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ABSTRACT

The Optimal Minimum Wage with Regulatory Uncertainty*

For two different regulatory standards, we examine the optimal minimum wage in a competitive labour market when the government is uncertain about supply and demand. Solutions are related to underlying supply and demand conditions, and the extent of uncertainty and of rationing efficiency. We show that regulatory uncertainty does not diminish the rationale for intervention, but may require a low minimum wage that may not bind. With expected earnings-maximization, greater uncertainty widens the range of parameter values for which a minimum wage should be set. With expected worker surplus-maximization and sufficiently efficient rationing, a minimum wage should always be set.

JEL Classification: J38, J31

Keywords: minimum wage, uncertainty, worker surplus

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1 Introduction

A statutory minimum wage has been a common feature of public policy for many years, and currently operates in 26 out of 34 OECD member countries. Some parts of the US also have 'living wage' laws, setting a minimum wage above the federal minimum, while in the UK the living wage will become a mandatory minimum from April 2016 for workers aged 25 or more. Although the effects of a minimum wage remain controversial, changes in the sectoral mix of employment in developed economies associated with globalization and the recent recession have aroused new interest in its use. Many manufacturing and construction jobs have been lost that are unlikely to be replaced, whereas there has been an expansion of relatively low-paid work, such as in wholesale and retail trade, and accommodation and food services (OECD, 2015).

When a minimum wage is introduced or raised some low-paid workers will receive higher wage rates, but there may be an overall negative impact on employment. Econometric evidence on this trade-off, which is reviewed by Neumark, Salas, and Wascher (2014), generally relates to the effects of small changes in a minimum wage. However, there is little evidence on the impact of larger changes, or for the introduction of a minimum wage. Thus, for example, a minimum wage law was rejected by Switzerland in 2014, but introduced by Germany in 2015, after years of heated debate about the size of the potential effects. In addition to the implications for employment, the fiscal impact and the effects on shadow activities and the incentive to leave education were among the issues discussed (IZA, 2014). Other potential considerations include the effects on non-wage worker benefits (Schmitt, 2013) and on the welfare of the poor, through increases in the prices.(MaCurdy, 2015).

In this paper, for two different regulatory standards, we examine the optimal minimum wage in a competitive labour market where the government is uncertain of supply and demand conditions. A straightforward measure that captures the wage-employment trade-off analytically is the total earnings from the minimum wage. Sobel (1999), for example, describes the maximization of such earnings as one of the most popularly-stated goals of minimum wage policy. However, this goal does not take into account workers' reservation wages, which reflect the value of unemployment benefits, leisure, and home production (Danziger, 2009). If reservation wages are netted out, the goal instead becomes the maximization of 'worker surplus', a term that was introduced by Marshall (1920) as a particular type of producer surplus.

We develop a simple model in which the government may be imperfectly informed about supply and demand conditions when it sets the minimum wage, though workers and firms have all the relevant information when employment takes place. This information structure may be interpreted to cover a situation where the minimum wage is fixed for a period over which a new set of market conditions may obtain. Alternatively, it could pertain to a single realization of a market in which both labour supply and labour demand are identically and independently distributed, and the government knows the parameters of the distributions. In practice, most OECD countries review minimum wage levels every year, or nearly every year, though the federal minimum in the United States was unchanged from September 1997 to July 2007 (see OECD, 2015).

Regulatory uncertainty stems from diverse sources, and is distinct from economic policy uncertainty, which relates to private agents' perceptions of potential government behaviour. Even under settled economic conditions, a government will face some uncertainty about labour market conditions, but sectoral adjustments in economic activity may make the fine-tuning of minimum-wage policy more challenging, and, historically, periods of lower growth have coincided with higher levels of uncertainty (Bloom, 2014). For simplicity, we consider regulatory uncertainty first over labour supply and then over labour demand, and we highlight some important differences of detail. In each case we analyze the optimal wage for the alternative objectives of expected earnings- and expected worker surplus-maximization.

Our choice of objective functions is not intended as a normative statement; but if, instead, the objective is to maximize expected welfare, a minimum wage would not be set. Nonetheless, our results for expected worker-surplus maximization would still hold qualitatively if we added expected profit into the objective function, but with a lower weight (Lee and Saez, 2012). The effect of a minimum wage on worker surplus is analyzed by Danziger (2009). In his model risk-averse workers face uncertainty about whether they will find a job, and the minimum wage that maximizes expected worker surplus is derived. Worker surplus, defined as the difference between the value of employment and the value of unemployment, has also played a prominent role in labour market matching models (see Jung and Kuhn, 2014, for a recent example).

When a minimum wage is set, if it binds, efficient rationing of employment would require that the available work goes to individuals with the lowest reservation wages amongst those willing to work at that wage. However, the minimum wage may prevent some workers with lower reservation wages from competing by accepting relatively low wages. Job allocation may be inefficient, with personal contacts or prejudice playing a role. Portugal and Cardoso (2006) and subsequent studies find that a higher minimum wage is associated with lower job turnover. One possible explanation is that, insofar as the higher minimum wage causes jobs to be kept by people who would move in a freer market, the rationing of jobs is inefficient.

Also, resources may have been expended to reach the efficient outcome (e.g., search costs or waiting).

Direct evidence on inefficiency of employment rationing is sparse, though Luttmer (2007) finds that no such inefficiency is associated with the 1990-91 rise in the US federal minimum wage. Nonetheless, Luttmer cautions that this evidence relates to a small increase in the minimum wage, and that inefficient allocation might nonetheless follow a large increase. Also, several recent papers consider the effect of rationing inefficiency in the context of price ceilings (see Glaeser and Luttmer, 2003; Davis and Kilian, 2011 and Bulow and Klemperer, 2012). Since the optimal minimum wage for expected worker surplus-maximization may be affected by rationing inefficiency, we parameterize it in our analysis.

We relate our solutions to the underlying supply and demand conditions, the degree of uncertainty the government faces, and the extent of rationing inefficiency. Specifically, with expected earnings-maximization, inefficiency of rationing has no impact on the level chosen for the minimum wage. For great enough uncertainty, a minimum wage should be set at a level which may or may not bind, depending on the resolution of the supply or demand uncertainty. Regulatory uncertainty does not undermine the case for setting a minimum wage, and, when the uncertainty relates to labour supply, it has no effect on the optimal level of the minimum wage. If, however, there is sufficiently large labour demand uncertainty, the minimum wage should be set at a conservative level that may turn out to be non-binding.

By not setting a high minimum wage, large potential declines in employment are avoided. By nonetheless setting a minimum wage, though a relatively low one, protection is provided against the most negative potential outcomes with regard to the free market wage. Moreover, we show that greater uncertainty actually widens the range of parameter values for which it is optimal to set a minimum wage.

If, instead, the objective is to maximize expected worker surplus, a minimum wage should always be set if the rationing of employment is efficient enough. If uncertainty is sufficiently small, the minimum wage should be at a level that is sure to bind. However, with greater uncertainty over either labour supply or labour demand, the minimum wage should be set at a lower level. As in the case of expected earnings-minimization, the optimal minimum wage may or may not bind, depending on the resolution of the uncertainty. Inefficiency of rationing complements uncertainty over supply or demand, in that it reduces the critical amount of uncertainty at which the government chooses a minimum wage that may or may not bind, rather than one that is sure to bind.

However, if inefficiency is sufficiently great, no minimum wage should be set. All employment at the minimum wage gives a higher worker surplus than would be obtained in a free market. But with inefficient rationing some of this employment is of workers who displace others who value employment more than themselves. If rationing is sufficiently inefficient, the loss in expected worker surplus due to such inefficient reallocations, together with the lower expected labour demand, can fully offset the aggregate expected benefits from the employment at the higher wage.

Our formulation is a partial equilibrium analysis of a low-wage segment of the labour market, for which our focus on perfect competition provides a particularly testing environment. Although a similar analysis may be undertaken for monopsony, the rationale for imposing the minimum wage to benefit low-income workers would then be stronger.

Existing theoretical literature on the minimum wage has not focused on the choice of the appropriate level, except in the context of optimal income tax theory, where the whole labour market is considered. The welfare impact of a minimum wage used in combination with an income tax in a competitive economy is analyzed in a stream of papers beginning with Allen (1987). In these models, the asymmetric information relates to the government's lack of knowledge of any individual's type, rather than to incomplete information about aggregate supply and demand. Lee and Saez (2012) derive sufficient conditions for it to be optimal to set a binding minimum wage, both with and without taxes and transfers. Furthermore, Danziger and Danziger (2015) show that a graduated minimum wage, in combination with an optimal income tax, can provide a Pareto improvement over an optimal income tax alone.

In Section 2 we formulate our model. In Sections 3 and 4 we consider the cases of supply uncertainty and demand uncertainty, respectively, and Section 5 concludes. Proofs not provided in the text are given in an appendix.

2 The Model

Consider a competitive labour market in which the government may be uncertain of supply and demand. We assume that here is no uncertainty on the part of private agents. Workers' aggregated reservation wages for supplying l units of labour are given by $\mathcal{S}(l,\phi) = (S+\phi)l + sl^2/2$, where $s > 0$, $S + \phi > 0$. The government regards ϕ as a random variable with zero mean ($E(\phi) = 0$), distributed according to a continuous differentiable c.d.f. $F(\phi)$ defined on a closed interval $[\phi_{\min}, \phi_{\max}]$. The inverse supply of labour is

$$\partial\mathcal{S}(l,\phi)/\partial l = S + \phi + sl \ .$$

Producers' gross revenue, net of non-labour costs, is given by $\mathcal{R}(l,\gamma) = (R+\gamma)l - rl^2/2$, where $r > 0$, $R + \gamma > 0$. The government regards γ as a random variable with zero mean

$(E(\gamma) = 0)$, distributed according to a continuous differentiable c.d.f. $G(\gamma)$ defined on a closed interval $[\gamma_{\min}, \gamma_{\max}]$. The inverse demand (marginal revenue product) for labour is

$$\partial\mathcal{R}(l,\gamma)/\partial l = R + \gamma - rl \ .$$

The respective supply and demand functions for labour are therefore

$$l^s(w,\phi) = (w - S - \phi)/s \text{ and } l^d(w,\gamma) = (R + \gamma - w)/r \ , \tag{1}$$

where w is the wage rate.

We assume that the two shocks ϕ and γ are independent and that the hazard rates $F'(\phi)/(1 - F(\phi))$ and $G'(\gamma)/(1 - G(\gamma))$ are strictly increasing.

The government chooses a minimum wage $\bar{w}$, which is announced to all private agents (firms and workers), who observe the realized values of ϕ and γ, and then generate labour supply and demand functions (1). Employment and production then take place. $\bar{w}$ is chosen to maximize the expectation of earnings $wl \equiv \Upsilon$ or worker surplus $wl - \mathcal{S}(l,\phi) \equiv \Omega$. In practice, the time dimension for a minimum wage varies between countries (for example, it is hourly in the UK, weekly in Malta and monthly in Belgium). Our analysis can be interpreted in any of these units.

Denote the ex-post free market wage (where $l^s(w) = l^d(w)$) by $w^*(\phi,\gamma)$ and the corresponding employment level by $l^*(\phi,\gamma)$. Thus,

$$w^*(\phi,\gamma) = [(S + \phi)r + (R + \gamma)s]/(r + s) \text{ and } l^*(\phi,\gamma) = (R + \gamma - S - \phi)/(r + s) \ . \tag{2}$$

To ensure a well-defined equilibrium employment ex post, we assume that $R+\gamma_{\min} > S+\phi_{\max}$. But, before the demand and supply shocks ϕ and γ are realized, the government views $w^*(\phi,\gamma)$ as a random variable with expected value

$$w_e^* = (Sr + Rs)/(r + s) \ . \tag{3}$$

A minimum wage rate $\bar{w}$ may or may not bind. If it binds, that is, if $\bar{w} > w^*$, then employment $l(\bar{w}) = l^d(w,\gamma)$, while if it does not bind, that is, if $\bar{w} \leq w^*$ the free-market equilibrium (2) obtains. Thus, if $l^s(w,\phi) > l^d(w,\gamma)$, employment is $l(\bar{w}) = l^d(w,\gamma)$; but if $l^s(w,\phi) \leq l^d(w,\gamma)$, employment is $l(\bar{w}) = l^*(\phi,\gamma)$.

We consider the minimum wage first for stochastic supply and then for stochastic demand. In principle, the model could also be developed with the two types of uncertainty together, but the interplay of constraints then makes the analysis intractable. The results for stochastic supply and demand separately are qualitatively similar, though, particularly for expected earnings-maximization, there are significant differences of detail. The outcome in each case depends on which of three ranges contains $\bar{w}$. In the 'high' range $\bar{w}$ is so high that it binds for all realizations of uncertainty. In the 'low' range $\bar{w}$ is so low that it is non-binding for all realizations of uncertainty. However, in the 'middle' range $\bar{w}$ may or may not bind, depending on the value of ϕ (for stochastic supply) or γ (for stochastic demand). We examine each of these ranges as a potential location for the optimal value of $\bar{w}$, and we derive and compare the results for both expected earnings-maximization and expected worker surplus-maximization.

If there is excess supply of labour at the minimum wage $\bar{w}$, employment is rationed. Insofar as rationing is inefficient, employment is not all allocated to the workers with the lowest reservation wages. We parameterize rationing efficiency for the case in which the government maximizes expected worker surplus by writing its objective function as a linear combination of the expected worker surplus for efficient rationing and for extreme inefficient rationing of employment among those who are willing to work at $\bar{w}$. The parameter $\alpha \in [0,1]$ captures the degree of rationing efficiency. For example, rationing is efficient if $\alpha = 1$ and extremely inefficient if $\alpha = 0$, while if $\alpha = 1/2$ the allocation of employment is random among those willing to work. (It is easily shown that a convex combination of the values of worker surplus for $\alpha = 1$ and $\alpha = 0$ equals the value obtained by setting α appropriately. In particular, with equal weights for $\alpha = 1$ and $\alpha = 0$, worker surplus is the same as when $\alpha = 1/2$.)

Any inefficiency affects worker surplus if $\bar{w} > w^*$. Then $l = l^d(\bar{w},\gamma) = (R+\gamma-\bar{w})/r$ and employment is rationed. If work is allocated efficiently, that is, to the workers whose reservation wages are lowest, worker surplus is

$$\begin{aligned}\Omega_\alpha(l^d(\bar{w},\gamma),\phi) &= \bar{w}l^d(\bar{w},\gamma) - [(S+\phi)l^d(\bar{w},\gamma) + s(l^d(\bar{w},\gamma))^2/2] \\ &= (R+\gamma-\bar{w})[2r(\bar{w}-S-\phi) - s(R+\gamma-\bar{w})]/2r^2 \ .\end{aligned}$$

But suppose instead that $l^d(\bar{w},\gamma)$ is allocated extremely inefficiently, i.e., to those workers with the highest reservation wages among those willing to work at wage $\bar{w}$. Then, employment $l^d(\bar{w},\gamma) = (R+\gamma-\bar{w})/r$ is along the highest part of the supply curve and below $\bar{w}$, i.e., from $l^s(\bar{w},\phi) - l^d(\bar{w},\gamma)$ to $l^s(\bar{w},\phi)$. At $l^s(\bar{w},\phi) - l^d(\bar{w},\gamma)$ worker surplus is $\bar{w} - S - \phi - s\left(l^s(\bar{w},\phi) - l^d(\bar{w},\gamma)\right)$ per unit of $l^d(\bar{w},\gamma)$, while at $l^s(\bar{w},\phi)$ it is zero. Taking the mean of these two values, and multiplying by $l^d(\bar{w},\gamma)$, gives total worker surplus,

$$\begin{aligned}\Omega_{1-\alpha}(l^d(\bar{w},\gamma),\phi) &= l^d(\bar{w},\gamma)\{\bar{w}-S-\phi-s[l^s(\bar{w},\phi)-(l^d(\bar{w},\gamma))]\}/2 \\ &= s\left(R+\gamma-\bar{w}\right)^2/2r^2 \ .\end{aligned}$$

For any $\alpha \in [0,1]$, aggregate worker surplus is therefore

$$\alpha\Omega_{\alpha}(l^d(\bar{w},\gamma),\phi) + (1-\alpha)\Omega_{1-\alpha}(l^d(\bar{w},\gamma),\phi) = \\ (R+\gamma-\bar{w})\left(2r\alpha\left(\bar{w}-S-\phi\right)-s\left(2\alpha-1\right)\left(R+\gamma-\bar{w}\right)\right)/2r^2 \equiv \Omega_I(\phi,\gamma)\ . \tag{4}$$

Our main qualitative results for expected worker surplus-maximization could be illustrated for the standard case of efficient rationing. However, our parameterization also allows us to determine a critical level of rationing efficiency, which depends on supply and demand slopes, above which these results carry over unchanged.

3 Stochastic Supply

We now assume that ϕ is a random variable that captures supply uncertainty, while demand is deterministic ($\gamma = 0$). The labour supply and demand schedules become $l^s(w,\phi) = (w - S - \phi)/s$ and $l^d(w) = (R-w)/r$. From (2), for a given realization of ϕ, the free market wage and employment are

$$w^*(\phi) = [(S+\phi)r + Rs]/(r+s) \text{ and } l^*(\phi) = \left(R-S-\phi\right)/(r+s)\ . \tag{5}$$

For a given minimum wage $\bar{w}$, the equation for $w^*(\phi)$ defines the specific value $\phi = \phi^*(\bar{w})$ at which the market clears:

$$\phi^*(\bar{w}) = (r+s)(\bar{w}-w_e^*)/r\ , \tag{6}$$

where w_e^* is given by (3).

Note that, from the definitions of $w^*(\phi)$ and $\phi^*(w)$, it follows that $prob(w^*(\phi) \le \bar{w}) = prob(\phi \le \phi^*(\bar{w})) = F(\phi^*(\bar{w}))$.

If the minimum wage $\bar{w}$ is set in the high range, where $\bar{w} \in (w^*(\phi_{\max}), R]$ (so that $\phi^*(\bar{w}) > \phi_{\max}$) it will bind for all values of ϕ; that is, $w = \bar{w}$, irrespective of the realization ϕ. If it is set in the low range, where $\bar{w} < w^*(\phi_{\min})$ (i.e., $\phi^*(\bar{w}) < \phi_{\min}$) it does not bind, whatever the value of ϕ, and so there is market clearance at $w = w^*(\phi)$. However, in the middle range where $\bar{w} \in [w^*(\phi_{\min}), w^*(\phi_{\max})]$ (so that $\phi^*(\bar{w}) \in [\phi_{\min}, \phi_{\max}]$) the effect of the minimum wage depends on the value of ϕ. If labour supply is relatively small at each wage w (when $\phi \in [\phi^*(\bar{w}), \phi_{\max}]$) the minimum wage does not bind ($w = w^*(\phi)$) whereas for a relatively large labour supply (when $\phi \in [\phi_{\min}, \phi^*(\bar{w})]$) the minimum wage binds and there is rationing ($w = \bar{w}$). For both expected worker surplus- and expected earnings-maximization, we consider the locally optimal minimum wage in each of these three ranges separately, and then combine the analyses to examine the global optimum.

3.1 Expected Earnings for Stochastic Supply

If the government's aim is to maximize the expected earnings $E(wl) \equiv E(\Upsilon)$, inefficiency of employment rationing has no effect on the choice of the minimum wage. Moreover, if there is no uncertainty the minimum wage should then be set such that the elasticity of labour demand is (minus) unity if and only if this minimum wage binds. Thus, with no uncertainty $R/2$ is the optimal minimum wage if $R/2 > w^*$, that is, if $R(r-s) - 2Sr > 0$, which requires that the demand slope r be sufficiently larger than the supply slope s. Otherwise, no minimum wage should be set. When there is supply uncertainty, however, we must consider the three potential ranges for the minimum wage specified above.

First, if the minimum wage is set in the high range, $\bar{w} > w^*(\phi_{\max})$, it binds for any $\phi \in [\phi_{\min}, \phi_{\max}]$ and employment is given by labour demand. If $R/2 > w^*(\phi_{\max})$ then $\bar{w} = R/2$ is the globally optimal minimum wage. But if $R/2 \leq w^*(\phi_{\max})$ then, for any realization ϕ, earnings would be greater at $\bar{w} = w^*(\phi_{\max})$ than at a higher minimum wage, and so the expected earnings-maximizing minimum wage is not in the high range.

Second, if the minimum wage is set in the low range, $\bar{w} < w^*(\phi_{\min})$, it does not bind for any realization ϕ, and so the free-market wage $w^*(\phi)$ will obtain. Any minimum wage in this range yields the same outcome as setting $\bar{w} = w^*(\phi_{\min})$.

Third, consider the middle range, $\bar{w} \in [w^*(\phi_{\min}), w^*(\phi_{\max})]$, so that the corresponding $\phi^*(\bar{w}) \in [\phi_{\min}, \phi_{\max}]$. In this range $\bar{w}$ may or not bind, depending on the realization ϕ. If $\phi \geq \phi^*(\bar{w})$, so that $\bar{w} \leq w^*(\phi)$, i.e., $\bar{w}$ does not bind, earnings are $\Upsilon\left(l^*(\phi)\right)$, as evaluated above for $\bar{w} < w^*(\phi_{\min})$. If $\phi < \phi^*(\bar{w})$, so that $\bar{w} > w^*(\phi)$, which is binding, there is excess supply and earnings are $\Upsilon(l^d(\bar{w}))$. Combining these two possibilities, expected earnings for $\bar{w}$ in the middle range are

$$E(\Upsilon) = \int_{\phi^*(\bar{w})}^{\phi_{\max}} \Upsilon\left(l^*(\phi)\right) f(\phi) d\phi + \int_{\phi_{\min}}^{\phi^*(\bar{w})} \Upsilon(l^d(\bar{w})) f(\phi) d\phi \equiv \Upsilon_s \ . \tag{7}$$

The first term in (7) is expected earnings conditional on $\bar{w} \leq w^*(\phi)$ (i.e., $\phi \geq \phi^*(\bar{w})$). Since $\bar{w}$ does not bind, then, from (5), $\Upsilon = l^*(\phi)w^*(\phi) = \left[(S+\phi)r + Rs\right]\left(R - S - \phi\right)/(r+s)^2$, and the conditional expectation of Υ is

$$\int_{\phi^*(\bar{w})}^{\phi_{\max}} \Upsilon\left(l^*(\phi)\right) f(\phi) d\phi = \frac{(Sr+Rs)(R-S)(1-F(\phi^*(\bar{w}))) - (2Sr + Rs - Rr)\,\epsilon_s^H(\phi^*(\bar{w})) - r\sigma_s^H(\phi^*(\bar{w}))}{(r+s)^2}\ , \tag{8}$$

where $\epsilon_s^H(\phi^*(\bar{w})) = \int_{\phi^*(\bar{w})}^{\phi_{\max}} \phi f(\phi) d\phi$ and $\sigma_s^H(\phi^*(\bar{w})) = \int_{\phi^*(\bar{w})}^{\phi_{\max}} \phi^2 f(\phi) d\phi$.

The second term in (7) is expected earnings conditional on $\bar{w} > w^*(\phi)$ (i.e., $\phi < \phi^*(\bar{w})$). In this case there is excess supply and demand binds, so that $\Upsilon = \bar{w} l^d(w) = \bar{w}(R - \bar{w})/r$.

The conditional expectation of Υ is then

$$\int_{\phi_{\min}}^{\phi^*(\bar{w})} \Upsilon(l^d(\bar{w}))f(\phi)d\phi = \frac{1}{r}\int_{\phi_{\min}}^{\phi^*(\bar{w})} \bar{w}(R-\bar{w})f(\phi)d\phi = \frac{1}{r}F(\phi^*(\bar{w}))\bar{w}(R-\bar{w}) \ . \tag{9}$$

Substituting into (7) from (8) and (9), and using (6), we find that for the middle range $\bar{w} \in [w^*(\phi_{\min}), w^*(\phi_{\max})]$,

$$\begin{aligned} \frac{d\Upsilon_s}{d\bar{w}} &= \frac{1}{r}(R-2\bar{w})F(\phi^*(\bar{w})) \ ; \\ \frac{d^2\Upsilon_s}{d\bar{w}^2} &= -\frac{2}{r}F(\phi^*(\bar{w})) + \frac{r+s}{r^2}(R-2\bar{w})F'(\phi^*(\bar{w})) \ . \end{aligned} \tag{10}$$

It is straightforward to show that expected earnings are continuous and differentiable at the boundaries of the middle range with the other ranges, as well as within each range (and this comment also applies for the other optimizations we undertake). We can now consider how the global optimum depends on which of the three parameter ranges contains $R/2$. We have seen that if $R/2$ is lower than the high range, Υ_s is increased by reducing $\bar{w}$ at least to $w^*(\phi_{\max})$, while if $R/2$ is above the low range then Υ_s is weakly greater if $\bar{w}$ is raised at least to $w^*(\phi_{\min})$. Therefore, if $R/2$ belongs to the middle range $[w^*(\phi_{\min}), w^*(\phi_{\max})]$, we can restrict attention to this range. From (10), $d\Upsilon_s/d\bar{w}$ is then positive at $\bar{w} = w^*(\phi_{\min})$ and negative at $\bar{w} = w^*(\phi_{\max})$. Since the f.o.c. $d\Upsilon_s/d\bar{w} = 0$ is satisfied at $\bar{w} = R/2$, at which the s.o.c. $d^2\Upsilon_s/d\bar{w}^2 < 0$ holds, our analysis of the three ranges leads to the following proposition.

Proposition 1 *With supply uncertainty, if $R/2 \geq w^*(\phi_{\min})$ the minimum wage that maximizes expected earnings $E(\Upsilon)$ is $\bar{w} = R/2$. But if $R/2 < w^*(\phi_{\min})$ no minimum wage should be set.*

This is a simple generalization of the solution when there is no uncertainty, with the constraint $R/2 \geq w^*$ replaced by $R/2 \geq w^*(\phi_{\min})$; i.e., the constraint $R(r-s) - 2Sr > 0$ is replaced by $R(r-s) - 2(S+\phi_{\min})r > 0$. With supply uncertainty, the optimal minimum wage, if it exists, is the same as without uncertainty. However, since $\phi_{\min} < 0$, the condition under which this minimum wage should be set is milder than when there is no uncertainty. Specifically, suppose that $R(r-s) - 2Sr < 0$, so that if there were no uncertainty there should be no minimum wage. When instead there is uncertainty, if also $\phi_{\min}$ is more negative than $[R(r-s) - 2Sr]/2r$, then the minimum wage should be set. Thus, the existence of uncertainty widens the range of parameter values for which a meaningful (i.e., potentially binding) minimum wage should be set.

The intuition underlying the impact of uncertainty is illustrated in Figure 1. If labour supply is certain, as given by $l^s(w, 0)$, then, since the intersection with labour demand $l^d(w)$

occurs above $R/2$, a minimum wage should not be set. Now suppose instead that labour supply is uncertain and that $\phi_{\min}$ is sufficiently negative that the lowest possible labour supply curve, $l^s(w, \phi_{\min})$, intersects $l^d(w)$ below $R/2$. Assume, as specified in Proposition 1, that the minimum wage is set at $R/2$. Then, for all ex-post realizations of uncertainty such that the supply curve cuts $l^d(w)$ above $R/2$, the minimum wage does not bind, and so has no effect. But for any realization such that supply cuts $l^d(w)$ below $R/2$, the minimum wage $\bar{w} = R/2$ binds and (by the standard unit-elasticity condition) maximizes earnings. Therefore, considering all possible realizations of ϕ together, the imposition of the minimum wage raises expected earnings, and this is because there is sufficient uncertainty.

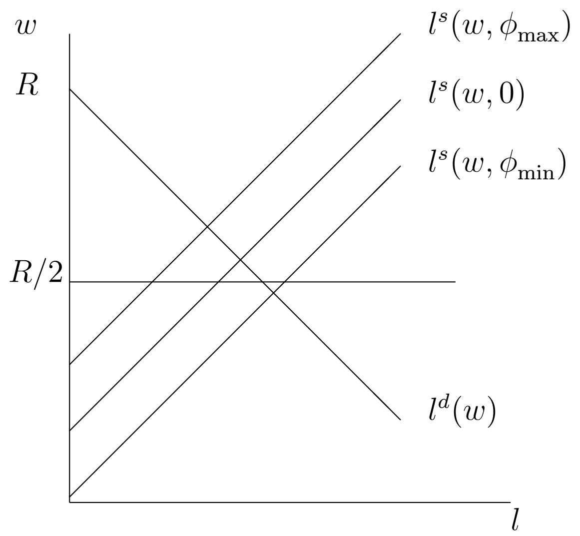

Figure 1: Expected earnings with stochastic supply

3.2 Expected Worker Surplus for Stochastic Supply

When expected worker surplus is maximized the solution depends on how far rationing is efficient. Again we examine three potential ranges for the minimum wage $\bar{w}$.

First, in the high range, $\bar{w} \in (w^*(\phi_{\max}), R]$, regardless of the realization ϕ, demand is a binding constraint. Therefore $l = l^d(\bar{w}, \phi) = (R - \bar{w})/r$ and employment is rationed. Total worker surplus is $\Omega_I(\phi)$, as given by (4) for $\gamma = 0$. Taking the expectation over all ϕ and differentiating with respect to $\bar{w}$, the minimum wage that satisfies the f.o.c. for maximizing expected worker surplus in this range is

$$\hat{w}(\alpha) = \frac{\alpha r(R + S) + (2\alpha - 1)sR}{2\alpha(r + s) - s} , \tag{11}$$

which is increasing in α.

However, some restrictions on parameter values are required for $E\left(\Omega_I(\phi)\right)$ to be concave at $\bar{w} = \hat{w}(\alpha)$ and for $\hat{w}(\alpha)$ to belong to the interval $(w^*(\phi_{\max}), R]$. Let ϕ_0 denote the value of ϕ at which $w^*(\phi) = \hat{w}(\alpha)$, i.e., from (5) and (11),

$$\phi_0 = \frac{(R-S)[\alpha(r+s)-s]}{2\alpha(r+s)-s} \ .$$

This defines the critical level of uncertainty for which (11) is a local optimum. It is also necessary that rationing is not 'too inefficient'. We show in the proof to the following lemma that the critical condition is that $\alpha > s/(r+s)$.

Lemma 1 *With supply uncertainty, if $\phi_{\max} < \phi_0$ and $\alpha > s/(r+s)$, the minimum wage that maximizes expected worker surplus for $\bar{w} \in (w^*(\phi_{\max}), R]$ is given by (11). If $\phi_{\max} \geq \phi_0$ the minimum wage that maximizes expected worker surplus cannot be strictly higher than $w^*(\phi_{\max})$.*

Thus, when supply uncertainty is small enough, in the sense that $\phi_{\max}$ is sufficiently small, and if rationing is efficient enough, there is a local maximum $\bar{w} = \hat{w}(\alpha) > w^*(\phi_{\max})$. For the given mean value ($\phi = 0$) variation of the lower bound may be accompanied by one or both of a redistribution of the mass and a variation of the upper bound. The cut-off level of uncertainty ϕ_0 is decreasing in α, so that the condition $\phi_{\max} < \phi_0$ is harder to satisfy if rationing is more efficient, ceteris paribus.

If instead a minimum wage is set in the low range $\bar{w} < w^*(\phi_{\min})$, then, regardless of the realization of ϕ, it does not bind. The free market wage and employment will be $w^*(\phi)$ and $l^*(\phi)$, respectively, as given by (5). Worker surplus is

$$\Omega\left(l^*(\phi)\right) = s\left(R-S-\phi\right)^2/2\left(r+s\right)^2 \ . \qquad (12)$$

which is the same for all $\bar{w} < w^*(\phi_{\min})$. Expected worker surplus is therefore the same for any minimum wage in this range (and the same as at $\bar{w} < w^*(\phi_{\min})$).

Finally, consider the middle range $\bar{w} \in [w^*(\phi_{\min}), w^*(\phi_{\max})]$. Parallel to (7) for expected earnings maximization, the maximand is now

$$E(\Omega) = \int_{\phi^*(\bar{w})}^{\phi_{\max}} \Omega\left(l^*(\phi)\right) f(\phi)d\phi + \int_{\phi_{\min}}^{\phi^*(\bar{w})} \Omega_I(\phi) f(\phi)d\phi \equiv \Omega_s \ . \qquad (13)$$

Substituting expressions for the two terms in (13), differentiating, and using (4), (6) and (12), we obtain

$$\frac{d\Omega_s}{d\bar{w}} = \frac{\alpha}{r}\epsilon_s^L(\phi^*(\bar{w})) + \frac{1}{r^2}F(\phi^*(\bar{w}))\left\{\left[2\alpha\left(r+s\right)-s\right]\left(R-\bar{w}\right)-\alpha r(R-S)\right\} \ ; \qquad (14)$$

$$\frac{d^2\Omega_s}{d\bar{w}^2} = -\frac{1}{r^2}F(\phi^*(\bar{w}))[2\alpha(r+s)-s] + \frac{1}{r^3}\left(R-\bar{w}\right)\left(r+s\right)\left[\alpha(r+s)-s\right]F'(\phi^*(\bar{w})) \ ,$$

where $\epsilon_s^L(\phi^*(\bar{w})) = \int_{\phi_{\min}}^{\phi^*(\bar{w})} \phi f(\phi) d\phi$.

Assume first that rationing is relatively efficient, i.e., $\alpha > s/(r+s)$. Since $\epsilon_s^L(\phi_{\min}) = F(\phi_{\min}) = 0$, $w^*(\phi_{\min})$ satisfies the f.o.c. $d\Omega_s/d\bar{w} = 0$ in this range. However, this is a local minimum, for Ω_s is convex at this point. In addition, we have seen that all $\bar{w} < w^*(\phi_{\min})$ result in the same levels of expected worker surplus as $\bar{w} = w^*(\phi_{\min})$ does. We show in the appendix that if $\phi_{\max} \geq \phi_0$ there is a unique optimal minimum wage in $[w^*(\phi_{\min}), w^*(\phi_{\max}))$. We have seen, however, that if $\phi_{\max} < \phi_0$, an optimal minimum wage cannot lie in the range $[w^*(\phi_{\min}), w^*(\phi_{\max})]$. Together with the findings in Lemma 1, this establishes the following result.

Proposition 2 *With supply uncertainty and $\alpha > s/(r+s)$, the minimum wage that maximizes expected worker surplus lies in the range $(w^*(\phi_{\min}), w^*(\phi_{\max})]$ if $\phi_{\max} \geq \phi_0$, but is given by (11) if $\phi_{\max} < \phi_0$.*

The proposition applies when rationing is sufficiently efficient. Then the location of the expected worker surplus-maximizing minimum wage depends on the extent of uncertainty. Specifically, if uncertainty is small enough (in the sense that $\phi_{\max} < \phi_0$, as in Lemma 1) the optimal minimum wage is $\hat{w}(\alpha)$. This minimum wage is in the high range and is sure to bind. If, however, uncertainty is greater, the optimal minimum wage is in the middle range and may or may not bind. In this case, if the minimum wage is ex-post binding, so that demand is a constraint, variation of the supply curve has no effect on employment. However, if ex post the minimum wage does not bind, greater supply uncertainty generates a smaller expected worker surplus. (Worker surplus for a given ϕ equals $(s/2)[(R-S-\phi)^2/(r+s)^2]$, which is decreasing in ϕ.) Since supply uncertainty has a negative effect on expected worker surplus if the minimum wage does not bind, and no effect if the minimum wage binds, it favours setting a low minimum wage that is more likely to bind.

Our analysis here rests on the simplifying assumption that ϕ is bounded above, and therefore that distinct high and middle ranges for $\bar{w}$ exist. If the assumption were dropped and if nonetheless the distribution had a centrally concentrated mass, the corresponding result would be that $\bar{w}$ would be set at a level at which it would bind with a high probability, rather than certainly. A similar comment applies to the other cases we consider below.

When $\phi_{\max} \geq \phi_0$, so that the optimal $\bar{w}$ lies in the middle range $(w^*(\phi_{\min}), w^*(\phi_{\max})]$, the wage $\hat{w}(\alpha)$ also belongs in this range. However, evaluating (14) at $\hat{w}(\alpha)$, we obtain

$$\frac{d\Omega_s(\hat{w}(\alpha))}{d\bar{w}} = \frac{\alpha}{r}\epsilon_s^L(\phi^*(\hat{w}(\alpha))) \leq 0 \ ,$$

which holds with equality only for $\phi_{\max} = \phi_0$ (or, $w^*(\phi_{\max}) = \hat{p}$). Therefore, if uncertainty is great enough ($\phi_{\max} \geq \phi_0$), the optimal minimum wage which lies in the middle range is strictly lower than $\hat{w}(\alpha)$, the optimal minimum wage with smaller uncertainty.

Note also that, from (14), evaluating $d\Omega_s/d\bar{w}$ at w_e^*:

$$\frac{d\Omega_s(w_e^*)}{d\bar{w}} = \frac{\alpha}{r}\epsilon_s^L(0) + \frac{1}{r^2}F(0)r[(r+s)\alpha - s]\frac{R-S}{r+s} \ .$$

Here, since $E(\phi) = 0$, the first term is negative, while the second is positive. It follows that an optimal minimum wage in the middle range may be greater or smaller than the *expected* market-clearing wage. Specifically, this minimum wage $\bar{w} \gtreqless w_e^*$ as $(r+s)\alpha\epsilon_s^L(0)+ F(0)\left[\alpha(r+s) - s\right](R-S) \gtreqless 0$.

Our qualitative results still obtain if some weight on expected profit is put in the objective function, though if a minimum wage is then optimal the weight on profit causes it to be lower, and so less likely to bind. A reworking of our analysis shows that the critical level of rationing efficiency becomes $\alpha > (s+\beta r)/(r+s)$ where $\beta \in [0,1)$ is the weight put on profit. With little uncertainty, the optimal value of $\bar{w}$ in the high range is then decreasing in the weight β. A greater weight on profit is associated with a more stringent requirement on the efficiency of rationing for a binding minimum wage to be optimal. These remarks also apply to the stochastic-demand case.

Another variation of the model - that would obtain with either of our objective functions - would be to allow for some non-compliance with the minimum wage law. Suppose that, if the minimum wage binds, then, with probability δ a firm complies with the minimum wage and with probability $1-\delta$ it sets the market wage. We can therefore think of a proportion δ complying and a proportion $1-\delta$ not. For workers in firms that comply, the expected worker surplus or expected earnings gain from a minimum wage would still apply, but for the others there would be no effect. Therefore, as in our model, there would be some conditions under which it would be optimal to set a minimum wage.

We now consider further the effect of rationing inefficiency.

Proposition 3 *With supply uncertainty, if $\alpha \leq s/(r+s)$, expected worker surplus is maximized by not setting a minimum wage.*

For $\alpha \leq s/(r+s)$, the market should be left unregulated, regardless of the degree of supply uncertainty. If, for example, labour supply and demand slopes are equal, a minimum wage should not be set if rationing is random or worse. Moreover, the smaller is the demand slope r, relative to the supply slope s, the greater is the rationing efficiency α required for a (possibly binding) minimum wage to be optimal.

The role of any rationing inefficiency can be seen by focusing on the case with no uncertainty, that is we consider the degenerate case with $\phi_{\min} = \phi_{\max} = 0$. In this case, the basic result - that to maximize worker surplus when there is no uncertainty a binding minimum wage should be set, provided rationing is efficient enough - would still hold with more general demand and supply functions. As an illustration, suppose rationing is efficient ($\alpha = 1$). Let $l^d(w)$ denote labour demand and $w^s(l)$ inverse labour supply, and assume that $\partial l^d(w)/\partial w < 0$ and $\partial w^s(l)/\partial w > 0$. When $w \geq w^*$, so that employment is demand-determined (with market clearance a special case), worker surplus is $\Omega = \int_0^{l^d(w)} (w - w^s(l))dl$. Using the envelope theorem, $d\Omega(w^*)/dw > 0$, i.e., a marginal increase in the wage from the free-market equilibrium raises worker surplus. A similar result is shown by Lee and Saez (2012) in their Proposition 1.

Figure 2 illustrates how worker surplus Ω is related to rationing efficiency when there is no uncertainty. With efficient rationing ($\alpha = 1$), Ω is maximized at a binding minimum wage ($\bar{w} = \hat{w}(1)$) greater than the market-clearing wage w^*. With less efficient rationing, but still with $\alpha > \alpha_0$, Ω is everywhere lower and is maximized at a lower minimum wage ($\bar{w} = \hat{w}(\alpha)$). For $\alpha < \alpha_0$, $d\Omega/d\bar{w} < 0$, so that no minimum wage is set and $\Omega = \Omega(w^*)$.

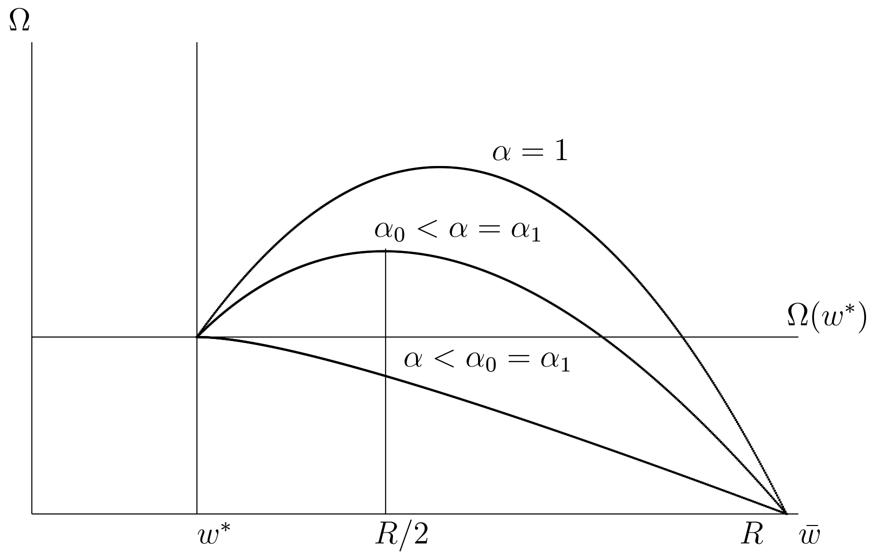

Figure 2: Worker surplus for different values of α ($R/2 > w^*$)

It is assumed in the figure that $R/2 > w^*$ (i.e., $R(r - s) - 2Sr > 0$), so that a minimum wage $\bar{w} = R/2$ should be set for earnings-maximization. To compare the solutions for the two objective functions, the value of α for the middle Ω-curve is chosen such that worker-surplus maximization occurs at $\bar{w} = R/2$. Using (11), $\hat{w}(\alpha) - R/2 \gtreqless 0$ as $\alpha \gtreqless Rs/(Sr + Rs) \equiv \alpha_1$. Thus, $\alpha = \alpha_1$ for the middle Ω-curve. The worker surplus-maximizing minimum wage exceeds that for the earnings maximization if $\alpha \in (\alpha_1, 1]$; but the ranking is reversed if $\alpha \in (\alpha_0, \alpha_1]$. However, if $R(r-s) - 2Sr \leq 0$, an earnings-maximizing government will not set a minimum wage, whereas a worker surplus-maximizing government will set a minimum wage if and only if $\alpha > \alpha_0$.

Intuitively, if $\alpha = 1$ the earnings-maximizing minimum wage balances the marginal gain of earnings per unit of labour from the last increment to the wage against the marginal loss of earnings from lower employment. Starting at this level of the minimum wage, if instead the objective is worker surplus-maximization then the marginal gain to worker surplus from the last increment to the wage is the same. But, because earnings below the reservation wage are excluded from the objective function, the marginal loss of worker surplus from the lower employment is smaller than the marginal loss of earnings. The minimum wage should therefore be raised further in the worker surplus-maximizing case.

However, compared to $\alpha = 1$, if $\alpha < 1$ the reservation wages of those employed are greater, and so the net gain in worker surplus from imposing the minimum wage is smaller. Therefore the minimum wage is set lower, and for sufficiently inefficient rationing ($\alpha < \alpha_1$) it is set below $R/2$.

4 Stochastic Demand

We now focus on the case of stochastic demand, with deterministic supply. There is some asymmetry with the converse case of Section 3 because each of the objective functions relates only to demand, and not to supply, for any given quantity. Nonetheless, our analysis in this section proceeds similarly to that for stochastic supply, and so some details are omitted or relegated to the appendix. We show that with expected earnings-maximization there is a qualitative difference between the results for the two types of uncertainty, whereas for expected worker surplus-maximization the results are largely similar to those for stochastic supply.

We assume that γ is a random variable that captures demand uncertainty, while ϕ is deterministic and equal to zero. For simplicity, our analysis focuses on a minimum wage that satisfies $\bar{w} > R + \gamma_{\min}$; that is, the government only considers a minimum wage that would result in positive employment and (therefore) positive worker surplus and earnings, regardless of the realization of the uncertainty.

Labour supply and demand are now $l^s(w) = (w - S)/s$ and $l^d(w) = (R + \gamma - w)/r$, respectively. The free market wage and employment are

$$w^*(\gamma) = [Sr + (R+\gamma)s]/(r+s) \ \text{ and } l^*(\gamma) = (R+\gamma - S)/(r+s) \ , \tag{15}$$

and, for a given minimum wage $\bar{w}$, (15) defines the specific value $\gamma = \gamma^*(\bar{w})$ at which the market clears,

$$\gamma^*(\bar{w}) = (r+s)(\bar{w} - w_e^*)/s \ . \tag{16}$$

We again distinguish three ranges of $\bar{w}$. In the low range $\bar{w} < w^*(\gamma_{\min})$ (so $\gamma^*(\bar{w}) < \gamma_{\min}$), $\bar{w}$ is non-binding for all $\gamma \in [\gamma_{\min}, \gamma_{\max}]$, and the market clears. In the high range $\bar{w} > w^*(\gamma_{\max})$ (so $\gamma^*(\bar{w}) > \gamma_{\max}$), and there is labour excess supply for all values of γ. Employment is determined by labour demand and is rationed. In the middle range $\bar{w} \in [w^*(\gamma_{\min}), w^*(\gamma_{\max})]$ (so $\gamma^*(\bar{w}) \in [\gamma_{\min}, \gamma_{\max}]$), and the effect of a minimum wage depends on the value of γ. For low demand (when $\gamma \in [\gamma_{\min}, \gamma^*(\bar{w})]$) there is excess supply and rationing, whereas for high demand (when $\gamma \in [\gamma^*(\bar{w}), \gamma_{\max}]$) $\bar{w}$ does not bind.

Note that, from the definitions of $w^*(\phi)$ and $\phi^*(w)$, it follows that $prob(w^*(\gamma) \leq \bar{w})$ $= prob(\gamma \leq \gamma^*(\bar{w})) = G(\gamma^*(\bar{w}))$.

4.1 Expected Earnings for Stochastic Demand

In our analysis of expected earnings maximization for uncertain supply there is a single level of $\bar{w}$ at which labour demand is unit elastic. However, with demand uncertainty, for any realization γ, unit elasticity obtains at $\bar{w} = (R+\gamma)/2$, that is, at a different level of $\bar{w}$ for each γ. This leads to a qualitative difference between the results for demand uncertainty and those for supply uncertainty. We again consider three ranges for the minimum wage $\bar{w}$.

If $\bar{w}$ is set in the high range, $\bar{w} > w^*(\gamma_{\max})$, then for any realization $\gamma \in [\gamma_{\min}, \gamma_{\max}]$ employment is given by labour demand. Expected earnings are then

$$E(\Upsilon) = \int_{\gamma_{\min}}^{\gamma_{\min}} \Upsilon(l^d(\bar{w}))g(\gamma)d\gamma = \frac{\bar{w}}{r}\int_{\gamma_{\min}}^{\gamma_{\min}} (R - \bar{w} + \gamma)\, g(\gamma)d\gamma = \frac{\bar{w}}{r}(R - \bar{w}) \ .$$

Hence, if $R/2 > w^*(\gamma_{\max})$, expected earnings are maximized at $\bar{w} = R/2 = E\left((R+\gamma)/2\right)$. If, however, $R/2 \leq w^*(\gamma_{\max})$, then $dE(\Upsilon)/d\bar{w} < 0$ in this range.

If, instead, the minimum wage is set in the low range, $\bar{w} < w^*(\gamma_{\min})$, then employment is given by labour supply at $w = \bar{w}$, and so earnings can be increased by raising $\bar{w}$ out of this range at least as far as $w^*(\gamma_{\min})$.

Thus, provided $R/2 \leq w^*(\gamma_{\max})$, $E(\Upsilon)$ is maximized in the middle range $\bar{w} \in [w^*(\gamma_{\min}),$ $w^*(\gamma_{\max})]$. In this range, parallel to (7), the maximand is

$$E(\Upsilon) = \int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \Upsilon\left(l^*(\gamma)\right) g(\gamma)d\gamma + \int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \Upsilon(l^d(\bar{w}))g(\gamma)d\gamma \equiv \Upsilon_d \ . \tag{17}$$

Then, as shown in the appendix,

$$\begin{aligned}\frac{d\Upsilon_d}{d\bar{w}} &= \frac{1}{r}(R-2\bar{w})G(\gamma^*(\bar{w}))+\frac{1}{r}\epsilon_d^L(\bar{w})\ ;\\ \frac{d^2\Upsilon_d}{d\bar{w}^2} &= -\frac{2}{r}G(\gamma^*(\bar{w}))-(s+r)\,\frac{\bar{w}(s-r)+rS}{rs^2}G'(\gamma^*(\bar{w}))\ ,\end{aligned} \tag{18}$$

where $\epsilon_d^L(\gamma^*(\bar{w}))=\int_{\gamma_{\min}}^{\gamma^*(\bar{w})}\gamma g(\gamma)d\gamma$.

If $R/2\leq w^*(\gamma_{\min})$, $d\Upsilon_d/d\bar{w}<0$ for all $\bar{w}\in(w^*(\gamma_{\min}),w^*(\gamma_{\max})]$. But we have seen that in this case $dE(\Upsilon)/d\bar{w}<0$ in the high range, while the minimum wage should always be raised out of the low range. Therefore, if $R/2\leq w^*(\gamma_{\min})$ it is optimal to set the minimum wage at $w^*(\gamma_{\min})$, i.e., in effect, there is no minimum wage.

Because $\epsilon_d^L(\gamma_{\min})=\epsilon_d^L(\gamma_{\max})=0$, if $R/2\in(w^*(\gamma_{\min}),\ w^*(\gamma_{\max}))$ then $d\Upsilon_d/d\bar{w}>0$ at $\bar{w}=\gamma_{\min}$ and $d\Upsilon_d/d\bar{w}<0$ at $\bar{w}=w^*(\gamma_{\max})$. Therefore a solution interior to the middle range, $\bar{w}\in(w^*(\gamma_{\min}),w^*(\gamma_{\max}))$ obtains. Since then $\epsilon_d^L(\gamma^*(\bar{w}))<0$, for $d\Upsilon_d/d\bar{w}=0$ it is necessary that $\bar{w}<R/2$, while if $R/2=w^*(\gamma_{\max})$ we have that $d\Upsilon_d/d\bar{w}=0$ at $\bar{w}=w^*(\gamma_{\max})$. We show in the appendix that the s.o.c. is satisfied, and so we have the following result.

Proposition 4 *With demand uncertainty and expected earnings maximization, if* $R/2\leq w^*(\gamma_{\min})$ *no minimum wage should be set, while if* $R/2\geq w^*(\gamma_{\max})$ *the minimum wage should be set at* $R/2$*. However, for* $R/2\in(w^*(\gamma_{\min}),w^*(\gamma_{\max}))$ *the optimal minimum wage* $\bar{w}\in(w^*(\gamma_{\min}),w^*(\gamma_{\max}))$ *and is less than* $R/2$*.*

Here, the condition for it to be optimal to set a minimum wage is that $R/2>w^*(\gamma_{\min})$, which, using (15), can be written as $R(r-s)-2(Sr+s\gamma_{\min})>0$. By a parallel argument to that made in relation to Proposition 1 for supply uncertainty, it follows that, with expected earnings maximization, the existence of demand uncertainty extends the set of parameter values for which it is optimal to set a minimum wage. However, in Proposition 4 the minimum wage is potentially below $R/2$, and so it is qualitatively different to the solution for supply uncertainty.

The intuitive rationale for this result can be seen from Figure 3. Suppose first that labour demand is known ($\gamma=0$), with intercept R. Given that labour supply $l^s(w)$ cuts labour demand $l^d(w,0)$ above $R/2$, a minimum wage should not be set. Now suppose instead, that labour demand is stochastic, and assume that $\gamma_{\min}$ is sufficiently negative for $l^d(w,\gamma_{\min})$ to intersect with $l^s(w)$ below $R/2$, as shown. If the minimum wage $\bar{w}=E\left((R+\gamma)/2\right)=R/2$ were set, then for all low demand realizations ($\gamma<0$), $(R+\gamma)/2$ would be lower than $R/2$, while for high demand realizations ($\gamma>0$) $(R+\gamma)/2$ would be higher than $R/2$. But, for any given realization γ, earnings are maximized at $w=(R+\gamma)/2$. Therefore, since for any realization $\gamma < 0$, a marginal reduction in $\bar{w}$ would be in the direction of $(R+\gamma)/2$, this would raise realized earnings. But for any realization $\gamma > 0$ the minimum wage would not bind and so the reduction in the minimum wage would have no effect on earnings. Thus, the expectation of earnings would be increased by the reduction in $\bar{w}$.

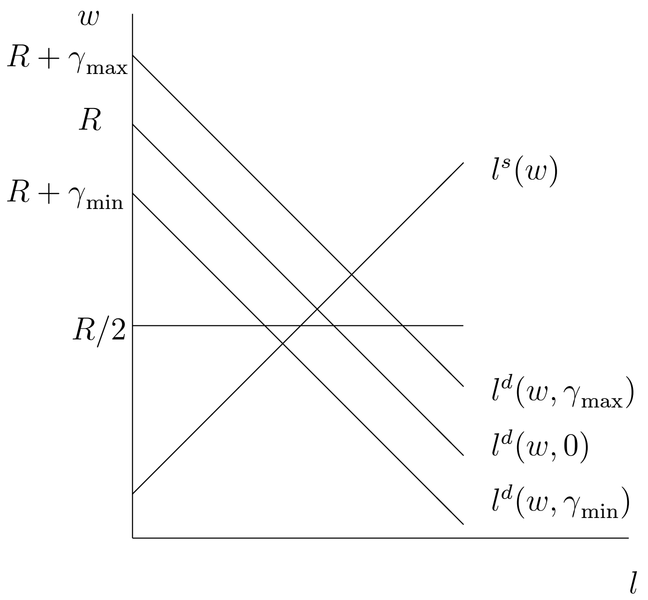

Figure 3: Expected earnings with stochastic demand

This illustrates two properties of expected-earnings maximization for stochastic labour demand. First, for values of the parameters (R, S, r, s) for which, with certainty, a minimum wage should not be set, if uncertainty, as represented by $\gamma_{\min}$ is sufficiently large, a minimum wage becomes optimal. Second, the optimal minimum wage in this case is less than $\bar{w} = E\left((R+\gamma)/2\right) = R/2$. This is consistent with our theme that uncertainty (weakly) leads to a lower optimal minimum wage.

4.2 Expected Worker Surplus for Stochastic Demand

When the government maximizes expected worker surplus with stochastic demand, the qualitative results are similar to when supply is stochastic, the main difference being that an extra condition must be imposed for concavity of the objective function when uncertainty is sufficiently large and rationing relatively efficient.

For $\bar{w}$ in the high range $\bar{w} > w^*(\gamma_{\max})$, demand $l^d(\bar{w}) = (R+\gamma-\bar{w})/r$ is a binding constraint, regardless of the realization γ. Expected worker surplus is $\Omega_I(\gamma)$, as given by setting $\phi = 0$ in (4). Let

$$\gamma_0 = \frac{(R-S)r[\alpha(r+s)-s]}{s[2\alpha(r+s)-s]} \text{ and } \gamma_1 = -\frac{\alpha r(R-S)}{2\alpha(r+s)-s} .$$

Parallel to the definition of ϕ_0 for supply uncertainty, γ_0 is the value of γ at which $w^*(\gamma) = \hat{w}(\alpha)$, while γ_1 is the value of γ at which $\gamma + S = \hat{w}(\alpha)$. Then, corresponding to Lemma 1 for supply uncertainty, we specify conditions under which (11) is a well-defined local optimum for $\bar{w} > w^*(\gamma_{\max})$. Lemma 2 parallels Lemma 1 except that it has the additional requirement that $\gamma_{\min} > \gamma_1$. As we restrict attention to $\bar{w} < R + \gamma_{\min}$, this requirement is needed so that $\hat{w}(\alpha) < R + \gamma_{\min}$.

Lemma 2 *With demand uncertainty, if $\gamma_{\max} < \gamma_0$, $\gamma_{\min} > \gamma_1$ and $\alpha > s/(r+s)$, the minimum wage $\bar{w} > w^*(\gamma_{\max})$ that maximizes expected worker surplus for $\bar{w} \in (w^*(\gamma_{\max}), R]$ is $\hat{w}(\alpha)$, as given by (11). If $\gamma_{\max} \geq \gamma_0$ or $\alpha \leq s/(s+r)$, the minimum wage that maximizes expected worker surplus cannot be strictly higher than $w^*(\gamma_{\max})$.*

Consider now a minimum wage in the low range $\bar{w} < w^*(\gamma_{\min})$. In this case, regardless of the realization of γ, $\bar{w}$ does not bind, and so the free market wage $w^*(\gamma)$ and employment $l^*(\gamma)$ obtain, as given by (15). Worker surplus is

$$\Omega\left(l^*(\gamma)\right) = s\left(R + \gamma - S\right)^2 / 2(r+s)^2 \ . \tag{19}$$

Expected worker surplus is therefore

$$s[(R-S)^2 + E(\gamma^2)]/2(r+s)^2 \equiv \Omega_U^d \ ,$$

which is independent of $\bar{w}$.

Finally, suppose $\bar{w} \in [w^*(\gamma_{\min}), w^*(\gamma_{\max})]$ (so that $\gamma^*(\bar{w}) \in [\gamma_{\min}, \gamma_{\max}]$). In this range, any $\bar{w} \leq w^*(\gamma)$ (i.e., $\gamma \geq \gamma^*(\bar{w})$), is non-binding and worker surplus is given by $\Omega\left(l^*(\gamma)\right)$ in (19). For $\bar{w} \geq w^*(\gamma)$ (i.e., $\gamma < \gamma^*(\bar{w})$) there is excess supply and worker surplus is $\Omega_I(\gamma)$, as given by (4) with $\phi = 0$. Expected worker surplus is therefore

$$E(\Omega) = \int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \Omega\left(l^*(\gamma)\right) g(\gamma) d\gamma + \int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \Omega_I(\gamma) g(\gamma) d\gamma \equiv \Omega_d \ . \tag{20}$$

Using (4) and (19), we then find that

$$\frac{d\Omega_d}{d\bar{w}} = \frac{1}{r^2}\left[\alpha(2s+r) - s\right] \epsilon_d^L(\gamma^*(\bar{w})) + \frac{1}{r^2} G(\gamma^*(\bar{w}))\{(R-\bar{w})[\alpha(r+2s) - s] - \alpha r(\bar{w} - S)\} \ ; \tag{21}$$

$$\frac{d^2\Omega_d}{d\bar{w}^2} = -\frac{1}{r^2} G(\gamma^*(\bar{w}))[2\alpha(s+r) - s] + \frac{1}{r^2 s^2}(\bar{w} - S) r\left(r+s\right)\left[\alpha(r+s) - s\right] G'(\gamma^*(\bar{w})) \ .$$

We can now derive the following proposition. The proof, which is given in the appendix, also includes analysis of the shape of Ω_d, which underlies the discussion below.

Proposition 5 *Consider the minimum wage $\bar{w}$ that maximizes expected worker surplus when there is demand uncertainty and $\alpha > s/(r+s)$. If $\gamma_{\max} \geq \gamma_0$, the optimal $\bar{w}$ lies in the range $(w^*(\gamma_{\min}), w^*(\gamma_{\max})]$; but if $\gamma_{\max} < \gamma_0$ and Ω_d is single-peaked, the optimal $\bar{w}$ is $\hat{w}(\alpha)$, as given by (11).*

Assuming that rationing is relatively efficient ($\alpha > s/(r+s)$), if uncertainty is large enough ($\gamma_{\max} \geq \gamma_0$), the optimal minimum wage is in the middle range $(w^*(\gamma_{\min}), w^*(\gamma_{\max})]$, and so may or may not bind. We show in the proof of the proposition that Ω_d must have an inflexion point in this range. If the inflexion point is unique, Ω_d is single-peaked and so the optimal minimum wage is also unique. A sufficient condition for this is for $d^2\Omega_d/d\bar{w}^2$ to be strictly monotonic on $(w^*(\gamma_{\min}), w^*(\gamma_{\max})]$. This is true, for instance, if γ is uniformly distributed.

If, however, uncertainty is smaller ($\gamma_{\max} < \gamma_0$), a sufficient condition for the optimal minimum wage to be $\bar{w} = \hat{w}(\alpha)$ in the high range (and certainly binding) is that Ω_d is single-peaked. (Without this condition we cannot rule out the possibility that Ω_d will be greater in the middle than in the high range.) Parallel to the result found for supply uncertainty, when $\gamma_{\max} \geq \gamma_0$, the wage $\hat{w}(\alpha)$ also belongs to the middle range, but it is weakly lower than the optimal minimum wage. (This can be seen by setting $\bar{w} = \hat{w}(\alpha)$ in (21): $d\Omega_d(\hat{w}(\alpha))/d\bar{w} = [\alpha(2s+r) - s]\epsilon_d^L(\gamma^*(\hat{w}(\alpha)))/r^2 \leq 0$.) Therefore, if uncertainty is great enough ($\phi_{\max} \geq \phi_0$), the optimal minimum wage which lies in the middle range is strictly lower than $\hat{w}(\alpha)$, the optimal minimum wage with smaller uncertainty.

Also, if $\gamma_{\max} \geq \gamma_0$ and Ω_d is single-peaked, evaluating $d\Omega_d/d\bar{w}$ at $\bar{w} = w_e^*$, the optimal minimum wage may be greater or smaller than the expected free market wage. The condition is

$$\bar{w} \gtreqless w_e^* \text{ as } (r+s)\left[\alpha(2s+r) - s\right]\epsilon_d^L(0) + G(0)r[\alpha(s+r) - s](R-S) \gtreqless 0 \ .$$

Finally, for low rationing efficiency, $\alpha \leq s/(r+s)$, a result parallel to that for supply uncertainty obtains.

Proposition 6 *With demand uncertainty, if $\alpha \leq s/(r+s)$, expected worker surplus is maximized by not setting a minimum wage.*

As before, this result does not depend on the amount of uncertainty.

5 Conclusion

Recognition of regulatory uncertainty might be expected to diminish the case for policy intervention such as a minimum wage. However, if the objective is to maximize expected earnings, such uncertainty expands the range of parameter values for which a minimum wage should be set, but has a weakly negative effect on the optimal level. This minimum wage may or may not bind ex post. Additionally, the qualitative impact of the uncertainty depends on whether the uncertainty relates to supply or demand. In the former case the optimal minimum wage is obtained by a simple modification of the unit-demand-elasticity rule that applies under certainty; while in the latter case unit demand elasticity does not play a direct role.

If the objective is to maximize expected worker surplus, a minimum wage should always be set, provided the efficiency of employment rationing is above a critical level. Uncertainty has no effect on whether a minimum wage should be used, but, as with expected earnings-maximization, if uncertainty is sufficiently great, the minimum wage should be set at a level that may or may not bind. This minimum wage is lower than the optimal minimum wage when there is no uncertainty, and may even be below the expected market-clearing level. Thus, our analysis shows that regulatory uncertainty has a weakly negative effect on the optimal minimum wage, but does not reduce the scope for intervention.

Our qualitative results would still apply if profit were added to the worker-surplus objective function, though with a lower weight. Also, future research might develop the analysis for more general demand and supply functions and for monopsony. We conjecture that the optimal minimum wage will be weakly higher than with perfect competition to keep up expected worker surplus or earnings for those realizations at which monopsonistic behaviour, but not competition, would be restrictive.

The main message of our paper, that regulatory uncertainty does not undermine the case for introducing of a minimum wage, but that may call for a conservative level to be set, bears on recent debates. For the minimum wage law recently rejected by Switzerland, the Kaitz index (the ratio of the minimum wage to median earnings) was 2/3, which, it was said, would have been the highest in the world (*Financial Times*, May 18, 2014). In contrast, a minimum wage with a Kaitz index of 1/2 was adopted by Germany. Although this puts the German minimum wage well above the OECD average, the rejection of a higher minimum wage in Switzerland and adoption of a somewhat lower one in Germany could be regarded as consistent with our analysis. However, the determination of an appropriately conservative level for the minimum wage remains an empirical question.

6 Appendix

Proof of Lemma 1. If $\alpha < 2s/(r+s)$ then $\hat{w}(\alpha) > R$, which is inconsistent with the range $(w^*(\phi_{\max}), R]$ (and gives an $E(\Omega_I(\phi))$-minimum). If $\alpha > s/2(r+s)$ and $\phi_{\max} \geq \phi_0$, $dE(\Omega_I(\phi))/d\bar{w} < 0$ for all $\bar{w} > w^*(\phi_{\max})$; also, $\hat{w}(\alpha) \leq w^*(\phi_{\max})$, which is inconsistent with the range $(w^*(\phi_{\max}), R]$. If $\alpha \in (s/2(r+s), s/(r+s))$, since $R > S$ and $\phi_{\max} > 0$, the condition $\phi_{\max} > \phi_0$ is trivially satisfied, and so $\hat{w}(\alpha) < w^*(\phi_{\max})$. However, if $\phi_{\max} < \phi_0$ and $\alpha > s/(r+s)$, as $d^2E(\Omega_I(\phi))/d\bar{w}^2 < 0$, (11) is a well-defined local maximum within the range $(w^*(\phi_{\max}), R]$. ■

Proof of Proposition 2. Here we show that whenever $\alpha > s/(r+s)$ and $\phi_{\min} \geq \phi_0$, there is a unique optimal $\bar{w} \in (w^*(\phi_{\min}), w^*(\phi_{\max})]$. The rest of the proof is provided in the text. ■

We substitute expressions for the two terms in (13) and then differentiate with respect to $\bar{w}$ to find the local optimum for $\bar{w} \in [w^*(\phi_{\min}), w^*(\phi_{\max})]$. First, using (4) and (12),

$$\int_{\phi^*(\bar{w})}^{\phi_{\max}} \Omega(l^*(\phi)) f(\phi) d\phi =$$
$$\frac{s}{2(r+s)^2}\left[(1-F(\phi^*(\bar{w}))(R-S)^2 - 2\epsilon_s^H(\phi^*(\bar{w}))(R-S) + \sigma_s^H(\phi^*(\bar{w}))\right] . \quad (22)$$

Then, using (4) and (12),

$$\int_{\phi_{\min}}^{\phi^*(\bar{w})} \Omega_I(\phi) f(\phi) d\phi =$$
$$\frac{R-\bar{w}}{2r^2}\left\{F(\phi^*(\bar{w}))[2\alpha r(\bar{w}-S) + (1-2\alpha)s(R-\bar{w})] - 2\alpha r \epsilon_s^L(\phi^*(\bar{w}))\right\} , \quad (23)$$

Substituting into (13) from (22) and (23), and using (6), we obtain (14).

Using (6) and (14), we can see that Ω_s is decreasing at $w^*(\phi_{\max})$ iff $\phi_{\max} \geq \phi_0$. Also, by Lemma 1, if $\alpha > s/(r+s)$ and $\phi_{\max} \geq \phi_0$ there is no candidate optimal $\bar{w} > w^*(\phi_{\max})$. Hence, if $\alpha > s/(r+s)$ and $\phi_{\max} \geq \phi_0$ the global optimal $\bar{w} \in (w^*(\phi_{\min}), w^*(\phi_{\max})]$. Furthermore, we can show that there is a unique $\bar{w}$ in this interval. For $\alpha > s/(r+s)$,

$$sign\frac{d^2\Omega_s}{d\bar{w}^2} = sign\left(\frac{F'(\phi^*(\bar{w}))}{F(\phi^*(\bar{w}))} - \frac{(2\alpha(r+s)-s)\,r}{(r+s)\,(\alpha(r+s)-s)}\frac{1}{R-\bar{w}}\right) .$$

By assumption, the hazard rate $F'(\phi)/(1-F(\phi))$ is strictly increasing on the interval $(w^*(\phi_{\min}), w^*(\phi_{\max})]$, and therefore $F'(\phi)/F(\phi)$ is strictly decreasing in this range.[1] As

[1] The strictly increasing hazard rate can be written, $\mathcal{F}''(\phi)(1-\mathcal{F}(\phi)) + (\mathcal{F}'(\phi))^2 = x > 0$. However, $sign\left(\frac{d}{d\phi}(\mathcal{F}'(\phi)/\mathcal{F}(\phi))\right) = sign\left(\mathcal{F}''(\phi)\mathcal{F}(\phi) - (\mathcal{F}'(\phi))^2\right) = sign\,(y)$. But since $x = \mathcal{F}''(\phi)(1-\mathcal{F}(\phi)) + (\mathcal{F}'(\phi))^2 > 0$, $\mathcal{F}''(\phi)\mathcal{F}(\phi) - (\mathcal{F}'(\phi))^2 = y < \mathcal{F}''(\phi)\mathcal{F}(\phi) + \mathcal{F}''(\phi)(1-\mathcal{F}(\phi)) = \mathcal{F}''(\phi) < 0$; i.e., $\mathcal{F}'(\phi)/\mathcal{F}(\phi)$ is strictly decreasing.

$1/(R-\bar{w})$ is strictly increasing in $\bar{w}$, it can be seen that Ω_s has a unique inflexion point at w_I^s. At $w^*(\phi_{\min})$, when $\alpha > s/(r+s)$, $d^2\Omega_s/d\bar{w}^2 > 0$, so that the function is convex for all $\bar{w} \in (w^*(\phi_{\min}), w_I^s)$. However, if $\bar{w} > w_I^s$, $1/(R-\bar{w})$ will take a lower value and $F'(\phi^*(\bar{w}))/F(\phi^*(\bar{w}))$ a higher one, so that $d^2\Omega_s/d\bar{w}^2 \leq 0$ for all $\bar{w} \in (w_I^s, w^*(\phi_{\max}))$. Hence, if $\alpha > s/(r+s)$ and $\phi_{\min} \geq \phi_0$, there is a unique optimal $\bar{w}$ in $(w^*(\phi_{\min}), w^*(\phi_{\max})]$.

Proof of Proposition 3. Assume $\alpha \leq s/(r+s)$. Then, from (14), $d\Omega_s(w^*(\phi_{\min}))/d\bar{w} = 0$ and $d^2\Omega_s(w^*(\phi_{\min}))/d\bar{w}^2 < 0$, so that $w^*(\phi_{\min})$ is a local maximum on $\bar{w} \in [w^*(\phi_{\min}), w^*(\phi_{\max})]$. Also, from (4), $dE\left(\Omega_I(\phi)\right)/d\bar{w} < 0$ for all $\bar{w} > w^*(\phi_{\max})$. Finally, any $\bar{w} \leq w^*(\phi_{\min})$ does not bind, and so gives the same outcome. Therefore $w^*(\phi_{\min})$ is a well-defined maximum in the range $[w^*(\phi_{\min}), w^*(\phi_{\max})]$, and any $\bar{w} < w^*(\phi_{\min})$ yields the same outcome, whereas Ω_s is lower for $\bar{w} > w^*(\phi_{\max})$. This proves the result. ■

Proof of Proposition 4. Here we consider the f.o.c. and s.o.c. for the middle range $\bar{w} \in [w^*(\gamma_{\min}), w^*(\gamma_{\max})]$. The rest of the proof is in the text.
In this range, if $\bar{w} \leq w^*(\gamma)$ (i.e., $\gamma \geq \gamma^*(\bar{w})$) $\bar{w}$ does not bind, so that, from (15), $\Upsilon = l^*w^* = \left[Sr + (R+\gamma)s\right]\left(R+\gamma-S\right)/(r+s)^2$. Expected earnings in this range are therefore

$$\int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \Upsilon\left(l^*(\gamma)\right) g(\gamma)d\gamma = \frac{1}{(r+s)^2}\int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \left[Sr + (R+\gamma)s\right]\left(R+\gamma-S\right) g(\gamma)d\gamma =$$

$$\frac{1}{(r+s)^2}\left[(Sr+Rs)(R-S)(1-G(\gamma^*(\bar{w}))) + (2Rs+Sr-sS)\epsilon_d^H(\gamma^*(\bar{w})) + s\sigma_d^H(\gamma^*(\bar{w}))\right] , \tag{24}$$

where $\epsilon_d^H(\gamma^*(\bar{w})) = \int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \gamma g(\gamma)d\gamma$ and $\sigma_d^H(\gamma^*(\bar{w})) = \int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \gamma^2 g(\gamma)d\gamma$. However, if $\bar{w} > w^*(\gamma)$ (i.e., $\gamma < \gamma^*(\bar{w})$) there is excess supply and demand binds, so that $\Upsilon = \bar{w}l^d(w) = \bar{w}(R+\gamma-\bar{w})/r$. Expected earnings are

$$\begin{aligned}\int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \Upsilon(l^d(\bar{w}))g(\gamma)d\gamma &= \frac{\bar{w}}{r}\int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \left(R-\bar{w}+\gamma\right) g(\gamma)d\gamma \\ &= \frac{\bar{w}}{r}\left[(R-\bar{w})G(\gamma^*(\bar{w})) + \epsilon_d^L(\bar{w})\right] .\end{aligned} \tag{25}$$

For $\bar{w} \in [w^*(\gamma_{\min}), w^*(\gamma_{\max})]$ $\bar{w}$ is set to maximize Υ_d in (17). Substituting from (24) and (25), and using (16), we obtain (18).
The second line of (18) can be written

$$\frac{d^2\Upsilon_d}{d\bar{w}^2} = -\frac{2G'(\gamma^*(\bar{w}))}{r}\left[\frac{G(\gamma^*(\bar{w}))}{G'(\gamma^*(\bar{w}))} + (s+r)\,\frac{\bar{w}(s-r)+rS}{2s^2}\right] .$$

If $s \geq r$ this is negative. But suppose $s < r$. Since $d\Upsilon_d/d\bar{w} > 0$ at $\bar{w} = \gamma_{\min}$ and $d\Upsilon_d/d\bar{w} < 0$ at $\bar{w} = w^*(\gamma_{\max})$, there can only be one turning point of Υ_d in the middle range unless $d^2\Upsilon_d/d\bar{w}^2 = 0$ at least three times in this range. But, given that the hazard rate $G'(\gamma^*(\bar{w}))/(1-G(\gamma^*(\bar{w}))$ is increasing in $\bar{w}$, so also is $G(\gamma^*(\bar{w}))/G'(\gamma^*(\bar{w}))$, while $(s+r)\,[\bar{w}(s-r)+rS]/2s^2$ is decreasing in $\bar{w}$. Therefore $d^2\Upsilon_d/d\bar{w}^2 = 0$ only once. It follows that Υ_d is concave in this range. ■

Proof of Lemma 2. If $\alpha > s/2(r+s)$ and $\gamma_{\max} > \gamma_0$, $\hat{w}(\alpha) < w^*(\gamma_{\max})$, which is inconsistent with the range $\bar{w} > w^*(\gamma_{\max})$, while $d\Omega_I(\gamma)/d\bar{w} < 0$ for $\bar{w} > w^*(\gamma_{\max})$. But when $\gamma_{\max} < \gamma_0$ and $\alpha > s/(r+s)$, $\hat{w}(\alpha)$ is a well-defined maximum, consistent with $\bar{w} > w^*(\gamma_{\max})$. To ensure that $\hat{w}(\alpha) < R + \gamma_{\min}$ is necessary that $\gamma_{\min} > \gamma_1$. ■

Proof of Proposition 5. First we derive (21). Using (19), the first term in (20) can be written

$$\int_{\gamma^*(\bar{w})}^{\gamma_{\max}} \Omega\left(l^*(\gamma)\right) g(\gamma) d\gamma = \frac{s[(1-G(\gamma^*(\bar{w})))(R-S)^2 + 2\epsilon_d^H(\gamma^*(\bar{w}))(R-S) + \sigma_d^H(\gamma^*(\bar{w}))]}{2(r+s)^2} \ . \tag{26}$$

Also, using (4), the second term in (20) can be written

$$\int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \Omega_I(\gamma) g(\gamma) d\gamma =$$
$$\frac{1}{2r^2}(R-\bar{w})[2\alpha r(\bar{w}-S) + (1-2\alpha)s\,(R-\bar{w})]G(\gamma^*(\bar{w})) +$$
$$\frac{1}{2r^2}\left\{[2\alpha r(\bar{w}-S) + 2(1-2\alpha)s(R-\bar{w})]\,\epsilon_d^L(\gamma^*(\bar{w})) + (1-2\alpha)s\sigma_d^L(\gamma^*(\bar{w}))\right\} \ , \tag{27}$$

where $\epsilon_d^L(\gamma^*(\bar{w})) = \int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \gamma g(\gamma) d\gamma$ and $\sigma_d^L(\gamma^*(\bar{w})) = \int_{\gamma_{\min}}^{\gamma^*(\bar{w})} \gamma^2 g(\gamma) d\gamma$.
Substituting (26) and (27) into (20), and using (16), we obtain (21).
As $\epsilon_d^L(\gamma_{\min}) = G(\gamma_{\min}) = 0$, $w^*(\gamma_{\min})$ satisfies the f.o.c. (21) for the middle range $\bar{w} \in [w^*(\gamma_{\min}), w^*(\gamma_{\max})]$. However, if rationing efficiency $\alpha > s/(s+r)$, $w^*(\gamma_{\min})$ gives a local minimum as the objective function is convex at this point. Therefore Ω_d is decreasing in $\bar{w}$ as it approaches $w^*(\gamma_{\min})$ from above, while, using (16) and (21), iff $\gamma_{\max} \geq \gamma_0$, Ω_d is decreasing at $w^*(\gamma_{\max})$. Also, we have seen that all $\bar{w} < w^*(\gamma_{\min})$ result in the same levels of Ω_d as $\bar{w} = w^*(\gamma_{\min})$, while, by Lemma 2, if $\alpha > s/(s+r)$ and $\gamma_{\max} \geq \gamma_0$, there is no candidate $\bar{w} > w^*(\gamma_{\max})$. Hence, if $\alpha > s/(s+r)$ and $\gamma_{\max} \geq \gamma_0$, the globally optimal $\bar{w}$ must belong to $(w^*(\gamma_{\min}), w^*(\gamma_{\max})]$.
Also, for $\bar{w} \in (w^*(\gamma_{\min}), w^*(\gamma_{\max})]$, if Ω_d has a unique inflexion point there is a unique optimal $\bar{w}$ in this range. A sufficient condition for this is that $d^2\Omega_d/d\bar{w}^2$ is strictly monotonic. When $\alpha > s/(s+r)$,

$$sign\frac{d^2\Omega_d}{d\bar{w}^2} = sign\{h_g\left(\gamma^*(\bar{w})\right)(R-\bar{w}) - \Delta\}$$

where $\Delta = (2\alpha(s+r)-s)\,s^2/\,(\alpha(s+r)-s)\,r(s+r)$ and $h_g(\gamma^*(\bar{w})) = G'(\gamma^*(\bar{w}))/(1-G(\gamma^*(\bar{w})))$ is the hazard rate of $G(\gamma^*(\bar{w}))$, which, by assumption, is strictly increasing. Since $h_g[\gamma^*(\bar{w})](R-$

$\bar{w}$) is monotonic in $\bar{w}$ over this range, $h_g(\gamma^*(\bar{w}))(R-\bar{w})-\Delta=0$ has a unique solution and so the inflexion point is unique and Ω_d is single-peaked on $\bar{w}\in(w^*(\gamma_{\min}),w^*(\gamma_{\max})]$.
If $\alpha>s/(s+r)$ but $\gamma_{\max}<\gamma_0$, then Ω_d is increasing at $w^*(\gamma_{\max})$. Provided Ω_d is single-peaked on $\bar{w}\in(w^*(\gamma_{\min}),w^*(\gamma_{\max})]$, then, given Lemma 2, $\bar{w}=\hat{w}(\alpha)$, as given by (11), is the global optimum. ■

Proof of Proposition 6. From (14), $d\Omega_d(w^*(\gamma_{\min}))/d\bar{w}=0$ and $d^2\Omega_d(w^*(\gamma_{\min}))/d\bar{w}^2<0$, so that $w^*(\gamma_{\min})$ is a local maximum on $\bar{w}\in[w^*(\gamma_{\min}),w^*(\gamma_{\max})]$. Also, from Lemma 2, $dE(\Omega_I(\gamma))/d\bar{w}<0$ for all $\bar{w}>w^*(\gamma_{\max})$. Finally, any $\bar{w}\leq w^*(\gamma_{\min})$ does not bind, so the gives the same outcome. Therefore $w^*(\gamma_{\min})$ is a well-defined maximum in the range $[w^*(\gamma_{\min}),w^*(\gamma_{\max})]$, and any $\bar{w}<w^*(\gamma_{\min})$ yields the same outcome, whereas Ω_d is lower for $\bar{w}>w^*(\gamma_{\max})$. This proves the result. ■

References

ALLEN, S. P. (1987): "Taxes, Redistribution, and the Minimum Wage: A Theoretical Analysis," *The Quarterly Journal of Economics*, 102(3).

BLOOM, N. (2014): "Fluctuations in Uncertainty," *Journal of Economic Perspectives*, 28(2).

BULOW, J., AND P. KLEMPERER (2012): "Regulated Prices, Rent Seeking, and Consumer Surplus," *Journal of Political Economy*, 120(1).

DANZIGER, E., AND L. DANZIGER (2015): "A Pareto-improving Minimum Wage," *Economica*, 82(326).

DANZIGER, L. (2009): "The Elasticity of Labor Demand and the Minimum Wage," *Journal of Population Economics*, 22(3).

DAVIS, L. W., AND L. KILIAN (2011): "The Allocative Cost of Price Ceilings in the US Residential Market for Natural Gas," *Journal of Political Economy*, 119(2).

GLAESER, E. L., AND E. F. LUTTMER (2003): "The Misallocation of Housing Under Rent Control," *The American Economic Review*, 93(4).

IZA (2014): *IZA Compact (March)*. Institute for the Study of Labor, Bonn.

LEE, D., AND E. SAEZ (2012): "Optimal Minimum Wage Policy in Competitive Labor Markets," *Journal of Public Economics*, 96(9).

LUTTMER, E. F. (2007): "Does the Minimum Wage Cause Inefficient Rationing?," *The BE Journal of Economic Analysis & Policy*, 7(1).

MACURDY, T. (2015): "How Effective Is the Minimum Wage at Supporting the Poor?," *Journal of Political Economy*, 123(2).

MARSHALL, A. (1920): *Principles of Economics*. London: Macmillan, 8th edition.

NEUMARK, D., J. I. SALAS, AND W. WASCHER (2014): "Revisiting the Minimum Wage-Employment Debate: Throwing Out the Baby with the Bathwater?," *Industrial & Labor Relations Review*, 67(3).

OECD (2015): *Employment Outlook 2015*. Paris: OECD Publishing.

Portugal, P., and A. R. Cardoso (2006): "Disentangling the Minimum Wage Puzzle: An Analysis of Worker Accessions and Separations," *Journal of the European Economic Association*, 4(5).

Schmitt, J. (2013): "Why Does the Minimum Wage Have No Discernible Effect on Employment?," *Center for Economic and Policy Research*, 22(1).

Sobel, R. S. (1999): "Theory and Evidence on the Political Economy of the Minimum Wage," *Journal of Political Economy*, 107(4).