Z expnds both $E(f)$ and $E_V(i)$, as well as $W(f)^+$ and $W(i)$, while a switch occurs out of S_V into S_I. However, a general increase in skill Y, with or without an associated increase in Z, diminishes $E_V(i)$ and $W(i)$, while a switch occurs into S_V from S_I. Suppose that both general education and on-the-job training would increase the stock of Y, whereas specialist management training is required to increase the stock of Z. This suggests that, if the aim is to reduce informality then, because of the role played by voluntary self-employment, general education and on-the-job training is more effective. Interestingly, we find that an increase in both skills, $\{Y, Z\}$, which may appear intuitively to favour entrepreneurship turns out only to expand set $E(f)$, while reducing it *in toto* (set $E(f) \cup E_V(i) \cup E_I(i)$). The payoff from formality increases more than that from informality, causing a substitution from informality to formality; but, in addition, as there are more formal wage jobs, fewer agents supply informal labour, so that w_i is driven up, discouraging informal entrepreneurship.

5 Concluding Comments

Existing theoretical models of urban labour markets in developing economies generally focus on a relatively restricted set of labour market states. In this paper we have attempted to formulate a parsimonious model that captures more fully the complexity that obtains in practice, and we have developed a simple diagrammatic interpretation of the model. Our analysis suggests the importance of underlying macroeconomic conditions in determining the effects of education and training on transitions of individuals between labour market states, and that these transitions may be non-monotonic. The role of the rationing scheme by which workers are selected for formal jobs is also highlighted. As an illustration, a numerical example is developed that generates results that correspond closely to Latin American experience. In this example, if the government wishes to reduce informality, reduction of the costs of formality is generally more effective than increasing the costs of informality, while education and training that improves the ability of individuals to produce and sell is more effective than increasing managerial skills.

These results are obtained from a highly stylized model. Among the factors missing are free labour provided by the family, and wealth and liquidity constraints that may hold back both self-employment and entrepreneurship. Also, it would be interesting to develop the model to include risk, for example so that diversification of family labour in different activities could be modeled. Finally, we might separate what we have called 'the ability to produce and sell' into two skills, with the ability to produce then affecting an agent's

productivity in wage employment.

Appendix

Proposition 1 Consider first the conditions under which wage employment is preferred. If $q/p > Q(w)$ then $C < B$. Since $A \leq y$, we have that $y < C(w) \Rightarrow A \leq y < C(w) < B(w)$; i.e., (6) is sufficient for (5) to be satisfied. If $q/p < Q(w)$ then $B(w) < C(w)$. To satisfy (5) and (6), we need either either $y < B(w)$ or $y \in (B(w), C(w))$ and $z < B(w)$ (since, $A \leq z$, so that $z < B(w)$ is sufficient for $A < B(w)$).

Now consider the conditions under which self-employment is preferred. We have seen that $y > C(w) \Rightarrow US > UW$, so now consider what is required for $US > UE$. First, suppose $A = z$; then, from (7), $UE > US$ if $A > [D(w)]^{\alpha} y^{1-\alpha} \equiv \tilde{z}(y)$. For this to be consistent with $A = z$ we require $y \geq \tilde{z}(y)$. Note that, for $y > 0$, $\tilde{z}(y)$ has a unique fixed point, $\tilde{z}(D(w)) = D(w)$, and that $\tilde{z}'(y) = (1-\alpha)[D(w)]^{\alpha} y^{-\alpha} > 0$, so that $\tilde{z}'(D(w)) = 1-\alpha < 1$ Since also $\tilde{z}''(y) = -\alpha(1-\alpha)[D(w)]^{\alpha} y^{-\alpha-1} < 0$, this implies that $y \gtrless D(w) \Leftrightarrow y \gtrless \tilde{z}(y)$. Hence, if $y > D(w)$, we have $US > UE \Leftrightarrow z < \tilde{z}(y)$. Alternatively, suppose $A = y$. Then, from (7) $y < D(w) \Rightarrow US > UE$.

Hence $US > UE$ if either (i) $y < D(w)$ or (ii) $y > D(w)$ and $z < \tilde{z}(y)$. Therefore $US > \max(UW, UE)$ when either (i) $y \in (\min(C(w), D(w)), D(w))$ or (ii) $y > \max(C(w), D(w))$ and $z < \tilde{z}(y)$. But also, from (5), (7) and (8) we have that $B(w) - D(w) \gtrless 0 \Leftrightarrow q/p \lessgtr Q(w)$. Therefore, since $B(w) - C(w) \gtrless 0$ as $q/p \gtrless Q(w)$, we have $q/p \gtrless Q(w) \Leftrightarrow C(w) \gtrless B(w) \gtrless D(w)$, and the conditions stated in the proposition under which self-employment is preferred follow. The conditions under which entrepreneurship is preferred then follow.

Lemma 1 First we find from (2) and (6)-(7) that $\hat{l}_p > 0$; $\hat{l}_q = 0$; $\hat{l}_w < 0$; $\hat{l}_v = 0$; $B_p(w) < 0$; $B_q(w) = 0$; $B_w(w) > 0$; $B_v(w) < 0$; $C_p(w) = 0$; $C_q(w) < 0$; $C_w(w) > 0$; $C_v(w) < 0$; $D_p(w) < 0$; $D_q(w) > 0$; $D_w(w) > 0$; $D_v(w) = 0$; $\tilde{z}_p < 0$; $\tilde{z}_q > 0$; $\tilde{z}_w > 0$; $\tilde{z}_v = 0$. Using Proposition 1, first we specify the supply of individuals to wage employment and entrepreneurship:

$$L^s = \begin{cases} \int_{\underline{y}}^{C} \int_{\underline{z}}^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_{\underline{y}}^{B} \int_{\underline{z}}^{\bar{z}} f(y,z)dzdy + \int_{B}^{C} \int_{\underline{z}}^{B} f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

$$E^s = \begin{cases} \int_{D}^{\bar{y}} \int_{\tilde{z}(y)}^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_{B}^{C} \int_{B}^{\bar{z}} f(y,z)dzdy + \int_{C}^{\bar{y}} \int_{\tilde{z}(y)}^{\bar{z}} f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

Inserting $\hat{l}(A)$ into each double integral in E^s we obtain labour demand, L^d. Using $A \equiv \min(y, z)$, this can be written

$$L^d = \begin{cases} \int_{D}^{\bar{y}} \int_{\tilde{z}(y)}^{y} \hat{l}(z) f(y,z)dzdy + \int_{D}^{\bar{y}} \int_{y}^{\bar{z}} \hat{l}(y) f(y,z)dzdy & q/p > Q(w), \\ \int_{B}^{C} \int_{B}^{y} \hat{l}(z) f(y,z)dzdy + \int_{B}^{C} \int_{y}^{\bar{z}} \hat{l}(y) f(y,z)dzdy & \\ + \int_{C}^{\bar{y}} \int_{\tilde{z}(y)}^{y} \hat{l}(z) f(y,z)dzdy + \int_{C}^{\bar{y}} \int_{y}^{\bar{z}} \hat{l}(y) f(y,z)dzdy & q/p < Q(w). \end{cases}$$

Differentiating L^s and L^d by (p, q, w, v) and using these inequalities, the lemma is obtained.

Proposition 2 From (9), $w = \underline{w} \Rightarrow W = \varnothing$; $w = \overline{w} \Rightarrow E = \varnothing$. Therefore, $L^s(\underline{w}) = 0$; $L^d(\overline{w}) = 0$. If $w = \underline{w}$, $UE\left(\overline{A}, \underline{w}\right) > US\left(\overline{A}\right) \Rightarrow E \neq \varnothing \Rightarrow L^d > 0$; and if $w = \overline{w}$, $UW(\overline{w}) > US\left(\underline{y}\right) \Rightarrow W \neq \varnothing \Rightarrow L^s > 0$. It follows that, if both $UE\left(\overline{A}, \underline{w}\right) > US\left(\overline{A}\right)$ and $UW(\overline{w}) > US\left(\underline{y}\right)$ the excess demand functions satisfy $L^d(\underline{w}) - L^s(\underline{w}) > 0$ and $L^d(\overline{w}) - L^s(\overline{w}) < 0$. Then, by the continuity of $L^d - L^s$, there must exist a $w^* \in (\underline{w}, \overline{w})$ such that $L^d(w^*) - L^s(w^*) = 0$. Additionally, if $w = \underline{w}$ then $UE(\underline{A}, \underline{w}) < US(\underline{A}) \Rightarrow S \neq \varnothing$, and if $w = \overline{w}$ then $UW(\overline{w}) < US(\bar{y}) \Rightarrow S \neq \varnothing$. The proposition follows.

Lemma 2 Writing labour supply and supply as $L^s(w, i)$ and $L^d(w, i)$, respectively, where $i = (p, q, v)$, when $w = w^*$, $dw/di = (L_i^d - L_i^s)/(L_w^s - L_w^d)$. Using Lemma 1 with this equation yields $dw/dp > 0$, $dw/dq \gtreqless 0$ and $dw/dv > 0$. Thus, (i) $dL/dp = L_w^d(dw/dp) + L_p^d = (L_p^d L_w^s - L_w^d L_p^s)/(L_w^s - L_w^d)$; from Lemma 1, $L_w^d - L_w^s < 0$ and if $q/p > Q(w)$, $L_p^s = 0$ and the result for dL/dp follows; (ii) $dL/dq = (L_q^d L_w^s - L_w^d L_q^s)/(L_w^s - L_w^d) < 0$; (iii) $dL/dv = (L_v^d L_w^s - L_w^d L_v^s)/(L_w^s - L_w^d)$ and the result in the lemma follows.

Borderline Preferences with Labour Market Segmentation To compare the utilities from the different activities we use (11) and (12).

Self employment versus wage employment. Since $w_f > w_i$, $UW_f > UW_i$. Thus, to consider first preferences, we compare US with UW_f. If the agent is rationed out of a formal job, second preferences matter, so we compare US with UW_i. Thus we obtain

$$US \gtreqless UW_j \text{ as } y \gtreqless \frac{1}{q}(w_j - v) \equiv C(w_j), \quad j = f, i.$$

Since $w_f > w_i$, $C(w_f) > C(w_i)$.

Entrepreneurship versus self employment. As an entrepreneur, an individual chooses formality if $A > \tilde{A}$, but informality otherwise. This gives two comparisons with self employment:

$$UE_j \gtreqless US \text{ as } A \gtreqless \frac{1}{p\hat{l}_j^{\alpha}}(qy + w_j \hat{l}_j) \equiv \tilde{z}_j(y), \quad j = f, i.$$

As in Section 2, denote the fixed points of $\tilde{z}_j(y)$ as $D(w_j)$, $j = f, i$; i.e., $\tilde{z}_j(D(w_j)) = D(w_j)$.

Entrepreneurship versus wage employment. With respect to the agent's first preference, we compare UE_i with UW_f, and if the agent is rationed out of a formal job, we compare UE_i with UW_i:

$$UE_i \gtreqless UW_j \text{ as } A \gtreqless \frac{1}{pl_i^{\alpha}}(w_j + w_i l_i - v) \equiv B(w_i, w_j), \quad j = f, i.$$

where the first argument of $B(.,.)$ is the wage paid as an entrepreneur and the second argument is the wage received as an employee.

We can now define corresponding values of $Q(.,.)$. $B(w_i, w_j) - C(w_j) \gtrless 0$ as $q/p \gtrless Q(w_i, w_j)$,where

$$Q(w_i, w_j) \equiv \frac{w_j - v}{l_i(w_j + w_i l_i^{\alpha} - v)}, \quad j = f, i.$$

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Figures

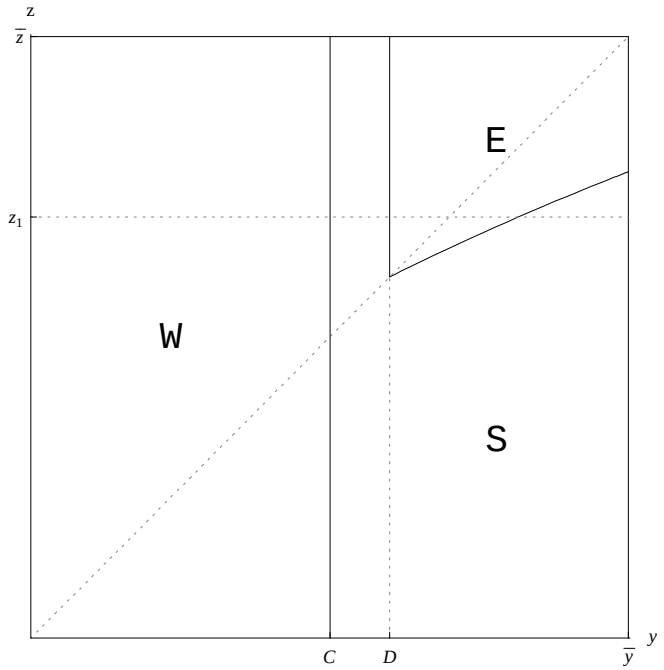

Figure 1(a): Labor allocation for $q/p > Q(w)$

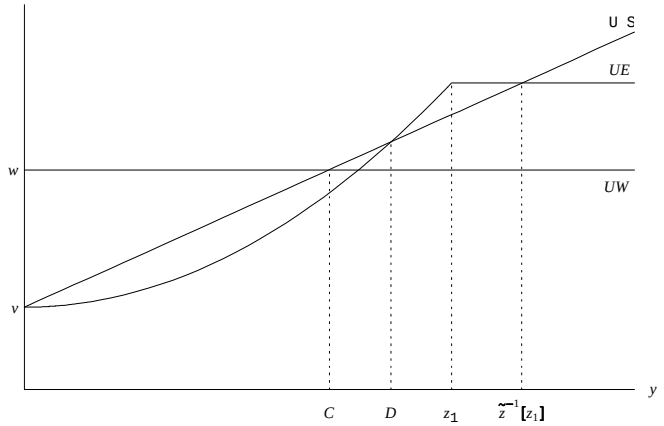

Figure 1(b): Utility for $q/p > Q(w)$

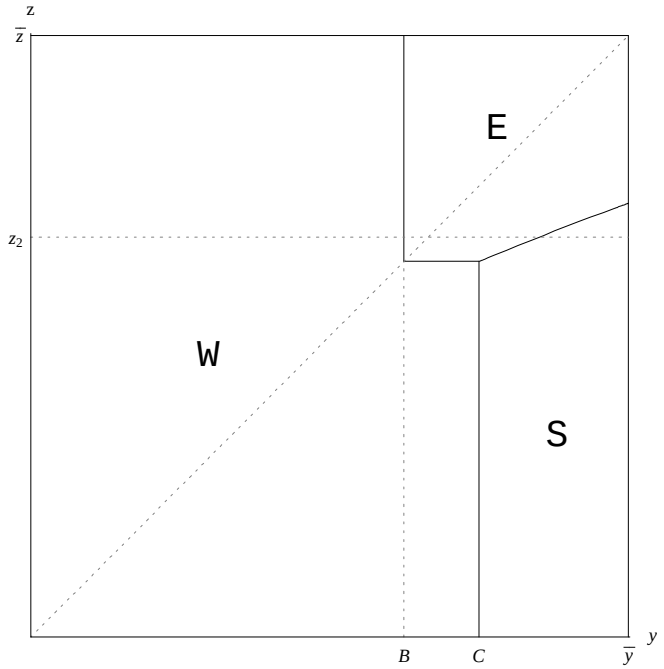

Figure 2(a): Labour allocation for $q/p < Q(w)$

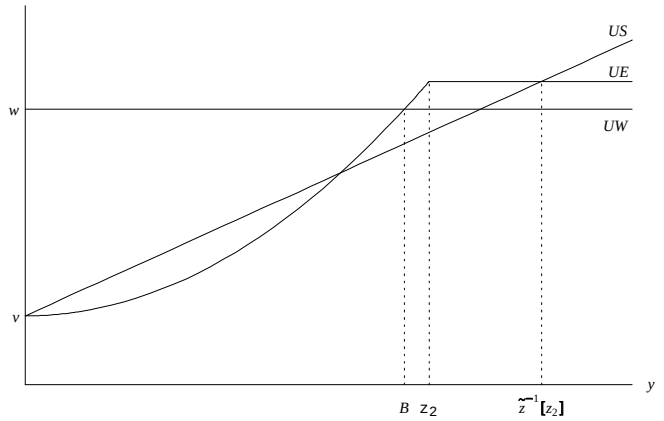

Figure 2(b): Utility for $q/p < Q(w)$

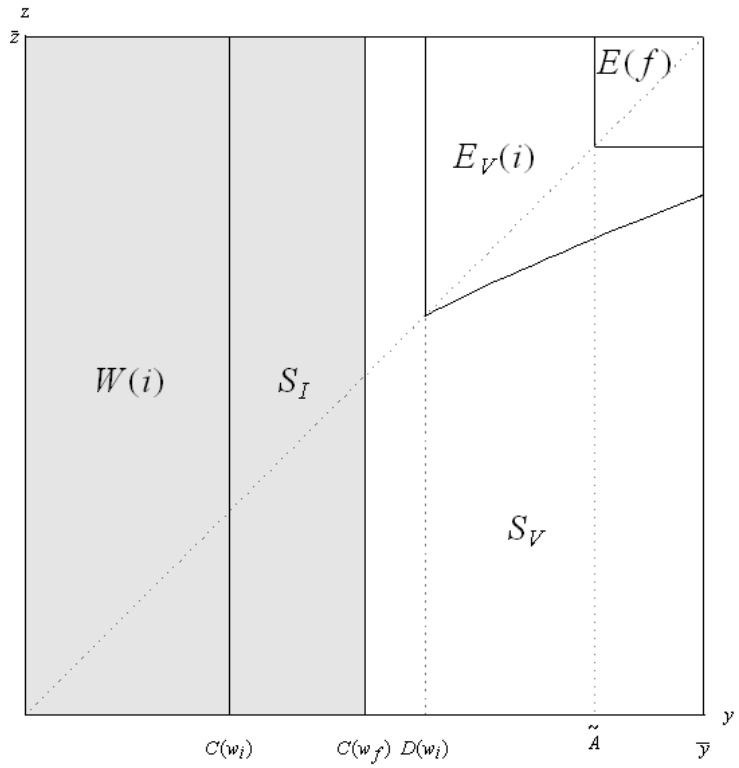

Figure 3(i): Labour allocation for $q/p > Q(w_i, w_f) > Q(w_i, w_i)$. Shaded area denotes the set $W(f)$.

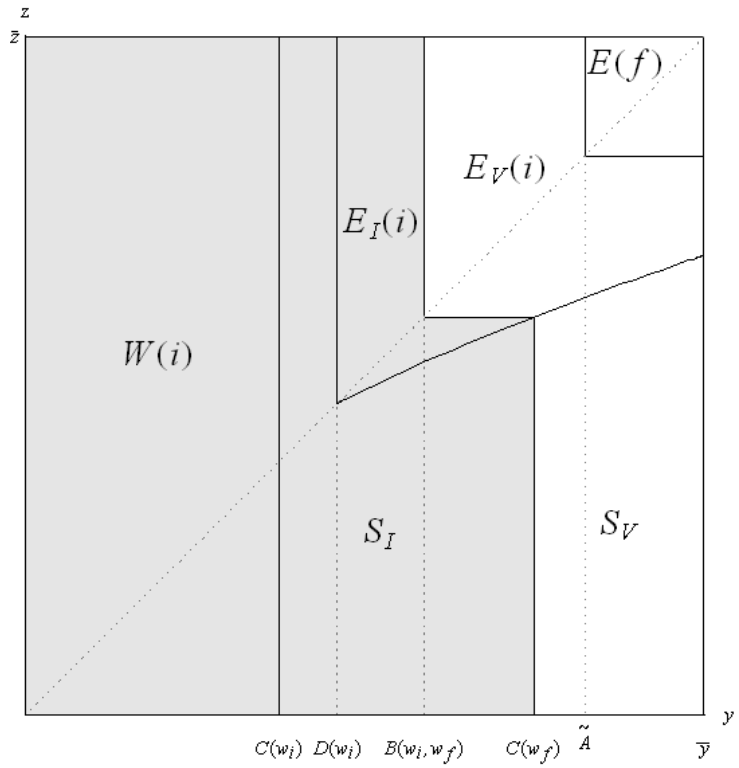

Figure 3(ii): Labour allocation for $Q(w_i, w_f) > q/p > Q(w_i, w_i)$. Shaded area denotes the set $W(f)$.

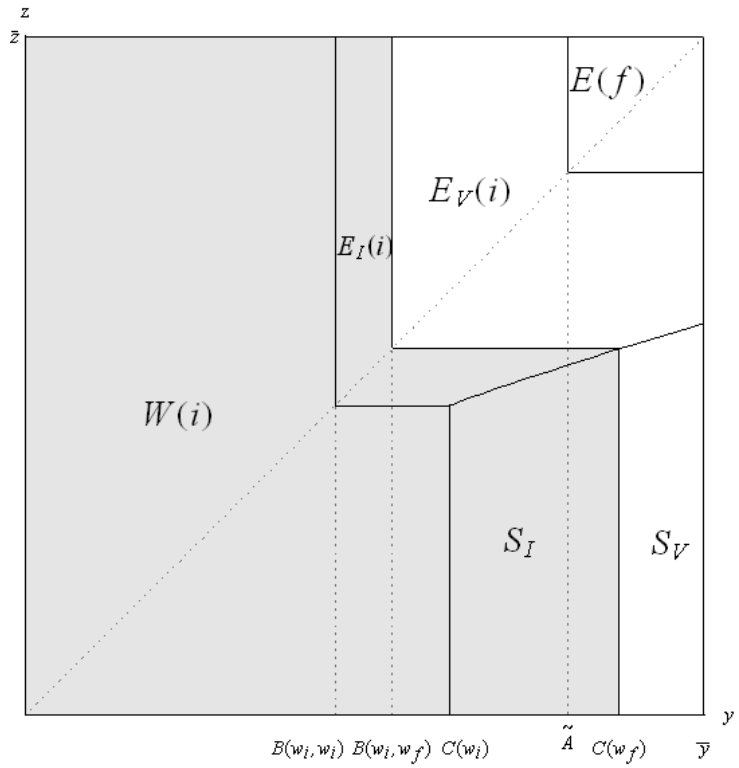

Figure 3 (iii): Labour allocation for $Q(w_i, w_f) > Q(w_i, w_i) > q/p$. Shaded area denotes the set $W(f)$.