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ABSTRACT

Optimal Taxation, Child Care and Models of the Household*

This paper presents for the first time the properties of optimal piecewise linear tax systems for two-earner households, based on joint and individual incomes respectively. A key contribution is the analysis of the interaction of second earner wage differences, variation in prices of bought-in inputs into household production in the form of child care, and domestic productivity differences as determinants of across-household heterogeneity in second earner labour supply. The analysis highlights the importance of the elasticity of substitution between parental and non-parental child care in determining the relationship between utility and income across households. A central result is that taking account of a richer and more realistic specification of household time use widens the set of cases in which individual taxation is welfare-superior to joint taxation.

NON-TECHNICAL SUMMARY

In this paper we find that individual taxation is superior to joint taxation on both equity and efficiency criteria when the analysis of the optimal taxation of couples is based on a labour supply model that recognises the empirical importance of the allocation of time to child care.

JEL Classification: J22, H21, H24, H31, D13

Keywords: optimal taxation, time allocation, household production, child care, inequality, labour supply

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1 Introduction

Real tax systems are almost universally of the piecewise linear kind, in which marginal tax rates are constant within but vary between a small number of specified income brackets. Yet there has been relatively little analysis of their optimal structure,¹ and none at all of the two-earner household case. This paper analyses the optimal two-bracket piecewise linear tax system for two earner households with the aim of bringing out the importance of the structural form of the underlying household model in determining the main features of this system.

Two central issues in the design of a piecewise linear tax system for two-earner households are the choice of tax base, whether individual or joint income, and the structure of the rate scale, in particular whether the marginal tax rates applying to successive income brackets should be strictly increasing, or whether over at least some income ranges they should be decreasing. In Apps, Long and Rees (2012) we refer to these as the "convex" and "nonconvex" cases respectively, to describe the types of budget sets in the space of gross income-net income/consumption to which they give rise. We show there that which of these structures is likely to be optimal depends closely on the distribution of wage or productivity types, and that given the actual empirical distributions, convex systems are very likely to yield welfare-superior results. In this paper, for our purposes it is sufficient to focus on the convex case, which is also analytically simpler to deal with.

By individual taxation we mean the case in which the two earners' incomes are taxed separately but according to the same tax schedule. This is in contrast to what we call "selective taxation", under which separate optimal tax schedules are found for primary and second earners respectively.² The main reason for constraining the rate schedules to be identical under individual taxation is that in practice, piecewise linear tax systems that are not joint are in fact overwhelmingly of the individual rather than selective kind.³ Moreover, if in-

¹The main references are Sadka (1976), Sheshinski (1989), Slemrod et al (1994), Dahlby (1998) and Apps, Long and Rees (2012), all of which deal only with single-person households. For further discussion of the literature see the last of these.

²Boskin and Sheshinski (1983) and Apps and Rees (1997), (2009) analyse this problem in the context of linear taxation. Apps and Rees (1999), (2009) also analyse the tax reform problem with two-earner households and household production. See also Alesina et al (2011). Schrøyen (2003), Brett (2007), Cremer et al. (2012) and Apps and Rees (2009) adopt the Mirrlees approach to optimal income taxation, imposing strong simplifications to make the analysis tractable (though Cremer et al. show in a very general model that joint taxation will be optimal only under very strong and unrealistic conditions). Kleven, Kreiner and Saez (2009) use a Mirrlees optimal tax framework to investigate the way in which the tax function defined on the primary earner's income should depend on the second earner's decision whether to work full time in the market or not to work in the market at all. The wage is the same for all second earners. This "optimal implicit participation tax" problem is a somewhat different issue to that analysed here, which concerns the tax schedule to be applied to the second earner's income when there is a non-degenerate distribution of second earner wage types and labour supply decisions vary also at the intensive margin.

³At the same time, it is easy to find examples of tax systems that contain selective elements. For example in Australia, a portion of family benefits is withdrawn on the basis of the second

dividual taxation yields higher social welfare than joint taxation under realistic assumptions, this result applies *a fortiori* to selective taxation, since removing the constraint that tax schedules must be identical cannot reduce the maximised value of social welfare and would be expected to increase it.

We carry out the analysis in two steps. First, we take the case of the standard labour supply model,⁴ labelled Model 1, and characterise the optimal piecewise linear tax systems for the cases of joint and individual taxation respectively. In this model each individual's time is divided between market work and leisure. In a single-person household, "leisure" can plausibly be seen as the direct consumption of one's own time, but this is not true for the two-person household, where typically partners specialise to differing extents in market and household production and exchange the resulting incomes or outputs.⁵ Viewed in this light, Model 1 can be seen as implicitly containing a household production system, but one with a very special assumption about productivity: across households every individual is equally productive in producing output, labeled "leisure", from one unit of non-market time. This assumption however lacks empirical support.⁶

In the second step we base the tax analysis on Model 2, which seeks to reflect the data on the time use and expenditure decisions of two-earner households with at least one pre-school child present. In such households parental child care is a major form of time use and bought in care, as a substitute at exogenously given prices, can be a large component of household expenditure. When these inputs to household production vary widely across households with the same wage rates and demographic characteristics, this can have significant implications for the nature of the across-household relationships among second earner labour supply, household income and utility possibilities. We bring out the importance of these to the design of optimal tax systems⁷ by showing that the properties of the optimal tax system and the comparison between joint and individual taxation are significantly affected by the choice of model and its assumptions on productivities and prices.

In Model 1, with standard stylised facts on the compensated labour supply elasticities of primary and second earners respectively, there are gains in effi-

earner's income. In Germany and the US, contributions to social security, which are effectively part of the tax system, vary with the income of the second earner. See Apps and Rees (2009), Ch 6, and Feldstein and Feenberg (1996).

⁴As explicated for example in Blundell and McCurdy (1999).

⁵See Apps (1982) for a model of intra-family production and exchange, and the effects of a gender gap in the "outside" wage on the intra-family terms of trade. See also Apps and Rees (1999).

⁶For further discussion of this point see Apps and Rees (2009), pp 88-96.

⁷Apps and Rees (1988), (1997), (2009) give extensive discussion of this in the context of linear taxation. Kleven et. al. (2009) and Immervoll et. al. (2011) take aspects of household production into account in their analyses of optimal taxation in the presence of second earner labour force participation decisions. In both these papers, second earners work either full time or not at all. In Kleven et. al. all face the same wage rate. In Immervoll et. al., for both earners only the participation decision is analysed, but there is a continuum of productivity types for each earner.

ciency in moving from optimal joint to optimal individual taxation.⁸ However, since this move tends to redistribute the tax burden from two-earner to single-earner households, and, in this model, household utility is strictly increasing with household income, the equity effects are adverse and may outweigh the efficiency gains. Under Model 2, these adverse effects are much weaker, or may actually be replaced by distributional improvements. In other words, the analysis of marginal rate progressive piecewise linear tax systems in the presence of a more plausible system of household production strengthens the case for individual taxation, even when not selective, still further.⁹

The crux of the issue is that the second earner labour supply decision is at the same time a decision about the substitution of bought in child care for parental care, the value of which is not included in household income. Therefore when these decisions vary substantially across households, income derived from market labour supply becomes an unreliable measure of a household's real standard of living. Individual taxation may improve distributional outcomes over joint taxation by imposing a lower tax burden on a household with a higher market labour supply at a given level of household income. We argue that the second model, as compared to the first, is a far better representation of actual household decision taking, and that the results that follow from it provide a much sounder basis for family tax policy.

2 Two Household Models

In Model 1 the two adults allocate time between market work and leisure,¹⁰ with individuals being equally productive across households in producing output from one unit of leisure time. In Model 2 we assume the primary earner allocates time to market work and leisure, while the second earner allocates time to market work and household production in the form of child care.¹¹ There is also a market child care input with the price varying exogenously across households. This price then becomes a dimension of "household type". The two adults in a household are designated as primary and second earners respectively, with the former receiving a strictly higher wage than the latter.¹²

The tax system pays households a uniform lump sum¹³ and taxes the labour incomes of the two earners according to a two-bracket piecewise linear rate

⁸This is to be expected given standard Ramsey-type considerations.

⁹Immervoll et al (2011) confirm this result empirically for a significant number of OECD countries.

¹⁰As for example in Boskin and Sheshinski (1983).

¹¹Nothing would be gained by having both parents consume leisure and contribute to household production. Although that would be more realistic, we think the assumption made here captures the salient aspects of reality - the differing margins of substitution facing primary and second earners - while keeping the model simple.

¹²This almost follows from the definition of "primary" and "second" earners, according to which the latter's income is by definition smaller. It simply rules out the possibility that the higher wage partner works sufficiently fewer hours that she has the lower income.

¹³Which could be thought of as a child benefit, though here it does not vary with the number of children.

schedule, which determines how the lump sum payment is funded. We consider the implications first, of taxing their joint income, and secondly, of taxing them separately but under the same schedule. In each case we characterise the optimal tax schedule and discuss its main properties. We then go on to set up a numerical model that allows us to compare the welfare levels, tax rates and the extent of redistribution under the two systems. We carry out this analysis for both models, in order to examine the implications for the comparison of the two types of tax system of a change in the source of across-household heterogeneity in second earner labour supply and income - from wage rates alone, as in the first model, to wage rates, productivities in household production and prices of the bought-in market child care input, as in the second.

2.1 Model 1

There is a composite market consumption good, x . Individuals face given gross wage rates w , representing their productivities in a linear aggregate production technology that produces x , and have earnings y from their labour supply. P types of primary and S types of second earners are defined by their wage rates, with $w_1 \in \{w_1^1, w_1^2, \dots, w_1^P\}$ and $w_2 \in \{w_2^1, w_2^2, \dots, w_2^S\}$, $w_2^1 < w_1^1$, $w_2^S < w_1^P$ and in every household $w_2 < w_1$. Subject to this restriction, household type is then defined by the pair (w_1, w_2) . Let h index these pairs (w_{1h}, w_{2h}) lexicographically so that, for any pair of indices h, h' ,

$$h > h' \Leftrightarrow w_{1h} > w_{1h'} \text{ or } w_{1h} = w_{1h'} \text{ and } w_{2h} > w_{2h'} \quad i = 1, 2, \quad h = 1, \dots, H$$

This convention determines how household welfare,¹⁴ labour supply and income will vary with h . Note that it does not imply that household income increases monotonically with h , since one household may have a higher primary wage than another but a sufficiently lower second wage that household income is lower.

The household's utility function¹⁵ is

$$u_h = x_h - \sum_{i=1}^2 u_i(l_{ih}) \quad h = 1, \dots, H \quad (1)$$

where the $u_i(\cdot)$ are identical across households for given i , strictly increasing and strictly convex in labour supplies l_{ih} . In the tax analysis later it will be more useful to work with gross labour earnings or incomes $y_{ih} = w_{ih}l_{ih}$, and so we rewrite the utility function as

$$u_h = x_h - \sum_{i=1}^2 u_i(y_{ih}/w_{ih}) = x_h - \sum_{i=1}^2 \psi_i(y_{ih}, w_{ih}) \quad h = 1, \dots, H \quad (2)$$

¹⁴Of course, only individuals, and not households, can have "welfare", but we use this term to refer to the set of feasible utility pairs that a household can choose from.

¹⁵The quasilinear and additively separable form assumed here, though special, is very convenient, since it eliminates income effects and greatly simplifies the presentation of the optimal tax formulas.

where the $\psi_i(\cdot)$ are strictly increasing and convex and possess the single-crossing property

$$\frac{\partial}{\partial w_{ih}} \left[\frac{\partial \psi_i}{\partial y_{ih}} \right] < 0 \quad i = 1, 2, \quad h = 1, \dots, H \quad (3)$$

This says that the higher the wage type, the lower the marginal effort cost to i of achieving a given increase in labour earnings.

The household budget constraint is given by

$$x_h \leq \sum_{i=1}^2 y_{ih} - T(y_{1h}, y_{2h}) \quad h = 1, \dots, H \quad (4)$$

where the tax function $T(y_{1h}, y_{2h})$ is further specified in Section 3 below.

To retain the assumption that preferences are identical across households, as is usually assumed in optimal tax analysis, we allow differences in preferences between primary and second earners within a household, but not between primary (resp. second) earners across households. We assume that the second earner has a greater preference for leisure to capture the fact that second earner hours are typically below primary earner hours at a given wage rate. Under this limited degree of preference variation, heterogeneity across households in second earner labour supply and income at a given primary earner wage is driven entirely by variation in the second earner wage, so that a household with low second earnings must have a low second wage. We adopt a model with this assumption because it underpins the commonly made argument for joint taxation, implying as it does redistribution from households with high to households with low second incomes.¹⁶

A serious difficulty with the assumption of identical second earner preferences across households is that the observed high degree of heterogeneity in labour supplies among second earners cannot be explained because wage rate variations in conjunction with empirical elasticity estimates are not large enough to do this. This contrasts with Model 2 in which differences in productivity and child care prices can provide a realistic explanation of variation of second earner labour supply, without having to introduce preference heterogeneity among second earners.

2.2 Model 2

Model 2 introduces a household production system which is not just empirically more relevant, but very importantly changes the nature of the relationship between income and a household's utility possibilities.¹⁷ In addition to the market consumption good x , household utility also depends on child care z , which is produced using the second earner's time input c , and a bought-in market child care input, b , according to a standard strictly quasiconcave and increasing production function

$$z_h = z(b_h, c_h; k_h) \quad (5)$$

¹⁶ Often called the "income splitting advantage".

¹⁷ As we have previously argued. See for example Apps and Rees (1988), (1997), (2009).

where $k \in \{k_1, k_2, \dots, k_R\}$ is a measure of the household's productivity, and can be thought of as depending positively and exogenously on its physical and human capital and on the quality of the child care good b . We define child care broadly, to denote not just physically looking after the child, but rather all the activities that contribute to the child's welfare and development of human capital. As noted earlier, for simplicity we assume that only the second earner supplies time to child care, while the primary earner divides his time between market work and leisure, as in Model 1.

In addition, we assume that the exogenously given price $p \in \{p_1, p_2, \dots, p_Q\}$ of b may be different for different households. Thus this model adds two further dimensions to household type, which now depends on the vector of variables (w_1, w_2, p, k) , with again $w_1 \in \{w_1^1, w_1^2, \dots, w_1^P\}$, $w_2 \in \{w_2^1, w_2^2, \dots, w_2^S\}$, $w_2^1 < w_1^1$, $w_2^S < w_1^P$ and in every household $w_2 < w_1$. We extend the previous method for defining the type index h by again taking a lexicographic ordering such that, for any pair h, h'

$$h > h' \Leftrightarrow w_{1h} > w_{1h'} \quad (6)$$

$$\text{or } w_{1h} = w_{1h'} \text{ and } w_{2h} > w_{2h'} \quad (7)$$

$$\text{or } w_{1h} = w_{1h'} \text{ and } w_{2h} = w_{2h'} \text{ and } p_h > p_{h'} \quad (8)$$

$$\text{or } w_{1h} = w_{1h'} \text{ and } w_{2h} = w_{2h'} \text{ and } p_h = p_{h'} \text{ and } k_h > k_{h'} \quad (9)$$

The vector (w_1^1, w_2^1, p_1, k_1) is denoted by $h = 1$ and (w_1^P, w_2^S, p_Q, k_R) by $h = H = P \times S \times Q \times R$. Thus, in this model, at any given primary earner wage rate, across-household heterogeneity is driven by child care price and productivity variation as well as by second earner's wage variation.

The household's utility possibilities increase *ceteris paribus* monotonically with increasing wage rates and productivity and decreasing child care price.¹⁸ However, the relationship between household income and utility possibilities is no longer necessarily positive or monotonic. It depends on exactly how *ceteris paribus* changes in a wage rate, productivity or child care price cause changes in labour supply, income and achieved household utility.

The household utility function is now given by

$$u_h = x_h - \psi_1(y_{1h}, w_{1h}) + \hat{u}(z_h) \quad h = 1, \dots, H \quad (10)$$

The $\hat{u}(\cdot)$ function is strictly increasing and strictly concave. For the second earner, the time spent in market work and child care must sum to the total time endowment, normalised at 1, and so we have

$$c_h + l_{2h} = 1 \quad h = 1, \dots, H \quad (11)$$

where l_{2h} is second earner market labour supply.

There is however a further time constraint: Although second earner time and bought in child care may not be perfect substitutes as inputs in producing

¹⁸ Assuming that some positive amount of child care is bought, which, given our broad definition of child care, seems a reasonable assumption.

child care, realistically it is the case that every hour the second earner spends at work requires an hour of child care, in which case $b_h = l_{2h}$. Recalling that $y_{2h} = w_{2h}l_{2h}$, we can use these time constraints to eliminate c_h and b_h and rewrite $\hat{u}(\cdot)$ as

$$\hat{u}[z(b_h, c_h; k_h)] \equiv \hat{u}[z(y_{2h}/w_{2h}, 1 - y_{2h}/w_{2h}; k_h)] \equiv -\psi_2(y_{2h}; w_{2h}, k_h) \quad (12)$$

Writing the household budget constraint as

$$x_h + p_h b_h = x_h + p_h y_{2h}/w_{2h} \leq \sum_{i=1}^2 y_{ih} - T(y_{1h}, y_{2h}) \quad h = 1, \dots, H \quad (13)$$

with $p_h \in \{p^1, p^2, \dots, p^Q\}$, we again have a model that can be used to derive the household's indirect utility function with the tax parameters as arguments.

3 Tax Functions

The tax functions $T(y_{1h}, y_{2h})$ are specified as follows.

Joint Taxation:

There is a two-bracket piecewise linear tax on total household labour earnings, the parameters of which are $(\alpha, \tau_1, \tau_2, \eta)$, where α is a uniform lump sum paid to every household, τ_1 and τ_2 are the marginal tax rates in the lower and upper brackets of the tax schedules, and η is the value of *joint earnings* defining the bracket limit. Thus the household tax function $T(y_{1h}, y_{2h}) \equiv T(y_h)$, with $y_h = \sum_{i=1}^2 y_{ih}$, is defined by:

$$T(y_h) = -\alpha + \tau_1 y_h \quad y_h \leq \eta \quad (14)$$

$$T(y_h) = -\alpha + \tau_2 y_h + (\tau_1 - \tau_2)\eta \quad y_h > \eta \quad h = 1, \dots, H \quad (15)$$

Individual Taxation:

There is a two-bracket piecewise linear tax system now applied to individual labour earnings, the parameters of which are (a, t_1, t_2, y) , where a is again a uniform lump sum paid to every household, t_1 and t_2 are the marginal tax rates in the lower and upper brackets, and y is the value of *individual earnings* defining the bracket. Thus the individual tax function $\hat{T}(y_{ih})$ is defined by:

$$\hat{T}(y_{ih}) = t_1 y_{ih} \quad y_{ih} \leq y \quad (16)$$

$$\hat{T}(y_{ih}) = t_2 y_{ih} + (t_1 - t_2)y \quad y_{ih} > y \quad h = 1, \dots, H \quad (17)$$

and the household tax function is $T(y_{1h}, y_{2h}) \equiv -a + \sum_{i=1}^2 \hat{T}(y_{ih})$. Note that this specification of the tax function implies that $\partial^2 T(y_{1h}, y_{2h}) / \partial y_{1h} \partial y_{2h} = 0$, and so does not allow the marginal tax rate paid by one earner in the household to depend on the income of the other.¹⁹ In what follows, as mentioned in the

¹⁹The analysis of optimal nonlinear taxation of couples shows that in general the marginal tax rate of one earner in the household will depend on the wage type of the other (see the references given in footnote 2). Thus restriction to a piecewise linear tax system implies sacrificing some social welfare in exchange for a more practicable and implementable tax system.

Introduction, we assume that we have the convex case, in which at the tax optima $\tau_1 < \tau_2$ and $t_1 < t_2$. Every household faces the same convex budget set.

4 Household Allocations

We present the analysis of the household's choice of consumption and wage earnings under each of the two alternative tax systems, first joint and then individual taxation, for Models 1 and 2 respectively.

4.1 Model 1

4.1.1 Joint Taxation

A household solves the problem

$$\max_{x_h, y_{ih}} u_h = x_h - \sum_{i=1}^2 \psi_i(y_{ih}, w_{ih}) \quad (18)$$

subject to a budget constraint determined by the tax system, as just described. We consider three cases which provide the results we require, the partial derivatives of the household's indirect utility function with respect to the tax parameters. We write below the constraints for each of these cases together with these derivatives.

Case 1. The household is at the optimum in the interior of the lower tax bracket. It therefore faces the budget constraint:

$$x_h = \alpha + (1 - \tau_1) \sum_i y_{ih} \quad (19)$$

and the first order conditions imply:

$$\frac{\partial \psi_i}{\partial y_{ih}} = 1 - \tau_1 \quad i = 1, 2, \quad (20)$$

giving the earnings supply functions $y_{ih}(\tau_1, w_{ih})$. The properties of the functions $\psi_i(\cdot)$ imply

$$\frac{\partial y_{ih}(\tau_1, w_{ih})}{\partial \tau_1} < 0, \quad i = 1, 2, \quad (21)$$

where, note, this is a compensated derivative.

We write the household indirect utility function as $v_h(\alpha, \tau_1; w_{1h}, w_{2h})$, with, by the Envelope Theorem,

$$\frac{\partial v_h}{\partial \alpha} = 1; \quad \frac{\partial v_h}{\partial \tau_1} = -y_h^* = - \sum_i y_{ih}(\tau_1, w_{ih}) \quad i = 1, 2, \quad (22)$$

Case 2. The household is effectively constrained at the bracket limit η , in the sense that it chooses $y_h = \eta$, but would prefer to increase its labour supply and

earnings if it would be taxed at the rate τ_1 , but not if it would be taxed at the rate τ_2 . We formulate its allocation problem by adding the constraint $y_h \leq \eta$, noting that this will be binding at the optimum.²⁰ We can write the first order conditions as

$$(1 - \tau_1) - \frac{\partial \psi_i}{\partial y_{ih}} - \mu_h = 0 \quad i = 1, 2, \quad (23)$$

$$y_h \leq \eta \quad \mu_h \geq 0 \quad \mu_h [y_h - \eta] = 0 \quad (24)$$

where μ_h is the multiplier associated with the constraint $y_h \leq \eta$.

We write the indirect utility function as $v_h(\alpha, \tau_1, \eta; w_{1h}, w_{2h})$, with, by the Envelope Theorem,

$$\frac{\partial v_h}{\partial \alpha} = 1; \quad \frac{\partial v_h}{\partial \tau_1} = -\eta; \quad \frac{\partial v_h}{\partial \eta} = (1 - \tau_1) - \frac{\partial \psi_i}{\partial y_{ih}} \geq 0 \quad (25)$$

Intuitively, the idea of the expression for $\partial v_h / \partial \eta$ is that a small relaxation of the constraint would increase consumption and utility at the rate $(1 - \tau_1)$, which exceeds for almost every individual the marginal cost of effort $\partial \psi_i / \partial y_{ih}$. In diagrammatic terms, the household is at the kink in its budget constraint at the bracket limit η . The term is zero only if i 's marginal rate of substitution happens to equal $(1 - \tau_1)$ at the kink. Note that condition (23) implies that the individuals' marginal effort costs are equalised also in this type of equilibrium.

Case 3. The household is in equilibrium in the interior of the upper income bracket. We therefore replace the previous budget constraint by

$$x_h \leq \alpha + (1 - \tau_2)y_h + (\tau_2 - \tau_1)\eta \quad (26)$$

and the first order conditions imply

$$\frac{\partial \psi_i}{\partial y_{ih}} = 1 - \tau_2 \quad i = 1, 2, \quad (27)$$

giving the earnings supply functions $y_{ih}(\tau_2, w_{ih})$. The properties of the functions $\psi(\cdot)$ imply

$$\frac{\partial y_{ih}(\tau_2, w_{ih})}{\partial \tau_2} < 0, \quad \frac{\partial y_{ih}(\tau_2, w_{ih})}{\partial w_{ih}} > 0 \quad i = 1, 2, \quad (28)$$

Writing the indirect utility function as $v_h(\alpha, \tau_1, \tau_2, \eta; w_{1h}, w_{2h})$ we now obtain

$$\frac{\partial v_h}{\partial \alpha} = 1; \quad \frac{\partial v_h}{\partial \tau_1} = -\eta; \quad \frac{\partial v_h}{\partial \tau_2} = -(y_h^* - \eta); \quad \frac{\partial v_h}{\partial \eta} = \tau_2 - \tau_1 > 0 \quad (29)$$

In all three cases, it follows from the properties of the function $\psi(\cdot)$ that $\partial v_h / \partial w_{ih} > 0$, $i = 1, 2$, $h = 1, \dots, H$.

Given these three cases, we define a partition $\{\mathcal{H}_0, \mathcal{H}_1, \mathcal{H}_2\}$ of the index set $\{1, 2, \dots, H\}$ as follows:

$$\mathcal{H}_0 = \{h \mid 0 \leq y_h^* < \eta\} \quad (30)$$

²⁰Case 1 can be thought of as the case in which this constraint is non-binding.

$$\mathcal{H}_1 = \{ h \mid y_h^* = \eta \} \quad (31)$$

$$\mathcal{H}_2 = \{ h \mid y_h^* > \eta \} \quad (32)$$

where y_h^* is the household's optimal income under the given tax structure. In all of what follows we assume that we are dealing with tax systems in which each of these subsets is non-empty. Total household gross and net income and therefore, in this model, household utility are increasing as we move from \mathcal{H}_0 to \mathcal{H}_1 to \mathcal{H}_2 , though these may not increase monotonically with h as pointed out earlier. Important points to note are that:

- τ_1 is a marginal tax rate for $h \in \mathcal{H}_0$ but defines an intra-marginal, non-distortionary tax for $h \in \mathcal{H}_1 \cup \mathcal{H}_2$
- A small increase in η has no effect for $h \in \mathcal{H}_0$, yields a net welfare gain for almost all $h \in \mathcal{H}_1$, and yields a lump sum income gain proportional to $(\tau_2 - \tau_1)$ for $h \in \mathcal{H}_2$ (recall we assume that $\tau_2 > \tau_1$)
- In effect, for purposes of the tax analysis the household can be treated as a single individual, given that at each level of household income individual earnings are chosen so as to equate marginal effort costs, i.e. to minimise the cost of supplying that level of earnings, because the budget constraint is defined only on total household income.²¹

4.1.2 Individual Taxation

With individual income as the tax base, and given that (by definition) the second earner's income is always below that of the primary earner, we can define six possible cases for the household equilibrium. In each case we present the earnings and indirect utility functions and partial derivatives of the latter with respect to the tax instruments.

Case 1: $y_{ih}^* < y$, $i = 1, 2$. In this case the household's budget constraint, earnings and indirect utility functions are identical to those in Case 1 of joint taxation.

Case 2: $y_{2h}^* < y = y_{1h}^*$. The results here are derived by imposing the constraint $y_{1h} \leq y$ on the problem and noting that it is binding at the optimum. Thus we have $y_{2h}^* = y_{2h}(t_1, w_{2h})$, and $v_h(a, t_1, y; w_{1h}, w_{2h})$, with

$$\frac{\partial v_h}{\partial a} = 1; \quad \frac{\partial v_h}{\partial t_1} = -(y + y_{2h}^*); \quad \frac{\partial v_h}{\partial y} = (1 - t_1) - \frac{\partial \psi_1}{\partial y_{1h}} \quad (33)$$

Case 3: $y_{ih}^* = y$, $i = 1, 2$. Here we impose the two constraints $y_{ih} \leq y$ and take them as both binding at the optimum, giving $v_h(a, t_1, y; w_{1h}, w_{2h})$ and

$$\frac{\partial v_h}{\partial a} = 1; \quad \frac{\partial v_h}{\partial t_1} = -2y; \quad \frac{\partial v_h}{\partial y} = 2(1 - t_1) - \sum_i \frac{\partial \psi_i}{\partial y_{ih}} \quad (34)$$

²¹To see this, note that we can solve the household's problem in two steps. First solve $\min_{y_{ih}} \sum_i \psi_i(y_{ih}, w_{ih})$ subject to $\sum_i y_{ih} \leq y_h$ for any given y_h , and define $\psi_h(y_h)$ as the value function of this problem. Then solve $\max_{x_h, y_h} x_h - \psi_h(y_h)$ subject to the relevant budget constraint in each case.

Case 4: $y_{2h}^* < y < y_{1h}^*$. In this case the budget constraint becomes

$$x_h \leq a + (t_2 - t_1)y + (1 - t_2)y_{1h} + (1 - t_1)y_{2h} \quad (35)$$

and we have $y_{1h}^* = y_{1h}(t_2, w_{1h})$, $y_{2h}^* = y_{2h}(t_1, w_{2h})$ and the indirect utility function $v_h(a, t_1, t_2, y; w_{1h}, w_{2h})$ with

$$\frac{\partial v_h}{\partial a} = 1; \quad \frac{\partial v_h}{\partial t_1} = -(y + y_{2h}^*); \quad \frac{\partial v_h}{\partial t_2} = (y_{1h}^* - y); \quad \frac{\partial v_h}{\partial y} = t_2 - t_1 \quad (36)$$

Case 5: $y_{2h}^* = y < y_{1h}^*$. We now have $y_{1h}^* = y_{1h}(t_2, w_{1h})$ and the indirect utility function $v_h(a, t_1, t_2, y; w_{1h}, w_{2h})$ with

$$\frac{\partial v_h}{\partial a} = 1; \quad \frac{\partial v_h}{\partial t_1} = -2y; \quad \frac{\partial v_h}{\partial t_2} = (y_{1h}^* - y); \quad \frac{\partial v_h}{\partial y} = t_2 - t_1 + (1 - t_1) - \frac{\partial \psi}{\partial y_{2h}} \quad (37)$$

Case 6: $y_{ih}^* > y$, $i = 1, 2$. This gives $y_{ih}^* = y_{ih}(t_2, w_{ih})$, $i = 1, 2$, and $v_h(a, t_1, t_2, y; w_{1h}, w_{2h})$ with

$$\frac{\partial v_h}{\partial a} = 1; \quad \frac{\partial v_h}{\partial t_1} = -2y; \quad \frac{\partial v_h}{\partial t_2} = \sum_i (y_{ih}^* - y); \quad \frac{\partial v_h}{\partial y} = 2(t_2 - t_1) \quad (38)$$

We define the partition of the index set corresponding to these six cases, $\{H_0, H_1, \dots, H_5\}$, as follows:

$$H_0 = \{ h \mid 0 \leq y_{ih}^*(t_1) < y, \ i = 1, 2 \} \quad (39)$$

$$H_1 = \{ h \mid y_{2h}^*(t_1) < y = y_{1h}^* \} \quad (40)$$

$$H_2 = \{ h \mid y_{ih}^* = y, \ i = 1, 2 \} \quad (41)$$

$$H_3 = \{ h \mid y_{2h}^*(t_1) < y < y_{1h}^*(t_2) \} \quad (42)$$

$$H_4 = \{ h \mid y_{2h}^* = y < y_{1h}^*(t_2) \} \quad (43)$$

$$H_5 = \{ h \mid y_{ih}^*(t_2) > y, \ i = 1, 2 \} \quad (44)$$

The obvious difference to the joint taxation case is that only in subsets H_0 and H_5 , where both the individuals in the household are in the interior of the same tax bracket, will the marginal rates of substitution between consumption and labour supply of primary and second earners be equalised. In all other cases they will not in general be the same, as each earner chooses their individually optimal earnings levels.

Contrasting the partition defined by (39)-(44) in this case with that in (30)-(32) for the joint taxation case makes clear the essential difference between joint and individual taxation. The latter implies a much finer partition into subsets reflecting likely differences in responsiveness of *individual* earnings (labour supply) decisions to tax rates, which is the source of the efficiency gains brought out by the analysis of optimal linear taxation²² and tax reform.²³ Lower wage

²²See Boskin and Sheshinski (1983). Note though that in the linear taxation case individual taxation must correspond to *selective* taxation as defined earlier.

²³See Apps and Rees (1999).

second earners, who empirically have relatively high labour supply elasticities, are able to choose incomes which place them in the lower tax bracket regardless of primary earner income. A further important difference is that under joint taxation, an increment in one earner's income may increase the tax rate both face, if it moves them into a higher bracket, but this cannot happen under individual taxation.

The equity effects of this finer matching of individuals with tax brackets are less easy to establish. Those households with lower second earner labour supplies tend to be made worse off by a switch from joint to individual taxation, since, to satisfy overall revenue neutrality, the tax burden on primary earners will be increased while that on second earners is reduced.²⁴ As the simulation results for Model 1 in Section 6 indicate, for sufficiently small differences in primary and second earner elasticities these equity effects can cause joint taxation to yield higher social welfare than individual taxation, contrary to the results presented in Boskin and Sheshinski (1983), which have become the "conventional wisdom" in this area.

4.2 Model 2

In applying this model to the optimal tax analysis, the key relationships are again the indirect utility function and its derivatives with respect to the tax parameters. The specifics of these will, as before, depend on whether we have individual or joint taxation. However, we can show that for Model 2 it is possible to write the expressions for the derivatives of indirect utility with respect to the tax parameters in each case in exactly the same form as for Model 1, despite the fairly radical differences in the underlying structural forms of the two models.²⁵ This leads to a considerable economy of effort in deriving and interpreting the optimal tax conditions, but, as we emphasise, this should not be at the cost of drawing the false conclusion that the results of the two models are "essentially" the same.

Thus, consider the Lagrange functions corresponding to the optimisation problems in the cases of Model 1 and Model 2 households respectively:

Model 1:

$$L_h = x_h - \sum_{i=1}^2 \psi_i(y_{ih}, w_{ih}) + \lambda_h \left[\sum_{i=1}^2 y_{ih} - T(y_{1h}, y_{2h}) - x_h \right] \quad (45)$$

²⁴For an analysis of this in the tax reform context see Apps and Rees (1999), (2009).

²⁵See also Sandmo (1990). This is because the "sufficient statistics" for the optimal taxes, in the sense of Chetty (2009), are just the derivatives of earnings/labour supplies with respect to the tax parameters, the marginal social utilities of incomes of the various household types and their proportions in the population. The "reduced forms" of the tax conditions are the same for the different structural models, but these structural differences do matter profoundly, not least in determining the basis for the empirical measurement of the reduced form parameters.

Model 2:

$$L_h = x_h - \psi_1(y_{1h}, w_{1h}) - \psi_2(y_{2h}; w_{2h}, k_h) + \lambda_h \left[\sum_{i=1}^2 y_{ih} - T(y_{1h}, y_{2h}) - x_h - p_h y_{2h} / w_{2h} \right] \quad (46)$$

for $h = 1, 2, \dots, H$. Since the tax parameters do not enter the utility functions in either problem, and p_h is taken as exogenously given throughout, by the Envelope Theorem the derivatives of the indirect utility functions $v_h(\alpha, \tau_1, \tau_2, \eta)$ and $v_h(a, t_1, t_2, y)$ will take the same form in each model whenever the tax function $T(y_{1h}, y_{2h})$ is also of the same form. Since these derivatives are all we use in the optimal tax analysis we obtain precisely the same general form of conditions on the tax parameters whether we take Model 1 or Model 2 as the household model. What is important however is that because of the underlying model structure, both the interpretation of the optimal tax conditions and their policy implications change fundamentally.

To see this, consider for example the first order conditions for the household's optimal allocation in the case under joint taxation in which the household is in the lower tax bracket. Then we have (recalling the definition of the function $\psi_2(\cdot)$ in this model)

$$\frac{\partial \psi_1}{\partial y_{1h}} = 1 - \tau_1 \quad h = 1, \dots, H \quad (47)$$

$$\hat{u}' \left[\frac{\partial z(b_h, c_h; k_h)}{\partial c_h} - \frac{\partial z(b_h, c_h; k_h)}{\partial b_h} \right] = (1 - \tau_1) w_{2h} - p_h \quad h = 1, \dots, H \quad (48)$$

where it is the condition (48) on the second earner's time use that is of main interest here. The right hand side of this condition makes clear that, far from being a "fixed cost of working", the cost of market child care is similar to a tax, since it must be paid for each hour of market labour supply. Thus both the decision to participate in the labour market and the choice of hours are important, and indeed inseparable. Variation in the price of bought in care will have an effect on the second earner's time allocation analogous to the net of tax wage rate, though, by the Envelope Theorem, its effect on household utility is proportional to the amount of child care bought in.²⁶

The left hand side of (48) gives the marginal value product of the second earner's time spent in child care net of that of bought in child care, thus giving the opportunity cost to the household of a diversion of a unit of the second earner's time to the labour market. This suggests that, even where the price of bought in child care is higher than the second earner's net of tax wage, she may still work in the market and buy in child care if it is sufficiently more productive than her own at the margin. Overall, since this condition determines the second earner's labour supply, it emphasises not only preferences (the marginal utility of child care \hat{u}' in terms of consumption) and the net of tax wage rate, but also the relative productivities of parental and market child care and the price of the latter, as the underlying determinants of labour supply elasticities that are

²⁶See the Lagrange function in (46) above.

relevant for the tax analysis. This gives a much richer theory of second earner labour supply than the standard "everything depends on preferences" model.

The indirect utility function derived from this model includes, as well as the relevant tax parameters, the vector of variables determining household type, $(w_{1h}, w_{2h}, k_h, p_h)$, and it is easy to see from this that the conditions under which households with the same joint income will have the same achieved utility level become far more stringent, and the closeness of the relationship between household income and utility becomes far weaker, due to the increasing steepness of the labour supply profile as the wage elasticity rises, as illustrated in Figure 2 of Section 6.

5 Optimal Tax Analysis

5.1 Joint Taxation

The planner solves

$$\max_{\alpha, \tau_1, \tau_2, \eta} W = \sum_{h=1}^H \phi_h S(v_h) \quad (49)$$

subject to the public sector budget constraint²⁷

$$\sum_{h \in \mathcal{H}_0} \phi_h \tau_1 y_h + \sum_{h \in \mathcal{H}_1} \phi_h \tau_1 \eta + \sum_{h \in \mathcal{H}_2} \phi_h [\tau_2 y_h + (\tau_1 - \tau_2) \eta] \geq \alpha \quad (50)$$

where ϕ_h is the proportion of households of type $h = 1, 2, \dots, H$, and $S(\cdot)$ is a strictly concave and increasing function expressing the planner's preferences over household utilities. From the first order conditions characterising the optimal tax parameters²⁸ we can derive:

Proposition 1: The optimal tax parameters satisfy the conditions:

$$\sum_{h=1}^H \phi_h (\sigma_h - 1) = 0 \quad (51)$$

$$\tau_1 = \frac{\sum_{\mathcal{H}_0} \phi_h (\sigma_h - 1) y_h^* + \eta \sum_{\mathcal{H}_1 \cup \mathcal{H}_2} \phi_h (\sigma_h - 1)}{\sum_{\mathcal{H}_0} \phi_h \partial y_h / \partial \tau_1} \quad (52)$$

$$\tau_2 = \frac{\sum_{\mathcal{H}_2} \phi_h (\sigma_h - 1) (y_h^* - \eta)}{\sum_{\mathcal{H}_2} \phi_h \partial y_h / \partial \tau_2} \quad (53)$$

$$\sum_{\mathcal{H}_1} \phi_h \left\{ \sigma_h \left[(1 - \tau_1) - \frac{\partial \psi}{\partial y_h} \right] + \tau_1 \right\} = -(\tau_2 - \tau_1) \sum_{\mathcal{H}_2} \phi_h (\sigma_h - 1) \quad (54)$$

²⁷We assume the aim of taxation is purely redistributive. Adding a non-zero revenue requirement would make no essential difference to the results.

²⁸Of course, exactly which households will be in which subsets is determined at the optimum, and depends on the values of the tax parameters. The following discussion characterises the optimal solution *given* the allocation of households to subsets that obtains at this optimum.

where y_h^* denotes household income at the optimum and σ_h is the marginal social utility of income to household h .

We first interpret and discuss those properties of these conditions which are common to both the models 1 and 2. In the following subsection we examine how the choice of model affects the interpretation of the conditions.

Condition (51) is familiar from linear tax theory: the optimal lump sum α equalises the average of the marginal social utilities of household income, σ_h , in terms of the numeraire, to the marginal cost of one unit of the lump sum, which of course is 1. Denoting the shadow price of the government budget constraint by λ , $\sigma_h \equiv S'(v_h)/\lambda$, and so the concavity of $S(\cdot)$ implies that σ_h falls with the utility level of the household. From now on we denote $\sigma_h - 1$ by δ_h . Then $\delta_h > (<)0$ according as household h is relatively worse (better) off than the average *in utility terms*.²⁹

The two conditions corresponding to the tax rates τ_1, τ_2 , are analogous to those obtained in optimal linear tax theory. The denominators are the average compensated derivatives of earnings with respect to the tax rates, and so give a measure of the marginal deadweight loss of the tax rate at the optimum, the efficiency cost of the tax. The numerators give the equity effects. The two terms in the numerator of (51) correspond to the two ways in which the lower bracket tax rate affects the contributions households make to funding the lump sum payment α . Given their optimal earnings y_h^* , the first term aggregates over subset \mathcal{H}_0 , which is the subset with relatively lower incomes, the effect of a marginal tax rate change on welfare net of its marginal contribution to tax revenue, all in terms of the numeraire. The second term reflects the fact that the lower bracket tax rate is effectively a lump sum tax on income earned by the two higher income brackets, \mathcal{H}_1 and \mathcal{H}_2 , since a change in this tax rate has only an intramarginal effect, changing the tax they pay at a rate given by η , while leaving their (compensated) labour supply unchanged.

Only the first of these two effects is present in the condition (53) corresponding to the second tax rate. The portion of the income of the households in the higher tax bracket that is taxed at the rate τ_2 is $(y_h^* - \eta)$, and so this weights the effect on social welfare net of the effect on tax revenue. Note that, unlike the case of linear income taxation, these numerator terms are not covariances, since the mean of σ_h over each of the subsets is not 1. However, intuitively they can still be thought of as measures of the strength of the relationship between the marginal social utility of income and household incomes, which determines the effectiveness of the tax rate on *income* in redistributing *utility* across households. In other words, the goal of taxation is to redistribute utility, but the available instruments are the lump sum payment and marginal tax rates on income, and so the strength of the relationship between the marginal social utility of income and income determines the effectiveness of the income tax system in redistributing utility.

²⁹ Whether this corresponds to the household having a relatively lower or higher income depends on which model, 1 or 2, underlies the analysis.

It is interesting to rewrite this numerator term as

$$\sum_{\mathcal{H}_2} \phi_h \delta_h y_h^* - \eta \sum_{\mathcal{H}_2} \phi_h \delta_h \quad (55)$$

where the second term is seen to be the negative of the second term in the numerator of (52), net of the lump sum tax contribution of the subset \mathcal{H}_1 . This suggests that the greater the contribution of the lump sum tax on upper income bracket households arising from the tax rate τ_1 , the smaller is the tax rate τ_2 , and so the smaller is the distortionary effect on labour supplies in this bracket, other things being equal.³⁰

Condition (54), the condition on the bracket limit η , has the following interpretation. The left hand side represents the marginal social benefit of a relaxation of the bracket limit. This consists first of all of the gain to all those households who are effectively constrained at η . The first term in brackets on the left hand side is the net marginal benefit to these consumers, weighted by their marginal social utilities of income. The second term is the rate at which tax revenue increases given the increase in gross income resulting from the relaxation of the bracket limit. The right hand side gives the marginal social cost of the relaxation. Since $(\tau_2 - \tau_1) > 0$ by assumption, all households $h \in \mathcal{H}_2$ receive a lump sum income increase at this rate and this is weighted by the deviation of the marginal social utility of income of these households from the average. As long as the sum of these deviations, weighted by the frequencies of the household types, is negative, the marginal cost of the bracket limit increase is a worsening in the equity of the income distribution. The condition then trades off the social value of the gain to households in \mathcal{H}_1 against the social cost of making households in \mathcal{H}_2 better off. If however the right hand term was not positive, then this condition could not be satisfied and this would make untenable the assumption that $(\tau_2 - \tau_1) > 0$, in other words, that the optimal piecewise linear tax system is indeed convex. We have ruled this possibility out by assumption, though strictly speaking it is an empirical question as to whether this is really the case.

5.2 Individual Taxation

The planner solves

$$\max_{a, t_1, t_2, y} \sum_{h=1}^H \phi_h S(v_h) \quad (56)$$

subject now to the public sector budget constraint

$$\sum_{\cup_{i=0}^2 H_i} \phi_h t_1 y_h + \sum_{\cup_{i=3}^4 H_i} \phi_h [t_2 y_{1h} + t_1 y_{2h} + (t_1 - t_2)y] + \sum_{H_5} \phi_h [t_2 y_h + 2(t_1 - t_2)y] \geq a \quad (57)$$

³⁰It is this tradeoff which can lead to the nonconvex case in which the upper bracket tax rate is optimally lower than that in the lower bracket. For further discussion see Apps, Long and Rees (2012).

where again $y_h = \sum_{i=1}^2 y_{ih}$. In what follows it will be useful to denote by μ_{ih} the term $(1 - t_1) - \partial\psi/\partial y_{ih}$, the value of a relaxation of the bracket limit to an individual at the kink in the budget constraint. Then from the first order conditions for an optimal solution³¹ we derive:

Proposition 2: The optimal tax parameters in the case of individual taxation are characterised by the following conditions.

$$\sum_{h=1}^H \phi_h \delta_h = 0 \quad (58)$$

$$t_1^* = \frac{\sum_{H_0} \phi_h \delta_h y_h^* + \sum_{H_1 \cup H_3} \phi_h \delta_h y_{2h}^* + y^* [\sum_{H_1 \cup H_3} \phi_h \delta_h + 2 \sum_{H_2 \cup H_4 \cup H_5} \phi_h \delta_h]}{\sum_{H_0} \phi_h \partial y_{1h} / \partial t_1 + \sum_{H_0 \cup H_1 \cup H_3} \phi_h \partial y_{2h} / \partial t_1} \quad (59)$$

$$t_2^* = \frac{\sum_{H_3 \cup H_4 \cup H_5} \phi_h \delta_h (y_{1h}^* - y^*) + \sum_{H_5} \phi_h \delta_h (y_{2h}^* - y^*)}{\sum_{H_3 \cup H_4 \cup H_5} \phi_h \partial y_{1h} / \partial t_2 + \sum_{H_5} \phi_h \partial y_{2h} / \partial t_2} \quad (60)$$

$$\sum_{H_1 \cup H_2} \phi_h (\sigma_h \mu_{1h} + t_1) + \sum_{H_2 \cup H_4} \phi_h (\sigma_h \mu_{2h} + t_1) = -(t_2 - t_1) \left[\sum_{H_3 \cup H_4} \phi_h \delta_h + 2 \sum_{H_5} \phi_h \delta_h \right] \quad (61)$$

The first condition, since it involves the entire population, is exactly as for joint taxation. The remaining three conditions have basically the same interpretation as before, but of course the relevant sums are now over subsets of individuals reflecting the partition defined in the previous section. In particular, both numerator and denominator of the expression for t_1^* contain terms corresponding to lower wage second earners in households with higher wage primary earners who are in the higher tax bracket. Such households may well have lower total incomes than households with both earners in the lower tax bracket, given sufficiently low second and high primary incomes, but paying more tax. The welfare interpretation of this will depend however on which of the two models is the basis for the analysis, as we further discuss below.

By comparing the denominators of the expressions in (52), (53), (59), and (60), and given the stylised fact that second earners' labour supplies are significantly more sensitive to net wage rate changes than those of primary earners, we see that as between the cases of joint and individual taxation, the denominators of the lower tax rate will tend to increase and those of the higher tax rate to fall as a result of the switch of second earners to the lower tax bracket. This implies, other things being equal, a fall in the lower bracket tax rate relative to that in the higher bracket, and so an increase in the progressivity of the tax system.

As Apps and Rees (1997) show, this change in the tax system, with more second earners being taxed at a lower marginal rate, also results in a reduction

³¹ Again, exactly which households will be in which subsets is determined at the optimum, and depends on the values of the tax parameters.

in aggregate deadweight losses associated with the tax system, on standard Ramsey grounds, and a shift in the burden of taxation from households with relatively high to households with relatively low second earner labour supplies. Intuitively, the higher tax rate must be raised to compensate for the fall in tax revenue resulting from taxing second earners at a lower rate, the burden of this falling on higher income primary earners. On balance, households with higher second earner labour supplies benefit from this change, households with low second earner labour supplies lose. This is the source of the equity effects of the change from joint to individual taxation. The evaluation of these in terms of the net change in social welfare again depends crucially on which of the two household models forms the basis of the analysis, and so we now discuss the results for each of the models in turn.

5.2.1 Model 1

As we pointed out earlier, a key property of this model, which it shares with the standard individual labour supply model, is that household utility increases whenever individual wage rates and therefore³² household incomes increase. Thus, in this model, the criteria of horizontal and vertical equity, which could be interpreted as requiring, respectively, equal tax burdens for equal incomes and tax burdens that increase with household income, appear to be met. Individual taxation, on the other hand, results in a violation of this notion of horizontal equity, since, as just pointed out, a two-earner household may well have a larger income than another but be paying less tax. This then suggests the intuition that, for this model, moving from joint to individual taxation involves an equity-efficiency trade-off.

It is of interest however to note that even in this model this intuition is not completely correct. Unlike the case of the standard individual labour supply model, household income here is not an exact measure of household utility, since households may have the same income but different achieved utility levels. There is therefore a violation of horizontal equity in this case, if this is interpreted, as it should be, in terms of household utility rather than income - households with different utility levels may pay the same amount of tax. In fact we can prove:

Proposition 3: For Model 1, households with the same incomes will necessarily have the same utility levels if and only if labour supply elasticities of primary and second earners are identical.

Proof: For simplicity, suppose we have a continuum of wage pairs (w_{1h}, w_{2h}) yielding at the household optimum equal household incomes $y_h = \sum_{i=1}^2 w_{ih}l_{ih}(w_{ih})$ and so within this subset

$$dw_{2h} = -\frac{l_{1h} + w_{1h}l'_{1h}(w_{1h})}{l_{2h} + w_{2h}l'_{2h}(w_{2h})}dw_{1h} \quad (62)$$

Given the indirect utility functions $v_h(w_{1h}, w_{2h})$ with derivatives $\partial v_h/\partial w_{ih} =$

³²Recall that in this model income effects are ruled out.

l_{ih} we have within this subset

$$dv_h = [l_{1h} - l_{2h} \frac{l_{1h} + w_{1h}l'_{1h}(w_{1h})}{l_{2h} + w_{2h}l'_{2h}(w_{2h})}]dw_{1h} \quad (63)$$

Then we have that $dv_h = 0$, so that all households within the subset are equally well off, if and only if at every wage pair primary and second earner labour supply elasticities are identical, that is, $e_1 = e_2$, where

$$e_i = w_{ih}l'_{ih}(w_{ih})/l_{ih} \quad i = 1, 2 \quad (64)$$

Since empirically we have $e_1 < e_2$, this implies that household utility is actually rising as we move through the subset of households with equal incomes by increasing the primary wage. Intuitively, as we increase the primary wage the second earner's wage has to be reduced to hold income constant, but, because of the higher labour supply elasticity of second earners, by proportionately less than the increase in the primary wage, and the net effect is to increase household utility.

If we start with a case in which all labour supply elasticities are identical, there would be no case for individual taxation on either efficiency or equity grounds. As we start raising second earner elasticities relative to those of primary earners, we not only bring into being potential efficiency gains from a switch to individual taxation, but also create inequities in the joint taxation system, which become greater, along with the efficiency gains, as the ratio of primary to second earner elasticities increases.

5.2.2 Model 2

The key point about joint taxation in Model 2 is that two households with the same income but widely different utility levels pay the same tax. Individual taxation may improve distributional outcomes over joint taxation by imposing a lower tax burden on a household with a higher market labour supply at a given level of household income. For this reason, it can be welfare improving if, say, two full time earners in a low- to medium-wage household pay less tax in total than one with two high wage earners and a lower total labour income. Thus a feature of individual as against joint taxation that is welfare reducing in the context of Model 1 becomes welfare increasing in Model 2. In the next section we use parametrised versions of Models 1 and 2 to illustrate this argument. The numerical analysis will also bring out clearly the importance of the elasticity of substitution between parental and bought in child care in determining the relationship between income and utility across households, and therefore the different equity properties of the two tax systems.

6 Numerical Analysis

We use specific functional forms and numerical values for the key parameters to explore how the relationship between the maximised values of social welfare under joint and individual taxation changes when we replace Model 1 by Model 2. In each case we solve the model for the optimal parameters of the tax system by maximising a social welfare function (SWF) of the form $[\sum_{i=1}^n v_i^{1-\pi}]^{1/(1-\pi)}$, with π a measure of inequality aversion.

6.1 Model 1

We choose as the household utility function the simple quasilinear form

$$u_h = x_h - \gamma_1(y_{1h}/w_{1h})^{\alpha_1} - \gamma_2(y_{2h}/w_{2h})^{\alpha_2} \quad h = 1, \dots, H \quad (65)$$

with $\gamma_i > 0$ and $\alpha_i = (1 + e_i)/e_i$ where e_i is the elasticity of labour supply with respect to the net wage. With preference variation between primary and second earners, but not across households, we calibrate

$$\gamma_i = (1/\alpha_i)(\bar{y}_i/\bar{l}_i)^{\alpha_i} \quad i = 1, 2 \quad (66)$$

where \bar{y}_i , \bar{w}_i , and \bar{l}_i are representative values of earnings, the gross wage rate and labour supply respectively.

As shown in Apps, Long and Rees (2011), of central importance to the results for the structure of optimal tax rates are the assumptions made on the wage distribution. The distribution indicated by household survey data for a number of the major OECD countries³³ is one in which wage rates of primary earners in full time work grow slowly and virtually linearly up to around the 80th percentile, and then increase sharply beyond the 90th percentile. To capture this kind of wage structure we generate a primary earner wage distribution by taking one million random draws from a Pareto distribution³⁴ with support $[20, \infty)$, a b -parameter of 3.5 and a 2% cut-off at the right tail, and calculate from these 100 primary wage rates. The resulting percentile distribution of the "primary wage" is shown graphically in Figure 1.

Figure 1 about here

Next we construct a distribution of the average wage of second earners in each primary wage percentile (labeled "average second wage" in Figure 1) as a proportion of the primary wage, beginning at 80% in percentile 1 and declining to 50% in the top percentile. We present two sets of simulation results. In Set 1 we construct 200 household types by associating with the primary wage in each percentile, consecutively, a lower and higher wage relative to the "average second wage". The higher second earner wage is taken from a distribution that is 25 per cent above the "average second wage" distribution. The lower wage is

³³See, for example, the data on full time primary earners in the 2009-10 ABS Household Expenditure Survey and the UK ONS 2010 Living Costs and Food Survey.

³⁴The cdf of a Pareto distribution for a variate x with Pareto index $a > 1$ and Pareto parameter $b = a/(a - 1)$ is given by $F(x) = 1 - (A/x)^a$ if $x \geq A$ and $F(x) = 0$ if $x < A$.

set at half the higher wage in each percentile. Figure 1 plots the wage profiles of these two types by percentiles of the primary wage. We label the household with the lower second wage “S1” and with the higher second wage, “S2”.

In Set 2 we expand the number of second earner wage types to 4. To each primary wage type we attach two lower and two higher second earner wage types respectively. We therefore have 400 household types. The first of the two higher second wages is the average second wage in the given primary wage percentile, and the second is set at 25% above the average second wage in each percentile. The two lower wages are set at 25% and 50% of the average second wage in each percentile.

In the simulations to follow we set \bar{w}_1 to the data mean for the primary earner wage distribution in Figure 1 and \bar{w}_2 to the data mean of the second earner wage distribution for each type. From (66) we can see that preference heterogeneity between primary and second earners can be introduced by varying e_i and/or by selecting different values of \bar{l}_2 . We set $e_1 = 0.1$ and $\bar{l}_1 = 2000$. These parameter values, together with the primary wage distribution, generate primary earner annual labour supplies that are broadly consistent with recent household survey data for countries such as the UK and Australia. The data for these countries also indicate that average second earner labour supply ranges between 50 and 60 per cent of that of primary earners (depending on the sample selection criteria), while the gap between the average second and primary wage is much smaller until the higher percentiles. To approximate these data we assume within household preference heterogeneity, as noted in Section 2.1, by setting $\bar{l}_2 = 0.5\bar{l}_2 = 1000$, implying that the second earner has a greater preference for leisure. We present results for e_2 taking the values 0.1, 0.2 and 0.3.

The effect of setting $\bar{l}_2 = 1000$ on the labour supplies of S1 and S2 in Set 1 is illustrated in Figure 2 for zero taxes. For $e_2 = 0.1$ the profiles are relatively close because changes in the wage have little effect on labour supply and the profiles are located around 1000 hours per annum. With a significantly higher elasticity of $e_2 = 0.5$ the gap between the S1 and S2 profiles is much larger and average second earner labour supply is higher due to steeply rising hours in the upper percentiles. Thus, at higher elasticities, very low second hours in the upper percentiles of the primary wage distribution can only be explained by a very low second wage or a low value of \bar{l}_2 . In other words, it becomes necessary to assume that at least some second earners have implausibly low wages or a much stronger preference for leisure than primary earners.

Figure 2 about here.

We present simulation results for three possible tax regimes: a linear tax with lump sum and single marginal tax rate; a two-bracket piecewise linear tax on joint income; and a two-bracket piecewise linear tax on individual incomes. In each of the latter cases, to solve for the optimal tax parameters we set an initial value of income for the tax bracket limit and solve for the optimal marginal tax rates. We then vary the bracket limit by steps of \$1000 to find the value that maximises the SWF overall.³⁵ At any set of tax parameters, the values of

³⁵Two methods were used: general grid search and global optimisation software. They gave

consumptions and earnings are of course those that maximise each household's utility subject to its budget constraint at that set of parameters.

Table 1 presents the results for Set 1 simulations, for two values of the inequality aversion parameter, $\pi = 0.1$ and 0.2 . Table 2 presents the results for Set 2 simulations, for the same values of π . The SWF for each of the second earner wage elasticities gives the social welfare ranking of the three tax regimes, given a primary earner wage elasticity of 0.1 in each case.

Tables 1 to 2 about here

In both tables we see that:

- Linear taxation is always dominated by a piecewise linear system, arguing therefore against a "flat tax". This is to be expected given the expressions that characterise the optimal tax rates in the theoretical analysis presented in the earlier sections and the steeply rising wage rates in the upper percentiles.
- When the labour supply elasticity of the second earner is sufficiently close³⁶ to that of the primary earner, joint taxation yields higher social welfare values than individual taxation, which must be due to the superiority of the former on equity grounds
- However, as the elasticity of the second earner rises relative to that of the primary earner, the efficiency gains, even given inequality aversion, start to outweigh whatever equity losses arise, and individual taxation increasingly dominates joint taxation.

The consistency of the results for the two sets of simulations shows that increasing the number of second earner wage rates at each primary earner wage rate from two to four, thus increasing the total number of household types from 200 to 400, leaves the SWF ranking of the three tax regimes essentially unchanged.

6.2 Model 2

For the main purpose of this paper it is sufficient to focus on non-wage sources of heterogeneity across households, and so we simplify by assuming perfect assortative matching - the second earner wage is a fixed proportion of the primary earner wage. The household's utility function is

$$u_h = x_h - \gamma_1 (y_{1h}/w_{1h})^{\alpha_1} + z_h^\kappa \quad h = 1, \dots, H \quad (67)$$

with $\kappa \in (0, 1)$. Child care is produced with the CES production function³⁷

$$z_h = k_h [\beta c_h^\rho + (1 - \beta) b_h^\rho]^{1/\rho} \quad h = 1, \dots, H \quad (68)$$

virtually identical results.

³⁶For example, in the simulations for Model 1 in Section 6 we find that, given a primary earner elasticity of 0.1 , for a second earner elasticity between 0.1 and 0.16 joint taxation is welfare superior to individual taxation.

³⁷The notation is as in Section 2 earlier.

where the parameter ρ determines the elasticity of substitution between second earner and bought in child care, $1/(1 - \rho)$. It is also useful to make the model more tractable by assuming only one source of non-wage heterogeneity, and so we assume that k_h is perfectly correlated with the second earner's wage rate.³⁸ On the other hand, at each wage rate the household faces a price of bought in child care that can vary across households. Thus the determinants of across household heterogeneity at a given second earner wage rate is the price of child care in conjunction with the elasticity of substitution, as determined by ρ . To allow for the fact that higher wage households are likely to buy child care of higher quality, we express the price of child care as a proportion of the second earner wage.

The analysis is based on the primary wage and average second earner wage distributions shown in Figure 1. The quality of bought-in care is assumed to match that of the second earner. We present simulation results for two degrees of price variation. In the first, the child care price varies by 10% above and below the second earner wage, and in the second, by 20% above and below the second earner wage. The resulting price profiles, and their relationship to the second wage, are shown graphically in Figure 3. Households facing the higher price are labelled H1 and those facing the lower price, H2.

Figure 3 about here

A key feature of Model 2 is that we obtain the kind of second earner labour supply heterogeneity observed in the data, after controlling for wage differences, when parental and bought in care are close substitutes. This is illustrated in Figure 4 for zero tax rates. For the close substitute case ($\rho = 0.9$),³⁹ an overall 20% difference in child care price implies a fourfold difference in market labour supply hours across the entire primary wage distribution,⁴⁰ while this falls sharply in the weak complements case ($\rho = -0.1$) and virtually disappears in the strong complements case ($\rho = -10.0$). The effects of an increasing wage alone, given the child care price, are virtually non-existent.⁴¹

The case labelled "H1-H2 substitutes", with $\rho = 0.9$, allows the price of bought in child care as a proportion of the second wage to fall from high to low uniformly with the wage, giving a steadily increasing second earner labour supply. In Model 1 this would be attributed entirely to the substitution of market work for leisure. The "H1-H2 substitutes+ben/tax" profile indicates the effect of subsidies that are offered at low wages, gradually fall to zero by the 50th percentile and thereafter switch to a rising tax rate on child care.

Figure 4 about here

The degree of substitutability between parental and bought in care is important in understanding the welfare comparison of joint and individual taxation.

³⁸This assumption is supported by studies that find child outcomes improve with maternal human capital and household income. For a survey see Almond and Currie (2011) and for a recent study see Lundborg et al. (2012).

³⁹For countries with a child care sector that essentially offers child minding rather than education, we can expect bought-in care and parental care to be close substitutes.

⁴⁰Which is that presented earlier in the analysis of Model 1.

⁴¹Recall that second earner time use is divided only between market work and child care, and that the quality chosen, and therefore the price of child care, increases with the wage.

Where they are strong complements, the labour supply of the second earner, as Figure 3 suggests, is virtually unaffected by the price of child care relative to the wage, and her labour income varies essentially with her wage rate. Thus household income reflects primarily wage type and is a good measure of household utility possibilities. However, in the more realistic case in which they are close substitutes, labour incomes can vary widely in response to a small variation in the child care price, at given wage rates. As a result household income is not a close or reliable measure of welfare because it omits the value of parental child care. Put differently, two households with the same labour incomes may be of widely differing wage types if the prices of market child care that they face differ, the more so, the greater the elasticity of substitution between domestic and market child care.

This is illustrated in Figure 5, which plots household income against the primary wage rate for the two household types. H1 incomes lie well below H2 incomes for all primary wages. However, when the additional value of parental child care in the H1 household is added to its income, there is virtually no difference between the resulting income profiles of the two household types.

Figure 5 about here

Tables 3a and 3b present the results of the optimal tax analysis for $\pi = 0.1$ and 0.2, respectively, and for $\kappa = 0.9$. The first panel in each table allows for the overall difference of 20% between high and low child care prices, the second for the overall 40% difference. In each case we see that individual taxation dominates joint taxation for the cases of close substitutes up to weak complements, while this is reversed when we move to the case of strong complements. This is because under strong complementarity, as argued above, time use choices are very insensitive to relative prices, and so there is virtually no labour supply heterogeneity across households with the same wage rates. The higher price of bought in child care in the H1 household simply makes it relatively worse off, thus supporting redistribution from H2 to H1 households. However, since strong complementarity is inconsistent with the observed heterogeneity in second earner labour supply, the results support the superiority of individual taxation.

Tables 3a and 3b about here

We extend the analysis by taking four rather than two possible relative prices for child care, increasing the total number of household types from 200 to 400. Again, as Table 4 shows, the results are qualitatively unchanged.

Table 4 about here.

7 Conclusions

This paper analyses the problem of optimal income taxation for two-earner households when the tax system is constrained to take the piecewise linear form that is typical of virtually all real-world tax systems. Its aim is to characterise the structure of the optimal tax system for the alternative tax bases of joint and individual incomes and to put forward the argument that the welfare superiority

of individual over joint taxation is substantially increased when we take a model of the household which, structurally speaking, is much closer to reality than those used up until now to explore these issues. The central point is that the positive monotonic relationship between household income and achieved utility which characterises both the single individual household model and the standard two-person model of household labour supply, here called Model 1, does not hold in an empirically more relevant setting, and this has important implications for the equity effects of the alternative tax systems. The numerical analysis of specific versions of the two models also brings out the importance of the elasticity of substitution between parental and bought in child care, together with the price of bought in care, in determining the across household relationship between household income and utility. This suggests new directions for the empirical work required to provide the basis for the design of real-world piecewise linear tax systems.

References

- [1] A Alesina, A Ichino and L Karabarbounis 2011, "Gender Based Taxation and the Division of Family Chores", *American Economic Journal: Economic Policy*, 3(2),1-40.
- [2] D Almond and J Currie, 2011, "Human Capital Development before Age Five" in O Ashenfelter and D Card (eds), *Handbook of Labor Economics*, Vol 4, Part B.
- [3] P F Apps, 1982, "Institutional Inequality and Tax Incidence", *Journal of Public Economics*, 18, 217-42.
- [4] P F Apps, N V Long and R Rees, 2012, "Optimal Piecewise Linear Income Taxation", *Journal of Public Economic Theory*, forthcoming. Available at: <http://ftp.iza.org/dp6007.pdf>
- [5] P F Apps and R Rees, 1988, "Taxation and the Household", *Journal of Public Economics*, 35, 355-69.
- [6] P F Apps and R Rees, 1997, "The Optimal Taxation of Couples", CES WP# 145, University of Munich
- [7] P F Apps and R Rees, 1999, "On the Taxation of Trade Within and Between Households", *Journal of Public Economics*, 75, 241-63.
- [8] P F Apps and R Rees, 2009, *Public Economics and the Household*, Cambridge: Cambridge University Press.
- [9] R Blundell and T MaCurdy, 1999, "Labor Supply: A Review of Alternative Approaches", in O Ashenfelter and D Card (eds), *Handbook of Labor Economics*, vol III. Amsterdam:North-Holland, 1560-1693.

- [10] M Boskin and E Sheshinski, 1983, "Optimal Tax Treatment of the Family: Married Couples", *Journal of Public Economics*, 20, 281-297.
- [11] C Brett, 2009, "Optimal Nonlinear Taxes for Families", *International Tax and Public Finance*, 14, 225-261.
- [12] R Chetty, 2009, "Sufficient Statistics for Welfare Analysis: A Bridge between Structural and Reduced Form Methods", *Annual Review of Economics*, 1, 451-488.
- [13] H Cremer, J-M Lozachmeur and P Pestieau, (2012), "Income Taxation of Couples and the Tax Unit Choice", *Journal of Population Economics*, forthcoming.
- [14] B Dahlby, 1998, "Progressive Taxation and the Marginal Social Cost of Public Funds", *Journal of Public Economics*, 67, 105-122.
- [15] M Feldstein and D R Feenberg, 1996, "The Taxation of Two-Earner Families", ch. 2 in M Feldstein and J Poterba (eds), *Empirical Foundations of Household Taxation*, Chicago: University of Chicago Press.
- [16] H Immervoll, H J Kleven, C T Kreiner and N Verdelin, 2011, "Optimal Tax and Transfer Programs for Couples with Extensive Labor Supply Responses", *Journal of Public Economics*, 95, 1485-1500.
- [17] H J Kleven, C T Kreiner and E Saez, 2009, "The Optimal Income Taxation of Couples", *Econometrica*, 77, 537-560.
- [18] P Lundborg, A Nilsson and D O Rooth, 2012, "Parental Education and Offspring Outcomes: Evidence from the Swedish Compulsory Schooling Reform", IZA DP# 6570.
- [19] E Sadka, 1976, "On Income Distribution, Incentive Effects and Optimal Income Taxation", *Review of Economic Studies*, 261-267.
- [20] A Sandmo, 1990, "Tax Distortions and Household Production", *Oxford Economic Papers*, 42, 78-90.
- [21] F Schrøyen, (2003), "Redistributive Taxation and the Household: The Case of Individual Filings", *Journal of Public Economics*, 87, 2527-2547.
- [22] E Sheshinski, 1989, "Note on the Shape of the Optimum Income Tax Schedule", *Journal of Public Economics*, 40, 201-215.
- [23] J Slemrod, S Yitzhaki, J Mayshar and M Lundholm, 1994, "The Optimal Two-Bracket Linear Income Tax", *Journal of Public Economics*, 53, 269-290.

Figure 1 Model 1 wage distributions

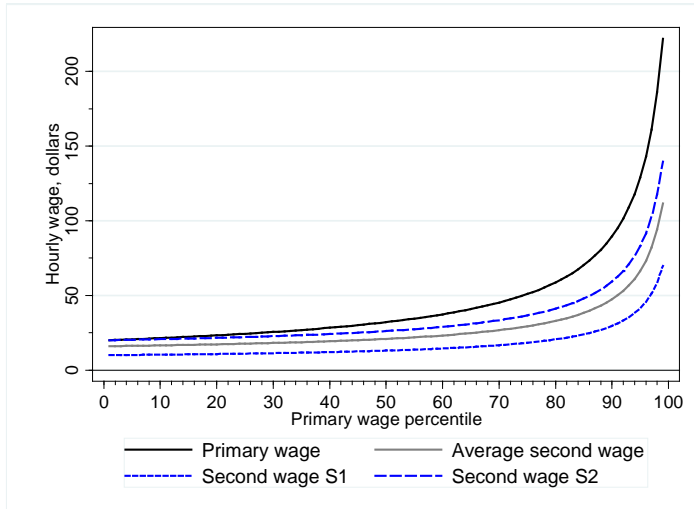


Figure 2 Model 1 labour supply heterogeneity

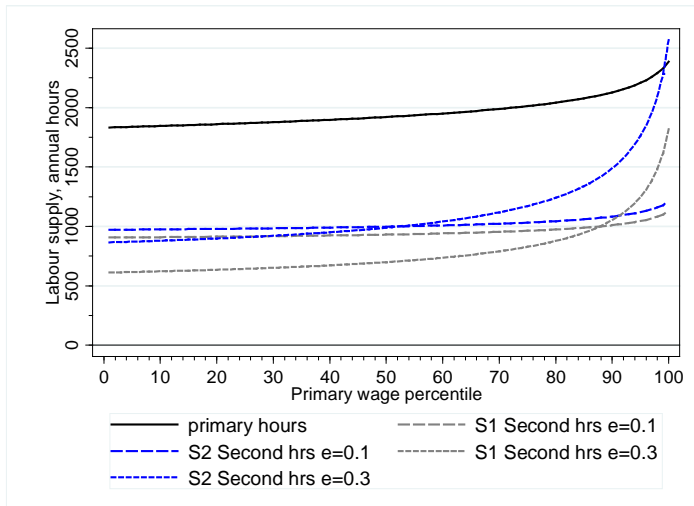


Table 1 Model 1, Set 1 simulations

π	ε_2	Tax system	τ_1, t_1	τ_2, t_2	α, a	Bracket*	SWF*10 ⁻³
0.1	0.1	linear	0.28	0.28	34295	-	40598
		pw individual	0.18	0.34	28085	80	40626
		pw joint	0.17	0.33	26704	73	40632
	0.2	linear	0.26	0.26	31938	-	40050
		pw individual	0.13	0.33	24471	75	40093
		pw joint	0.12	0.31	23 201	65	40087
	0.3	linear	0.24	0.24	29636	-	39634
		pw individual	0.12	0.31	22733	77	39687
		pw joint	0.11	0.28	21478	64	39670
0.2	0.1	Linear	0.40	0.40	48107	-	83677
		pw individual	0.27	0.46	39529	80	83793
		pw joint	0.29	0.43	39958	74	83799
	0.2	Linear	0.37	0.37	44570	-	82425
		pw individual	0.20	0.45	34379	76	82612
		pw joint	0.18	0.43	32491	65	82589
	0.3	Linear	0.34	0.34	41143	-	81461
		pw individual	0.18	0.43	32378	75	81695
		pw joint	0.16	0.40	30435	63	81624

* Income percentile of bracket point

Table 2 Model 1, Set 2 simulations

π	ε_2	Tax system	τ_1, t_1	τ_2, t_2	α, a	Bracket*	SWF*10 ⁻³
0.1	0.1	Linear	0.28	0.28	33552	-	85791
		pw individual	0.18	0.34	27522	80	85850
		pw joint	0.19	0.34	27646	77	85859
	0.3	Linear	0.24	0.24	28579	-	82842
		pw individual	0.12	0.32	22606	76	82954
		pw joint	0.11	0.29	21152	63	82918
0.2	0.1	Linear	0.40	0.40	47066	-	194690
		pw individual	0.27	0.46	38728	80	194957
		pw joint	0.30	0.44	39881	77	194967
	0.3	Linear	0.35	0.35	40790	-	187525
		pw individual	0.18	0.44	31793	75	188064
		pw joint	0.16	0.41	29735	62	187901

* Income percentile of bracket point

Figure 3 Wage and child care price distributions

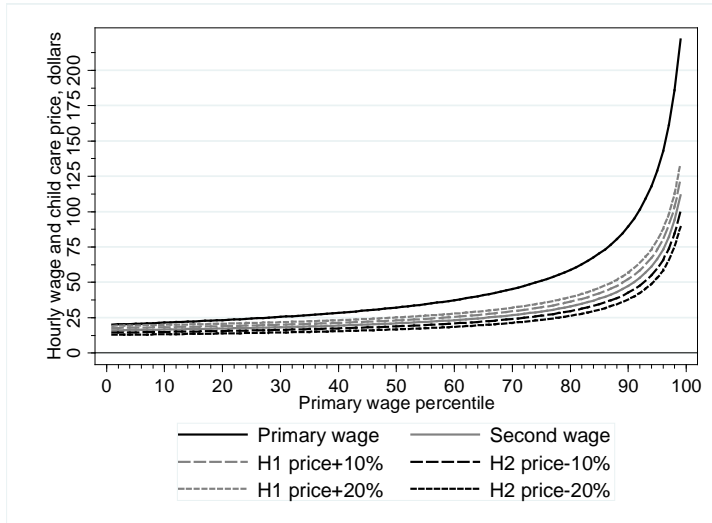


Figure 4 Secondary earner labour supply profiles

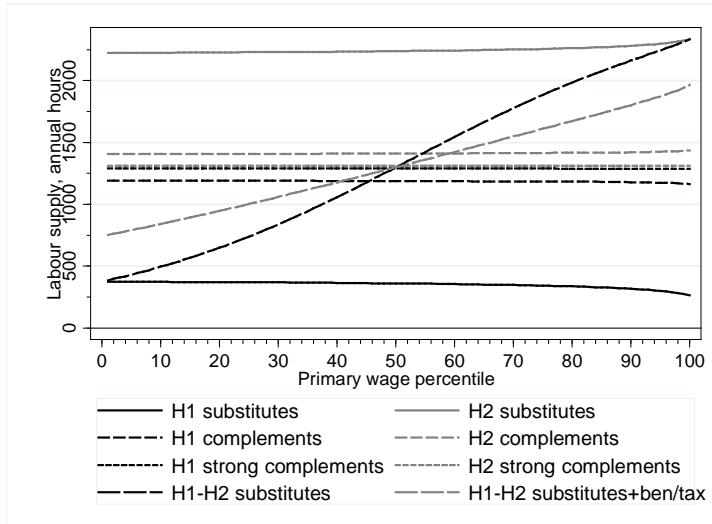


Figure 5 Household income and parental child care

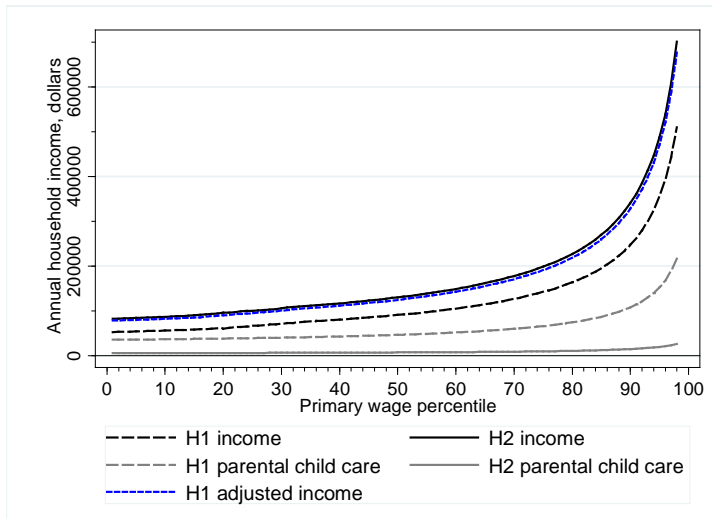


Table 3a Model, 2 Set 1 simulations, $\pi = 0.1$

Price, p	ρ	Tax system	τ_1, t_1	τ_2, t_2	α, a	Bracket*	SWF*10 ⁻³
$p=\pm 10\%w_2$	0.9	Individual	0.04	0.47	18871	90	60022
		Joint	0.03	0.04	5702	47	59760
	-0.1	Individual	0.18	0.47	36687	91	59397
		Joint	0.11	0.19	22470	67	59308
	-10.0	Individual	0.27	0.47	51464	88	59539
		Joint	0.18	0.46	41169	80	59584
$p=\pm 20\%w_2$	0.9	Individual	0.10	0.32	17408	98	61393
		Joint	0.10	0.11	16619	89	61344
	-0.1	Individual	0.07	0.47	22729	89	59649
		Joint	0.07	0.29	21275	73	59534
	-10.0	Individual	0.25	0.43	47995	86	59498
		Joint	0.18	0.43	40009	80	59537

* Income percentile of bracket point

Table 3b Model 2, Set 1 simulations, $\pi = 0.2$

Price, p	ρ	Tax system	τ_1, t_1	τ_2, t_2	α, a	Bracket*	SWF*10 ⁻³
$p=\pm 10\%w_2$	0.9	Individual	0.09	0.52	24149	92	127049
		Joint	0.10	0.11	16949	78	126487
	-0.1	Individual	0.19	0.47	38027	91	122566
		Joint	0.21	0.40	40159	79	121956
	-10.0	Individual	0.38	0.47	66077	88	123185
		Joint	0.34	0.47	61544	81	123266
$p=\pm 20\%w_2$	0.9	Individual	0.10	0.64	25838	94	126397
		Joint	0.13	0.14	21420	49	125584
	-0.1	Individual	0.08	0.47	24837	89	122902
		Joint	0.07	0.29	21275	73	122455
	-10.0	Individual	0.28	0.46	57058	72	123492
		Joint	0.13	0.46	41983	56	123560

* Income percentile of bracket point

Table 4 Model 2, Set 2 simulations

π, ρ	ρ	Tax system	τ_1, t_1	τ_2, t_2	α, a	Bracket*	SWF*10 ⁻³
$\pi=0.1$ $\rho=\pm 10\%w_2,$ $\pm 20\%w_2$	0.9	Individual	0.04	0.47	18846	90	131015
		Joint	0.06	0.07	9797	99	130679
	-0.1	Individual	0.08	0.47	24639	89	128480
		Joint	0.07	0.20	18298	67	128212
	-10.0	Individual	0.25	0.43	47995	87	128533
		Joint	0.18	0.43	40009	80	128616
$\pi=0.2$ $\rho=\pm 10\%w_2,$ $\pm 20\%w_2$	0.9	Individual	0.06	0.47	21329	90	297116
		Joint	0.06	0.09	10542	85	294089
	-0.1	Individual	0.08	0.47	24639	89	291539
		Joint	0.16	0.30	31613	73	290063
	-10.0	Individual	0.21	0.46	48262	71	293825
		Joint	0.21	0.47	48894	65	293933

* Income percentile of bracket point