

IZA DP No. 7232

**The Changing Size Distribution of U.S. Trade Unions and  
Its Description by Pareto's Distribution**

John Pencavel

February 2013

# The Changing Size Distribution of U.S. Trade Unions and Its Description by Pareto's Distribution

**John Pencavel**

*Stanford University  
and IZA*

Discussion Paper No. 7232  
February 2013

IZA

P.O. Box 7240  
53072 Bonn  
Germany

Phone: +49-228-3894-0

Fax: +49-228-3894-180

E-mail: [iza@iza.org](mailto:iza@iza.org)

Any opinions expressed here are those of the author(s) and not those of IZA. Research published in this series may include views on policy, but the institute itself takes no institutional policy positions. The IZA research network is committed to the IZA Guiding Principles of Research Integrity.

The Institute for the Study of Labor (IZA) in Bonn is a local and virtual international research center and a place of communication between science, politics and business. IZA is an independent nonprofit organization supported by Deutsche Post Foundation. The center is associated with the University of Bonn and offers a stimulating research environment through its international network, workshops and conferences, data service, project support, research visits and doctoral program. IZA engages in (i) original and internationally competitive research in all fields of labor economics, (ii) development of policy concepts, and (iii) dissemination of research results and concepts to the interested public.

IZA Discussion Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. A revised version may be available directly from the author.

## **ABSTRACT**

### **The Changing Size Distribution of U.S. Trade Unions and Its Description by Pareto's Distribution**

The size distribution of trade unions in the United States and changes in this distribution are documented. Because the most profound changes are taking place among very large unions, these are subject to special analysis by invoking Pareto's distribution. This represents a new application of this distribution. Extensions to trade union wealth and to Britain are broached. The role of the public sector in these changes receives particular attention. A simple model helps account both for the logarithmic distribution of union membership and for the contrasting experiences of public and private sector unions since the 1970s.

JEL Classification: J51

Keywords: trade union membership, public-sector unions, Pareto's distribution

Corresponding author:

John Pencavel  
Department of Economics  
Stanford University  
Stanford, CA 94305-6072  
USA  
E-mail: [pencavel@stanford.edu](mailto:pencavel@stanford.edu)

THE CHANGING SIZE DISTRIBUTION OF U.S. TRADE UNIONS AND  
ITS DESCRIPTION BY PARETO'S DISTRIBUTION

John Pencavel\*

**I. Introduction**

In Economics, in the search for empirical regularities, patterns in the size distribution of certain key variables - incomes, wealth, firms, cities, and macroeconomic shocks - have long been an area of active research. Decades ago the size distribution of trade unions was among these areas of research.<sup>1</sup> However, in the discussion over the retreat of unionism since the 1970s, the issue of the size distribution of unions has been largely neglected. Yet, over this period, there have been important changes in the structure of unionism, changes that perhaps have been sensed by the political system but that have been somewhat overlooked by scholars. Between 1974 and 2007, there were 101 fewer labor organizations so that, notwithstanding the drop in union membership, the average size of U.S. unions rose: the number of members per union grew from 114 thousands in 1974 to 180 thousands in 2007.<sup>2</sup> The changes in the size distribution are linked to the growth of a few very large unions.

---

\* I thank George Bulman, Andres Drenik, and Rebecca Sachs for their help in preparing this manuscript. I benefited from the comments of two anonymous referees, of Lawrence Kahn, and of participants at a Workshop on Trade Unions at the University of Mainz. Melvin Reder provided insightful criticism on an earlier draft. This paper is dedicated to Mel for his friendship over the years.

<sup>1</sup> For instance, see Estey (1966), Hart and Phelps Brown (1957), Simpson (1972), and Windmuller (1981).

<sup>2</sup> These numbers cover both employee associations and conventional labor unions. An Appendix provides information on the sources for the data used in this paper. The data are available at <http://siepr.stanford.edu/?q=/system/files/shared/pubs/papers/12-011.pdf> in the appendix of an earlier draft.

The principal purpose of this paper is to document these changes in the size distribution of U.S.-based unions. Because of the importance of large unions in these developments and because Pareto's distribution has been proposed as a convenient and compact depiction of the degree of concentration of other variables in Economics, I examine the case for invoking Pareto's distribution to describe the increasing conglomeration of union membership into a smaller number of very large unions. Pareto's distribution is well-known to those who study the personal income distribution where its performance in describing the distribution of high incomes has been assessed very favorably.<sup>3</sup> It has also been invoked to depict the distribution of employment among firms and the distribution of population across cities. Although there is a large literature on the effectiveness of Pareto's distribution in describing the size distribution of these variables, Pareto's distribution has not been used before to characterize the size distribution of large labor unions. Hence another goal of this paper is to contribute to the body of research on Pareto's distribution.

Why should scholars care about whether unions have become more concentrated into a smaller number of large unions? Some might ask the analogous question about the distribution of personal incomes: why should we care about whether the distribution of incomes has become increasingly concentrated among a small number of families or individuals? An answer to this second question is similar to an answer to the question about unions: we should care because of the links between the political process and the concentration of incomes or union members. As shown later in this paper, the distribution of union membership resembles the distribution of wealth among unions so that, just as a greater fraction of aggregate income accruing to a few individuals allows

---

<sup>3</sup> Thus Atkinson, Piketty, and Saez (2011) write "A number of the top income studies conclude that the Pareto approximation works remarkably well today....." even though these authors go on to show that "...Pareto coefficients vary substantially over time and across countries".

these individuals to have a disproportionate impact on political decisions, so the greater concentration of the net wealth of unions implies that a smaller number of decision-makers in the union movement can command more resources to sway the political system. This argument will be revisited in the Conclusions.

This paper first describes the distribution of the number of unions by their membership size and the distribution of union members by the size of unions. Then the changes in these size distributions since the 1970s are reported. Because very large unions figure prominently in these changes, in seeking a concise depiction of developments among the large unions, the performance of Pareto's distribution to characterize these changes is reviewed. Extensions of Pareto's distribution in the study of unionism are broached in Section V which is devoted to the distribution of unions by their net worth and to the distribution of unions by membership in Britain. In Section VI, a simple model is sketched that yields a logarithmic density function of national union membership and that helps to account for the growth of public-sector unions and for the contraction of private-sector unions.

The focus in this paper is on national unions. This is not the organization that many union members see as their principal link with their union. Their connection is with the union at the local level where workplace grievances are handled and where the shop steward is the face of unionism. However, the national union plays a critical role in the typical union member's work life. Normally, it is the national union that needs to approve any strikes and that provides strike benefits. Although many collective bargaining contracts are decentralized and negotiated between the management at a firm or establishment and the union local or the regional union bodies, the national union often

contributes advice and material support.<sup>4</sup> “The principal locus of political and economic power in the American union movement has long been the national unions” (Rees (1988) p. 23). In this paper, “unions” and “labor organizations” refer to employee associations as well as conventional trade unions.

## **II. The Size Distribution of U.S. Trade Unions in the 1970s**

In the 1970s, unionism appeared to be well-entrenched in U.S. labor markets. About 20 million workers were members of unions and they represented almost one-quarter of wage and salary workers. Over eight thousand union representation elections were conducted each year. Three-quarters of all union members in the 1970s were in the private sector<sup>5</sup> and, as indicated in Table 1, four of the five largest unions drew most of their membership from employees in the private sector. Among the largest unions in 1974, only the National Education Association had a substantial representation among workers in the public sector.

There were many unions with relatively few members that, in aggregate, represented only a small fraction of all union members and there were a small number of unions with many members that accounted for a substantial portion of all union members. Thus Table 2 shows that, in 1974, those labor organizations with half-a-million members or more (rows 5 and 6) made up fewer than 6 percent of all unions but 50 percent of all union members belonged to these unions. At the other end of the distribution, in 1974, three-quarters of all U.S. unions consisted of those where each had

---

<sup>4</sup> Some unions’ procedures require the national union to approve the negotiated settlement to ensure it conforms to some wider patterns. Many unions also have bodies intermediate (often regional) between the local and the national union that coordinate activities among their locals

<sup>5</sup> 1977 was the first year that the Current Population Survey included members of employee associations in its count of members of labor unions. In that year, 74.2 percent of labor union and employee association members were employed in the private sector.

fewer than 100,000 members (rows 1, 2, and 3 of Table 2), but merely 10 percent of all union members were members of these unions.

A visual impression of these frequency distributions is instructive so Figures 1 and 2 present diagrams of the densities in 1974. Figure 1 pictures the percentage distribution of the number of unions by the size of the unions where each union size class is one hundred thousand members up to the open-ended class of one million members or more. The dominance of small unions in the count of all unions is evident. The percent distribution of union members by union size (using the same size classes as in Figure 1) in Figure 2 is dominated by the bar corresponding to unions with one million or more members. The highest bar in Figure 1 is the smallest size class and the highest bar in Figure 2 is the largest size class.

The size distribution of unions resembles the size distribution of firms in that most firms (or establishments) are small but most workers are employed in a relatively few large firms.<sup>6</sup> Some may assume that this resemblance of the size distribution of unions to the size distribution of firms is natural and expected. After all, some may reason, unions deal with and respond to firms so should we not expect the distribution of unions to mirror the distribution of firms or establishments?

Of course, there is a link between union membership and employment, but the reasoning in the previous paragraph ignores the fact that most U.S. workers were and are not covered by collective bargaining contracts. Hence a mapping from the size distribution of firms to the size distribution of unions has to account for an intervening variable, the incidence of unionism across firms of different

---

<sup>6</sup> Seminal contributions to the literature on the size distributions of firms, industries, and establishments include Hart and Prais (1956), Simon and Bonini (1958), and Quandt (1966 b). In this literature, size is measured in different ways including employment, assets, market valuation on a stock exchange, and profit. International comparisons are presented in OECD (1996).

sizes. In addition, many collective bargaining contracts in the U.S. are decentralized and are negotiated between a firm or establishment and the union local, not the national union. Furthermore, some unions in the United States are organized on an occupational basis with members in different industries while other unions are centered on an industry (or closely related industries) and embrace different types of workers in various occupations. In short, it is not straightforward to move from employment by firm size to membership by size of national union.

A different line of reasoning to account for the resemblance between the size distribution of unions and the size distribution of firms draws upon the fact that, in the sense in which Economics conceives of the firm, a union is a firm. A union employs factors, combines and organizes these factors in a manner that is not knowingly wasteful of resources, and supplies the resulting services to members who pay regular dues and fees. These activities are exactly those that economists ascribe to the firm. Of course, the union is not characterized as maximizing net revenues, but then nor does every conventional firm maximize its net revenues. However, interpreting the size distribution of members of national unions as an evident deduction from the size distribution of employment by firms overlooks the fact that union members are not employees of the national unions. In a real sense, union members are customers of the services provided by the union so that a closer analogy with the size distribution of firms would be provided if firm size were measured by the number of the customers of each firm.

### **III. Developments since the 1970s**

By the first decade of this century, the standing of unions in U.S. labor markets looked more precarious than it appeared three decades earlier. Union membership was three-quarters of its 1970s value and union representation elections in 2006-09 were less than one-quarter of their number in

1976-79. Public-sector unionism had become more important: the percent of all union members who were employees in the private sector fell from three-quarters in the late 1970s to one-half in the years 2007-10.<sup>7</sup>

The structure of unionism in the 1970s described above - most unions were small and most union members belonged to a relative handful of very large unions - has become more pronounced. As shown in Table 2, by 2007, more than 70 percent of all union members belonged to organizations each of whose membership consisted of half-a-million members or more. The five organizations with at least one million members accounted for 45 percent of all members yet they represented less than 5 percent of all unions. Seventy-two percent of unions consisted of organizations where each had less than 100,000 members and yet, in aggregate, they had only 10 percent of all members.

According to Table 3, between 1974 and 2007, those unions with one million or more members were the only size class recording an increase in the number of members, an increase of 2.6 million members. Figures 1 and 2 present a visual expression of the changing size distribution. In these figures, for each size class specified on the horizontal axis, there are two columns: the unshaded column on the left of each pair represents the year 1974 and the column (shaded in black) on the right of each pair relates to the year 2007. In Figure 1, the height of each bar represents the number of unions in each size class as a percentage of unions in all size classes. The 2007 bars indicate a slight reduction in the percent of unions in the two smallest size classes and a small increase in the percent of unions in the largest size class, but these changes do not alter the pattern that most unions are small. Figure 2 expresses the percentage of union members in each size class. For every size class

---

<sup>7</sup> From the membership data collected by the Current Population Surveys, the share of union members who were employed in the private sector was 49.95 percent averaged over 2007-10.

below 500,000, the column for 2007 is lower than the column for 1974; for the two size classes at and above 500,000, the columns for 2007 are higher than the columns for 1974: in 2007, a larger fraction of union members belong to unions with memberships of half-a-million or more than in 1974. These figures express the percentage distribution of unions and the percentage distribution of union members by size classes that differ by a constant number of members, namely, 100,000. A different organization specifies size classes that differ in membership by a constant proportion as in Figure 3 and 4. Here the lower threshold of membership in each class is twice the lower membership threshold in the previous class.<sup>8</sup> In Figure 3, the percentages of unions in the three largest size classes are greater in 2007 than in 1974 and the percentages of unions in the three smallest size classes are lower in 2007 than in 1974. Figure 4 depicts the percentage distribution of union members by the sizes of unions and the skewed distribution of union membership is more pronounced in 2007 than in 1974. The large increase since 1974 in the percent of union members in the largest size class is conspicuous. In 2007, this frequency distribution's modal class is the largest class with well over a million members.

This increasing concentration of total union membership in the largest of unions is associated with the growing importance of public-sector unionism. As shown in Table 1, the four private-sector unions listed among the five largest unions in 1974 all experienced declines in their membership to 2007 and the single public-sector union among the five largest unions increased its membership, more than doubling in size. Whereas in 1974 four of the five largest unions drew most of their members from the private sector, most members of the three largest unions in 2007 are public-sector

---

<sup>8</sup> The analysis later in this paper that examines differences among unions in the logarithm of membership inspires this organization of cells whose memberships bear the same ratio from one to another.

employees.<sup>9</sup> The public-sector unions tend to represent more skilled and highly paid employees than the workers represented by the private-sector unions so the growth of public-sector unionism implies a growth in the clout of the better paid unionised workers.<sup>10</sup> Among public-sector union members, it has been those who are State employees whose relative (and absolute) importance has grown.<sup>11</sup>

The increase in concentration in unionism has come about because unionism in the public sector is more concentrated than that in the private sector and a larger share of union members are in the public sector than in the private sector in 2007 than in 1978.<sup>12</sup> In addition, the concentration of union membership has increased within the private sector and within the public sector.<sup>13</sup>

How has this greater concentration in union membership in recent decades come about? In some

---

<sup>9</sup> Many of these unions had some members employed in the private sector and some members in the public sector. The private/public employment distinction in the text concerns what is believed to be the sector where most members are employed. Table 1 may understate the importance of public sector unionism: the sixth largest union in 2007 is the American Federation of Teachers (AFT) with 832,100 members, another union with members principally employed in the public sector.

<sup>10</sup> According to the Current Population Survey, the median weekly earnings of full-time unionised wage and salary workers in the public sector exceeded the earnings of the corresponding unionised workers in the private sector by 12 percent in 2011: <http://www.bls.gov/news.release/pdf/union2.pdf>.

<sup>11</sup> The percent of all public sector union members who are State employees rose from 18.7 percent in 1983 to 25.7 percent in 2007 while the fractions who are Federal and Local government employees fell. Union members who are local government employees represented 62.2 percent of all public sector union members in 2007. This information is taken from <http://unionstats.gsu.edu/> maintained by Barry Hirsch and David MacPherson.

<sup>12</sup> Union membership in the public sector as a fraction of all union membership was 26.2 percent in 1978 and 48.2 percent in 2007.

<sup>13</sup> Merging information from the Directory of Labor Organizations on membership in particular unions with the information from the Current Population Survey on total union membership in a given sector, the membership of the four largest private-sector unions as a fraction of the membership of all private-sector unions was 41.5 percent in 1978 and 50.9 percent in 2007. The membership of the four largest public-sector unions as a fraction of the membership of all public-sector unions was 75.0 percent in 1978 and 93.2 percent in 2007.

cases, increasing concentration has been the consequence of vigorous organizing campaigns undertaken by some of the larger organizations to build up their membership and representation while smaller unions have tended to have been less active in this regard. In addition, there has been a continual process of amalgamation. Mergers between unions are by no means new and Figure 5 plots the number of union mergers per year (averaged over five year periods) from 1900 to 2007. Merger activity was especially animated in the 1980s.

Often this process of amalgamation has not involved unions of approximately the same size but unions whose memberships were quite different in size. The literature on mergers of unions sometimes draws a distinction between mergers that take the form of amalgamations and those of absorptions though the line distinguishing absorptions from amalgamations is “often faint”.<sup>14</sup> Both because of mergers and because of organizing campaigns, unions that already had large memberships in the 1970s have tended to become larger both absolutely and relatively.<sup>15</sup>

The amalgamations over the past forty years have resulted in the demise of several celebrated unions. For instance, the members of the International Typographical Union, a union that traced its origins to early printing guilds, were absorbed into the Teamsters and the Communications Workers of America in the 1980s. The International Ladies’ Garment Workers Union (ILGWU), a union involved in celebrated strikes a century ago, joined in 1995 with the Amalgamated Clothing and Textile Workers Union to form the Union of Needletrades, Industrial and Textile Employees

---

<sup>14</sup> Windmuller (1981), p. 53.

<sup>15</sup> Five unions have been particularly involved in mergers: the Communications Workers of America (CWA), the Service Employees International Union (SEIU), the United Food and Commercial Workers (UFCW), the International Association of Machinists and Aerospace Workers (IAM), and the United Steel Workers (USW).

(UNITE). The Brotherhood of Sleeping Car Porters, a union of predominantly Black workers, merged in 1978 with the Brotherhood of Railway and Airline Clerks which, in turn, merged with other unions in the 1980s to form the Transportation-Communications International Union.

Years ago, the issue of the changing size distribution of labor organizations prompted a number of analyses. For instance, using the fraction of total union membership belonging to the largest unions as measures of concentration, Marten Estey (1966) examined data from 1897 to 1962. He found different trends within the ranks of the ten largest unions though his “major empirical finding” was that concentration in the two largest unions had declined since the beginning of the century. He contrasted this with the British experience of increasing concentration.

Subsequently Windmuller (1981) examined the distribution of union membership in the U.S. and eight other countries from the 1950s to the 1970s and found a tendency toward greater concentration over these two decades.<sup>16</sup> The information in Figures 2 and 4 suggests that Windmuller’s conclusion endures to the present. Using the type of indicators that Estey constructed - the percent of total membership who are members of the largest unions - Table 4 indicates that these reveal a marked increase in concentration in recent decades.

Although there has been an evident trend toward concentration in union membership, it must not be thought that small unions no longer exist. The seven unions with fewer than 500 members in 2007 are all listed in the 1975 Directory for 1974. By contrast, all but two of the ten smallest unions in 1974 have been swallowed up by larger organizations or they have disappeared.<sup>17</sup> The

---

<sup>16</sup> Note that Windmuller omitted employee associations in his analysis for the U.S.

<sup>17</sup> We may underestimate the incidence of small unions in the public sector because the Labor Management Reporting and Disclosure Act does not require organizations exclusively representing local or state public sector workers to file.

endurance of unions of substantially different sizes suggests there are no meaningful economies or diseconomies of scale in supplying services to union members.

#### **IV. Pareto's Distribution**

The concentration of the union structure and the growth of the largest unions may be taken up in a less impressionistic and more organized fashion if the distribution in the sizes of unions followed a compact systematic pattern. This has led some researchers to draw upon the lognormal distribution to describe the size distribution of unions,<sup>18</sup> but I explore a different method to characterize the size distribution of large labor unions: Pareto's distribution. Its use to depict the pattern of high incomes is well known where it relates the logarithm of the percentage of observations whose income is larger than a given value to the logarithm of that value. It has other applications in Economics including the size distribution of large firms and the size distribution of large cities. I consider the case for the same relationship to describe the membership of large unions. The key parameter of Pareto's distribution provides a convenient summary gauge of concentration and students of inequality may find it instructive to learn whether this distribution applies to the membership of large unions.

The issue immediately presents itself of defining "large" unions. In other areas of economics where Pareto's distribution has been applied, the consequences of different thresholds for defining "large" have sometimes been investigated and I consider this here in its application to "large" labor organizations. I begin with an analysis of Pareto's distribution using union membership in 2007.

---

<sup>18</sup> For instance, see Hart and Phelps Brown (1957) and Simpson (1972) who examined the structure of British unionism.

## Union Membership in 2007

Suppose “large” unions in 2007 are those with at least 200,000 members. When the threshold is established at this level, there are 21 labor organizations in this set. These unions constituted almost a fifth (precisely 18.9 percent) of all the unions and over four-fifths (precisely 83.1 percent) of all the reported membership in the Directory. Let  $M_i$  be the number of members belonging to union  $i$  and suppose  $S_i$  is the percentage of unions with membership greater than union  $i$ 's membership.<sup>19</sup> The values of  $S_i$  constitute the survivor function and it is the complement of the cumulative distribution function of membership.<sup>20</sup> Pareto's Law maintains that, beyond a certain threshold (initially, a level corresponding to membership of 200,000 in 2007 is specified), the survivor function of union membership is  $S(M_i) = e^{-\lambda} (M_i)^{-\alpha}$  or, in logarithms,

$$(1) \quad \ln S_i = \lambda - \alpha \ln M_i \quad ,$$

where  $\lambda > 0$  and  $\alpha > 0$  are parameters.<sup>21</sup>  $\alpha$  is sometimes called Pareto's coefficient. The special case of  $\alpha = 1$  is named Zipf's Law or the “rank-size” rule. In the study of high incomes,  $\alpha$  is often estimated between 1.5 and 2.5; in its application to the size distribution of large firms,  $\alpha$  is estimated between 0.89 and 1.06; and in its application to the distribution of large cities in a country,  $\alpha$  is

---

<sup>19</sup> Unions are ordered from the smallest to the largest so, among these 21 unions, the largest union is ranked 21 (RANK = 21). Because we intend to form logarithms of the variables, to avoid trying to take the logarithm of zero (!) and to ensure the largest union is represented in the data, in forming  $S_i$ , we add unity to the number of unions. That is, in the set of the 21 largest unions in 2007,  $S_i$  is defined for union  $i$  as  $[(22 - \text{RANK}_i)/22]100$ . This means  $S_i = [(22 - 21)/22] = 4.545$  for the largest union and  $S_i = [(22 - 1)/22]100 = 95.545$  for the smallest union among this set of 21 unions.

<sup>20</sup> The cumulative function is non-decreasing and the survivor function must be non-increasing.

<sup>21</sup> For values of  $M_i > 200$  thousand, the density function is  $f(M_i) = e^{-\lambda} \alpha (M_i)^{-(1+\alpha)}$ . When plotted on linear axes, the density function looks approximately like an upper case L with a monotonic negative slope. This is how the frequency distribution of unions by membership appears in 2007 for those unions with membership greater than 200,000 members.

estimated between 0.729 and 1.963.<sup>22</sup>

$\alpha$  provides an indicator of the degree of concentration of the values of the variable among those observations to which Pareto's distribution is applied: higher values of  $\alpha$  denote less concentration because the larger the value of  $\alpha$ , the steeper the decline of the survivor function and the larger the range of values of the survivor function for a given difference in union membership. Indeed, under certain conditions,  $\alpha$  can be mapped into other indicators of inequality with higher values of  $\alpha$  associated with less inequality.<sup>23</sup> A useful attribute of Pareto's distribution is that, if  $\alpha > 1$ , the average membership of those unions equal to and above a minimum value, say,  $M_N$ , equals  $\alpha (\alpha - 1)^{-1} M_N$ . Or the ratio of the average membership to  $M_N$  is the constant  $\alpha (\alpha - 1)^{-1}$  so that lower values of  $\alpha$  imply a greater ratio of average membership to  $M_N$ .

An impression of the relationship between the variables in equation (1) is provided by the scatter diagram, Figure 6, which plots  $\ln S_i$  against  $\ln M_i$  for the 21 unions with membership no less than 200,000 in 2007. The largest union had 3,167,612 members and the smallest union included in

---

<sup>22</sup>The values for high incomes come from Atkinson, Piketty, and Saez (2011) and from Pareto (1965) who reported estimates of  $\alpha$  values ranging from 1.35 in England in 1879-80 to 2.10 in Uri in 1887. The values for large firms are drawn from Axtell (2001) and Fujiwara et al. (2008). The values for city size are reported in Rosen and Resnick (1980) and Soo (2005). For other applications of Pareto's distribution, see Clauset, Shalizi, and Newman (2009), Gabaix (2009), and Steindl (1965).

<sup>23</sup> When comparisons are made among distributions with the same mean, if  $\alpha > 1$  and if  $G$  stands for Gini's coefficient,  $\alpha = (1 + G)(2G)^{-1}$ . Chipman (1974) uses Atkinson's concept of the equally distributed equivalent income to write a social welfare function in terms of  $\alpha$  and  $\lambda$ . Aitchison and Brown (1954) relate values of  $\alpha$  to Lorenz' Curve. If  $\alpha \leq 1$ , the distribution's population moments do not exist though those of the sample observations are well-defined. If  $\lambda = (M_N)^\alpha$  where  $M_N$  is a fixed low level (perhaps the minimum observed level), equation (1) may be written  $\ln S_i = \alpha \ln (M_N / M_i)$  and the size distribution is described by one parameter,  $\alpha$ .

this scatter diagram had 229,248 members.<sup>24</sup> The negative relationship between  $\ln S_i$  and  $\ln M_i$  in Figure 6 does not appear to be exactly linear although the deviations from linearity may be random and inconsequential. To what extent is this relationship satisfactorily described as linear? Is this a good fit by some criterion? Are the deviations from the fitted relationship randomly distributed? These issues will be taken up by estimating the parameters of Pareto's distribution by the method of least-squares. From equation (1), departures from precise log-linearity may be accommodated by adding a stochastic term:

$$(1.1) \quad \ln S_i = a_0 + b_0 \ln M_i + e_{0i}$$

where  $a_0$  and  $b_0$  are parameters to be estimated with the interpretation that  $a_0 = \lambda$  and  $b_0 = -\alpha$  and where  $e_{0i}$  represents the value of  $i$ 's residual from the fitted line.<sup>25</sup>

The equation above describes the nature of the association between two variables. In terms of Figure 6, there is no necessary reason to minimize the sum of squared residuals in a vertical direction instead of in a horizontal direction. Therefore, rearrange equation (1) as follows:

$$(2) \quad \ln M_i = \lambda \alpha^{-1} - \alpha^{-1} \ln S_i,$$

a stochastic version of which is

---

<sup>24</sup> In Figure 6,  $S_i$  is expressed not as a percentage but as a fraction. Therefore,  $S_i$  is 0.04545 and  $\ln(S_i) = -3.091$  for the largest union and  $S_i$  is 0.9545 and  $\ln(S_i) = -0.0466$  for the smallest union in this set. The difference between expressing values of  $S_i$  as a fraction and as a percentage is simply the value of the intercept. The slope of the relationship,  $-\alpha$ , is unaffected.

<sup>25</sup> The use of least-squares is common in applications of Pareto's distribution although other techniques have been proposed. See Gabaix and Ibragimov (2009), Nishiyama and Osada (2004), and Quandt (1966a). In subsequent work, I hope to take up the consequences of alternative methods of estimating  $\alpha$ , but in this initial application of Pareto's distribution to union membership, I ask whether the inferences from the conventional estimating method of estimating  $\alpha$  concur with those using other indicators of concentration such as the concentration ratios calculated by Estey (1966) and reported in Table 4.

$$(2.1) \quad \ln M_i = a_1 + b_1 \ln S_i + e_{1i} \quad ,$$

where  $a_1 = \lambda / \alpha$ ,  $b_1 = -\alpha^{-1}$ , and  $e_{1i}$  is an additive disturbance.

Equation (1.1) is estimated first to the 21 largest unions in 2007 (the unions with no less than 200,000 members) and the ordinary least-squares estimates are reported in the first row of Table 5. For these unions, Pareto's coefficient is estimated to be 1.13 and 95 percent of the variance in the dependent variable is removed by the single right-hand variable,  $\ln M$ . In Table 5, the rows from 2 to 5 expand the number of unions to which Pareto's coefficient is estimated. As the membership threshold is lowered, so Pareto's coefficient is estimated to be lower. This decline in the implied value of  $\alpha$  as the threshold is lowered for inclusion into the set of unions to which equation (1.1) is estimated is consistent with the notion that the concentration of union members into the largest unions appears more salient as smaller and smaller unions are added to the data. It illustrates the sensitivity of Pareto's coefficient to the set of observations to which it is applied and, in particular, to the threshold value that defines inclusion into the set. When the threshold defining unions included into the estimation set is more than 10,000 members, the estimated value of Pareto's coefficient is 0.62, almost one-half its value when the threshold is at more than 200,000 members.

The ordinary least-squares estimates in Table 6 of the inverse specification (2.1) imply values of  $\alpha$  that are similar to those from estimating equation (1.1) in Table 5. In view of the high values of the  $R^2$  statistics, this is not surprising: the observations lie close to the fitted line.

Reconsider the least-squares estimates of equation (1.1) fitted to the 21 largest unions in 2007 as reported in the first line of Table 5. When the estimated residuals from this regression are ordered by the size of the union, the null hypothesis that they are random and display no first-order serial correlation is rejected at conventional levels of significance by familiar tests. In the literature on

estimating Pareto's distribution, this finding is far from unprecedented. Would different inferences about Pareto's coefficient follow if a procedure were applied that recognizes this pattern in the residuals? For this, the residuals were assumed to follow a first-order autoregressive pattern and the autoregressive parameter was estimated along with  $a_0$  and  $b_0$ . Cochrane and Orcutt's familiar iterative technique was used for this purpose. With the same 21 largest unions in 2007, the consequences for the estimated parameters are shown in Table 7. The estimates of Pareto's coefficient from estimating equation (1.1) are similar - though not identical - to those in Table 5 that do not recognize such serial correlation. In the case of the inverse specification equation (2.1), when allowing for a first-order serial correlation parameter, the estimates of  $\alpha$  in Table 7 differ more from those in Table 6 : the estimated value of  $\alpha$  in Table 7 barely changes as the threshold size of unions is lowered.

As is well known, the finding of autocorrelated residuals in a cross-section regression may signal a functional form specification error. Indeed, in some previous work on estimating Pareto's distribution, equation (1) has been augmented with a second-order term in  $\ln M_i$  and this formulation is investigated here:

$$(3.1) \quad \ln S_i = a_2 + b_2 \ln M_i + c_2 (\ln M_i)^2 + e_{2i} \quad ,$$

where  $e_{2i}$  is an additive disturbance. This represents a departure from Pareto's distribution. The least-squares estimates from fitting equation (3.1) to the 2007 largest unions applying different threshold sizes are reported in Table 8. In all cases, the estimates of  $b_2$  are positive and those of  $c_2$  negative indicating the relationship is concave from below (broadly consistent with the scatter diagram in Figure 6). The addition of the quadratic term often results in a statistically significant improvement in the goodness of fit. This result is sometimes found in applications of Pareto's

distribution to firm sizes and to city sizes.<sup>26</sup> Although the second-order term constitutes a strict rejection of log-linearity, a less drastic reaction to this finding is to view Pareto's log-linearity as a useful first-order approximation to the relationship and not to discard Pareto's rule unconditionally.

### **Union Membership in Other Years**

Having estimated Pareto's distribution to the 21 unions with at least 200,000 members in 2007, consider how these estimates contrast with those fitted to a comparable set of unions in other years and consider what these estimates in other years imply. What constitutes a "comparable" set of unions? In the literature on fitting Pareto's distribution to observations on other variables, the "comparable" sets have been defined in different ways. For instance, in the research that estimates Pareto's distribution to large cities in different countries, the sets of cities in these countries are sometimes defined by the same number of cities, sometimes defined by the same minimum city size, and sometimes defined by the same fraction of all cities. By analogy, this would call for estimating Pareto's distribution to large unions in different years by specifying the same number of unions in these years as in 2007 (namely, the 21 largest unions) or by specifying the same minimum threshold level of union members as in 2007 (namely, 200,000 members) or by specifying the same percentage of all unions as in 2007 (namely, the largest 18.9 percent of all unions).

To pursue these different selection criteria, consider data on union membership for a year in the 1970s, a year in the 1960s, and, to provide a longer-term perspective, consider using data on the size distribution of unions in 1939 and in 1920. The implications of these three selection criteria for the set of unions to which the stochastic equations above are estimated are given in Table 9. Evidently, these three selection criteria result in a different number of unions being defined as "large"

---

<sup>26</sup>For example, see Ijiri and Simon (1974) on firm sizes and Soo (2005) on city sizes.

in a given year.

The least-squares estimates of  $\alpha$  and the corresponding  $R^2$  values from estimating equation (1.1) to the different years by the three different criteria are reported in Table 10. There is a tendency for the estimate of Pareto's coefficient to increase as the equation is fitted to unions in earlier years with a value of about 1.53 in 1920 and a value of 1.13 in 2007.<sup>27</sup> This implies less concentration among large unions in earlier years.

How may this difference between 1.53 and 1.13 be illustrated? As noted earlier, a property of Pareto's distribution is that, provided  $\alpha > 1$ , for any union with membership  $M^*$ , the average membership of those unions whose membership equals or exceeds  $M^*$  is  $[\alpha (\alpha - 1)^{-1}] M^*$ . This implies that a lower value of  $\alpha$  increases the gap between  $M^*$  and the average-sized union among those unions with membership above  $M^*$ . In this sense, a lower  $\alpha$  signifies greater inequality in membership among the larger unions. Suppose  $M^*$  equals 200,000. Then the average size of unions with membership equal to or greater than 200,000 is 577,359 when  $\alpha = 1.53$  and the average union size is three times this number (precisely, 1,738,462) when  $\alpha = 1.13$ . This difference in  $\alpha$  reflects a considerable difference in inequality among these large unions. Expressed differently, ostensibly "small" differences in  $\alpha$  have "large" implications for concentration.

Thus the decline from 1920 to the present in the value of Pareto's coefficient is consistent with the interpretation that, among the largest unions, the concentration of union membership has tended to increase over time. Even though the concentration ratios in Table 4 are designed to indicate concentration among all unions and not just the largest unions, the increases in the concentration

---

<sup>27</sup> This is also apparent when each equation is estimated after accounting for first-order serial correlation in the residuals. The  $R^2$  values range from a low of 0.882 to a high of 0.967.

ratios from 1968 to 2007 in Table 4 are consonant with the decline in the estimated values of Pareto's coefficient between these years.

To this point, Pareto's coefficient has been estimated to unions in different years. Suppose these observations on the membership of large unions in different years are pooled and Pareto's distribution is fitted to the pooled observations. What is the value of Pareto's coefficient when applied to these pooled data when the intercept of the fitted line is permitted to differ by year (that is, permitting  $a_0$  in equation (1.1) to vary by year)? The answer is reported in Table 11 for each of the three sets of "large" unions specified in Table 9. The estimates of Pareto's coefficient range from 1.04 to 1.31 with the coefficient higher in absolute value as the number of observations falls.<sup>28</sup>

Again, in describing the distribution of large unions in the U.S., Pareto's coefficient is not an iron constant but it varies within a relatively narrow range over the years. However, as indicated earlier in this section, "small" differences in the coefficient may have "large" consequences. To compare the estimate of  $\alpha = 1.04$  with  $\alpha = 1.31$ , recall that, for any union with membership  $M^*$ , the average membership of unions whose membership equals or exceeds  $M^*$  is  $[\alpha(\alpha - 1)^{-1}] M^*$ . Again, if  $M^*$  equals 200,000, the average size of unions with membership equal to or greater than 200,000 is 5,200,000 when  $\alpha = 1.04$  and the average size of unions with membership equal to or greater than 200,000 is 845,161 when  $\alpha = 1.31$ . The former is six times the value of the latter.

---

<sup>28</sup>Applying conventional F-distribution tests, the hypothesis that the year-specific intercept terms may be eliminated is rejected for the observation sets consisting of 105 unions and 162 unions, but not for the observation set of 82 unions. In this last case, when fitting Pareto's distribution without these year-specific intercepts, the estimate of Pareto's coefficient is 1.302 with an estimated standard error of 0.040.

## V. Two Extensions to the Use of Pareto's Distribution Describing Trade Unions

To this point, Pareto's distribution has been applied to the membership of large U.S. unions. Two questions arise. First, how do the estimates of Pareto's coefficient change if the size of unions is measured by something other than the number of members? Second, because the history and structure of U.S. unionism are singular, how does Pareto's distribution fare as a description of the size distribution of larger unions in other countries?<sup>29</sup> This section is devoted to a treatment of these questions.

### Assets

There has long been an interest in the degree to which union membership is concentrated in a relatively small number of unions. However, there are other indicators of size and, for purposes such as the ability to finance long protracted disputes and for mounting legislative campaigns, the wealth of unions may be more consequential than the number of members. A convenient documentation of union finances in the United States is Leo Troy and Neil Sheflin's Union Sourcebook compiled in 1985.<sup>30</sup> From this, Table 12 presents the size distribution of net assets of national unions in 1982. It is similar to the size distribution of membership in that most unions have relatively meagre assets but a small number of unions account for a large share of all net assets

---

<sup>29</sup> This is a suitable place to recognize that a number of national unions based in the United States describe themselves as 'international' because their contracts with U.S. firms cover these firms' workplaces in Canada or Puerto Rico or the Panama Canal Zone. These members outside the U.S. were once identified in earlier issues of the Directory and in 1978 they constituted seven percent of the membership of all U.S.-based unions. This information is not available in recent issues of the Directory. In the analysis above, these members outside the U.S. have been included with members in the U.S.

<sup>30</sup> Financial information on unions is not available in the published Directory of Labor Organizations.

owned by national unions. Thus more than one-half of all unions have less than one million dollars in net assets and 3.6% of unions have assets greater than one hundred million dollars. These few wealthy unions hold almost three-quarters of all union net assets. Wealthy unions tend also to be unions with large membership. Of the ten unions with the highest net assets in 1982, six are also ranked in the top ten by membership and the remaining four unions are ranked seventeenth or above in membership.

### **Britain**

For many years, British unionism has been characterized by the dominance of a few “general” unions that draw their membership from many different industries and that account for a large fraction of total union membership.<sup>31</sup> In this respect, the growth of certain very large unions in the United States such as the Service Employees International Union, the Teamsters, and Steel Workers Union<sup>32</sup> follows the pattern set earlier by British unionism. Another similarity between American and British unionism is the decline in the extent of unionism in recent decades.

At about the same time that Troy and Sheflin (1985) published their Union Sourcebook for the United States, a U.K. Department of Employment Research Paper by Paul Willman and Timothy Morris (1986) reported research into the membership and finances of large unions in Britain.<sup>33</sup> Hence

---

<sup>31</sup> Hart and Phelps Brown (1957) reported that, in 1954, the six biggest unions accounted for almost one-half of all union members.

<sup>32</sup> The full name of the Steel Workers Union signifies its breadth: the United Steel, Paper and Forestry, Rubber, Manufacturing, Energy, Allied Industrial and Service Workers International Union.

<sup>33</sup> Similar to the U.S. data used above, the British data are derived from reports that British unions are required by law to file. From the information provided, what the British authors call “net worth” (assets minus liabilities) corresponds to what the U.S. researchers call “net assets”. There will surely be differences in details between the British and the U.S. data if only because the reporting forms are different and there are likely to be different accounting conventions.

an Anglo-American comparison can be made of Pareto's coefficient both for membership and for net assets that applies to the same period.

The British study lists the membership of the 56 largest unions but the net worth of only the 20 largest (wealthiest) unions. As in the United States, unions in Britain with relatively large memberships tend to be unions ranked high in net worth: of the British unions ranked in the top twenty by net worth, eighteen were in the top twenty ranked by membership. All of the British unions ranked in the top twenty by net worth had more than 100,000 members. Nevertheless, the shape of the distribution of British unions by net worth differs from the shape of the distribution of British unions by membership: the coefficient of variation for membership exceeds that for net worth, membership is more skewed than net worth, and the authors report a value of kurtosis for membership that is twice the value for kurtosis for net worth.

Once again, when making comparisons of Pareto's coefficient to different sets of observations, the issue arises of how to define "large" unions. In this case, the issue is more or less "solved" by the fact that information is available on no more than the 20 largest British unions defined by net worth. For Britain, information on the membership of British unions is available for the 56 largest unions but, to facilitate comparison with the U.S. in column A of Table 10, Pareto's distribution is fitted to the membership on the 21 largest British unions.

First, compare the estimates for Pareto's coefficient when applied to U.S. union membership with the estimates when applied to the net assets of U.S. unions. In one case, unions are ordered by their membership and, in the other case, unions are ordered by their net assets. As already mentioned, there is a overlap in the set of large unions by membership and the set of large unions by net assets but this intersection is not complete. Analogous to equations (1.1) and (3.1) above, the following

regression equations are specified to observations on these unions ordered by their net assets, where  $K_i$  denotes the net assets of union  $i$  and  $e_{3i}$  and  $e_{4i}$  are stochastic residuals:

$$(1.2) \quad \ln S_i = a_3 + b_3 \ln K_i + e_{3i}$$

$$(3.2) \quad \ln S_i = a_4 + b_4 \ln K_i + c_4 (\ln K_i)^2 + e_{4i}$$

The least-squares estimates of equations (1.2) and (3.2) applied to the 21 largest unions in the 1980s in the United States are given in Table 13. The upper panel of Table 13 reports that Pareto's coefficient is 0.85 when fitted to the distribution of the assets of the 21 wealthiest U.S. unions. This compares with an estimate of 1.28 for Pareto's coefficient when fitted to the distribution of membership of the 21 largest U.S. unions. The lower value of Pareto's coefficient when fitted to the distribution of net assets in the U.S. than the value of Pareto's coefficient when fitted to the distribution of membership means that a given proportional difference in membership spans a larger fraction of unions than the same proportional difference in net assets. Expressed differently, in the U.S., there is more concentration among the larger unions in their assets than in their membership. This may reflect the fact that the unions with large memberships are mainly white-collar higher-earning workers.

The corresponding least-squares estimates of equations (1.2) and (3.2) for British unions are given in Table 14. For Britain, as for the U.S., Pareto's coefficient has a lower absolute value for net worth than that for membership. In addition, with respect to membership, Pareto's coefficient is lower in absolute value for Britain than its estimate for the U.S. implying greater concentration of membership among the larger unions in Britain than among the larger U.S. unions. The opposite is the case for net worth where Pareto's coefficient for Britain is larger than that for the U.S.

The consequences of estimating a quadratic relationship in membership and net worth are presented in the lower panels of Tables 13 and 14. In all four cases, the signs of the point estimates imply a concave from below relationship. For both countries, the improvement in fit for the membership equation by adding the second-order term is statistically significant. The relevance of the second-order term is also suggested for net worth in Britain. This is not the case for the distribution of assets in the U.S.

This extension of Pareto's distribution shows that it has potential in helping to understand the size distribution of unions along dimensions other than membership and that it may be useful as a summary description of differences in the concentration of unions across countries.

## **VI. A Simple Model**

The empirical results reported above prompt two questions: first, why have unions grown in the public sector and those in the private sector contracted; second, why does the size distribution of trade unions follow, as a first approximation, a log-linear density function such as Pareto's distribution? Here a simple model is sketched that addresses both questions and that suggests relevant variables. The model is a simple adaptation of a union optimizing an expression for its welfare. In particular, consider a single union local and suppose its objectives  $U$  over wages  $w$  and employment  $E$  take the following special form:

$$(4) \quad U(w, E) = (w - v) E^\theta$$

where the positive parameter  $\theta$  gauges the weight on wages vis-à-vis employment in the union's welfare and where  $v$  is some comparison or reference wage such that  $w > v$ .

The union local's pursuit of its objectives is constrained by the following semi-logarithmic trade-off between employment ( $E$ ) and wages:

$$(5) \quad E = \exp(\rho + \eta w + \mu r + \delta p)$$

where  $r$  is the price of physical capital and  $p$  stands for any variable that shifts the employment-wage trade-off rightwards and enhances the union's employment-wage opportunities. Increases in  $p$  raise  $E$  so  $\delta > 0$ . In the private sector,  $p$  may represent the prices of goods that compete in the product market for the good produced by this union local. In the public sector,  $p$  may stand for the government agency's allotted budget.  $\eta$  is negative while  $\mu$  is positive when capital and labor are substitute inputs and  $\mu$  is negative when capital and labor are complementary inputs. The effects of other variables on  $E$  are embodied in  $\rho$ .

In this bilateral monopoly setting, the union local is assumed to act as a monopolist seller of labor and posts a wage that maximizes  $U(w, E)$  in equation (4) subject to management selecting employment according to (5).<sup>34</sup> The optimal wage is a multiple of  $v$  and the resulting implied expression for employment is

$$(6) \quad E = \exp(\rho - \theta^{-1} + \eta v + \mu r + \delta p)$$

Higher values of  $\theta$  (corresponding to a greater relative weight placed on employment in the union's objectives) imply greater employment.

Assume union membership in the union local is proportional to employment. Then an expression for local union membership,  $m$ , is arrived at:

$$(7) \quad m = \exp(\gamma + \eta v + \mu r + \delta p)$$

---

<sup>34</sup> Of course, there is no suggestion that the assumption of this particular solution to the bilateral monopoly problem rules out other such solutions. Thus, imaginatively, one referee derived a similar equation to equation (6) but by setting up an expression for Nash's bargaining model. This is not the place to return to the question of the appropriate way to model union-management bargaining.

where  $\gamma$  incorporates  $\rho$ ,  $\theta^{-1}$ , and the factor of proportionality between employment and union membership.

Suppose this is one of  $N$  locals that belong to the national union and suppose each local faces the same constraints and possesses the same objectives. This is certainly a strong assumption, but the differences among the union locals within a given national union are likely to be narrower than the differences in the union locals across different national unions. Then aggregating equation (7) over all the  $N$  union locals yields  $M : \sum_i m_i = M$ , the membership in the national union which is

$$(8) \quad M = N [\exp (\gamma + \eta v + \mu r + \delta p )]$$

Write the logarithm of this national union membership equation for the  $k$  th national union

$$(9) \quad \ln M_k = \ln N_k + (\gamma_k + \eta_k v_k + \mu_k r_k + \delta_k p_k)$$

and, according to equation (9), holding constant the number of union locals in each national union, absolute differences in the values of the right-hand side variables in parentheses are transformed into proportionate differences in national union membership. Because general linear functions of random variables tend toward normality in the limit (even when the components may not be normal),

$\ln M_k$  tends to be distributed as normal and  $M_k$  to be lognormal. A logarithmic distribution for national union membership - as implied by Pareto's distribution - is derived. According to (8), the precise form of the logarithmic distribution is not a constant but will vary as the objectives of the constituent local unions differ and as their employment-wage trade-offs differ.

In addition to providing an explanation for Pareto's distribution, this simple model can also help to understand the different experiences of membership in private and public sector unions since the 1970s. Write equation (9) for a representative national union in sector  $k$  in year 0 and in a subsequent year  $t$  and then first difference the equations:

$$(10) \quad \Delta \ln M_k = \Delta \ln N_k + \eta_k \Delta v_k + \mu_k \Delta r_k + \delta_k \Delta p_k$$

where  $\Delta \ln M_k = (\ln M_k)_t - (\ln M_k)_0$ ,  $\Delta \ln N_k = (\ln N_k)_t - (\ln N_k)_0$ ,  $\Delta v_k = v_{kt} - v_{k0}$ ,  $\Delta r_k = r_{kt} - r_{k0}$ , and  $\Delta p_k = p_{kt} - p_{k0}$ . In forming this first difference equation, the parameters  $\eta_k$ ,  $\gamma_k$ ,  $\mu_k$ , and  $\delta_k$  have been assumed to be unchanged. Now consider applying equation (10) to a representative public-sector national union and to a representative private-sector union where year 0 is 1974 and year  $t$  is 2007.

An examination of the large unions in 1974 and 2007 in Table 1 suggests that an important difference between the large private-sector unions and the large public-sector unions is that the typical large private-sector union in 1974 was one involved in producing tradeables whereas the typical large public-sector union in 1974 and in 2007 does not produce something that is traded across international borders. The contraction of the large private-sector unions after 1974 accompanied a large growth in imports of the durable goods made by the workers in these unions, a growth stimulated by a reduction in their effective prices. In other words, for the typical large private-sector union after 1974,  $\Delta p_k$  assumed a large negative value: competition from cheaper foreign imports made the product markets for these U.S.-produced goods much more competitive and exacting. By contrast, the members of many public-sector unions were employed in delivering services sheltered from rigorous competition. Indeed, at least until the last few years, rising state and local government budgets boosted the employment of unionized public sector workers.

In addition, in a number of cases, the typical member of a large public-sector union is a skilled or semi-skilled worker while the typical member of the large private-sector union in 1974 was a blue-collar worker with easily acquired skills. There is evidence that, in many instances, skilled labor and capital tend to be complementary inputs in production while less skilled workers and capital are often

substitute inputs.<sup>35</sup> Given this, the drastic reduction in the real price of computers and peripheral equipment since the 1970s<sup>36</sup> has had the effect of reducing the employment of unskilled workers and increasing the employment of skilled workers. That is, in terms of equation (10),  $\Delta r_k < 0$  and, for the private sector blue-collar unions  $\mu_k$  is positive while for the public sector white-collar unions  $\mu_k$  is negative. The reduction in the price of an important type of capital has helped the growth of public sector unions and contributed to the contraction of private sector unions of blue-collar workers.

In this way, a simple form of the familiar model of trade union behavior can help understand both the logarithmic distribution function of union membership and the contrasting experiences of the large public and large private-sector unions since the 1970s.<sup>37</sup>

## VII. Conclusions

The information in this paper points to an increase in the concentration of membership in national unions in the United States since the 1970s and this concentration has come about as the heart of U.S. unionism has shifted from the private to the public sector. What have been called the “labor conglomerates”(Chitayat (1979)) are situated principally in the public sector.

How important is concentration in national unionism in the U.S.? One way to assess this is to invoke standards from the product market which has accorded special attention to Hirschman-

---

<sup>35</sup> See, for instance, Berndt and Christensen (1974).

<sup>36</sup> Consider the price index for computers, software, and communication published by the Bureau of Economic Analysis and deflated by the producer price index. Setting the value of this to 100 in the year 2000, its value in 1976 was over 831 and its value in 2007 was 61.3 .

<sup>37</sup> The contrasting environments over the past fifty years or so of the private-sector and public-sector union sketched in this section are those highlighted by Reder (1988).

Herfindahl's index (HHI). In its work in assessing the product market effects of mergers, the U.S. Department of Justice routinely uses values of HHI to classify product markets into unconcentrated markets, moderately concentrated markets, and highly concentrated markets. In 2007, the value for HHI for all the national labor organizations studied in this paper was 570, a value that would categorize these unionized labor markets as "unconcentrated". However, because the value of HHI for these national unions was 330 in 1978, the change in HHI between 1978 to 2007 was 240 points, an increase that would raise "competitive concerns" if experienced in moderately concentrated or highly concentrated markets.<sup>38</sup>

This paper has drawn on Pareto's distribution to characterize the size distribution of large trade unions and changes in the size distribution of large trade unions. In providing a summary statement of the distribution and its changes, the performance of Pareto's distribution has been mixed. Even though goodness of fit statistics are usually well over 90 percent, the description provided by strict log-linearity is sometimes inferior to one that is a concave from below. Moreover, the deviations from log-linearity are often not random.

Even if flawed, Pareto's distribution has provided a useful summary description of the changes in the concentration of unions in the U.S. over time and its movements over time match those used by Estey (1966) to track the concentration of union membership. The further application of Pareto's distribution to trade unions is well worth exploration. It appears to provide a convenient

---

<sup>38</sup> See U.S. Department of Justice and Federal Trade Commission, Horizontal Merger Guidelines, issued: August 19, 2010, especially pages 19-20. If  $\theta_j$  is union  $j$ 's membership as a fraction of total union membership, then HHI is the sum over all unions of  $(\theta_j)^2$ . Its values range from a minimum of  $n^{-1}$  where  $n$  is the number of unions to unity with higher values indicating greater concentration. The values of HHI in this paragraph are those when market shares (the fraction of total membership in each union) are expressed in percentages.

description of the size distribution of unions by their wealth and to have potential for its application to unions in other countries. At the same time, the estimates of Pareto's coefficient are sensitive to the choice of observations to which to fit the equation and the lack of strict criteria to guide the choice implies that there is ample room for the researcher's prior beliefs and, through the publication process, there is ample room for the profession's prior beliefs to influence the selection of estimates that are reported and disseminated.

A model of unionism has been adapted to generate a skew distribution such as Pareto's or the lognormal. Although it makes demanding assumptions, it falls into a class of statistical models in which random shocks asymptotically tend towards a normal distribution.

Does the trend towards greater concentration in U.S. unionism matter? We know that many outcomes are associated with differences in the sizes of firms: there are systematic earnings, fringe benefit, working conditions, and turnover differences by size of firm.<sup>39</sup> Are there differences in the labor market outcomes of union members between those who belong to large unions and those who belong to small unions? This question may be amenable to research by imaginatively combining the files on labor organizations at the Office of Labor-Management Standards with those on employment and compensation in the Census Bureau's Business Dynamics so that the effects of firm size on labor market outcomes may be distinguished from any effects associated with union size.<sup>40</sup>

Windmuller (1981) conjectured that increasing concentration "...tends to diminish direct

---

<sup>39</sup> See, for instance, Brown, Hamilton, and Medoff (1990). We also know that the outcomes of representation elections are correlated with the size of electoral unit (Farber (2001)).

<sup>40</sup> An anonymous referee suggested combining the union files with the Census Bureau's Longitudinal Employment-Household Dynamics data which may well be a worthwhile line of investigation if issues of confidentiality can be dealt with.

member participation in union affairs and to produce more representative, and thus more indirect, patterns of union government”. Because most negotiated union collective bargaining contracts involve local and regional unions, the implications of greater concentration among national unions may have more consequences for the pressure group activities of unions as they try to influence legislative and executive bodies. Because there is a positive correlation between the membership of unions and their net worth, a union movement concentrated in a smaller number of large unions implies a union movement in which much of its wealth is allocated by a smaller number of decision-makers. Unions representing workers in the public sector now constitute many of the very large labor organizations. Much has been written on “union voice”, but less on what these voices are saying. Increasingly the union voice heard by government is the voice of white-collar public-sector workers and the voice of lower-paid private-sector workers has been dulled.

A serious concern is that a more concentrated union movement that is dominated by public-sector unions may further politicize unionism. That is, the focus of union activity will be less on attending to grievances and to the conditions of union members at their place of work and more on issues that are the province of politics. After all, the managers and supervisors of public sector workers ultimately report to politicians which makes the links between unions of public-sector employees and politicians more visible and palpable than those of unions representing private-sector employees. Unions have always been involved in politics so this would be a change of degree, not of kind, but it is an important change because, ultimately, more politicized unionism will not help the typical union worker.<sup>41</sup>

---

<sup>41</sup> This view derives from observation of several episodes. One draws upon the experience of Britain in the 1960s and 1970s when a politicized union movement (with unions of public sector workers at the helm) prompted General Elections where the electorate were asked “Who runs the country?”

This judgment - that a more politicized union movement is an undesirable development - does not hold when people lack the mechanisms to alter their government peacefully. In other countries, unions have contributed significantly in converting autocratic regimes into more decent and civilized societies. But in a democracy, a politicized union movement is another step toward government run by pressure groups in which the great mass of citizens are disempowered and the classic union activities of protecting the interests of employees at their place of work take a backseat to service that is political in nature.

This harks back to Dicey's classic dilemma expressed a century ago.<sup>42</sup> The principle that working people should have the freedom to form associations that represent and guard their interests would seem to be an intrinsic feature of a liberal society. But, if these associations exploit this principle to procure entitlements that enhance their interests at the expense of others, a new base of authority and influence is created that, at best, feathers its nest and, at worst, challenges the jurisdiction of the state. A balance is needed between promoting the principle of free association and avoiding the creation of a mischievous organization. In due course, this may become the issue presented by the increasing concentration of members in a smaller number of unions and by the

---

After indecision and wavering, the British electorate plumped for a government that substantially trimmed the sails of unions and especially those of unions of public sector workers. This episode is not singular but it is an example of unions refocusing their energies on a political agenda and using their human and financial resources to prevail upon government. In time, this induces a backlash that leaves unions with substantially less clout and influence.

<sup>42</sup> In 1912, Dicey wrote, "In England, as elsewhere, trade unions and strikes, or federations of employers and lock-outs;...in the United States, the efforts of Mercantile Trusts to create for themselves huge monopolies....[S]ome forms of association force upon public attention the practical difficulty of so regulating the right of association that its exercise may neither trench upon each citizen's individual freedom nor shake the supreme authority of the State" (Dicey, pp. 331-2 of 2008 edition).

dominance of the union movement by public sector employees. This is not a prediction; it is a concern, a source of apprehension.

## REFERENCES

- Aitchison, J., and J.A.C. Brown 1954. On Criteria for Descriptions of Income Distribution. *Metroeconomica*, 6(3): 88-107.
- Ashack, Elizabeth A., Major Union Mergers, Alliances, and Disaffiliations, 1995-2007, posted September 2008. *Compensation and Working Conditions* U.S. Bureau of Labor Statistics <http://stats.bls.gov/opub.cwc/cb20090919ar01p1.htm>
- Atkinson, Anthony B., Piketty, Thomas, and Saez, Emmanuel. March 2011. Top Incomes in the Long Run of History. *Journal of Economic Literature*, 49(1) : 3-71.
- Axtell, Robert L. September 2001. Zipf Distribution of U.S. Firm Sizes. *Science*, 293 : 1818-20.
- Berndt, Ernst R. and Laurits R. Christensen. June 1974. Testing for the Existence of a Consistent Aggregate Index of Labor Inputs *American Economic Review*, 64(3) : 391- 404 .
- Brown, Charles, James Hamilton, and James Medoff, 1990. *Employers Large and Small*, Cambridge: Harvard University Press.
- Chaison, Gary N. October 1980. A Note on Union Merger Trends, 1900-1978. *Industrial and Labor Relations Review*, 34 (1) : 114-20.
- Chipman, John S. November 1974. The Welfare Ranking of Pareto Distributions. *Journal of Economic Theory*, 9 : 275-82 .
- Chitayat, Gideon. 1979. *Trade Union Mergers and Labor Conglomerates*. New York: Praeger.
- Clauset, Aaron, Cosma Rohilla Shalizi, and M.E.J. Newman. 2009. Power-Law Distributions in Empirical Data. *SIAM Review*. 51. : 661-703.

Dicey, A.V. 1914. *Lectures on the Relationship between Law and Public Opinion in England during the Nineteenth Century*, originally published: London. Macmillan. ; new edition Liberty Fund, inc., Indiana, 2008.

Estey, Marten S..August 1966. Trends in Concentration of Union Membership, 1897-1962. *Quarterly Journal of Economics*. 80(3): 343-60 .

Farber, Henry S. January 2001. Union Success in Representation Elections: Why Does Unit Size Matter?. *Industrial and Labor Relations Review*. 54(2): 329-48.

Fujiwara, Yoshi, Corrado Di Guilmi, Hideaki Aoyama, Mauro Gallegati, Wataru Souma. April 2004. Do Pareto-Zipf and Gibrat Laws Hold True? An Analysis with European Firms. *PhysicaA: Statistical Mechanics and Its Applications*, 335 (1-2) : 197-216 .

Gabaix, Xavier, June 2009. Power Laws in Economics and Finance *Annual Review in Economics and Finance*, 1, : 255-93.

Gabaix, Xavier, and Rustam Ibragimov. May 2009. Rank -  $\frac{1}{2}$  : A Simple Way to Improve the OLS Estimation of Tail Exponents. Unpublished manuscript.

Hart, P.E., and E.H.Phelps Brown. March 1957. The Sizes of Trade Unions: A Study in the Laws of Aggregation. *Economic Journal* 67 (265) : 1-15 .

Hart, P.E., and S.J.Prais . 1956. The Analysis of Business Concentration: A Statistical Approach. *Journal of the Royal Statistical Society Series A (General)* . 119 (2) : 150-91 .

Ijiri, Yuji, and Herbert.A.Simon. March-April 1974. Interpretations of Departures from the Pareto Curve Firm-Size Distributions. *Journal of Political Economy*. 82(2), Part I : 315-31.

Nishiyama, Yoshihiko, and Susumu Osada. January 2004. Statistical Theory of Rank Size Rule Regression under Pareto Distribution. *Interfaces for Advanced Economic Analysis, Kyoto University, Discussion Paper No.009.*

OECD. 1996. Size Distribution of Output and Employment: A Data Set for Manufacturing Industries in Five OECD Countries, 1960-1990. *OECD Working Paper. IV (74), Paris .*

Pareto, Vilfredo. 1965. *Écrits sur la Courbe de la Répartition de la Richesse .* Genève.

Quandt, Richard E. [1966a] Old and New Methods of Estimation and the Pareto Distribution. *Metrika . 10(1) : 55-82 .*

Quandt, Richard E. [1966b]. On the Size Distribution of Firms. *American Economic Review . 56(3): 416-32.*

Reder, Melvin W. Spring 1988. The Rise and Fall of Unions: The Public Sector and the Private. *Journal of Economic Perspectives. 2(2): 89-110.*

Rees, Albert [1989] *The Economics of Trade Unions.* Chicago: University of Chicago Press. third edition .

Rosen, Kenneth T. And Mitchel Resnick, 1980. The Size Distribution of Cities: An Examination of the Pareto Law and Primacy. *Journal of Urban Economics 8 : 165-186 .*

Simon, Herbert A., and Charles P. Bonini September 1958. The Size Distribution of Business Firms. *American Economic Review . 48(4) : 607-17 .*

Simpson, D.H. November 1972. An Analysis of the Size of Trade Unions. *British Journal of Industrial Relations . 1 (3) : 382-91 .*

Soo, Kwok Tong, May 2005. Zipf's Law for Cities: A Cross-Country Investigation. *Regional Science and Urban Economics . 35(3) : 239-63 .*

Steindl, Josef, *Random Processes and the Growth of Firms: A Study of the Pareto Law* . New York: Hafner .

Troy, Leo, and Neil Sheflin. 1985. *U.S. Union Sourcebook: Membership, Finances, Structure, Directory*. Industrial Relations Data and Information Services. NJ. First edition.

Willman, Paul, and Timothy Morris, 1988. *The Finances of British Trade Unions 1975-1985*. [U.K.] Department of Employment Research Paper No.62, 1988

Windmuller, John P.. October 1981. Concentration Trends in Union Structure: An International Comparison. *Industrial and Labor Relations Review*. 35(1): 43-57 .

## APPENDIX NOTE ON DATA SOURCES

The household Current Population Survey (CPS) has collected annual information on union membership since 1973. In 1977, the question collecting this information was amended to include employee associations in addition to unions. The CPS information does not identify the union to which the worker belongs.

Membership by labor organization is reported to the Office of Labor-Management Standards at the U.S. Department of Labor by private sector unions (and those covering the U.S. Postal Service) as required by the Labor Management Reporting and Disclosure Act (the Landrum-Griffin Act) of 1959. Organizations representing other Federal workers are required by the Civil Service Reform Act of 1978 to supply similar reports. Organizations exclusively representing local or state public sector workers are not required to file. The information was published every two years by the BLS until September 1980 when BLS Bulletin 2079 reported data for 1978. Public and professional employee associations were first included in the BLS Directory in 1968. The Bureau of National Affairs published their Directory in 1982 and Leo Troy and Neil Sheflin published their Union Sourcebook in 1985.

The values of aggregate union and employee association membership from the CPS usually fall short of the summation of the membership reported by unions to the Office of Labor-Management Standards. This is the case even though some unions are not required to file reports and a few unions are listed without their membership being reported. The common assumption is that unions tend to overstate their membership by not deleting from their records people who have discontinued paying their dues.

In this paper, the 2007 data are from the Directory of U.S. Labor Organizations 2008 Edition,

edited by Court Gifford, the Bureau of National Affairs, Arlington, Virginia. The 1983 membership data are from Leo Troy and Neil Sheflin, Union Sourcebook: Membership, Structure, Finance, Directory First Edition 1985, Industrial Relations Data Information Services, West Orange, NJ. The 1978 data are from the Directory of National Unions and Employee Associations, 1979, U.S. Department of Labor Bureau of Labor Statistics, 1980, Bulletin 2079. The 1974 data are from the Directory of National Unions and Employee Associations, 1975, U.S. Department of Labor Bureau of Labor Statistics, 1977, Bulletin 1937. The 1968 data are from the Directory and National and International Unions in the United States 1969, U.S. Department of Labor Bulletin 1665, BLS 1970. The 1939 data used in this paper are from Leo Troy, Trade Union Membership, 1897-1962, National Bureau of Economic Research, Occasional Paper 92, New York 1965. The 1920 data are from Leo Wolman, Ebb and Flow in Trade Unionism, NBER, New York, 1936.

Table 1

The Five Largest Unions in 1974 and in 2007 (Membership in Thousands)

<u>Full Name of Union in 1974</u>	<u>Rank</u>	<u>Membership</u>	
	<u>in 1974</u>	in 1974	in 2007
International Brotherhood of Teamsters, Chauffeurs, Warehousemen & Helpers of America (IBT)	1	1,973.3	1,398.6
International Union of Automobile, Aerospace & Agricultural Implement Workers of America (UAW)	2	1,544.9	538.4
National Education Association (NEA)	3	1,470.2	3,167.6
United Steelworkers of America (USW)	4	1,300.0	730.9
International Brotherhood of Electrical Workers (IBEW)	5	991.2	697.9
<u>Full Name of Union in 2007</u>	<u>Rank</u>	<u>Membership</u>	
	<u>in 2007</u>	in 1974	in 2007
National Education Association (NEA)	1	1,470.2	3,167.6
Service Employees International Union (SEIU)	2	550.0	1,575.5
American Federation of State County, & Municipal Employees (AFSCME)	3	648.2	1,470.1
International Brotherhood of Teamsters (IBT)	4	1,973.3	1,398.6
United Food & Commercial Workers International Union (UFCW)*	5	1,175.9	1,304.1

\*United Food & Commercial Workers International Union did not exist in 1974. It was created in June 1979 through the merger of the Retail Clerks International Association and the Amalgamated Meat Cutters and Butcher Workmen of North America. In 1974, the Retail Clerks reported their membership as 650,876 and the Meat Cutters as 525,000. The entry in the table of 1,175.9 is the sum of these two membership figures in thousands. After 1979, a number of other unions joined with the UFCW.

Table 2

The Distribution of Unions by the Number of their Members in 1974 and 2007

row	size of union (number of members)	1974		2007	
		percent of all unions	percent of all members	percent of all unions	percent of all members
1	< 1,000	12.7	0.04	9.0	0.02
2	from 1,000 to 9,999	24.1	0.87	22.5	0.50
3	from 10,000 to 99,999	38.2	8.21	40.5	9.54
4	from 100,000 to 499,999	19.3	37.01	16.2	19.45
5	from 500,000 to 999,999	3.8	23.89	7.2	25.93
6	≥ 1,000,000	1.9	25.99	4.5	44.58

Table 3

Changes in the Number of Unions and in the Number of Members (in thousands) between 1974 and 2007 by Size Class

size of union in thousands of members	number in 2007 minus number in 1974	
	number of unions	number of members in thousands
< 1	- 17	- 4.8
from 1 to 4.9	- 20	- 64.8
from 5 to 9.9	- 6	- 46.6
from 10 to 24.9	- 13	- 187.0
from 25 to 49.9	- 19	- 577.1
from 50 to 99.9	- 4	- 282.2
from 100 to 199.9	- 13	- 1,949.6
from 200 to 299.9	- 5	- 1,107
from 300 to 399.9	- 1	- 251.4
from 400 to 499.9	- 4	- 1,756.1
from 500 to 999.9	0	- 593.4
≥ 1,000	+ 1	+ 2,627.8
Total	- 101	- 4,195.1

Table 4  
 Percent of All Union Members Who Are Members of the Ten, Five, and Two Largest Unions,  
 Selected Years 1920 - 2007

<u>year</u>	<u>membership of the ten largest unions as a percent of total union membership</u>	<u>membership of the five largest unions as a percent of total union membership</u>	<u>membership of the two largest unions as a percent of total union membership</u>
1920	43.85	28.70	14.98
1939	36.84	17.88	14.29
1968	43.82	28.79	14.72
1974	45.43	30.09	14.54
1978	47.13	30.46	14.85
1983	48.10	31.60	14.70
2007	62.40	44.58	23.71

Table 5  
Estimates of Equation (1.1) Fitted to Largest Unions in 2007

$$(1.1) \quad \ln S_i = a_0 + b_0 \ln M_i + e_{0i}$$

row	size of unions	number of unions	estimated $a_0$ (s.e.)	estimated $b_0$ (s.e.)	implied $\alpha$	$R^2$
1	> 200,000	21	10.894 (0.395)	-1.126 (0.061)	1.13	0.947
2	> 100,000	31	8.890 (0.284)	-0.883 (0.047)	0.88	0.923
3	> 50,000	46	7.873 (0.167)	-0.784 (0.030)	0.78	0.938
4	> 25,000	60	7.239 (0.120)	-0.723 (0.023)	0.72	0.943
5	> 10,000	76	6.407 (0.101)	-0.615 (0.021)	0.62	0.919

Table 6  
Estimates of Equation (2.1) Fitted to the Largest Unions in 2007

$$(2.1) \quad \ln M_i = a_1 + b_1 \ln S_i + e_{1i}$$

size of unions	number of unions	estimated $a_1$ (s.e.)	estimated $b_1$ (s.e.)	implied $\alpha$	$R^2$
> 200,000	21	9.503 (0.172)	-0.841 (0.046)	1.19	0.947
> 100,000	31	9.749 (0.211)	-1.045 (0.056)	0.96	0.923
> 50,000	46	9.749 (0.174)	-1.196 (0.046)	0.84	0.938
> 25,000	60	9.731 (0.158)	-1.305 (0.042)	0.77	0.943
> 10,000	76	9.931 (0.193)	-1.490 (0.052)	0.67	0.919

Table 7

Estimates of Equations (1.1) and (2.1) Fitted to the Largest Unions in 2007 After Accounting for First-Order Serial Correlation in the Residuals

$$(1.1) \quad \ln S_i = a_0 + b_0 \ln M_i + e_{0i}$$

size of unions	number of unions	estimated $a_0$ (s.e.)	estimated $b_0$ (s.e.)	implied $\alpha$
> 200,000	21	11.050 (0.659)	-1.149 (0.098)	1.15
> 100,000	31	9.378 (0.594)	-0.967 (0.090)	0.97
> 50,000	46	8.415 (0.433)	-0.890 (0.069)	0.89
> 25,000	60	7.893 (0.327)	-0.852 (0.055)	0.85
> 10,000	76	7.265 (0.324)	-0.795 (0.054)	0.80

$$(2.1) \quad \ln M_i = a_1 + b_1 \ln S_i + e_{1i}$$

size of unions	number of unions	estimated $a_1$ (s.e.)	estimated $b_1$ (s.e.)	implied $\alpha$
> 200,000	21	9.260 (0.250)	-0.761 (0.072)	1.31
> 100,000	31	9.010 (0.325)	-0.761 (0.092)	1.31
> 50,000	46	8.871 (0.388)	-0.760 (0.099)	1.32
> 25,000	60	8.722 (0.382)	-0.765 (0.084)	1.31
> 10,000	76	8.720 (0.407)	-0.760 (0.078)	1.32

Table 8  
Estimates of Equation (3.1) Fitted to the Largest Unions in 2007

$$(3.1) \quad \ln S_i = a_2 + b_2 \ln M_i + c_2 (\ln M_i)^2 + e_{2i} \quad ,$$

size of unions	number of unions	estimated $a_2$ (s.e.)	estimated $b_2$ (s.e.)	estimated $c_2$ (s.e.)	$R^2$
> 200,000	21	0.331 (2.401)	2.112 (0.732)	-0.245 (0.055)	0.975
> 100,000	31	0.344 (0.862)	1.987 (0.287)	-0.235 (0.023)	0.983
> 50,000	46	2.666 (0.480)	1.117 (0.173)	-0.167 (0.015)	0.984
> 25,000	60	3.744 (0.290)	0.655 (0.112)	-0.128 (0.010)	0.984
> 10,000	76	4.086 (0.125)	0.440 (0.055)	-0.109 (0.006)	0.987

Table 9

The Number of Unions and their Sizes in Five Years According to Different Selection Criteria

A. Selecting the 21 Largest Unions in each Year

year	number of unions included	percent of all unions	percent of all members	size of smallest union included	size of largest union included
2007	21	18.9	83.1	229,248	3,167,612
1978	21	10.1	66.6	284,329	1,923,896
1968	21	10.4	61.9	283,155	1,755,025
1939	21	10.2	55.1	78,600	495,000
1920	21	13.5	63.7	73,600	393,600

B. Selecting all Unions with 200,000 or more Members in each Year

year	number of unions included	percent of all unions	percent of all members	size of smallest union included	size of largest union included
2007	21	18.9	83.1	229,248	3,167,612
1978	27	13.0	73.8	200,000	1,923,896
1968	27	13.4	68.0	200,000	1,755,025
1939	7	3.4	30.9	201,500	495,000
1920	3	1.9	21.5	330,800	393,600

C. Selecting the Largest 18.9 Percent of all Unions in each Year

year	number of unions included	percent of all unions	percent of all members	size of smallest union included	size of largest union included
2007	21	18.9	83.1	229,248	3,167,612
1978	39	18.9	82.6	154,242	1,923,896
1968	38	18.9	71.1	144,682	1,755,025
1939	39	18.9	70.3	43,500	495,000
1920	25	18.9	68.9	60,400	393,600

Table 10

Point Estimates of Pareto's Coefficient and Goodness of Fit Statistics from Fitting Equation (1.1) to the Largest Unions in Different Years, "Large" by Three Different Criteria

$$(1.1) \ln S_i = a_0 + b_0 \ln M_i + e_{0i}$$

year	estimates of $\alpha$				values of $R^2$			
	by criterion.....			average	by criterion.....			average
	..... A	..... B	..... C		..... A	..... B	..... C	
2007	1.13	1.13	1.13	1.13	0.947	0.947	0.947	0.947
1978	1.28	1.16	1.10	1.18	0.907	0.917	0.944	0.923
1968	1.43	1.13	1.23	1.26	0.958	0.954	0.964	0.959
1939	1.46	1.70	1.32	1.49	0.948	0.882	0.967	0.932
1920	1.55		1.51	1.53	0.955		0.962	0.959
average	1.37	1.28	1.26	1.32	0.943	0.925	0.957	0.944

#### Notes to Table 10

The values in column A correspond to fitting equation (1.1) to the 21 largest unions in each year.

The values in column B correspond to fitting equation (1.1) to unions with 200,000 or more members in each year. (There are only three unions satisfying this criterion in 1920.)

The values in column C correspond to fitting equation (1.1) to the largest 18.9 percent of all unions in each year.

Table 11  
 Estimating Pareto's Coefficient to the Observations on Unions from 1920 to 2007  
 Allowing the Intercept to Vary by Year

$$\ln(S_{it}) = \eta_t + \alpha \ln(M_{it}) + u_{it}$$

<u>Each Year's Selection Criterion</u>	<u>total number of observations</u>	<u>estimated <math>\alpha</math></u> (s.e.)	<u>implied <math>\alpha</math></u>	<u><math>R^2</math></u>
21 Largest Unions	105	-1.123 (0.057)	1.12	0.877
Unions with $\geq 200,000$ Members	82	-1.313 (0.042)	1.31	0.932
Largest 18.9 % of Unions	162	-1.036 (0.035)	1.04	0.904

Table 12  
Distribution of Net Assets of U.S. Trade Unions in 1982

net assets in thousands of \$s	number of national unions	percent of all national unions	percent of all unions' net assets
≤ 10	32	16.8	-0.05
10.001 - 100	27	14.1	0.04
100.001 - 1,000	42	22.0	0.60
1,000.001 - 1,500.0	33	17.3	3.39
1,500.001 - 10,000.0	15	7.9	3.73
10,000.001 - 25,000.0	20	10.5	10.61
25,000.001 - 50,000.0	12	6.3	12.60
50,000.001 - 100,000.0	3	1.6	5.96
100,000.001 - 250,000.0	5	2.6	37.60
> 250,000.0	2	1.0	35.52

Table 13

Estimates of Equations (1.1), (1.2), (3.1), and (3.2) Fitted to the Membership and Net Assets of the 21 Largest U.S. Unions in 1982 (Net Assets) and in 1983 (Membership)

$$(1.1) \quad \ln S_i = a_0 + b_0 \ln M_i + e_{0i}$$

$$(1.2) \quad \ln S_i = a_3 + b_3 \ln K_i + e_{3i}$$

membership equation (1.1)				net assets equation (1.2)			
estimates of.....		implied $\alpha$	$R^2$	estimates of.....		implied $\alpha$	$R^2$
$a_0$ (s.e.)	$b_0$ (s.e.)			$a_3$ (s.e.)	$b_3$ (s.e.)		
11.600 (0.540)	-1.278 (0.087)	1.28	0.920	13.086 (0.387)	-0.848 (0.035)	0.85	0.969

$$(3.1) \quad \ln S_i = a_2 + b_2 \ln M_i + c_2 (\ln M_i)^2 + e_{2i} \quad ,$$

$$(3.2) \quad \ln S_i = a_4 + b_4 \ln K_i + c_4 (\ln K_i)^2 + e_{4i}$$

membership equation (3.1)				net assets equation (3.2)			
estimates of.....			$R^2$	estimates of.....			$R^2$
$a_2$ (s.e.)	$b_2$ (s.e.)	$c_2$ (s.e.)		$a_4$ (s.e.)	$b_4$ (s.e.)	$c_4$ (s.e.)	
-10.356 (3.096)	5.768 (0.990)	-0.560 (0.079)	0.979	5.187 (5.261)	0.532 (0.917)	-0.060 (0.039)	0.973

Table 14

Estimates of Equations (1.1), (1.2), (3.1), and (3.2) Fitted to the 20 Largest British Unions by Net Worth in 1985 and the 21 Largest British Unions by Membership in 1984

$$(1.1) \quad \ln S_i = a_0 + b_0 \ln M_i + e_{0i}$$

$$(1.2) \quad \ln S_i = a_3 + b_3 \ln K_i + e_{3i}$$

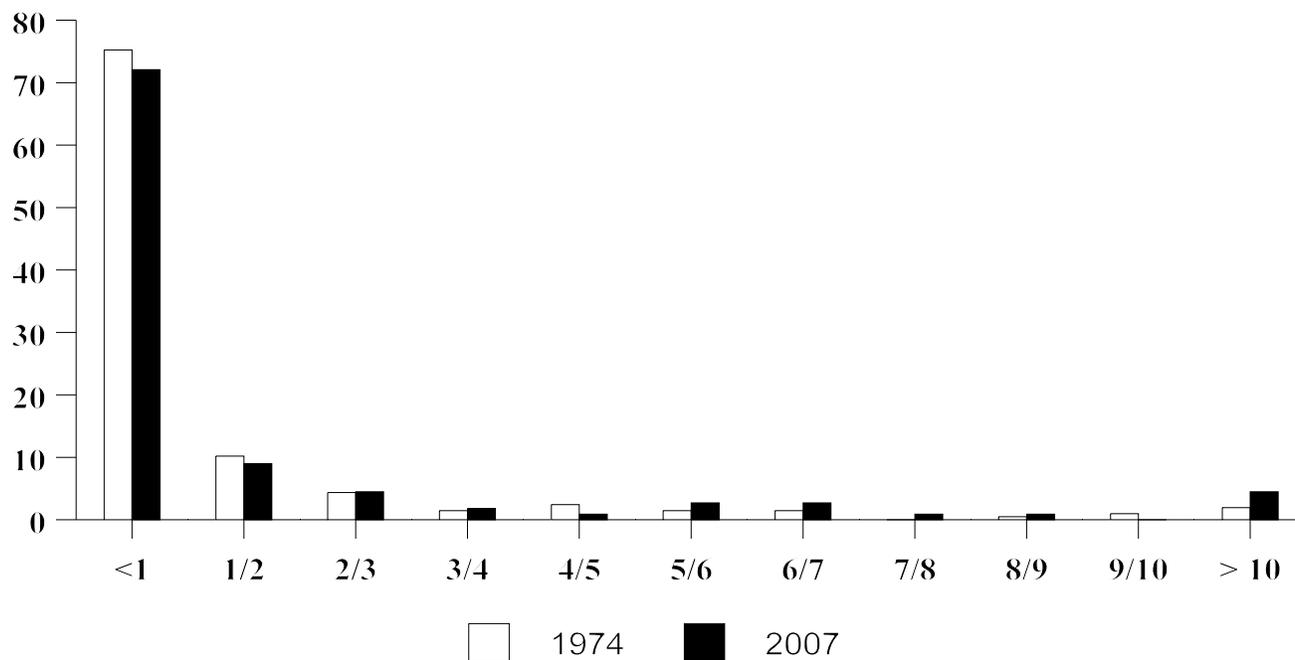
membership equation (1.1)				net worth equation (1.2)			
estimates of.....		implied $\alpha$	$R^2$	estimates of.....		implied $\alpha$	$R^2$
$a_0$ (s.e.)	$b_0$ (s.e.)			$a_3$ (s.e.)	$b_3$ (s.e.)		
10.313 (0.281)	-1.153 (0.048)	1.15	0.968	12.691 (0.747)	-0.955 (0.079)	0.96	0.891

$$(3.1) \quad \ln S_i = a_2 + b_2 \ln M_i + c_2 (\ln M_i)^2 + e_{2i} \quad ,$$

$$(3.2) \quad \ln S_i = a_4 + b_4 \ln K_i + c_4 (\ln K_i)^2 + e_{4i}$$

membership equation (3.1)				net worth equation (3.2)			
estimates of.....			$R^2$	estimates of.....			$R^2$
$a_2$ (s.e.)	$b_2$ (s.e.)	$c_2$ (s.e.)		$a_4$ (s.e.)	$b_4$ (s.e.)	$c_4$ (s.e.)	
3.451 (2.224)	1.162 (0.747)	-0.192 (0.062)	0.979	-15.892 (3.103)	5.125 (0.658)	-0.321 (0.035)	0.982

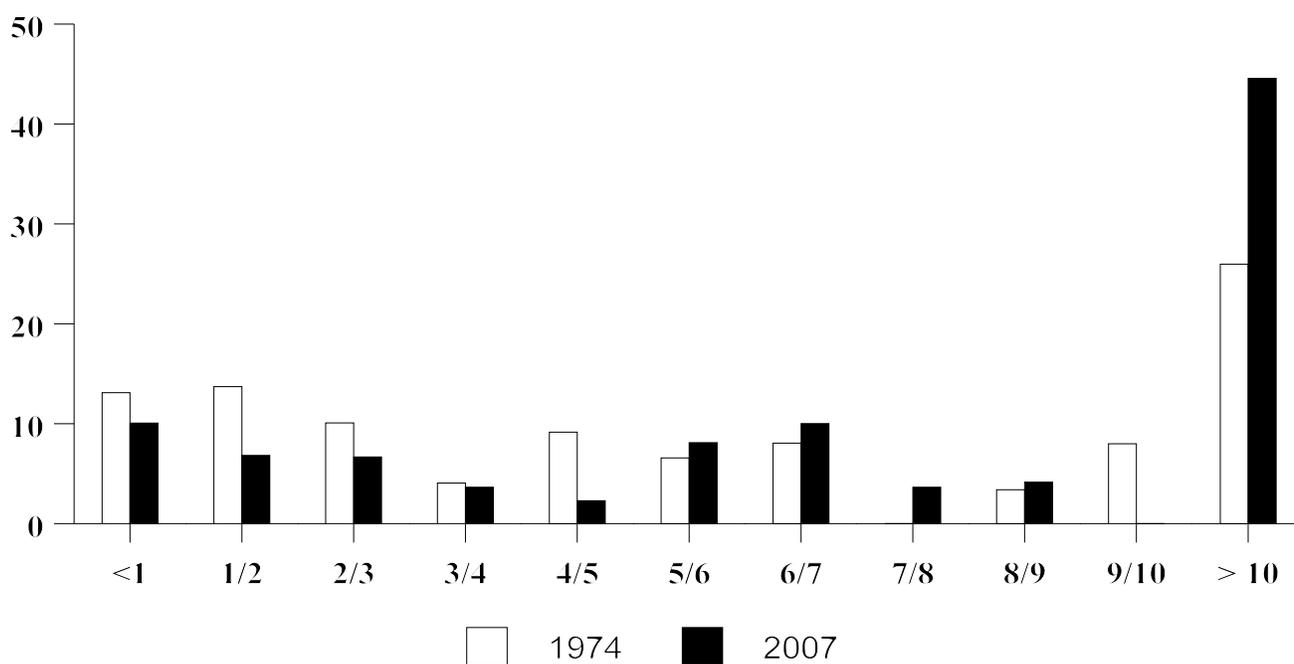
Figure 1  
 Percent Distribution of the Number of Unions by Size of Union in 1974 and 2007 : Size Classes  
 in Intervals of One Hundred Thousand Members



The horizontal axis measures membership in labor organizations by hundred thousand of members. Thus < 1 means less than 100,000, 1/2 means from 100,000 to 199,999, and so on.

The vertical axis measures the total number of unions in the size class as a percentage of the total number of unions in all size classes. The white columns describe the year 1974 and the black columns describe the year 2007.

Figure 2  
 Percent Distribution of Union Membership by Size of Union in 1974 and 2007: Size  
 Classes in Intervals of One Hundred Thousand Members

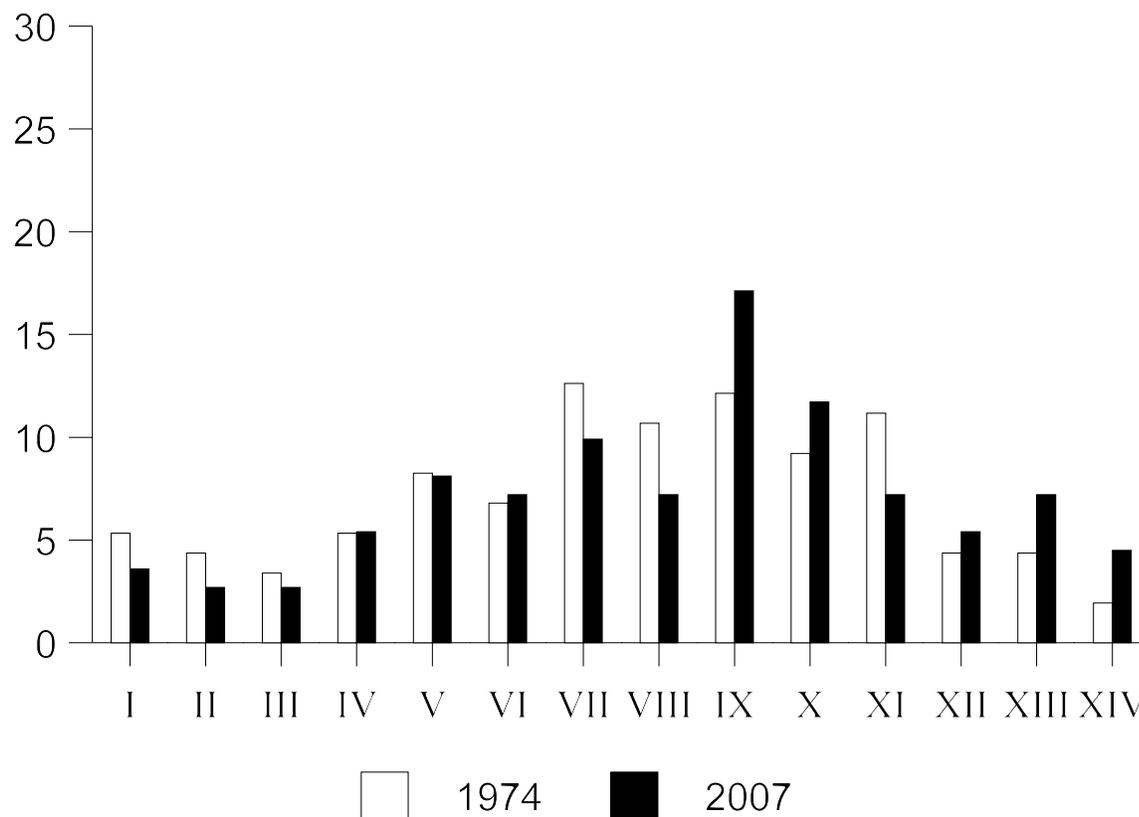


The horizontal axis measures membership in labor organizations by hundred thousand of members. Thus “< 1” means less than 100,000, “1/2” means from 100,000 to 199,999, and so on in 100,000 intervals until the largest class of 1,000,000 or more denoted “> 10” .

The vertical axis measures the membership in unions in the size class as a percentage of total union membership in all size classes.

The white columns describe the year 1974 and the black columns describe the year 2007.

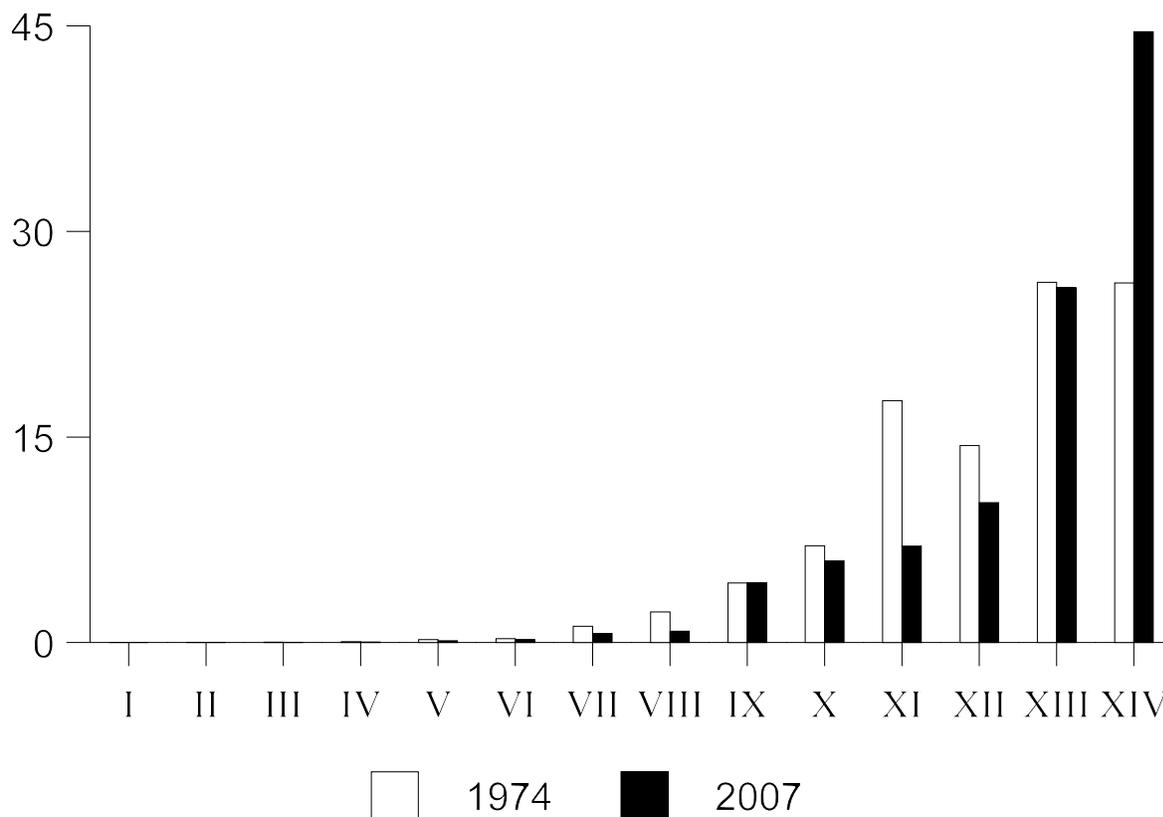
Figure 3  
Percent Distribution of the Number of Unions by Size of Union in 1974 and 2007: Size Classes in Constant Ratios



The vertical axis measures the number of unions in the size class as a percentage of the total number of unions in all size classes. The black columns describe the year 2007 and the white columns describe the year 1974. The horizontal axis measures membership in labor organizations where each size class bears a constant ratio from class to adjacent class: class I means < 250 members; class II means 250-499 members; class III means 500-999 members; class IV means 1,000-1,999 members; class V means 2,000-3,999 members; class VI means 4,000-7,999 members; class VII means 8,000-15,999 members; class VIII means 16,000-31,999 members; class IX means 32,000-63,999 members; class X means 64,000-127,999 members; class XI means 128,000-255,999 members; class XII means 256,000-511,999 members; class XIII means 512,000-1,023,999 members; and class XIV means  $\geq 1,024,000$  members.

Figure 4

Percent Distribution of Union Membership by Size of Union in 1974 and 2007: Size Classes in Constant Ratios



The vertical axis measures the membership in unions in the size class as a percentage of total union membership in all size classes. The black columns describe the year 2007 and the white columns describe the year 1974. The horizontal axis measures membership in labor organizations where each size class bears a constant ratio from class to adjacent class: class I means < 250 members; class II means 250-499 members; class III means 500-999 members; class IV means 1,000-1,999 members; class V means 2,000-3,999 members; class VI means 4,000-7,999 members; class VII means 8,000-15,999 members; class VIII means 16,000-31,999 members; class IX means 32,000-63,999 members; class X means 64,000-127,999 members; class XI means 128,000-255,999 members; class XII means 256,000-511,999 members; class XIII means 512,000-1,023,999 members; and class XIV means  $\geq 1,024,000$  members.

Figure 5  
Average Number of Mergers of Labor Unions per Year, 1900-2007



With the exception of 2007, each year on the horizontal axis denotes the beginning of a five year period so, for instance, 1900 means the average annual number of union mergers during the years 1900-04. The observation for 2007 is the average number of union mergers for 2005-07. Observations from 1900 to 1959 are drawn from Chaison (1980) and observations from 1960 to 2007 are from Ashack (2008).

Figure 6  
Pareto's Law for Labor Organizations with at least 200,000 Members in 2007 ?

